

Hyperbolic P-Y Criterion for Cohesive Soils

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ABSTRACT

Drilled shafts have been frequently used as a foundation to support lateral loads. The p-y method of analysis has been widely used for predicting the behavior of laterally loaded drilled shafts. The existing p-y criteria for cohesive soils are divided into soft or stiff clays, on the basis of a limited number of lateral load test results. Currently, there is no p-y criterion developed for cohesive intermediate geomaterial. In this paper, a hyperbolic equation for p-y curve is presented for cohesive soils and intermediate geomaterials. Based on 3-D FEM simulation results, a new empirical equation is presented for calculating the initial tangent to p-y curve. The proposed hyperbolic p-y criterion is verified by using the results of six full-scale lateral load tests on fully instrumented drilled shafts with diameters ranging from 0.76 m to 1.83 m in the geo-medium ranging from soft clays to intermediate geomaterial. The proposed hyperbolic p-y criterion is shown to be capable of predicting the loaddeflection and bending moments of the laterally loaded shafts for the six cases studied in this paper.

KEYWORDS: p-y criterion, Cohesive soils; Intermediate geomaterial, Lateral loads, Drilled shaft.

1. INTRODUCTION

Drilled shaft foundations are commonly used to resist axial and lateral loads applied to structures, or to stabilize slopes. There is a number of different approaches for analyzing the behavior of laterally loaded drilled shafts, and the most widely employed is the p-y approach developed by Reese and his coworkers. The p-y method is based on the numerical solution of beam-on-elastic foundation, where the structural behavior of a drilled shaft is modeled as a beam and the soil-shaft interaction is represented by discrete, non-linear springs characterized by p-y curves.

A number of p-y curves for clays have been developed by Matlock (1970), Reese and Welch (1975), Evans and Duncan (1982), Gazioglu and O'Neill (1984), Dunnivant and O'Neill (1989), Hsiung and Chen (1997)

and Yang and Liang (2005). These existing p-y curves were developed based on a limited number of model or field lateral load tests on piles or drilled shafts. Furthermore, Yang and Liang (2005) have shown that the existing Reese and Welch (1975) and Gazioglu and O'Neill (1984) p-y criteria do not work well for the lateral load tests that they studied. The accuracy and applicability of the existing p-y curves for clays can be further validated or improved by using additional lateral load test results that have become available since their original developments.

The advancement of computer technology has made it possible to study lateral soil-drilled shaft interaction problems with the rigorous finite element method. Brown and Shie (1990) have conducted a series of three-dimensional finite element analyses on the behavior of single pile and pile group by using an elastic soil model. Brown and Shie derived p-y curves from finite element analysis results and provided some comparison with the

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empirical design procedures in use. Bransby and Springman (1999) utilized the two-dimensional finite element method to find load-transfer relationships for translation of an infinitely long pile through undrained soil for a variety of soil-constitutive models. Yang and Jeremić (2002) performed a finite element study on pile behavior in layered elastic-plastic soils under lateral loads and generated p-y curves from the finite element results. It was found that the finite element results generally agree well with centrifuge data.

The objective of this paper is to present a unified p-y criterion for cohesive soils and intermediate geomaterials by using hyperbolic mathematical formulation. The 3-D finite element parametric study results of laterally loaded drilled shafts in clay are used to develop empirical equations for calculating two important parameters in the hyperbolic p-y formulation: initial tangent to p-y curve, K_i , and ultimate resistance, P_{ult} . A total of six full-scale fully instrumented lateral load test results are used to validate the proposed p-y criterion in predicting the load-deflection and bending moment of the drilled shafts under lateral loads.

The use of a hyperbolic function, given by Equation (1), as a p-y curve for sand, has been demonstrated by Kim et al. (2004) in their study of model tests in the Nakdong River sand.

$$p = \frac{y}{\frac{1}{K_i} + \frac{y}{P_{ult}}} \quad (1)$$

p=force per unit shaft length (F/L),

y=lateral displacement of shaft (L),

K_i =initial subgrade reaction modulus of soil (F/L²)

and P_{ult} =the ultimate reaction force per unit length of shaft (F/L).

In this paper, the hyperbolic function is suggested for constructing p-y curves for both soft and stiff clays as well as for intermediate geomaterial under short-term static loading. The methods to compute K_i and P_{ult} are based on FEM simulation results and are discussed in the sections below.

2. BACKGROUND

Given below is a brief review of existing equations for calculating the two key parameters in the hyperbolic p-y function.

Initial Tangent to P-y Curve

By fitting the subgrade reaction solution with the continuum elastic solution for the beam on an elastic foundation, Vesic (1961) provided an elastic solution for the modulus of subgrade reaction, K_i as follows:

$$K_i = \frac{0.65E}{1-\nu^2} \left[\frac{ED^4}{E_p I_p} \right]^{1/12} \quad (2)$$

where E = modulus of elastic materials, ν = Poisson's ratio, D = beam width, and $E_p I_p$ = flexural rigidity of beam.

Bowles (1988) suggested to double the value of K_i in Equation (2) for piles under lateral loading, since the pile would have contact with soils on both sides. However, in reality, soils do not fully contact with the piles when lateral loads are applied. Based on field test data, Carter (1984) modified Vesic's equation as follows to account for the effect of pile diameter:

$$K_i = \frac{1.0ED}{(1-\nu^2)D_{ref}} \left[\frac{ED^4}{E_p I_p} \right]^{1/12} \quad (3)$$

where the reference pile diameter, $D_{ref} = 1.0$ m, $E_p I_p$ = flexural rigidity of the piles or drilled shafts. As noted in FEM Modeling Section in this paper, the effect of Poisson's ratio on K_i by Carter (1984) seems to contradict to the FEM parametric study results.

Ultimate Resistance, P_{ult}

Based on a wedge failure mechanism, Reese (1958) provided an equation for estimating P_{ult} , near the ground surface as follows:

$$P_{ult} = (2S_u + \gamma'z + \frac{2.83S_u z}{D})D \quad (4)$$

Table 1. Maximum Bending Moment at Different Loading Level for the Six Cases.

Site	Lateral Load (kN)	Max. Bending Moment (kN.m)			Prediction Error	
		Proposed	Reese/Matlock	Measured	Proposed	Reese/Matlock
LOR-6	1601	3101	3470	1750	0.77	0.98
	1868	3812	4243	2691	0.42	0.58
	2313	5014	5487	4586	0.09	0.20
	2580	5676	6171	5918	0.04	0.04
Slat Lake	53	48	55	50	0.04	0.10
	98	93	114	90	0.03	0.26
CUY-90	1335	4678	5624	2427	0.93	1.32
	1780	6881	8194	4312	0.60	0.90
	2225	9165	10992	9619	0.05	0.14
	2670	11488	13952	12105	0.05	0.15
	2759	11973	14537	13000	0.08	0.12
	3560	16699	20339	29063	0.43	0.30
CDOT	22	37	38	28	0.33	0.37
	45	77	78	58	0.33	0.34
	67	118	119	91	0.29	0.30
	90	159	162	124	0.28	0.31
	134	236	253	190	0.24	0.33
	180	320	352	351	0.09	0.00
	224	406	448	431	0.06	0.04
	269	494	544	521	0.05	0.04
	343	651	705	694	0.06	0.02
	394	756	812	776	0.03	0.05
JEF-152	89	736	750	790	0.07	0.05
	165	1349	1501	1452	0.07	0.03
	267	2206	2284	2407	0.08	0.05
	334	2759	2901	2917	0.05	0.01
	400	3310	3527	3598	0.08	0.02
	476	3951	4173	4236	0.07	0.01
	538	4467	4884	4970	0.10	0.02
	614	5103	5552	5411	0.06	0.03
	689	5736	6223	6045	0.05	0.03
	761	6327	6944	6468	0.02	0.07
WAR-48	178	200	203	163	0.23	0.25
	445	560	568	407	0.38	0.40
	667	900	907	667	0.35	0.36
	890	1220	1268	1190	0.03	0.07
	1112	1539	1647	1684	0.09	0.02
	1334	1860	2041	2051	0.09	0.01
	1512	2100	2421	2264	0.07	0.07
	1690	2340	2784	2416	0.03	0.15

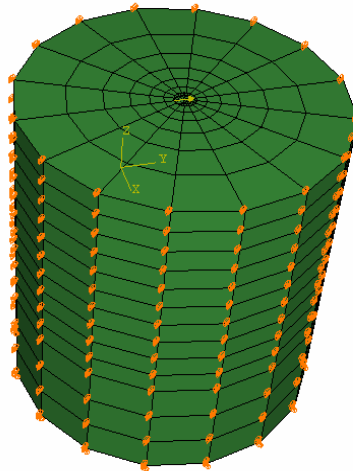


Fig. 1: 3-D FEM Model.

