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We study all modes of hyperon nonleptonic decay and consider the CP-odd observables which result. Explicit calculations are provided in the Kobayashi-Maskawa, Weinberg-Higgs, and left-right-symmetric models of CP nonconservation.

I. INTRODUCTION

Hyperon decays provide tests of CP violation which appear promising as signals of the possible $\Delta S=1$ CP nonconservation which is present in many models. Recently two of us discussed these tests and calculated Ξ decay parameters as an example.[1] The purpose of this paper is to present the formalism and theoretical predictions for all of the possible hyperon decay modes, using a variety of the major theories of CP violation.

The goal of work such as this is to uncover systems where new CP-violating effects can be measured. New tests are needed if we are to differentiate the various models of CP violation.[2] Hyperon decays are valuable in this respect because they provide a measure of $\Delta S=1$ CP nonconservation. In kaons, $\Delta S=2$ effects (e.g., the box diagram) appear to be dominant. However, many models also have $\Delta S=1$ CP-odd effects and it would be important to observe these. Hyperons are readily produced, and their decays can be studied in high-precision experiments so that they are likely to be useful.

Some theories, such as the superweak model[3,4] and the models[5] where very heavy neutral Higgs bosons produce the CP violation, have no $\Delta S=1$ CP-odd effects and hence will not produce the signals discussed in this paper. In the Kobayashi-Maskawa (KM) model,[6] the penguin diagram[7] will produce $\Delta S=1$ effects. This generates a nonvanishing value of the kaon decay parameter ϵ'. Although it has not yet been observed, the present generation of experiments is expected to uncover ϵ' if the model is correct. In hyperon decays the penguin diagram will generate CP-odd effects at the order $20\epsilon'$ (the factor of 20 occurs because ϵ' contains an extra suppression factor of this magnitude due to the $\Delta I=\frac{3}{2}$ character of the observable). In the Weinberg-Higgs model,[8] it is the $\Delta S=2$ effects which are small, while charged-Higgs-boson exchange produces a large CP-odd $\Delta S=1$ signal. This model has the largest signals in the processes which we study in this paper, with phases of order 10^{-3}. The left-right-symmetric model[9,10] also has $\Delta S=1$ CP nonconservation. The version which we analyze—that with the "isoconjugate structure"—will have signals at about the same level as the KM model.

The outline of the paper is as follows. Section II contains the isospin decomposition of all the hyperon decays and gives the formulas for the CP-odd observables. In Secs. III, IV, and V, we provide calculations of the weak phases in the KM, Weinberg-Higgs, and left-right-symmetric models, respectively. Finally the conclusion, Sec. VI, tabulates the observables and discusses the results.

II. ANALYSIS OF OBSERVABLES

In this section, we discuss some general CP properties in hyperon decays. Hyperon decays proceed into both S-wave (parity-violating) and P-wave (parity-conserving) final states with amplitudes S and P, respectively. We write the amplitude as

$$\mathrm{Amp}(B^a\to B^b\pi^c)=S(B^a_c)+P(B^a_c)\sigma\cdot\mathbf{q}\ . \tag{2.1}$$

The experimental observables[11] are the total rate Γ, and the decay parameters α, β, and γ which govern the decay-angular distribution and the polarization of the final baryon. Among α, β, and γ, only two are independent as they are related by

$$\begin{aligned}&\alpha^2+\beta^2+\gamma^2=1\ ,\\ &\alpha=2\,\mathrm{Re}S^*P/(|S|^2+|P|^2)\ ,\\ &\beta=2\,\mathrm{Im}S^*P/(|S|^2+|P|^2)\ .\end{aligned} \tag{2.2}$$

Similar observables for antihyperon decays are $\bar\Gamma$, $\bar\alpha$, $\bar\beta$, and $\bar\gamma$:

$$\begin{aligned}&\bar\alpha=2\,\mathrm{Re}\bar S^*\bar P/(|\bar S|^2+|\bar P|^2)\ ,\\ &\bar\beta=2\,\mathrm{Im}\bar S^*\bar P/(|\bar S|^2+|\bar P|^2)\ ,\end{aligned} \tag{2.3}$$

α and β are usually parametrized as

$$\beta=(1-\alpha^2)^{1/2}\sin\phi\ . \tag{2.4}$$

α and ϕ are more closely related to experimental data and are essentially uncorrelated. The present status[12] of the measurements is summarized in Table I.

