

HYPER MV-DEDUCTIVE SYSTEMS OF HYPER MV-ALGEBRAS

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ABSTRACT. The notions of (weak) hyper MV-deductive systems and (weak) implicative hyper MV-deductive systems are introduced, and several properties are investigated. Relations among hyper MV-deductive systems, weak hyper MV-deductive systems, implicative hyper MV-deductive systems and weak implicative hyper MV-deductive systems are discussed. A characterization of a hyper MV-deductive system is provided. A condition for a weak hyper MV-deductive system to be a weak implicative hyper MV-deductive system is given.

1. Introduction

In this paper, we introduce the notions of (weak) hyper MV-deductive systems and (weak) implicative hyper MV-deductive systems, and we investigate several properties. We discuss relations among hyper MV-deductive systems, weak hyper MV-deductive systems, implicative hyper MV-deductive systems and weak implicative hyper MV-deductive systems. We give a characterization of a hyper MV-deductive system. We give a condition for a weak hyper MV-deductive system to be a weak implicative hyper MV-deductive system.

2. Preliminaries

Definition 2.1 ([1]). A *hyper MV-algebra* is a non-empty set M endowed with a hyper operation “ \oplus ”, a unary operation “ $*$ ” and a constant “ 0 ” satisfying the following axioms:

- (a1) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$,
- (a2) $x \oplus y = y \oplus x$,
- (a3) $(x^*)^* = x$,
- (a4) $(x^* \oplus y)^* \oplus y = (y^* \oplus x)^* \oplus x$,
- (a5) $0^* \in x \oplus 0^*$,
- (a6) $0^* \in x \oplus x^*$,

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(a7) $x \ll y, y \ll x \Rightarrow x = y$

for all $x, y, z \in M$, where $x \ll y$ is defined by $0^* \in x^* \oplus y$.

For every subsets A and B of M , we define

$$A \ll B \Leftrightarrow (\exists a \in A) (\exists b \in B) (a \ll b),$$

$$A \oplus B := \bigcup_{a \in A, b \in B} a \oplus b.$$

We also define $0^* := 1$ and $A^* := \{a^* \mid a \in A\}$.

Proposition 2.2 ([1]). *Every hyper MV-algebra M satisfies the following statements:*

- (b1) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$,
- (b2) $0 \ll x, x \ll 1$,
- (b3) $x \ll x$,
- (b4) $x \ll y \Rightarrow y^* \ll x^*$,
- (b5) $A \ll B \Rightarrow B^* \ll A^*$,
- (b6) $A \ll A$,
- (b7) $A \subseteq B \Rightarrow A \ll B$,
- (b8) $x \ll x \oplus y, A \ll A \oplus B$,
- (b9) $(A^*)^* = A$,
- (b10) $0 \oplus 0 = \{0\}$,
- (b11) $x \in x \oplus 0$,
- (b12) $y \in x \oplus 0 \Rightarrow y \ll x$,
- (b13) $y \oplus 0 = x \oplus 0 \Rightarrow x = y$

for all $x, y, z \in M$ and for all subsets A, B and C of M .

3. Hyper MV-deductive systems

In what follows, let M denote a hyper MV-algebra unless otherwise specified.

Definition 3.1. A nonempty subset D of M is called a *weak hyper MV-deductive system* of M if it satisfies:

- (d1) $0 \in D$,
- (d2) $(\forall x, y \in M)((x^* \oplus y)^* \subseteq D, y \in D \Rightarrow x \in D)$.

Definition 3.2. A non-empty subset D of M is called a *hyper MV-deductive system* of M if it satisfies (d1) and

- (d3) $(\forall x, y \in M)((x^* \oplus y)^* \ll D, y \in D \Rightarrow x \in D)$.

Example 3.3. Let $M := \{0, a, b, 1\}$ be a set with a hyper operation “ \oplus ” and a unary operation “ $*$ ” which are given in the following Cayley tables:

\oplus	0	a	b	1
0	{0}	{0, a}	{0, a, b}	{0, a, b, 1}
a	{0, a}	{0, a}	{0, a, b, 1}	{0, a, b, 1}
b	{0, a, b}	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}
1	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}

x	x^*
0	1
a	b
b	a
1	0

Then $(M, \oplus, *, 0)$ is a hyper MV-algebra. It is easily checked that $D_1 = \{0, a\}$, $D_2 = \{0, b\}$ and $D_3 = \{0, a, b\}$ are weak hyper MV-deductive systems of M .