Based on the flow-around failure theory for clay at a great depth, P_{ult} for in-depth clay is given as:

$$p_{ult} = 11S_u D \quad (5)$$

Later, Matlock (1970) suggested computing P_{ult} by using the smaller one of the values given by the equations below:

$$p_{ult} = \left(3 + \frac{\gamma'}{S_u} z + \frac{J}{D} z \right) S_u D \quad (6)$$

$$p_{ult} = 9S_u D \quad (7)$$

where γ' = average effective unit weight from ground surface to the depth z under consideration, z = depth of the p-y curve, S_u = undrained shear strength at depth z , D = diameter of the drilled shaft, $J = 0.5$ for a soft clay and $J=0.25$ for a medium clay. A value of 0.5 is frequently used for J .

3. FEM MODELING

Initial Tangent K_i

The commercial ABAQUS finite element program is used for modeling the soil-drilled shaft interaction, as shown in Fig. 1. The drilled shaft is modeled as a

cylinder with elastic material properties. The solid elements C3D15 and C3D8 available in ABAQUS were used to develop mesh representation for shaft and clay, respectively. Surface interface technique is employed to simulate the soil-shaft interface. Since the determination of initial tangent to p-y curve constitutes the primary objective of this part of FEM study, only the elastic response of clay is the main concern of this part of the parametric study.

Effect of Elastic Modulus of Soils, E_s

The effect of elastic modulus of soils, E_s , is studied by varying it from 5,000 kPa to 35,000 kPa. Other pertinent parameters are kept as constant as follows: $D=1\text{m}$, $E_p=2.0 \times 10^7$ kPa, $\nu_s=0.3$. The FEM computed results are plotted in Fig. 2 (a), from which one can see that a power function fits the relationship between K_i and elastic modulus of the soil.

Effect of Shaft Diameter, D

The effect of shaft diameter is investigated by varying the diameter from 1 m to 4 m. Other pertinent parameters are kept as constant as follows: $E_p=2.0 \times 10^7$ kPa, $E_s=2.0 \times 10^4$ kPa, $\nu_s=0.3$. The K_i generated at a depth of 2 m is shown in Fig. 2 (b). It can be seen that K_i increases linearly with the increase of the shaft diameter.

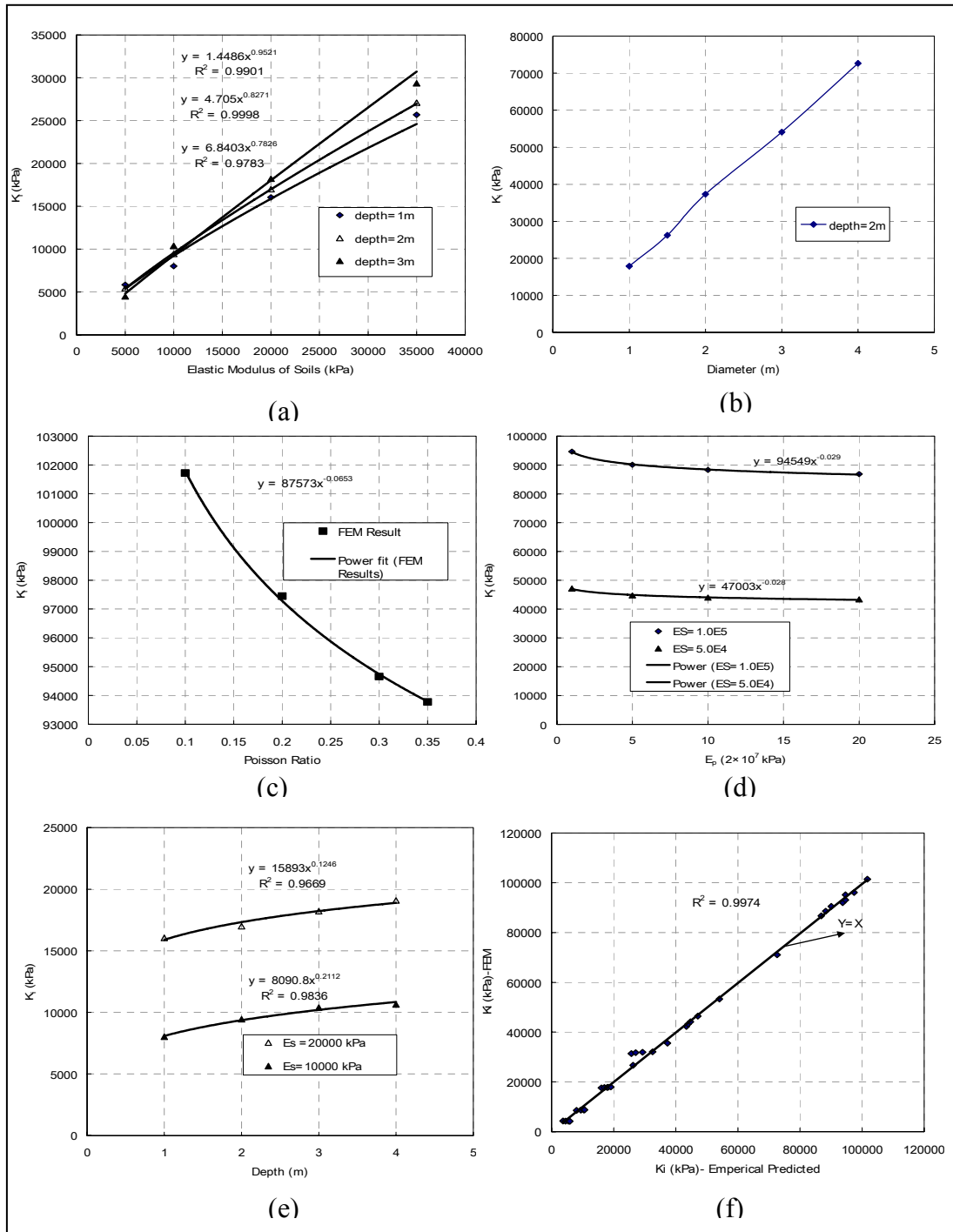


Fig. 2: Effects of Shaft and Soils Parameters on K_s .

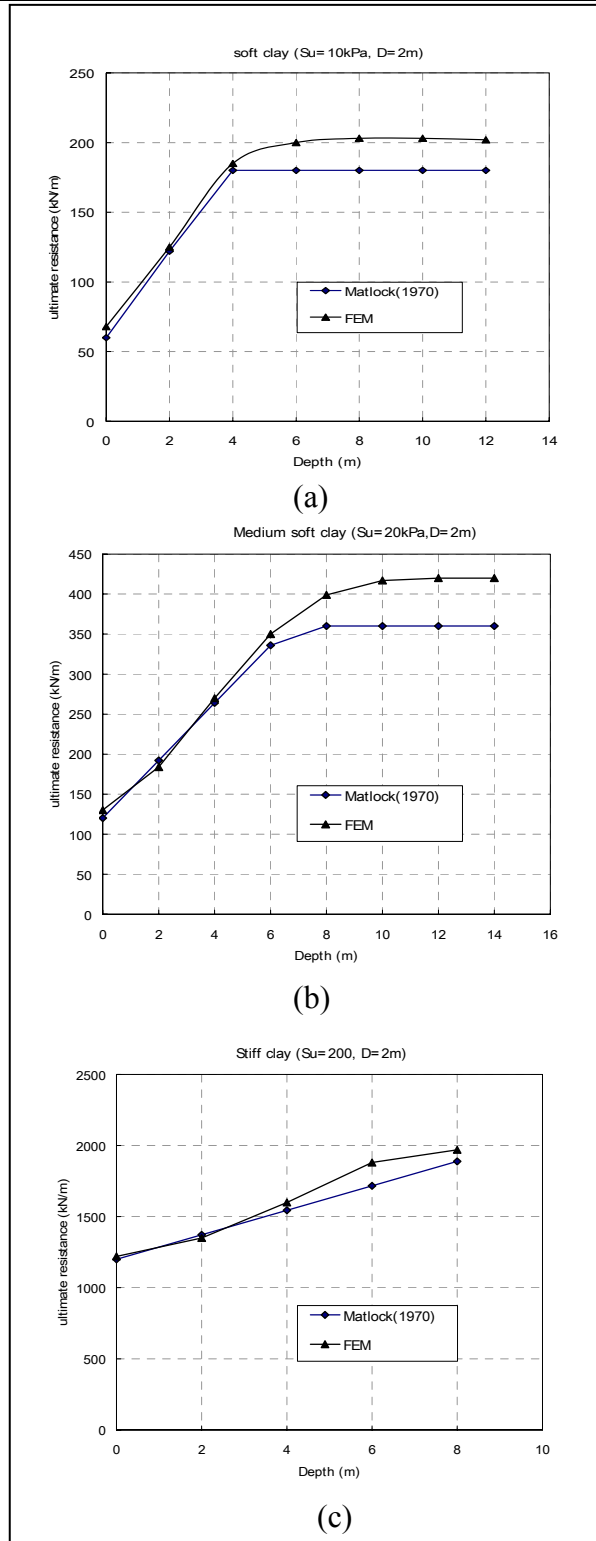


Fig. 3: Ultimate Resistances vs. Depth.