We define some observables which vanish in the limit of CP conservation:

TABLE I. Experimental measurements of α and ϕ.

	α	ϕ
$\Lambda^0 \to n\pi^0$	0.642 ± 0.013	$-6.5°\pm3.5°$
$\Lambda^0 \to p\pi^-$	0.642 ± 0.013	$-6.5°\pm3.5°$
$\Xi^0 \to \Lambda^0\pi^0$	-0.413 ± 0.022	$20.7°\pm11.7°$
$\Xi^- \to \Lambda^0\pi^-$	-0.434 ± 0.015	$2.0°\pm5.7°$
$\Sigma^- \to n\pi^-$	-0.0681 ± 0.0077	$10.3°\pm4.6°$
$\Sigma^+ \to p\pi^0$	-0.979 ± 0.016	$35.8°\pm33.7°$
$\Sigma^+ \to n\pi^+$	0.068 ± 0.013	$167.3°\pm20.1°$

$$\Delta=\frac{\Gamma-\bar{\Gamma}}{\Gamma+\bar{\Gamma}}\,,\quad A=\frac{\Gamma\alpha+\bar{\Gamma}\bar{\alpha}}{\Gamma\alpha-\bar{\Gamma}\bar{\alpha}}\,,\quad B=\frac{\Gamma\beta+\bar{\Gamma}\bar{\beta}}{\Gamma\beta-\bar{\Gamma}\bar{\beta}}\,,\quad C=\frac{\beta}{\alpha}-\left[\frac{\beta}{\alpha}\right]_{CPC}, \tag{2.5}$$

where $(\beta/\alpha)_{CPC}$ is equal to β/α in the limit of CP conservation. Δ and C have been discussed before[1,13] for some of the hyperon decay modes. Note that Δ, A, and B are quantities which can be measured directly. The test implied by the quantity C is less useful because it requires an experimentally measured quantity β/α to be compared to a value calculated as if CP violation were absent. The latter requires high-precision knowledge of the strong-interaction phase shifts, and therefore is unlikely to be known to the desired accuracy. We include it as a test primarily for reasons of completeness. In Ξ decay, there is another quantity $(\beta/\alpha)_{\Xi^0}-(\beta/\alpha)_{\Xi^-}$ which is a direct test of CP, independent of any knowledge of the phase shifts. This occurs because there is a unique isospin of the final state in Ξ decay.

The quantities A and B are quoted with $\Gamma\alpha$ and $\Gamma\beta$ as variables rather than α and β. This allows simpler formulas for these asymmetries and may be easier to measure. To first order in the weak-interaction phases

$$A=\frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}}+\Delta\,,\quad B=\frac{\beta+\bar{\beta}}{\beta-\bar{\beta}}+\Delta\,. \tag{2.6}$$

Let us now study the quantities Δ, A, B, and C in a little more detail. We parametrize the decay amplitudes as follows:

$$S=\sum_i S_i e^{i(\delta_i^S+\phi_i^S)}\,,\quad P=\sum_i P_i e^{i(\delta_i^P+\phi_i^P)}\,. \tag{2.7}$$

Where S_i and P_i are real, i runs over all possible amplitudes for different final isospin states and change in isospin ΔI, δ_i is the (strong-) final-state interaction phase, and ϕ_i is the weak-interaction phase. In this notation, the antihyperon decay amplitudes are

$$\bar{S}=-\sum_i S_i e^{i(\delta_i^S-\phi_i^S)}\,,\quad \bar{P}=\sum_i P_i e^{i(\delta_i^P-\phi_i^P)}\,. \tag{2.8}$$

Since the weak phases are small compared to 1, we keep only the lowest-order terms in ϕ_i. We have

$$\Delta=-2\frac{\sum_{i>j}[S_iS_j\sin(\delta_i^S-\delta_j^S)\sin(\phi_i^S-\phi_j^S)+P_iP_j\sin(\delta_i^P-\delta_j^P)\sin(\phi_i^P-\phi_j^P)]}{\sum_i(S_i^2+P_i^2)+2\sum_{i>j}[S_iS_j\cos(\delta_i^S-\delta_j^S)+P_iP_j\cos(\delta_i^P-\delta_j^P)]}\,, \tag{2.9a}$$

