HYPERCOMPLEX AUTO- AND CROSS-CORRELATION OF COLOR IMAGES

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Abstract

Autocorrelation and cross-correlation have been defined and utilized in signal and image processing for many years, but not for color or vector images. In this poster we present for the first time a definition of correlation applicable to color images, based on quaternions or hypercomplex numbers. We have devised a visualization of the result using the polar form of a quaternion in which color denotes quaternion eigenaxis and phase, and a grayscale image represents the modulus.

1. Introduction

As part of our work on Fourier transforms of color images using hypercomplex, or quaternion Fourier transforms [1], we have considered the problem of defining the autocorrelation of a color image, and the cross-correlation of two color images. We show in this poster that there is a natural extension of the definition of correlation from the case based on complex numbers to one based on hypercomplex numbers or quaternions (discovered by Hamilton in 1843 [2]), and we present a visualization of the hypercomplex result.

2. Hypercomplex numbers

A quaternion (Cartesian):

$$q = a + ib + jc + kd$$

a, b, c, d real,
i, j, k complex:

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ij = k \quad jk = i \quad ki = j$$
$$ji = -k \quad kj = -i \quad ik = -j$$

Conjugate and modulus:
$\overline{q} = a - ib - jc - kd$
$|q| = \sqrt{a^2 + b^2 + c^2 + d^2}$
If $a = 0$ the quaternion is *pure*, and if $|q| = 1$ it is a *unit* quaternion.
Scalar/vector form:

$$q = S(q) + V(q)$$

where $S(q) = a$, and $V(q) = ib + jc + kd$.

3. Color image representation

A color image in RGB color space may be represented using hypercomplex numbers by encoding the red, green and blue components of the RGB value as a pure quaternion:

$$f(x, y) = ir(x, y) + jg(x, y) + kb(x, y)$$

where $r(x, y)$ is the red component of the color image and similarly for the green and blue components. The reason for choosing this representation is that the RGB values represent a 3-space vector (a point in RGB space), as does the pure quaternion.

4. Hypercomplex correlation formula

We use the standard definition of cross-correlation of two images e.g. [3], but both images and the result are hypercomplex, and the conjugate is hypercomplex:

$$r(n, m) = \sum_{p=0}^{N-1} \sum_{q=0}^{M-1} f(p, q)\overline{g((p - n) \bmod N, (q - m) \bmod M)}$$

Our current implementation is based on explicit evaluation and is $O(N^2 M^2)$.
We preprocess the images to subtract the DC value (essential to obtain meaningful phase).

5. Visualization

Quaternion polar form:

$$q = |q|e^{\mu\Phi}$$

μ is a unit pure quaternion, and $0 \leq \Phi \leq \pi$. μ is the *eigenaxis* and Φ is the *eigenangle* or *phase*:

$$\mu = V(q)/|V(q)|$$

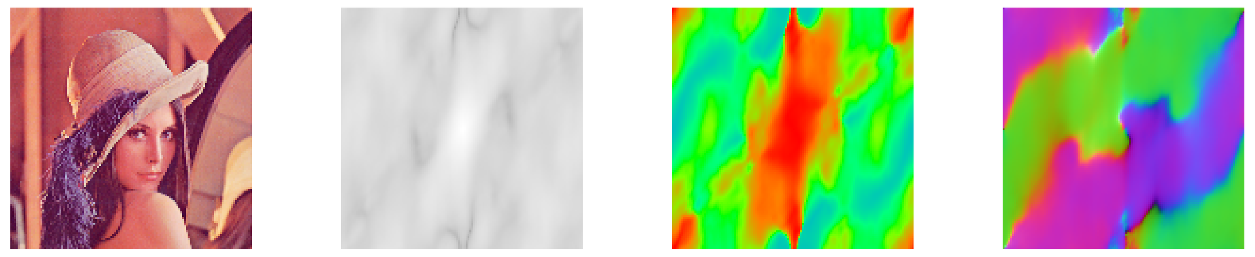
$$\Phi = \tan^{-1} \frac{|V(q)|}{S(q)}$$