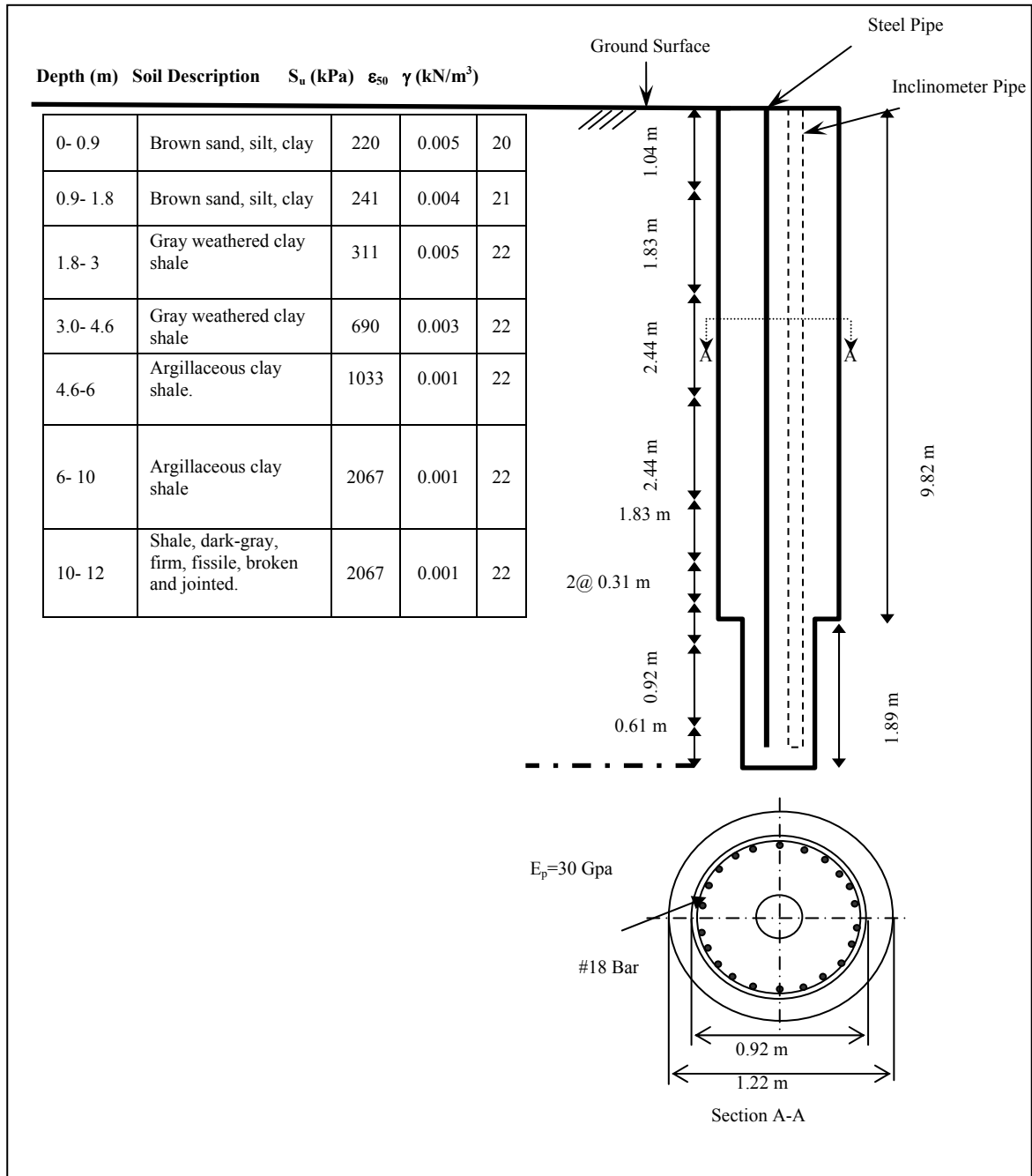


Fig. 4: Soil Profile and Shaft Dimension at Ohio LOR-6 Test Site.

Effect of Poisson's Ratio of Soils, ν_s

The Poisson's ratio for cohesive soil is varied from 0.1 to 0.35, while other pertinent parameters are kept as constant as follows: $D=1.0$ m, $E_p=2.0 \times 10^7$ kPa, $E_s=1.0 \times 10^5$ kPa. The K_i generated at a depth of 1 m is shown in Fig. 2 (c), where a power function can fit the data well. It is noted that in Carter's empirical equation (Carter, 1984), an increase in Poisson's ratio leads to an increase in K_i . The present FEM results show that K_i decreases with an increase in Poisson's ratio.

Effect of Elastic Modulus of Shafts, E_p

The effect of elastic modulus of shafts, E_p , is studied by varying it from 2.0×10^7 kPa to 4.0×10^8 kPa, while other parameters are kept as constant as follows: $D=1$ m, $E_s=1.0 \times 10^5$ kPa and 5.0×10^4 kPa, $\nu_s=0.3$. Fig. 2 (d) shows the relationship between K_i and E_p . It can be seen that a power function fits very well the relationship between K_i and E_p .

Effect of Depth, Z

The relationship between initial tangent of p-y curve and depth is also investigated by varying the depth from 1.0 m to 4.0 m. Other pertinent parameters are kept as constant as follows: $D=1$ m, $E_p=2.0 \times 10^7$ kPa, $\nu_s=0.3$. The FEM results are computed for two values of E_s (10,000 kPa and 20,000 kPa) as shown in Fig. 2 (e), from which one can see that a power function can also fit the relationship between K_i and the depth.

Suggested Empirical Equation for K_i

Based on the parametric study presented in the previous section, the following conclusions may be drawn:

- K_i and the following variables exhibit a power relationship: the modulus of soils, the modulus of a shaft, depth and Poisson's ratio of a soil.
- K_i increases linearly with an increase in shaft diameter.

A regression analysis on data from the FEM parametric study is carried out. An equation for predicting initial modulus of subgrade reaction is fitted to match K_i values obtained from the FEM parametric

study. As shown in Fig. 2 (f), the empirical equation can be derived as follows.

$$K_i = 0.943 (Z / Z_{ref})^{0.016} (D / D_{ref}) \nu_s^{-0.078} E_s^{1.036} E_p^{-0.031} \quad (8)$$

where

$$Z_{ref} = 1.0 \text{ m and } D_{ref} = 1.0 \text{ m.}$$

Ultimate Resistance, P_{ult}

The strength of saturated clay is usually characterized by undrained shear strength, S_u ; therefore, clay is modeled as a simple Von Mises material in the FEM simulation.

The finite element model used to obtain the ultimate resistance is the same as the one used to obtain K_i . The drilled shaft with a diameter of 2 m is modeled as an elastic material, while the undrained shear strength of cohesive soils is varied from 10 kPa to 200 kPa. The ultimate resistances computed from finite element analyses are shown in Fig. 3 (a) to (c) for three representative values of S_u . It can be seen that the results of finite element analyses agree generally well with those deduced from the Matlock's method. The ultimate resistance of in-depth clay from FEM is generally larger than $9S_u D$, but smaller than $11S_u D$. Therefore, $10S_u D$ is adopted in this paper. The ultimate resistance of clays can be calculated by using the smaller one of the values given by the following equations:

$$p_{ult} = \left(3 + \frac{\gamma'}{S_u} z + \frac{J}{D} z \right) S_u D \quad (9)$$

$$p_{ult} = 10 S_u D \quad (10)$$

4. CASE STUDIES

Six lateral load tests are employed to validate the proposed p-y criterion for cohesive soils by comparing the measured and predicted load-deflection curves, and the deflection vs. depth curves.