$$A=-\left[\frac{\beta}{\alpha}\right]_{CPC}\frac{\sum_{i,j}S_iP_j\sin(\delta_j^P-\delta_i^S)\sin(\phi_j^P-\phi_i^S)}{\sum_{i,j}S_iP_j\sin(\delta_j^P-\delta_i^S)}\,, \tag{2.9b}$$

$$B=\left[\frac{\alpha}{\beta}\right]_{CPC}\frac{\sum_{i,j}S_iP_j\cos(\delta_j^P-\delta_i^S)\sin(\phi_j^P-\phi_i^S)}{\sum_{i,j}S_iP_j\cos(\delta_j^P-\delta_i^S)}\,, \tag{2.9c}$$

$$C=\left[\frac{\beta}{\alpha}\right]_{CPC}(B-A)\,, \tag{2.9d}$$

$$\left[\frac{\beta}{\alpha}\right]_{CPC}=\frac{\sum_{i,j}S_iP_j\sin(\delta_j^P-\delta_i^s)}{\sum_{i,j}S_iP_j\cos(\delta_j^P-\delta_i^S)}\,. \tag{2.9e}$$

We are now ready to study specific hyperon decays. We will study $\Lambda^0\to p\pi^-$, $\Lambda^0\to n\pi^0$, $\Sigma^+\to n\pi^+$, $\Sigma^+\to p\pi^0$, $\Sigma^-\to n\pi^-$, $\Xi^-\to\Lambda^0\pi^-$ and $\Xi^0\to\Lambda^0\pi^0$ using the parametrization of Overseth and Pakvasa.[14] There S_{ij}, P_{ij} correspond to $S_{2\Delta I,2I}$, $P_{2\Delta I,2I}$, and δ_{2I} and $\delta_{2I,1}$ for S- and P-wave amplitudes, respectively. In Λ decay we find for $\Lambda^0\to p\pi^-$

$$S(\Lambda^0_-)=-(\tfrac{2}{3})^{1/2}S_{11}e^{i(\delta_1+\phi_1^S)}+(\tfrac{1}{3})^{1/2}S_{33}e^{i(\delta_3+\phi_3^S)}\ ,$$
$$P(\Lambda^0_-)=-(\tfrac{2}{3})^{1/2}P_{11}e^{i(\delta_{11}+\phi_1^P)}+(\tfrac{1}{3})^{1/2}P_{33}e^{i(\delta_{11}+\phi_3^P)}\ , \qquad (2.10)$$

and for $\Lambda^0\rightarrow n\pi^0$

$$S(\Lambda^0_0)=(\tfrac{1}{3})^{1/2}S_{11}e^{i(\delta_1+\phi_1^S)}+(\tfrac{2}{3})^{1/2}S_{33}e^{i(\delta_3+\phi_3^S)}\ ,$$
$$P(\Lambda^0_0)=(\tfrac{1}{3})^{1/2}P_{11}e^{i(\delta_{11}+\phi_1^P)}+(\tfrac{2}{3})^{1/2}P_{33}e^{i(\delta_{33}+\phi_3^P)}\ . \qquad (2.11)$$

Experimentally we know that $\Delta I=\frac{3}{2}$ amplitudes are much smaller than $\Delta I=\frac{1}{2}$ amplitudes. If we work to first order in the $\Delta I=\frac{3}{2}$ amplitudes and the weak-interaction phases, we find

$$\Delta(\Lambda^0_-)=\sqrt{2}\frac{S_{33}}{S_{11}}\sin(\delta_3-\delta_1)\sin(\phi_3^S-\phi_1^S)\ , \qquad (2.12a)$$

$$A(\Lambda^0_-)=-\tan(\delta_{11}-\delta_1)\sin(\phi_1^P-\phi_1^S)\left[1+\frac{1}{\sqrt{2}}\frac{S_{33}}{S_{11}}\left[\frac{\cos(\delta_{11}-\delta_3)}{\cos(\delta_{11}-\delta_1)}-\frac{\sin(\delta_{11}-\delta_3)}{\sin(\delta_{11}-\delta_1)}\frac{\sin(\phi_1^P-\phi_3^S)}{\sin(\phi_1^P-\phi_1^S)}\right]\right.$$
$$\left.+\frac{1}{\sqrt{2}}\frac{P_{33}}{P_{11}}\left[\frac{\cos(\delta_{31}-\delta_1)}{\cos(\delta_{11}-\delta_1)}-\frac{\sin(\delta_{31}-\delta_1)}{\sin(\delta_{11}-\delta_1)}\frac{\sin(\phi_3^P-\phi_1^S)}{\sin(\phi_1^P-\phi_1^S)}\right]\right]\ , \qquad (2.12b)$$