Example 3.4. Consider the hyper MV-algebra M which is described in Example 3.3. Then $D_1 = \{0, a\}$ is a hyper MV-deductive system of M .

Example 3.5. Let $M := \{0, a, b, 1\}$ be a set with a hyper operation “ \oplus ” and a unary operation “ $*$ ” which are given in the following Cayley tables:

\oplus	0	a	b	1	x	x^*
0	{0}	{0, a}	{0, b}	{0, a, b, 1}	0	1
a	{0, a}	{a}	{0, a, b, 1}	{0, a, b, 1}	a	b
b	{0, b}	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}	b	a
1	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}	1	0

Then $(M, \oplus, *, 0)$ is a hyper MV-algebra. It is easily checked that $D_1 = \{0, a\}$ is a hyper MV-deductive systems of M . But $D_2 = \{0, b\}$ is not a hyper MV-deductive system of M , since $(a^* \oplus b)^* = \{0, a, b, 1\} \not\subseteq D_2$, $b \in D_2$ but $a \notin D_2$.

Example 3.6. Let $M = \{0, a, b, 1\}$ be a set with a hyper operation “ \oplus ” and a unary operation “ $*$ ” which are given in the following Cayley tables:

\oplus	0	a	b	1	x	x^*
0	{0}	{0, a, b}	{0, b}	{0, a, b, 1}	0	1
a	{0, a, b}	{0, 1}	{0, a, b, 1}	{0, a, b, 1}	a	b
b	{0, b}	{0, a, b, 1}	{b}	{0, a, b, 1}	b	a
1	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}	{0, a, b, 1}	1	0

Then $(M, \oplus, *, 0)$ is a hyper MV-algebra, but $D := \{0, a, 1\}$ is not a weak hyper MV-deductive system of M , since $(b^* \oplus a)^* = (a \oplus a)^* = \{0, 1\} \subseteq D$ and $b \notin D$. We can easily check that $D_1 := \{0, x\}$, $D_2 := \{0, b, x\}$ and M are weak hyper MV-deductive systems of M for all $x \in M$.

Proposition 3.7. Let M be a hyper MV-algebra in which $0 \in x \oplus y$ for all $x, y \in M$. Let D be a subset of M such that $0 \in D$ and $1 \notin D$. Then D is a weak hyper MV-deductive system of M .

Proof. For any $x, y \in M$, assume that $x \notin D$ and $y \in D$. Since $0 \in x^* \oplus y$, we have $1 = 0^* \in (x^* \oplus y)^*$, which shows that $(x^* \oplus y)^*$ is not contained in D . Therefore D is a weak hyper MV-deductive system of M . □

Theorem 3.8. Let M be a hyper MV-algebra satisfying the following condition:

$$(3.1) \quad (\forall x, y \in M) (x \in x \oplus y)$$

Then every subset of M containing 0 is a weak hyper MV-deductive system of M .

Proof. Let D be a subset of M containing 0, and let $x, y \in M$ be such that $(x^* \oplus y)^* \subseteq D$ and $y \in D$. Assume that $x \notin D$. Then $x^* \in x^* \oplus y$ by (3.1),

and so $x = (x^*)^* \in (x^* \oplus y)^*$ by (a3). Hence $(x^* \oplus y)^*$ is not contained in D , which is a contradiction. Therefore $x \in D$, proving that D is a weak hyper MV-deductive system of M . \square

A hyper MV-algebra satisfying (3.1) can have a subset containing 0 which is not a hyper MV-deductive system of M . See the following example.

Example 3.9. Consider a hyper MV-algebra $M := \{0, a, b, 1\}$ which is described in Example 3.5. It can be shown that M satisfies (3.1). The set $D := \{0, b\}$ is not a hyper MV-deductive system of M , since $(a^* \oplus b)^* = \{0, a, b, 1\} \ll D$ and $a \notin D$.