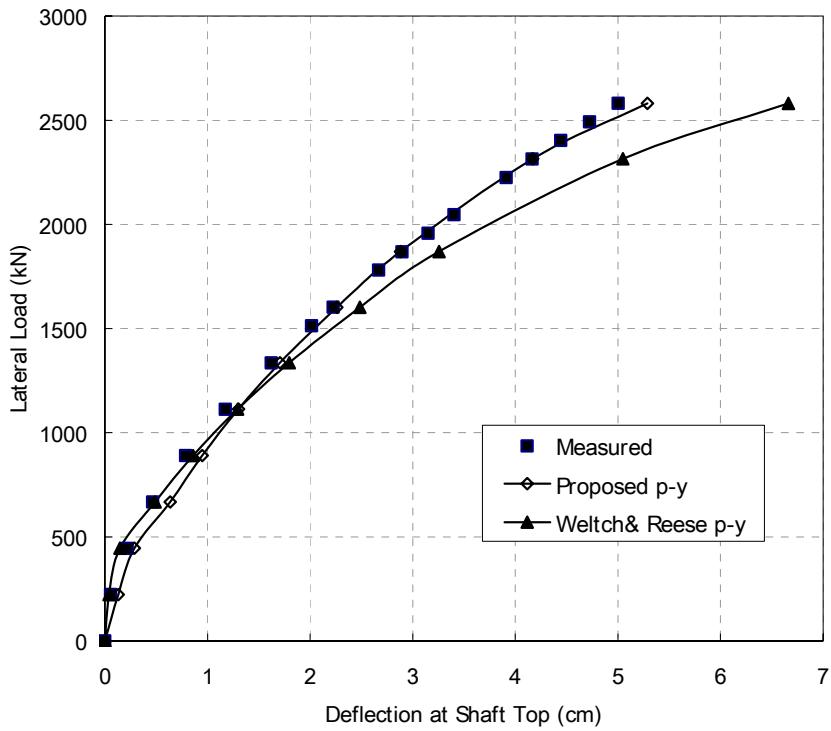


Fig. 5: Comparison of Predicted and Measured Load-Deflections of Ohio LOR-6 Test.

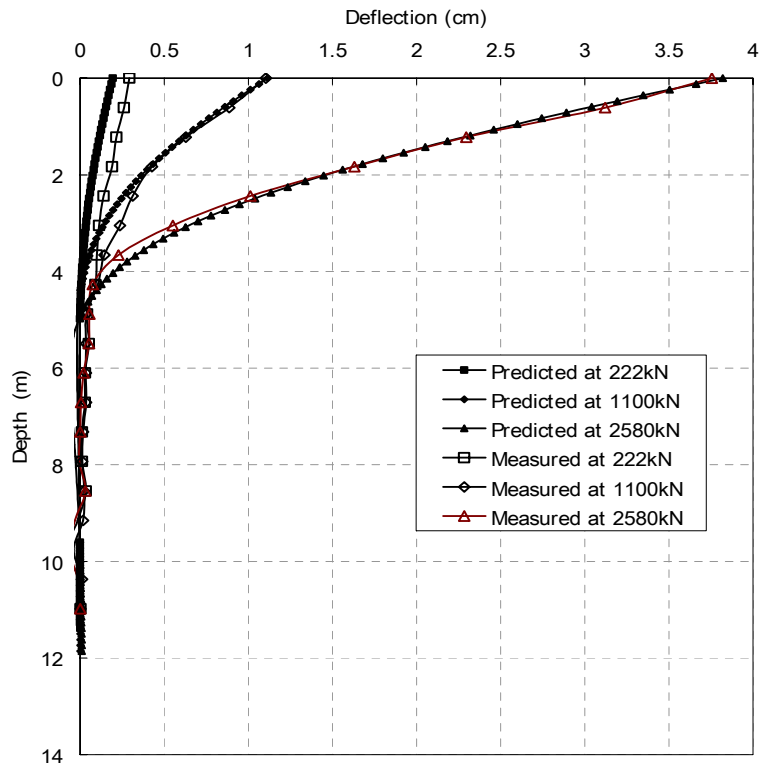


Fig. 6: Comparison of Predicted and Measured Deflections vs. Depth of Ohio. LOR-6 Test.

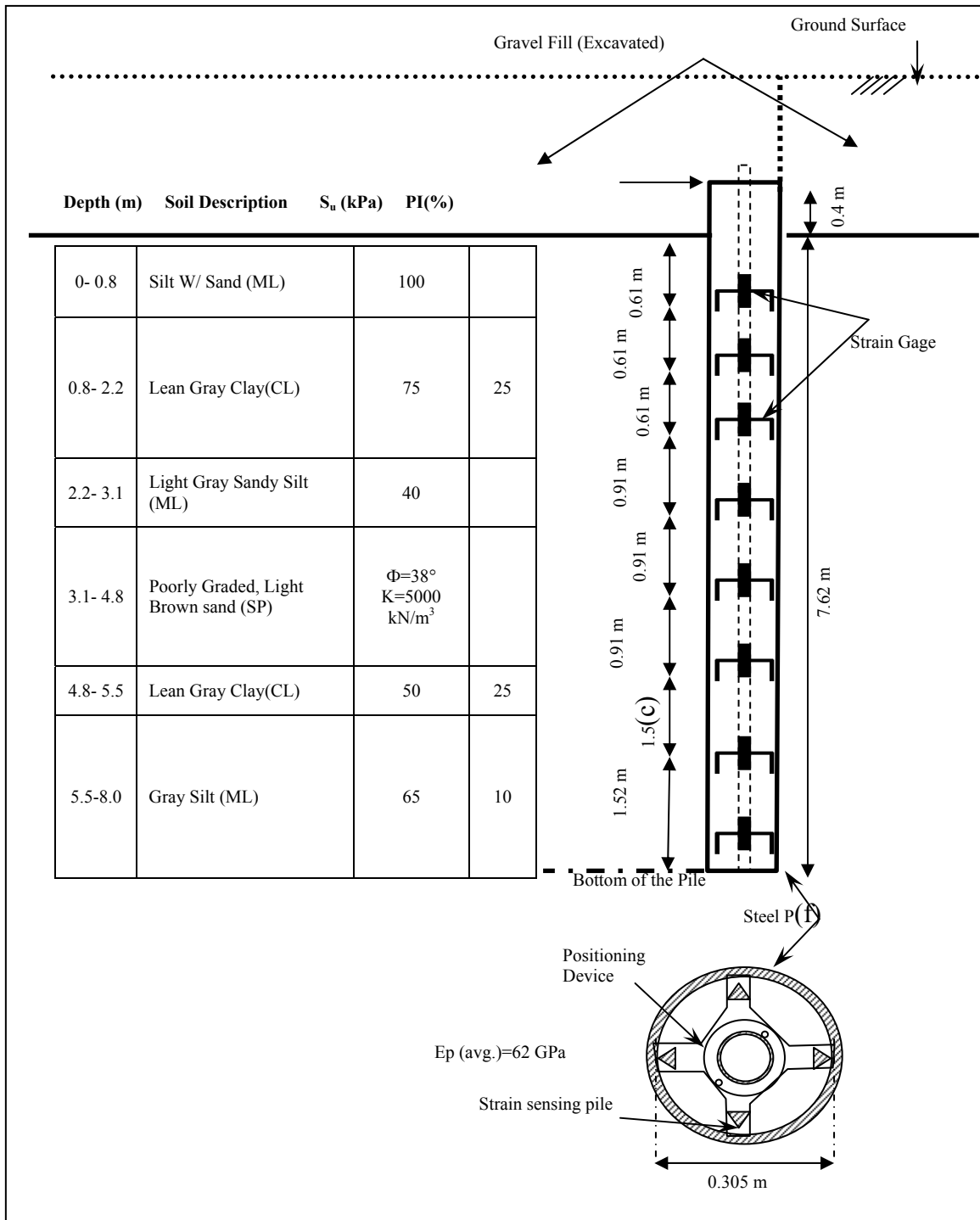


Fig. 7: Soil Profile and Pile Detail at Salt Lake International Airport Test (After Rollins et al., 1998).

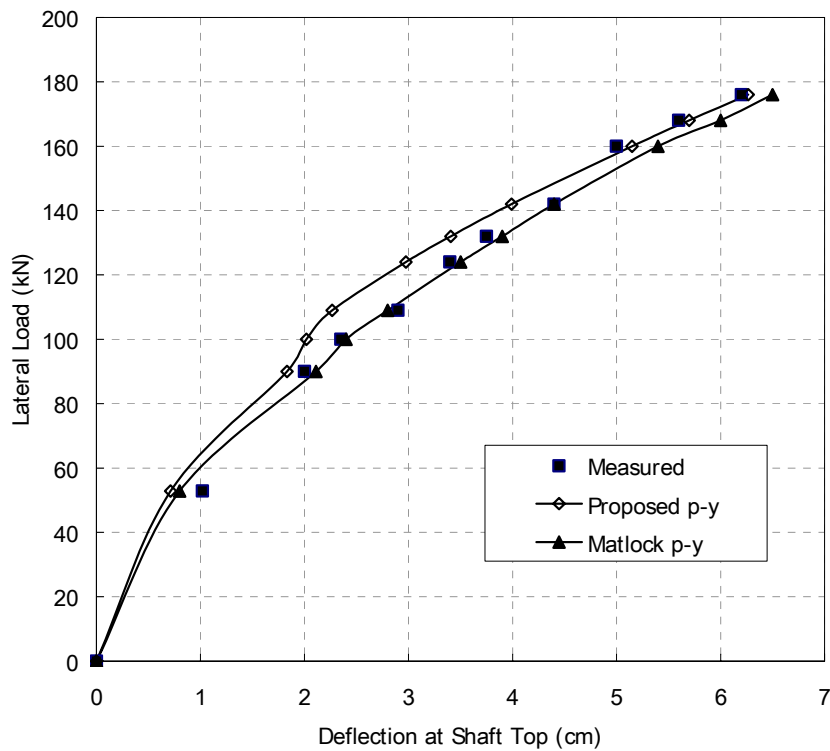


Fig. 8: Comparison of Predicted and Measured Load-Deflection Curves of Salt Lake International Airport.