$$B(\Lambda^0_-)=\cot(\delta_{11}-\delta_1)\sin(\phi_1^P-\phi_1^S)\left[1+\frac{1}{\sqrt{2}}\frac{S_{33}}{S_{11}}\left[\frac{\sin(\delta_{11}-\delta_3)}{\sin(\delta_{11}-\delta_1)}-\frac{\cos(\delta_{11}-\delta_3)}{\cos(\delta_{11}-\delta_1)}\frac{\sin(\phi_1^P-\phi_3^S)}{\sin(\phi_1^P-\phi_1^S)}\right]\right.$$
$$\left.+\frac{1}{\sqrt{2}}\frac{P_{33}}{P_{11}}\left[\frac{\sin(\delta_{31}-\delta_1)}{\sin(\delta_{11}-\delta_1)}-\frac{\cos(\delta_{31}-\delta_1)}{\cos(\delta_{11}-\delta_1)}\frac{\sin(\phi_3^P-\phi_1^S)}{\sin(\phi_1^P-\phi_1^S)}\right]\right]\ , \qquad (2.12c)$$

$$C(\Lambda^0_-)=\tan(\delta_{11}-\delta_1)\left[1+\sqrt{2}\frac{S_{33}}{S_{11}}\frac{\sin(\delta_3-\delta_1)}{\sin2(\delta_{11}-\delta_1)}+\sqrt{2}\frac{P_{33}}{P_{11}}\frac{\sin(\delta_{11}-\delta_{31})}{\sin2(\delta_{11}-\delta_1)}\right][B(\Lambda^0_-)-A(\Lambda^0_-)]\ . \qquad (2.12d)$$

To lowest order in the $\Delta I=\frac{3}{2}$ amplitude, we have

$$\Delta(\Lambda^0_0)=-\tfrac{1}{2}\Delta(\Lambda^0_-)\ ,$$
$$\Delta(\Lambda^0_0)=A(\Lambda^0_-)\ ,$$
$$B(\Lambda^0_0)=B(\Lambda^0_-)\ ,$$
$$C(\Lambda^0_0)=C(\Lambda^0_-)\ . \qquad (2.13)$$

We see that for Λ decay only one decay mode needs to be studied. We choose to study $\Lambda^0\rightarrow p\pi^-$. The strong phases are $\delta_1=6.0°$, $\delta_3=-3.8°$, $\delta_{11}=-1.1°$, and $\delta_{31}=-0.7°$ (Ref. 15) with uncertainties of the order 1°.

The decay amplitudes for $\Xi^-\rightarrow\Lambda^0\pi^-$ are

$$S(\Xi^-_-)=S_{12}e^{i(\delta_2+\phi_{12}^S)}+\tfrac{1}{2}S_{32}e^{i(\delta_2+\phi_{32}^S)}\ ,$$
$$P(\Xi^-_-)=P_{12}e^{i(\delta_{21}+\phi_{12}^P)}+\tfrac{1}{2}P_{32}e^{i(\delta_{21}+\phi_{32}^P)}\ , \qquad (2.14)$$

while for $\Xi^0\rightarrow\Lambda\pi^0$

$$S(\Xi^0_0)=\frac{1}{\sqrt{2}}(S_{12}e^{i(\delta_{21}+\phi_{12}^S)}-S_{32}e^{i(\delta_2+\phi_{32}^S)})\ ,$$
$$P(\Xi^0_0)=\frac{1}{\sqrt{2}}(P_{12}e^{i(\delta_{21}+\phi_{12}^P)}-P_{32}e^{i(\delta_{21}+\phi_{32}^P)})\ . \qquad (2.15)$$