Using (b7) and (d3), we have the following theorem.

Theorem 3.10. *Every hyper MV-deductive system is a weak hyper MV-deductive system.*

The following example shows that the converse of Theorem 3.10 is not valid.

Example 3.11. Consider a hyper MV-algebra M which is described in Example 3.3. Then $D_3 := \{0, a, b\}$ is a weak hyper MV-deductive system of M (see Example 3.3). We know that

$$(1^* \oplus a)^* = (0 \oplus a)^* = \{0, a\}^* = \{b, 1\} \ll D$$

and $a \in D$, but $1 \notin D$. This proves that D_3 is not a hyper MV-deductive system of M .

Proposition 3.12. *Every hyper MV-deductive system D of M have the following condition:*

$$(3.2) \quad (\forall x, y \in M) (x \ll y, y \in D \Rightarrow x \in D).$$

Proof. Let $x, y \in M$ be such that $x \ll y$ and $y \in D$. Then $0 \in (x^* \oplus y)^*$, and so $(x^* \oplus y)^* \ll D$ by (d1) and (b3). It follows from (d3) that $x \in D$. \square

Combining (b2) and Proposition 3.12, we have the following corollary.

Corollary 3.13. *Let D be a hyper MV-deductive system of M . If D contains the element 1, then $D = M$.*

By Corollary 3.13, we know that any proper subset of M containing 1 cannot be a hyper MV-deductive system of M .

Proposition 3.14. *Let D be a hyper MV-deductive system of M . For every subsets A and B of M , if $B \subseteq D$ and $(A^* \oplus B)^* \ll D$, then $A \ll D$.*

Proof. We have $(A^* \oplus B)^* = \bigcup_{a \in A, b \in B} (a^* \oplus b)^* \ll D$, and so there exists $x \in (a^* \oplus b)^*$ for some $a \in A$ and $b \in B$, and there exists $y \in D$ such that $x \ll y$. It follows that $(a^* \oplus y)^* \ll D$. Since $b \in B \subseteq D$, by (d3) we obtain $a \in D$. Therefore $A \ll D$. \square

Remark 3.15. (1) In Proposition 3.14, it is not necessary to mention that $A \subseteq D$. To show this, consider a hyper MV-algebra $M = \{0, a, b, 1\}$ which is given in Example 3.5. Let $A := \{0, a, b\}$ and $B := \{0\}$. Note that $D := \{0, a\}$ is a hyper MV-deductive system of M and $(A^* \oplus B)^* = \{0, a, b, 1\} \ll D$, but A is not contained in D .

(2) In Proposition 3.14, if we use $B \ll D$ instead of $B \subseteq D$, then the result does not hold. Consider a hyper MV-algebra $M = \{0, a, b, 1\}$ which is given in Example 3.5. It can be shown that $D := \{0, a\}$ is a hyper MV-deductive system of M . If we take $A := \{b\}$ and $B := \{0, b\}$, then $B \ll D$ and $(A^* \oplus B)^* = \{0, a, b, 1\} \ll D$, but $A \ll D$ is not valid.

Proposition 3.16. *Every weak hyper MV-deductive system D of M satisfies the following condition:*

$$(3.3) \quad (\forall A, B \subseteq M) ((A^* \oplus B)^* \subseteq D, B \subseteq D \Rightarrow A \subseteq D)$$

Proof. For all $a \in A$ and $b \in B$, we have $(a^* \oplus b)^* \subseteq (A^* \oplus B)^* \subseteq D$ and $b \in B \subseteq D$. It follows from (d2) that $a \in D$. \square

Corollary 3.17. *Every weak hyper MV-deductive system D of M satisfies the condition:*

$$(3.4) \quad (\forall A, B \subseteq M) ((A^* \oplus B)^* \subseteq D, B \subseteq D \Rightarrow A \ll D)$$

Proof. Straightforward. \square

Remark 3.18. In Proposition 3.16, the condition $B \subseteq D$ can not be replaced by $B \ll D$. In fact, in Example 3.6, we see that if $A = \{b\}$, $B = \{a\}$ and $D = \{0, 1\}$, then D is a weak hyper MV-deductive system of M and $B \ll D$. Also, $(A^* \oplus B)^* = (b^* \oplus a)^* = \{0, 1\} \subseteq D$, but $A \not\subseteq D$.