In addition to deflection prediction, the maximum moments of drilled shafts under different loading levels predicted by using LPILE based on the proposed p-y criterion and other existing p-y criteria are compared with the corresponding maximum moments based on strain gage readings. The moment prediction errors, defined as the absolute value of moment difference divided by the measured moments, are summarized in Table 1.

Ohio LOR-6 Test (Liang, 1997)

The soils at the test site are composed primarily of silt clay, underlain by gray clay shale. The soil profile at the LOR-6 test site is presented in Fig. 4.

The undrained shear strength of clay is correlated to undrained elastic modulus as follows (U.S. Army Corps of Engineers, 1990):

$$E_s = K_c S_u \tag{11}$$

where K_c is a correlation factor. The value of K_c as a

function of the overconsolidation ratio and plasticity index, PI, is estimated to be 500.

The geometry and dimension of test shaft as well as the instrumentation layout are shown in Fig. 4. The diameter of the shaft in clay is 1.22 m, while the diameter of the shaft socket in the shale is 0.92m. Inclinometer casing and strain gages were installed inside the shaft.

Fig. 5 shows the comparison of the predicted and measured load-deflection curves. It can be seen that the predicted load-deflection using the proposed hyperbolic p-y criterion agrees very well with the measured values. The Reese and Welch (1975) p-y criterion overpredicts the deflections. A comparison of the predicted and measured deflections vs. depth is shown in Fig. 6.

The average maximum bending moment prediction errors using the proposed p-y criterion is 0.33 as it can be calculated from Table 1. It is less than 0.45, the average prediction error by using Reese and Welch p-y criterion (1975).

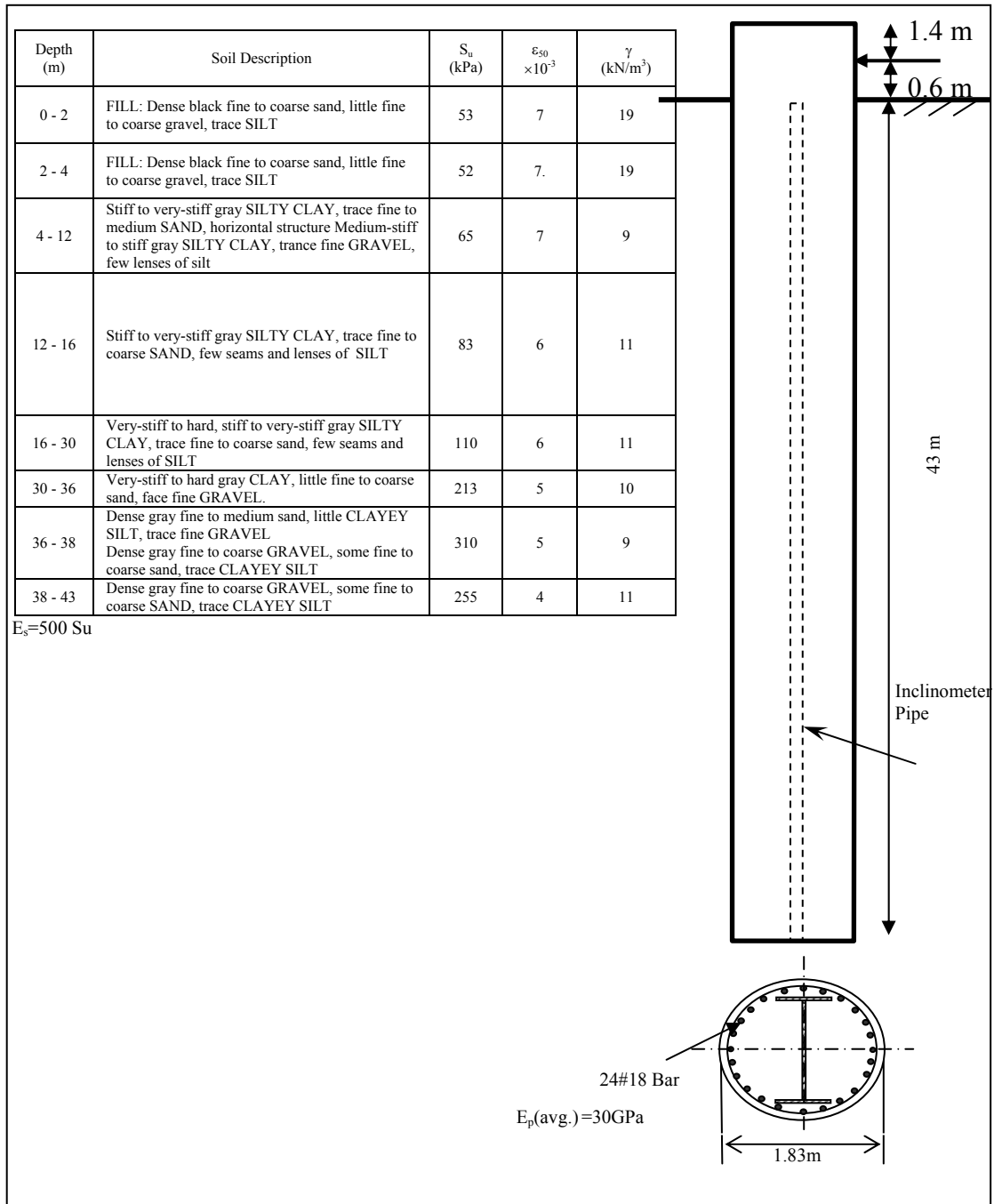


Fig. 9: Soil Profile and Shaft Dimension at CUY-90 Test Site.

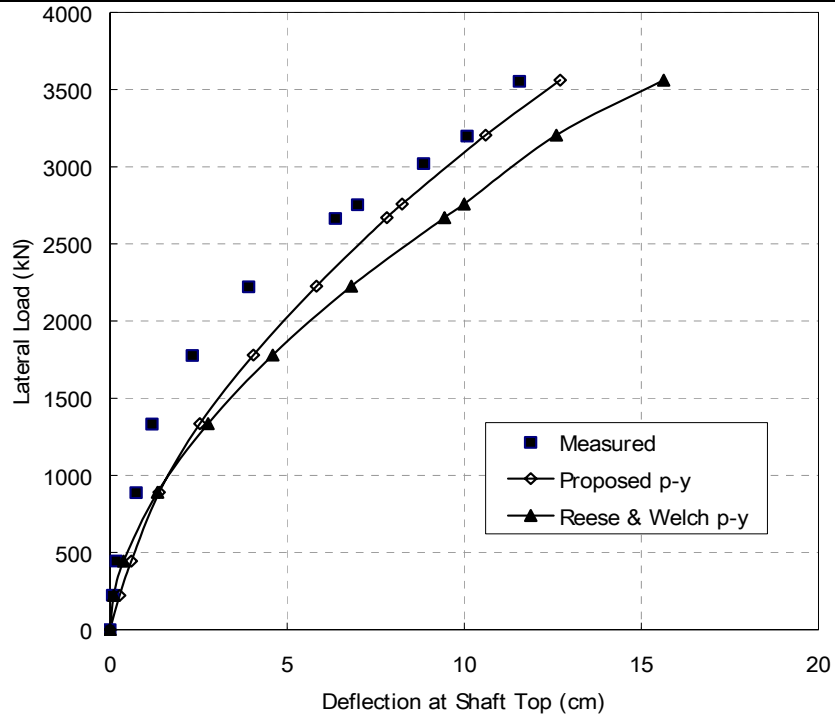


Fig. 10: Comparison of Predicted and Measured Load-Deflection Curve of Ohio CUY-90 Test.