The discussion goes completely parallel to that for Λ decay and we find

$$\Delta(\Xi^-_-)=0\ , \qquad (2.16a)$$

$$A(\Xi^-_-)=-\tan(\delta_{21}-\delta_2)\left[\sin(\phi_{12}^P-\phi_{12}^S)+\frac{1}{2}\frac{P_{32}}{P_{12}}\sin(\phi_{32}^P-\phi_{12}^P)+\frac{1}{2}\frac{S_{32}}{S_{12}}\sin(\phi_{12}^S-\phi_{32}^S)\right]\ , \qquad (2.16b)$$

$$B(\Xi^-_-)=\cot(\delta_{21}-\delta_2)\left[\sin(\phi_{12}^P-\phi_{12}^S)+\frac{1}{2}\frac{P_{32}}{P_{12}}\sin(\phi_{32}^P-\phi_{12}^P)+\frac{1}{2}\frac{S_{32}}{S_{12}}\sin(\phi_{12}^S-\phi_{32}^S)\right]\ , \qquad (2.16c)$$

$$C(\Xi^-_-)=[1+\tan^2(\delta_{21}-\delta_2)]\left[\sin(\phi_{12}^P-\phi_{12}^S)+\frac{1}{2}\frac{P_{32}}{P_{12}}\sin(\phi_{32}^P-\phi_{12}^P)+\frac{1}{2}\frac{S_{32}}{S_{12}}\sin(\phi_{12}^S-\phi_{32}^S)\right]\ . \qquad (2.16d)$$

Again if $\Delta I=\frac{3}{2}$ contributions are treated to lowest order, we have

$$\Delta(\Xi_0^0)=0\ ,\quad A(\Xi_0^0)=A(\Xi_-^-)\ ,$$
$$B(\Xi_0^0)=B(\Xi_-^-)\ ,\quad C(\Xi_0^0)=C(\Xi_-^-)\ , \tag{2.17}$$
$$\left[\frac{\beta}{\alpha}\right]_{\Xi_0^0}-\left[\frac{\beta}{\alpha}\right]_{\Xi_-^-}=\frac{3}{2}[1+\tan^2(\delta_{21}-\delta_2)]\times\left[\frac{S_{32}}{S_{12}}\sin(\phi_{12}^S-\phi_{32}^S)+\frac{P_{32}}{P_{12}}\sin(\phi_{32}^P-\phi_{12}^P)\right]\ .$$

The equality $\Delta(\Xi_0^0)=\Delta(\Xi_-^-)=0$ is exact. This is because there is only one final-state isospin. Phase shifts for Ξ-π scattering have not been measured experimentally yet. Nath and Kumar[16] calculate $\delta_2=-18.7°$ and $\delta_{21}=-2.7°$ whereas Martin[17] obtains $\delta_{21}=-1.2°$. In our estimates we use $\delta_{21}=-2.7°$.

In the case of Σ decay, the decay amplitudes are, for $\Sigma^-\to n\pi^-$,

$$S(\Sigma_-^-)=S_{13}e^{i(\delta_3+\phi_{13}^S)}+(\tfrac{2}{3})^{1/2}S_{33}e^{i(\delta_3+\phi_{33}^S)}\ ,$$
$$P(\Delta_-^-)=P_{13}e^{i(\delta_{31}+\phi_{13}^P)}+(\tfrac{2}{3})^{1/2}P_{33}e^{i(\delta_{31}+\phi_{33}^P)}\ , \tag{2.18}$$

while for $\Sigma^+\to p\pi^0$,

$$S(\Sigma_0^+)=\frac{\sqrt{2}}{3}(S_{11}e^{i\phi_{11}^S}-\frac{1}{2}S_{31}e^{i\phi_{31}^S})e^{i\delta_1}+\frac{\sqrt{2}}{3}[S_{13}e^{i\phi_{13}^S}-2(\tfrac{2}{5})^{1/2}S_{33}e^{i\phi_{33}^S}]e^{i\delta_3}\ ,$$
$$P(\Sigma_0^+)=\frac{\sqrt{2}}{3}(P_{11}e^{i\phi_{11}^P}-\tfrac{1}{2}P_{31}e^{i\phi_{31}^P})e^{i\delta_{11}}+\frac{\sqrt{2}}{3}[P_{13}e^{i\phi_{13}^P}-2(\tfrac{2}{5})^{1/2}P_{33}e^{i\phi_{33}^S}]e^{i\delta_{31}}\ , \tag{2.19}$$

and the $\Sigma^+\to n\pi^+$ amplitudes are

$$S(\Sigma_+^+)=-\tfrac{2}{3}(S_{11}e^{i\phi_{11}^S}-\tfrac{1}{2}S_{31}e^{i\phi_{31}^S})e^{i\delta_1}+\tfrac{1}{3}[S_{13}e^{i\phi_{13}^S}-2(\tfrac{2}{5})^{1/2}S_{33}e^{i\phi_{33}^S}]e^{i\delta_3}\ ,$$
$$P(\Sigma_+^+)=-\tfrac{2}{3}(P_{11}e^{i\phi_{11}^P}-\tfrac{1}{2}P_{31}e^{i_{31}^P})e^{i\delta_{11}}+\tfrac{1}{3}[P_{13}e^{i\phi_{13}^P}-2(\tfrac{2}{5})^{1/2}P_{33}e^{i\phi_{33}^P}]e^{i\delta_{31}}\ . \tag{2.20}$$