Modulus: log grayscale:

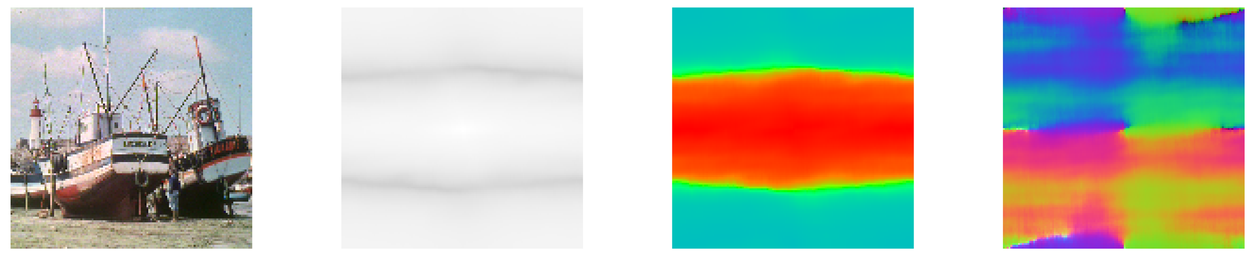
$$M = \frac{\log(1 + |q|)}{\log(1 + K)}$$

where K is the largest modulus in the image.
Phase: hue of IHS color space [4].
Eigenaxis: using unit RGB vectors centered at mid-gray.

6. Results – autocorrelation of natural images



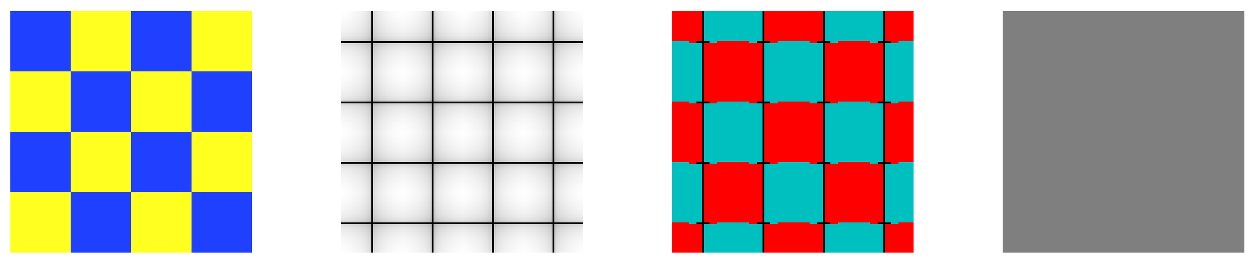
Autocorrelation of the 'Lena' image (128 × 128 pixels). Left to right: original image, modulus, phase, eigenaxis.



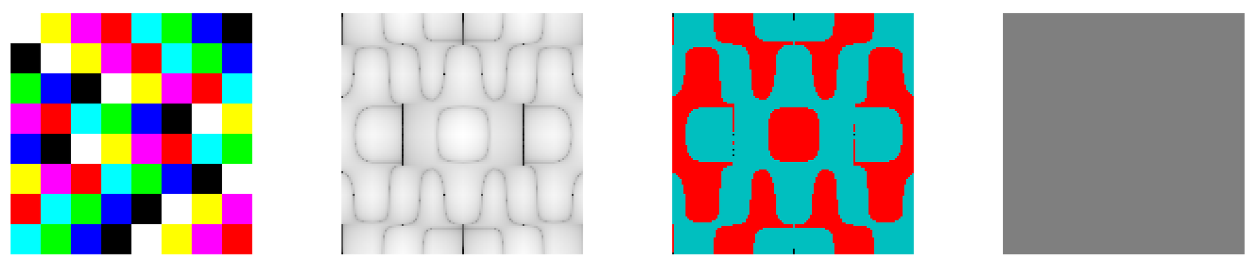
Autocorrelation of the 'Boat' image (128 × 128 pixels). Left to right: original image, modulus, phase, eigenaxis. (Original images from the USC-SIPI image database.)