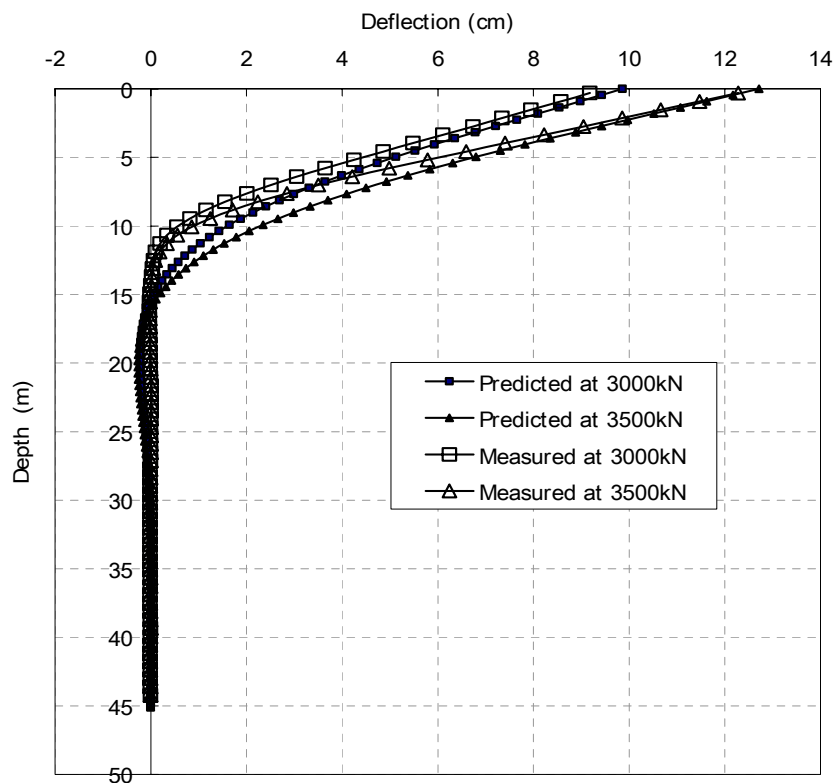


Fig. 11: Comparison of Predicted and Measured Deflections vs. Depth of Ohio CUY-90 Test.

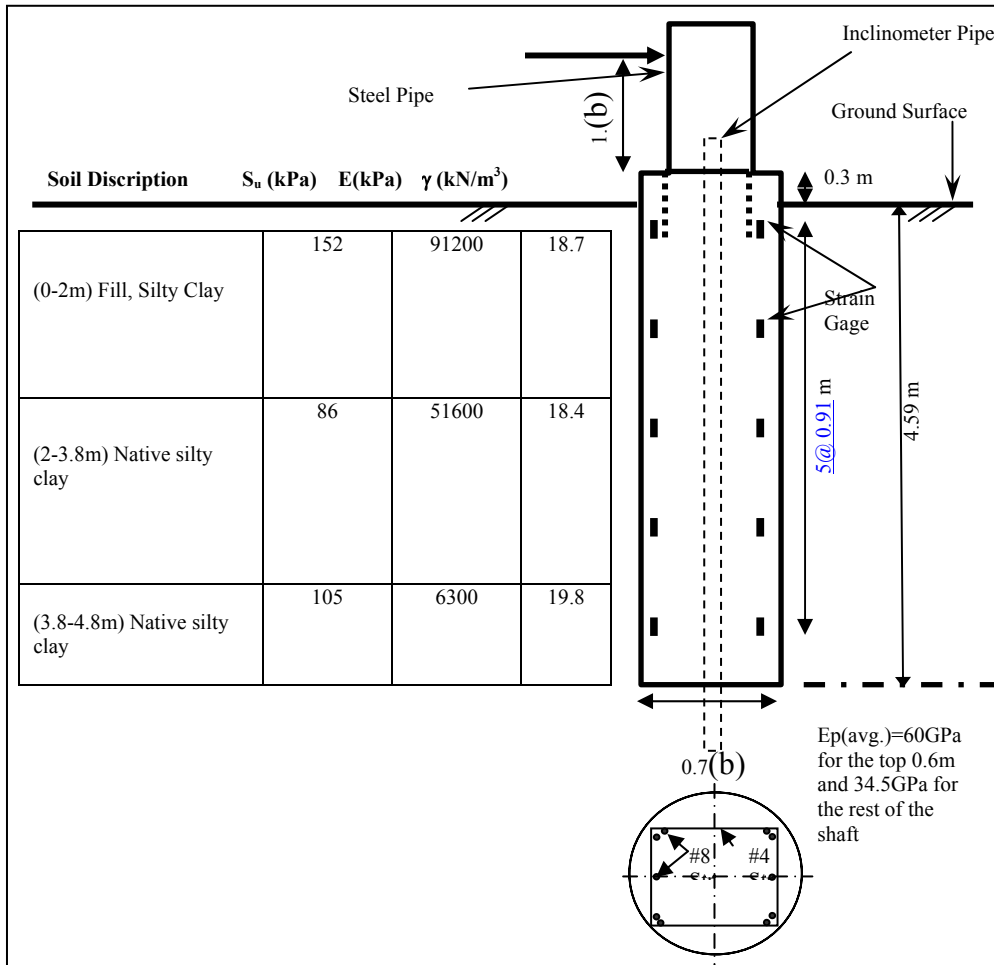


Fig. 12: Soil Profile and Shaft Dimension at Colorado I-225 Clay Site.

Salt Lake International Airport Test (Rollins et al., 1998)

The test pile was 0.305m I.D. closed-end steel pipe with a 9.5-mm wall thickness driven to a depth of approximately 9.1 m. The elastic modulus of the steel was 200 GPa, and the minimum yield stress was 331 MPa. Prior to conducting the lateral pile load testing, inclinometer casing and strain gauges were placed inside the pile. The pile was then filled with concrete. The compressive strength and the elastic modulus of the concrete at the time of testing were 20.7 MPa and 17.5 GPa, respectively.

The soil profile at the test site as well as the pile instrumentation detail are presented in Fig. 7. The soils

near the ground surface are clays and silts with undrained shear strength typically between 25 kPa and 50 kPa. Some layers showed undrained shear strength of 100 kPa. The underlying cohesionless soil layer consists of poorly graded medium-grained sands and silty sands. Based on EM 1110-1-1904, for PI=25, E_s of cohesive soils can be estimated to be 1000 S_u .

The proposed hyperbolic criterion and the Matlock (1970) criterion are used to generate two sets of p-y curves for the clay layers. Reese et al. (1975) p-y criterion is used to generate the p-y curves for the sand layers. The LPILE analysis results by using these p-y curves are compared with the measured load-deflection curve in Fig. 8. The proposed p-y curves provides good

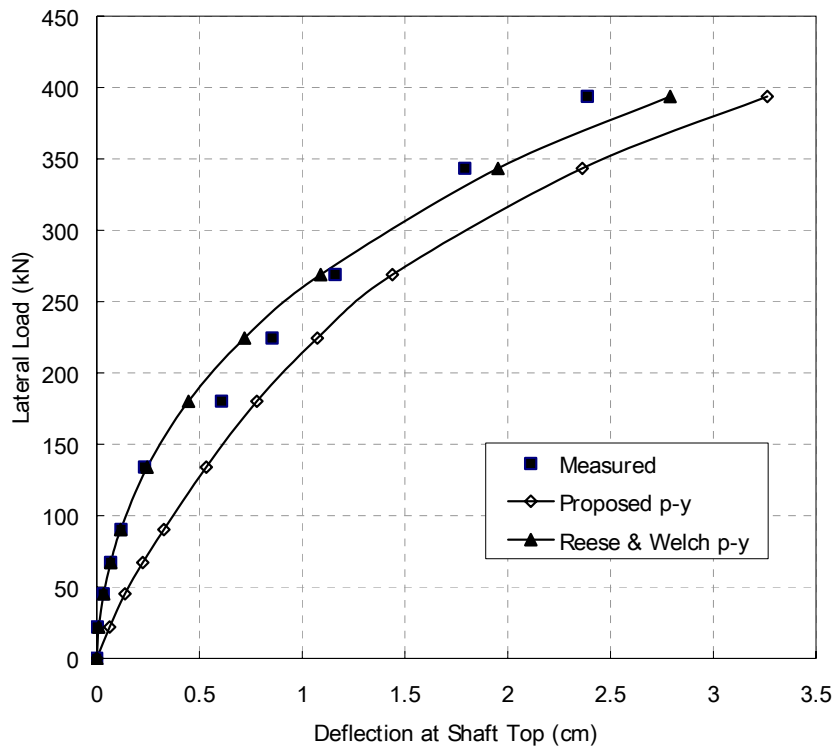


Fig. 13: Comparison of Predicted and Measured Load-Deflection Curves of CDOT Clay Site.

match with the measured load-deflection curve. Also, its bending moment prediction errors are less than that of Matlock (1970) p-y criterion prediction.