We notice that

$$S_{11}e^{i\phi_{11}^S}-\tfrac{1}{2}S_{31}e^{i\phi_{31}^S}$$

and

$$P_{11}e^{i\phi_{11}^P}-\tfrac{1}{2}P_{31}e^{i\phi_{31}^P}$$

always appear together, we redefine them as $\bar{S}_1e^{i\bar{\phi}_1^S}$ and $\bar{P}_1e^{i\bar{\phi}_1^P}$, respectively. The strong phases are $\delta_1=9.4°$, $\delta_3=-10.1°$, $\delta_{11}=-1.8°$, and $\delta_{31}=-3.5°$ (Ref. 15) again with uncertainties of the order of 1°. Calculating the decay asymmetries we obtain for $\Sigma^-\to n\pi^-$,

$$\Delta(\Sigma_-^-)=0\ ,$$
$$A(\Sigma_-^-)=-\tan(\delta_{31}-\delta_3)A\ ,$$
$$B(\Sigma_-^-)=\cot(\delta_{31}-\delta_3)A\ , \tag{2.21}$$
$$C(\Sigma_-^-)=[1+\tan^2(\delta_{31}-\delta_3)]A\ ,$$

where

$$A=\sin(\phi_{13}^P-\phi_{13}^S)\frac{\left[1+\left[\frac{2}{5}\right]^{1/2}\frac{S_{33}}{S_{13}}\frac{\sin(\phi_{13}^P-\phi_{33}^S)}{\sin(\phi_{13}^P-\phi_{13}^S)}+\left[\frac{2}{5}\right]^{1/2}\frac{P_{33}}{P_{13}}\frac{\sin(\phi_{33}^P-\phi_{13}^S)}{\sin(\phi_{13}^P-\phi_{13}^S)}+\frac{2}{5}\frac{S_{33}P_{33}}{S_{13}P_{13}}\frac{\sin(\phi_{33}^P-\phi_{33}^S)}{\sin(\phi_{13}^P-\phi_{13}^S)}\right]}{\left[1+\left[\frac{2}{5}\right]^{1/2}\frac{S_{33}}{S_{13}}+\left[\frac{2}{5}\right]^{1/2}\frac{P_{33}}{P_{13}}+\frac{2}{5}\frac{S_{33}P_{33}}{S_{13}P_{13}}\right]}\ . \tag{2.22}$$

For $\Sigma^+\to p\pi^0$ and $\Sigma^+\to n\pi^+$, we notice that

$$S_{13}e^{i\phi_{13}^S}-2(\tfrac{2}{5})^{1/2}S_{33}e^{i\phi_{33}^S}$$

and

$$P_{13}e^{i\phi_{13}^P}-2(\tfrac{2}{5})^{1/2}P_{33}e^{i\phi_{33}^P}$$

appear together, and defining them as $\bar{S}_3e^{i\bar{\phi}_3^S}$ and $\bar{P}_3e^{i\bar{\phi}_3^P}$, respectively, we have

$$\bar{S}_3=S_{13}-2(\tfrac{2}{5})^{1/2}S_{33}\ ,\quad \bar{P}_3=P_{13}-2(\tfrac{2}{5})^{1/2}P_{33}$$

and

$$\bar{\phi}_3^S=\frac{S_{13}\phi_{13}^S-2(\frac{2}{5})^{1/2}\phi_{33}^S}{\bar{S}_3}\ ,\quad \bar{\phi}_3^S=\frac{P_{13}\phi_{13}^P-2(\frac{2}{5})^{1/2}\phi_{33}^P}{\bar{P}_3}\ .$$