We give a characterization of a hyper MV-deductive system.

Theorem 3.19. *Let D be a non-empty subset of M . Then D is a hyper MV-deductive system of M if and only if it satisfies (d1) and*

$$(e1) \quad (\forall x, y \in M) ((x^* \oplus y)^* \cap D \neq \emptyset, y \in D \Rightarrow x \in D).$$

Proof. Assume that D is a hyper MV-deductive system of M . Let $x, y \in M$ be such that $y \in D$ and $(x^* \oplus y)^* \cap D \neq \emptyset$. Then there exists $a \in (x^* \oplus y)^* \cap D$. Since $a \ll a$ by (b3), we get $(x^* \oplus y)^* \ll D$. It follows from (d3) that $x \in D$.

Conversely, suppose that D satisfies (d1) and (e1). Let $x, y \in M$ be such that $y \in D$ and $(x^* \oplus y)^* \ll D$. Then there exist $a \in (x^* \oplus y)^*$ and $b \in D$ such that $a \ll b$. Hence $0^* \in a^* \oplus b$, and so $0 \in (a^* \oplus b)^*$. Since $0 \in D$, we have $0 \in (a^* \oplus b)^* \cap D$, i.e., $(a^* \oplus b)^* \cap D \neq \emptyset$. It follows from (e1) that $x \in D$. Therefore D is a hyper MV-deductive system of M . \square

Proposition 3.20. *Every hyper MV-deductive system D of M satisfies the following condition:*

$$(3.5) \quad (\forall A \subseteq M) (\forall b \in D) ((A^* \oplus b)^* \cap D \neq \emptyset \Rightarrow A \cap D \neq \emptyset)$$

Proof. Let $A \subseteq M$ and $b \in D$ be such that $(A^* \oplus b)^* \cap D \neq \emptyset$. Then there exists $x \in D$ such that $x \in (a^* \oplus b)^*$ for some $a \in A$. Thus $(a^* \oplus b)^* \cap D \neq \emptyset$. Since D is a hyper MV-deductive system, it follows from Theorem 3.19 that $a \in D$ so that $A \cap D \neq \emptyset$. \square

Theorem 3.21. *For any hyper MV-deductive system D of M , the following are equivalent:*

- (i) $(\forall x, y \in M) ((x^* \oplus y)^* \ll D)$.
- (ii) $(\forall x, y \in M) ((x^* \oplus y)^* \cap D \neq \emptyset)$.

Proof. (i) \Rightarrow (ii). Assume that (i) is valid. Then there exist $a \in (x^* \oplus y)^*$ and $b \in D$ such that $a \ll b$, that is, $0^* \in a^* \oplus b$. Thus $0 \in (a^* \oplus b)^* \subseteq ((x^* \oplus y) \oplus b)^*$, and so $((x^* \oplus y) \oplus b)^* \cap D \neq \emptyset$. It follows from (b9) and Proposition 3.20 that $(x^* \oplus y)^* \cap D \neq \emptyset$.

(ii) \Rightarrow (i). Straightforward. \square

4. Implicative hyper MV-deductive systems

Definition 4.1. A non-empty subset D of M is called a *weak implicative hyper MV-deductive system* of M if it satisfies (d1) and

$$(d4) \quad (\forall x, y, z \in M) (((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \subseteq D, z \in D \Rightarrow x \in D).$$

Example 4.2. Let $M := \{0, a, b, 1\}$ be a set with the hyper operation “ \oplus ” and the unary operation “ $*$ ” which are given in the following Cayley tables:

\oplus	0	a	b	1
0	{0}	{0, a}	{b}	{b, 1}
a	{0, a}	{0, a}	{b, 1}	{b, 1}
b	{b}	{b, 1}	{b, 1}	{b, 1}
1	{b, 1}	{b, 1}	{b, 1}	{b, 1}

x	x^*
0	1
a	b
b	a
1	0

Then $(M, \oplus, *, 0)$ is a hyper MV-algebra. For every $x \in M$, the set $\{0, x\}$ is a weak implicative hyper MV-deductive system of M . Also, $D_1 := \{0, b, 1\}$ is a weak implicative hyper MV-deductive system of M . We know that $D := \{0, a, b\}$ is not a weak implicative hyper MV-deductive system of M , since

$$((1^* \oplus b) \oplus (a^* \oplus 1)^*)^* = \{0, a\} \subseteq D$$

and $b \in D$, but $1 \notin D$.