Ohio CUY-90 Test (Liang, 2000)

The soils near the ground surface are mainly composed of silts, soft to stiff clays. Deposits in great depth are shale with trace of gravel. The test shaft is 45.0 m long with an embedded length of 43 m. The lateral load was applied at a point 0.6 m above the ground surface. The diameter of the test shaft is 1.83 m, reinforced with 24 bars, 57mm in diameter, and a built-up beam. Fig. 9 shows the soil profile at the site as well as the details of reinforcement inside the shaft.

Fig. 10 and Fig. 11 show the predicted and measured load-deflection curves and the deflection-depth curves, respectively. It can be seen that the proposed p-y criterion can predict the load-deflection of the drilled shaft much

better than the Reese and Welch (1975) p-y criterion.

At five different loading levels out of the six shown in Table 1, the maximum bending moment prediction errors calculated based on the proposed p-y criterion results are less than those calculated based on the Reese and Welch (1975) p-y criterion.

Colorado I-225 Test (Nusairat et al., 2004)

The soil profiles at the test site with the soil parameters interpreted from pressuremeter test as well as the instrumentation of the test shaft are presented in Fig.12.

The shaft is relatively short, with embedment length of 4.59 m. Therefore, the resistance of the shaft tip is a significant component of the overall lateral resistance. The comparisons between the predicted and measured load-deflection curves are shown in Fig. 13. A discrepancy is observed, which may have been contributed by the shear resistance at the base of the shaft .

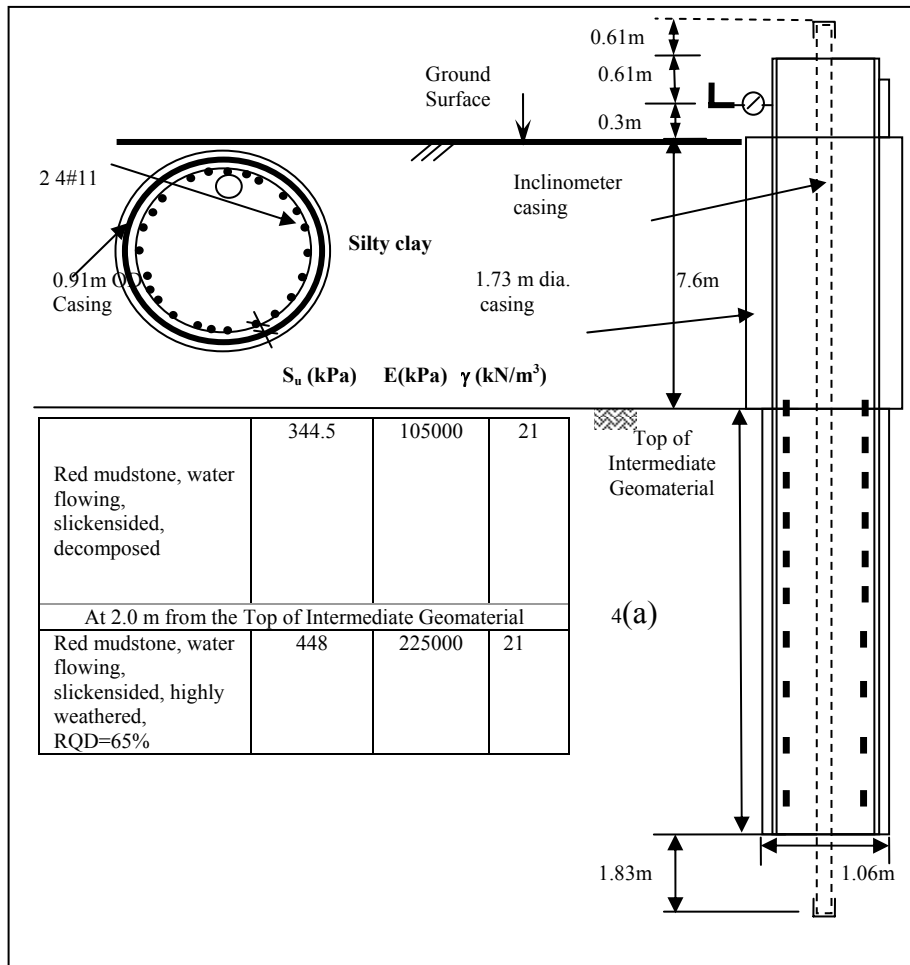


Fig. 14: Soil Profile and Shaft Dimension at Ohio JEF-152 Test Site.

This is another case proving that the maximum bending moment predicted using the proposed p-y criterion matches the measured value better than the prediction using Reese & Welch (1975) p-y criterion as calculated in Table 1.

Ohio JEF-152 Test (Nusairat et al., 2006)

The soil profile and the undrained shear strength and modulus of elasticity of the soil interpreted from pressuremeter test are summarized in Fig. 14. The depth to the intermediate geomaterial is 7.6 m from the ground surface as shown in Fig. 14, where the instrumentation of the test shaft is also presented.

The proposed hyperbolic p-y criterion and Reese and

Welch (1975) p-y curves for stiff clay are used to generate the p-y curves as input in the LPILE program to predict the load-deflection curve of the test shaft. Fig. 15 shows the predicted and measured load-deflection curves. The predicted and measured deflection vs. depth curves are compared in Fig. 16. The comparison of the predicted and measured maximum bending moment is given in Table 1. It can be seen that the proposed p-y criterion allows LPILE program to predict the measured data very well.

Ohio WAR-48 Test (Nusairat et al., 2006)

The test site is located on SR 48 over Clear Creek in Warren County, Ohio. The test shafts were part of retaining wall supporting the roadway. The drilled shafts

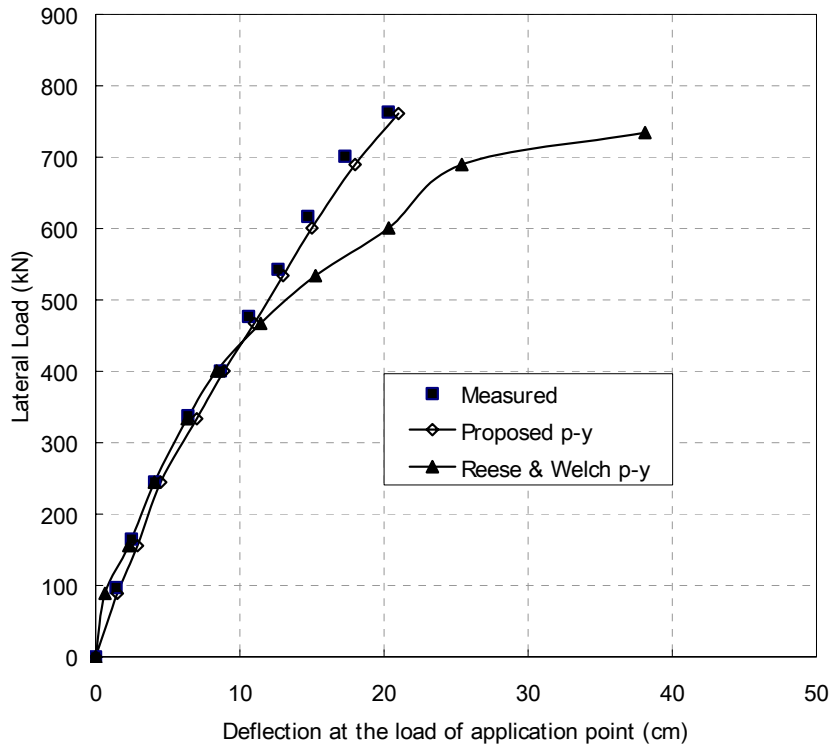


Fig. 15: Comparison of Predicted and Measured Load Deflection Curves of Ohio JEF-152 Test.

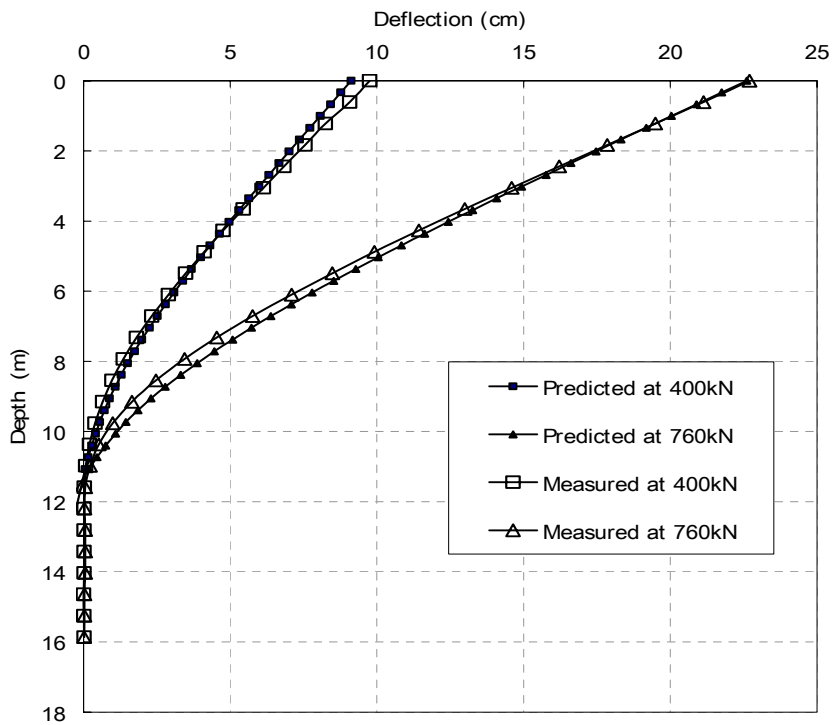


Fig. 16: Comparison of Predicted and Measured Deflection vs. Depth Curves of Ohio JEF-152 Test.