In this notation, we find, for $\Sigma^+\to p\pi^0$:

$$\Delta(\Sigma_0^+)=-2\left[\frac{\bar S_1\bar S_3\sin(\delta_1-\delta_3)\sin(\bar\phi_1^S-\bar\phi_3^S)+\bar P_1\bar P_3\sin(\delta_{11}-\delta_{31})\sin(\bar\phi_1^P-\bar\phi_3^P)}{\bar S_1^{\,2}+\bar S_3^{\,2}+\bar P_1^{\,2}+\bar P_3^{\,2}+2\bar S_1\bar S_3\cos(\delta_1-\delta_3)+2\bar P_1\bar P_3\cos(\delta_{11}-\delta_{31})}\right], \tag{2.23a}$$

$$\begin{aligned}A(\Sigma_0^+)=-\left[\frac{\beta}{\alpha}\right]_{CPC}(\Sigma_0^+)\Bigg[&\sin(\delta_{11}-\delta_1)\sin(\bar\phi_1^P-\bar\phi_1^S)+\frac{\bar P_3}{\bar P_1}\sin(\delta_{31}-\delta_1)\sin(\bar\phi_3^P-\bar\phi_1^S)+\frac{\bar S_3}{\bar S_1}\sin(\delta_{11}-\delta_3)\sin(\bar\phi_1^P-\bar\phi_3^S)\\&+\frac{\bar S_3\bar P_3}{\bar S_1\bar P_1}\sin(\delta_{31}-\delta_3)\sin(\bar\phi_3^P-\bar\phi_3^S)\Bigg]\Bigg/\Bigg[\sin(\delta_{11}-\delta_1)+\frac{\bar P_3}{\bar P_1}\sin(\delta_{31}-\delta_1)\\&+\frac{\bar S_3}{\bar S_1}\sin(\delta_{11}-\delta_3)+\frac{\bar S_3\bar P_3}{\bar S_1\bar P_1}\sin(\delta_{31}-\delta_3)\Bigg],\end{aligned} \tag{2.23b}$$

$$\begin{aligned}B(\Sigma_0^+)=\left[\frac{\alpha}{\beta}\right]_{CPC}(\Sigma_0^+)\Bigg[&\cos(\delta_{11}-\delta_1)\sin(\bar\phi_1^P-\bar\phi_1^S)+\frac{\bar P_3}{\bar P_1}\cos(\delta_{31}-\delta_1)\sin(\bar\phi_3^P-\bar\phi_1^S)+\frac{\bar S_3}{\bar S_1}\cos(\delta_{11}-\delta_3)\sin(\bar\phi_1^P-\bar\phi_3^S)\\&+\frac{\bar S_3\bar P_3}{\bar S_1\bar P_1}\cos(\delta_{31}-\delta_3)\sin(\bar\phi_3^P-\bar\phi_3^S)\Bigg]\Bigg/\Bigg[\cos(\delta_{11}-\delta_1)+\frac{\bar P_3}{\bar P_1}\cos(\delta_{31}-\delta_1)\\&+\frac{\bar S_3}{\bar S_1}\cos(\delta_{11}-\delta_3)+\frac{\bar S_3\bar P_3}{\bar S_1\bar P_1}\cos(\delta_{31}-\delta_3)\Bigg],\end{aligned} \tag{2.23c}$$

$$C(\Sigma_0^+)=\left[\frac{\beta}{\alpha}\right]_{CPC}(\Sigma_0^+)[B(\Sigma_0^+)-A(\Sigma_0^+)], \tag{2.23d}$$

where from the data we find $(\beta/\alpha)_{CPC}(\Sigma_0^+)=0.033$.

For $\Sigma^+\to n\pi^+$, we have

$$\Delta(\Sigma_+^+)=4\frac{\bar S_1\bar S_3\sin(\delta_1-\delta_3)\sin(\bar\phi_1^S-\bar\phi_3^S)+\bar P_1\bar P_3\sin(\delta_{11}-\delta_{31})\sin(\bar\phi_1^P-\bar\phi_3^P)}{4\bar S_1^{\,2}+\bar S_3^{\,2}+4\bar P_1^{\,2}+\bar P_3^{\,2}-4\bar S_{11}\bar S_3\cos(\delta_1-\delta_3)-4\bar P_1\bar P_3\cos(\delta_{11}-\delta_{31})}, \tag{2.24a}$$