Definition 4.3. A non-empty subset D of M is called an *implicative hyper MV-deductive system* of M if it satisfies (d1) and

$$(d5) \quad (\forall x, y, z \in M) (((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \ll D, z \in D \Rightarrow x \in D).$$

Example 4.4. In a hyper MV-algebra $M := \{0, a, b, 1\}$ in Example 4.2, $\{0, a\}$ is an implicative hyper MV-deductive system of M . But $\{0, b\}$ is not an implicative hyper MV-deductive system of M , since

$$((1^* \oplus b) \oplus (a^* \oplus 1)^*)^* = \{0, a\} \ll \{0, b\}.$$

Theorem 4.5. *Every implicative hyper MV-deductive system is a weak implicative hyper MV-deductive system.*

Proof. Let D be an implicative hyper MV-deductive system of M and let $x, y, z \in M$ be such that $z \in D$ and $((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \subseteq D$. Using (b7), we have $((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \ll D$. It follows from (d5) that $x \in D$. Therefore D is a weak implicative hyper MV-deductive system of M . \square

The following example shows that the converse of Theorem 4.5 need not be true in general.

Example 4.6. Consider a hyper MV-algebra $M := \{0, a, b, 1\}$ which is given in Example 3.3. Then it can be easily shown that every subset of M containing 0 is a weak implicative hyper MV-deductive system of M , but $D := \{0, a, 1\}$ is not an implicative hyper MV-deductive system of M , since

$$((b^* \oplus 0) \oplus (0^* \oplus b)^*)^* = M \ll \{0, a, 1\}.$$

Theorem 4.7. *Every implicative hyper MV-deductive system is a hyper MV-deductive system.*

Proof. Let D be an implicative hyper MV-deductive system of M and let $x, y \in M$ be such that $y \in D$ and $(x^* \oplus y)^* \ll D$. Then there exist $a \in (x^* \oplus y)^*$ and $z \in D$ such that $a \ll z$. Using (b2), (b9) and (b11), we obtain

$$a^* \in a^* \oplus 0 \subseteq ((x^* \oplus y)^*)^* \oplus (0^* \oplus x)^* = (x^* \oplus y) \oplus (0^* \oplus x)^*,$$

and so $a \in ((x^* \oplus y) \oplus (0^* \oplus x)^*)^*$. Hence $((x^* \oplus y) \oplus (0^* \oplus x)^*)^* \ll D$ which implies from (d5) that $x \in D$. Therefore D is a hyper MV-deductive system of M . \square

The following example shows that the converse of Theorem 4.7 need not be true in general.

Example 4.8. Consider a hyper MV-algebra $M := \{0, a, b, 1\}$ which is given in Example 3.3. Then $D := \{0, a\}$ is a hyper MV-deductive system of M . Since

$$((1^* \oplus 0) \oplus (0^* \oplus 1)^*)^* = M \ll \{0, a\},$$

$D = \{0, a\}$ is not an implicative hyper MV-deductive system of M .

The following example shows that a weak implicative hyper MV-deductive system may not be a weak hyper MV-deductive system.

Example 4.9. Consider a hyper MV-algebra $M := \{0, a, b, 1\}$ which is described in Example 3.6. Then $D := \{0, a, 1\}$ is a weak implicative hyper MV-deductive system of M , but it is not a weak hyper MV-deductive system of M , since $(b^* \oplus a)^* = \{0, 1\} \subseteq D$.

We give a condition for a weak hyper MV-deductive system to be a weak implicative hyper MV-deductive system.

Theorem 4.10. *Let D be a weak hyper MV-deductive system of M satisfying the following condition:*

$$(4.1) \quad (\forall x, y \in M) ((x^* \oplus (y^* \oplus x)^*)^* \subseteq D \Rightarrow x \in D)$$

Then D is a weak implicative hyper MV-deductive system of M .