- The modulus, eigenaxis and phase images are displayed with zero shift at the center.
- The DC component was subtracted from all pixels.
- The autocorrelation (and visualization) give sensible results.
- The center pixel of the autocorrelation images (corresponding to zero shift) has a phase value of zero (red) and an undefined eigenaxis (represented by mid-gray) as well as maximum modulus.
- Symmetry is apparent in phase and eigenaxis.

7. Results – autocorrelation of test images

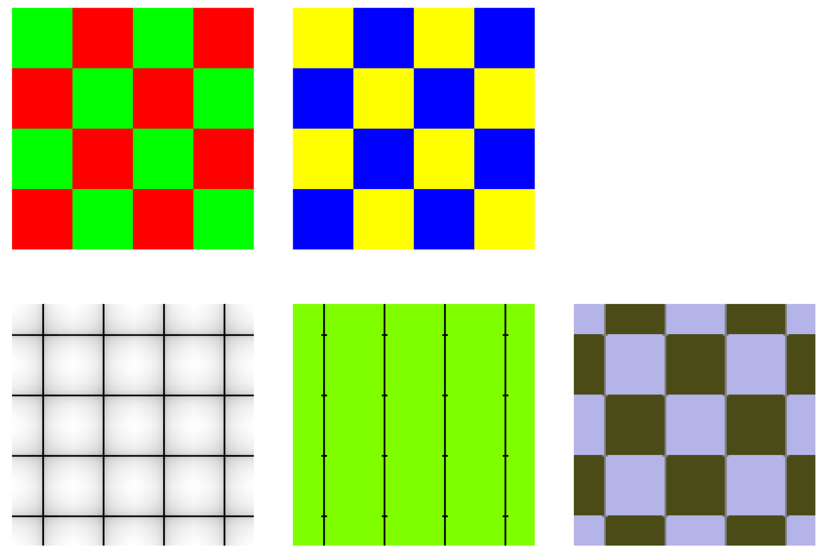


Autocorrelation of checkerboard image with blue (32, 64, 255) and yellow (255, 255, 32) squares. Left to right: original image; modulus; phase, eigenaxis.



Autocorrelation of colour blocks image. Left to right: original image; modulus; phase, eigenaxis.

8. Results – cross-correlation



Cross-correlation of two checkerboard images. Top row: original images; bottom row; left to right: modulus; phase; eigenaxis.

9. Conclusions

- Auto- and cross-correlation can be extended to color images using hypercomplex numbers.
- Worth pursuing a hypercomplex Fourier transform realization (for practical speed and numeric accuracy).
- Cross-correlation could be utilized in applications like: object location, image registration *using color images.*

References

[1] S. J. Sangwine and T. A. Ell. The discrete Fourier transform of a colour image. In J. M. Blackledge, editor, *Second IMA Conference on Image Processing: Mathematical Methods, Algorithms and Applications, De Montfort University, Leicester, UK 22-25 September 1997*, pages –, Chichester, 1998. Horwood Publishing for Institute of Mathematics and its Applications. In Press.

[2] William R. Hamilton. *Elements of Quaternions*. Longmans, Green and Co., London, 1866.

[3] R. C. Gonzalez and R. E. Woods. *Digital Image Processing*. Addison-Wesley, Reading, MA, third edition, 1992. Reprinted with corrections 1993.

[4] H. Palus. Representations of colour images in different colour spaces. In S. J. Sangwine and R. E. N. Horne, editors, *The Colour Image Processing Handbook*, Optoelectronics Imaging and Sensing, chapter 4, pages 67–90. Chapman and Hall, London, 1998.

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