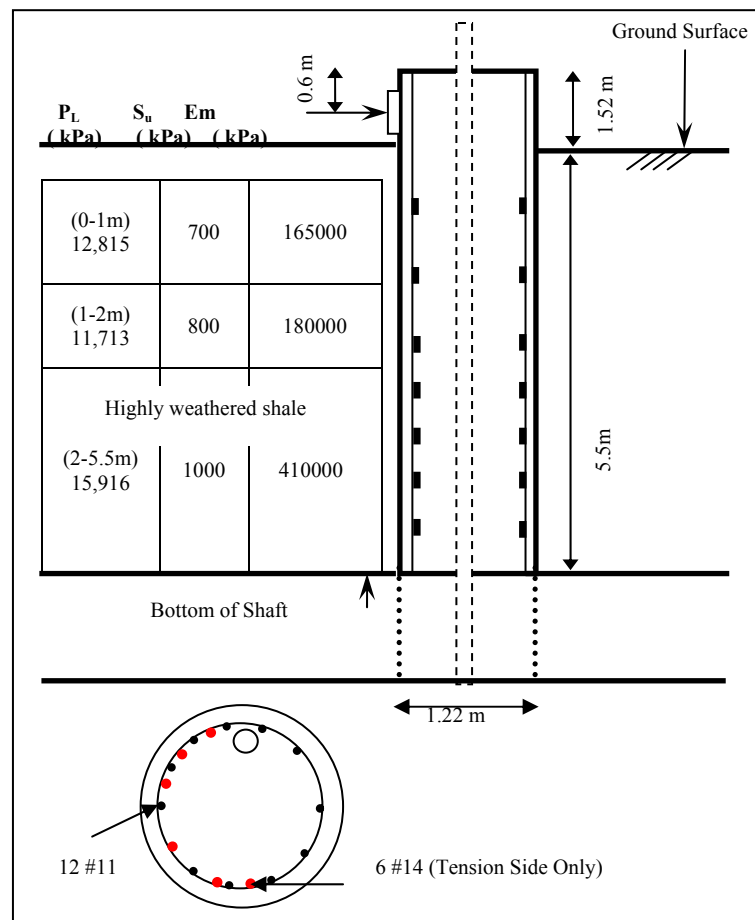


Fig. 17: Soil Profile and Shaft Dimension at Ohio War-48 Test Site.

were excavated by using the “dry” method, with temporary casings for excavation support. The total length of the drilled shaft is about 7.02 m, with 1.52 m extension above the ground surface for load application. The subsurface stratigraphy encountered during drilling of the 1.22 m diameter shafts consisted of shale from the ground surface to the bottom of the shafts.

Field rock dilatometer tests were conducted at this site. The limiting pressure, undrained shear strength and the modulus of elasticity, interpreted from the dilatometer tests, are summarized in Fig. 17 together with instrumentation details. The reinforcement of the shaft was 12 -35 mm and 6 -44.4 mm bars with a concrete cover of 7.6 cm.

The lateral loads were applied 0.6 m below the top of the shaft in increments of 222.5 kN. The maximum lateral load reached was 1780 kN.

For comparison, the proposed hyperbolic criterion, Reese and Welch (1975) stiff clay criterion and Reese (1997) weak rock criterion were used to generate p-y curves. The load-deflection curves predicted by LPILE program with these p-y curves are shown in Fig. 18. Fig. 19 shows the predicted and measured deflection vs. depth. It can be seen that the proposed hyperbolic criterion provides the best match.

The maximum bending moment prediction by using both p-y criteria are almost the same as it can be seen in Table 1.

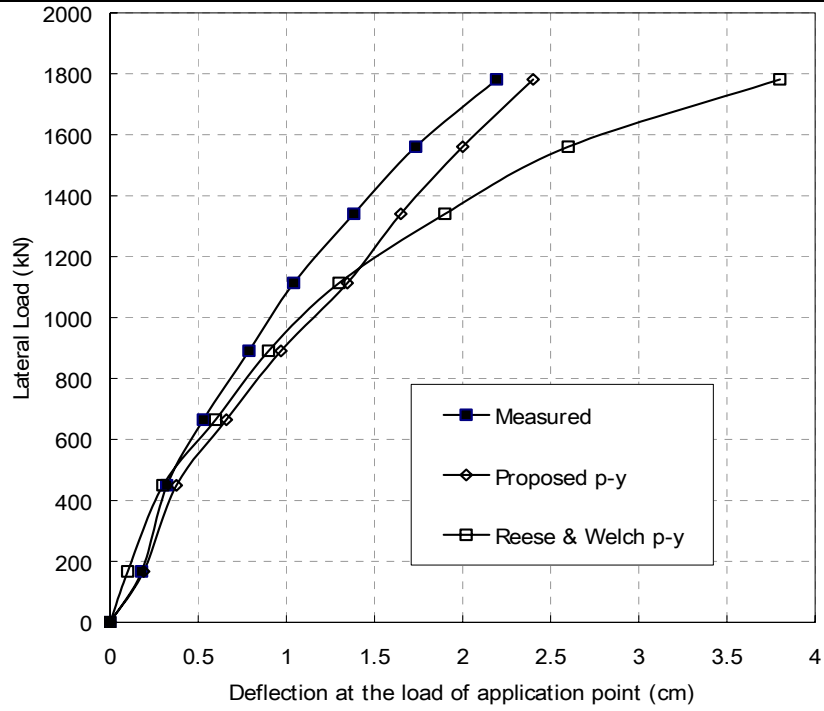


Fig. 18: Comparison of Predicted and Measured Load Deflection of Ohio WAR-48 Test.

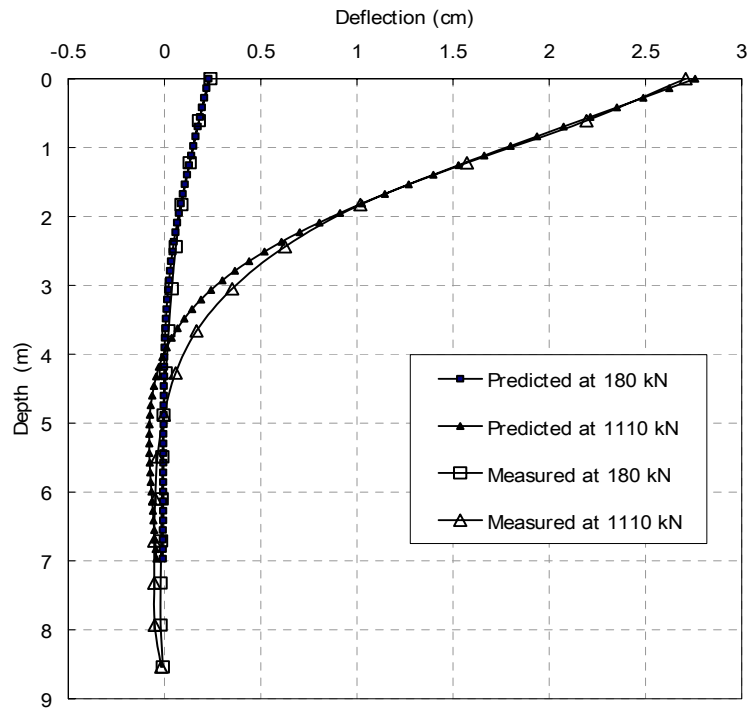


Fig. 19: Comparison of Predicted and Measured Deflection vs. Depth Curves of Ohio WAR-48 Test.

5. CONCLUSIONS

A hyperbolic p-y criterion is developed for cohesive soils and intermediate geomaterial. The 3-D FEM simulation results are used to develop a new empirical equation to calculate initial tangent to p-y curve, K_i . The

proposed p-y criterion is verified based on comparisons with six full-scale lateral load test results conducted in soft clay to intermediate cohesive geomaterial. The proposed p-y criterion is shown to be capable of predicting the behavior of these test shafts under lateral loading.

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