Proof. Let $x, y, z \in M$ be such that $z \in D$ and $((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \subseteq D$. Then

$$(((x^* \oplus (y^* \oplus x)^*)^*)^* \oplus z)^* = ((x^* \oplus (y^* \oplus x)^*) \oplus z)^* \subseteq D.$$

Using Proposition 3.16, we obtain $(x^* \oplus (y^* \oplus x)^*)^* \subseteq D$ which implies from (4.1) that $x \in D$. This shows that D is a weak implicative hyper MV-deductive system of M . \square

We provide a condition for a weak implicative hyper MV-deductive system to be a weak hyper MV-deductive system, as follows:

Theorem 4.11. *Assume that M satisfies the following condition:*

$$(4.2) \quad (\forall x \in M) (|(0^* \oplus x)^*| = 1 = |(x^* \oplus 0)^*|).$$

Then every weak implicative hyper MV-deductive system of M is a weak hyper MV-deductive system of M .

Proof. If M satisfies (4.2), then $(0^* \oplus x)^* = \{0\}$ and $(x^* \oplus 0)^* = \{x\}$ for all $x \in M$. Hence $(0^* \oplus A)^* = \{0\}$ and $(A^* \oplus 0)^* = A$ for every non-empty subset A of M . Let $x, y \in M$ be such that $y \in D$ and $(x^* \oplus y)^* \subseteq D$. Then

$$((x^* \oplus y) \oplus (0^* \oplus x)^*)^* = ((x^* \oplus y) \oplus 0)^* = (x^* \oplus y)^* \subseteq D,$$

which implies from (d4) that $x \in D$. Therefore D is a weak hyper MV-deductive system of M . \square

Theorem 4.12. *Every implicative hyper MV-deductive system D of M satisfies the following condition:*

$$(e2) \quad (\forall x, y, z \in M) (((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \cap D \neq \emptyset, z \in D \Rightarrow x \in D).$$

Proof. Assume that D is an implicative hyper MV-deductive system of M . Let $x, y, z \in M$ be such that $z \in D$ and $((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \cap D \neq \emptyset$. Then there exists $a \in ((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \cap D$. Since $a \ll a$ by (b3), we have $((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \ll D$. Using (d5), we obtain $x \in D$, which proves (e2). \square

Theorem 4.13. *If a non-empty subset D of M satisfies (d1) and (e2), then D is a hyper MV-deductive system of M .*

Proof. Let $x, y \in M$ be such that $y \in D$ and $(x^* \oplus y)^* \cap D \neq \emptyset$. Then there exists $a \in M$ such that $a \in (x^* \oplus y)^*$ and $a \in D$. Using (b2), (b9) and (b11), we have $a^* \in a^* \oplus 0 \subseteq (x^* \oplus y) \oplus (0^* \oplus x)^*$ and so $a \in ((x^* \oplus y) \oplus (0^* \oplus x)^*)^*$. Hence $((x^* \oplus y) \oplus (0^* \oplus x)^*)^* \cap D \neq \emptyset$, which implies from (e2) that $x \in D$. Using Theorem 3.19, we conclude that D is a hyper MV-deductive system of M . \square

Theorem 4.14. *Let D be a non-empty subset of M . If D satisfies (d1) and (e2), then D is an implicative hyper MV-deductive system of M .*

Proof. Let $x, y, z \in M$ be such that $((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \ll D$ and $z \in D$. Then there exist $a, b \in M$ such that $a \in ((x^* \oplus z) \oplus (y^* \oplus x)^*)^*$, $b \in D$ and $a \ll b$, i.e., $0 \in (a^* \oplus b)^*$. It follows that $(a^* \oplus b)^* \cap D \neq \emptyset$. By applying Theorems 3.19 and 4.13 we obtain that $a \in D$. Thus $((x^* \oplus z) \oplus (y^* \oplus x)^*)^* \cap D \neq \emptyset$, which implies from (e2) that $x \in D$. Therefore D is an implicative hyper MV-deductive system of M . \square

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