$$\begin{aligned}\Delta(\Sigma_+^+)=-\left[\frac{\beta}{\alpha}\right]_{CPC}(\Sigma_+^+)\Bigg[&\sin(\delta_{11}-\delta_1)\sin(\bar\phi_1^P-\bar\phi_1^S)-\frac{1}{2}\frac{\bar P_3}{\bar P_1}\sin(\delta_{31}-\delta_1)\sin(\bar\phi_3^P-\bar\phi_1^S)-\frac{1}{2}\frac{\bar S_3}{\bar S_1}\sin(\delta_{11}-\delta_3)\sin(\bar\phi_1^P-\bar\phi_3^S)\\&+\frac{1}{4}\frac{\bar S_3\bar P_3}{\bar S_1\bar P_1}\sin(\delta_{31}-\delta_3)\sin(\bar\phi_3^P-\bar\phi_3^S)\Bigg]\Bigg/\Bigg[\sin(\delta_{11}-\delta_1)-\frac{1}{2}\frac{\bar P_3}{\bar P_1}\sin(\delta_{31}-\delta_1)-\frac{1}{2}\frac{\bar S_3}{\bar S_1}\sin(\delta_{11}-\delta_3)\\&+\frac{1}{4}\frac{\bar S_3\bar P_2}{\bar S_1\bar P_1}\sin(\delta_{31}-\delta_3)\Bigg],\end{aligned} \tag{2.24b}$$

$$\begin{aligned}B(\Sigma_+^+)=\left[\frac{\alpha}{\beta}\right]_{CPC}(\Sigma_+^+)\Bigg[&\cos(\delta_{11}-\delta_1)\sin(\bar\phi_1^P-\bar\phi_1^S)-\frac{1}{2}\frac{\bar P_3}{\bar P_1}\cos(\delta_{31}-\delta_1)\sin(\bar\phi_3^P-\bar\phi_1^S)-\frac{1}{2}\frac{\bar S_3}{\bar S_1}\cos(\delta_{11}-\delta_3)\sin(\bar\phi_1^P-\bar\phi_3^S)\\&+\frac{1}{4}\frac{\bar S_3\bar P_3}{\bar S_1\bar P_1}\cos(\delta_{31}-\delta_3)\sin(\bar\phi_3^P-\bar\phi_1^S)\Bigg]\Bigg/\Bigg[\cos(\delta_{11}-\delta_1)-\frac{1}{2}\frac{\bar P_3}{\bar P_1}\cos(\delta_{31}-\delta_1)-\frac{1}{2}\frac{\bar S_3}{\bar S_1}\cos(\delta_{11}-\delta_3)\\&+\frac{1}{4}\frac{\bar S_3\bar P_2}{\bar S_1\bar P_1}\cos(\delta_{31}-\delta_3)\Bigg],\end{aligned} \tag{2.24c}$$

$$C(\Sigma_+^+)=\left[\frac{\beta}{\alpha}\right]_{CPC}(\Sigma_+^+)[B(\Sigma_+^+)-A(\Sigma_+^+)]. \tag{2.24d}$$

By assuming that the values given in Ref. 18 are for the real parts of the amplitudes, we find $(\beta/\alpha)_{CPC}(\Sigma_+^+)=3.5$. In the above expressions, we use exact formulas because all contributions are of the same order of magnitude.

The remaining task is to calculate weak-interaction phases ϕ_i's which in turn depend on the model for CP violation. We proceed to estimate them in the following sections for different models.

III. KOBAYASHI-MASKAWA MODEL

In the standard model with three generations, the interaction of the quarks with the charged gauge bosons may contain a phase which can generate CP violation. In the kaon sector the box diagram provides the dominant CP-odd effect, i.e., the mixing of K_1^0 and K_2^0. In hyperon decay the box diagram does not contribute as it is $\Delta S=2$, and the CP nonconservation is contained in the $\Delta S=1$ penguin interactions. The operator which is implied by this is

$$\mathscr{L}_{CPV}=i\frac{G_F}{2\sqrt{2}}\sin\theta_1\cos\theta_1 \mathrm{Im}C_5 \bar{d}t^A\gamma_\mu(1+\gamma_5)\times s\bar{q}t^A\gamma^\mu(1-\gamma_5)q\ , \tag{3.1}$$

where t^A are the Gell-Man SU(3) matrices acting on the colors of quarks. The strength of the interaction, $\mathrm{Im}C_5$, has been calculated by Gilman and Wise[19] to be

$$\mathrm{Im}C_5=-0.1\sin\theta_2\sin\theta_3\sin\delta\ . \tag{3.2}$$

Present bounds on the KM angles force this to satisfy

$$\mathrm{Im}C_5\leq 2\times 10^{-4}\ , \tag{3.3}$$

but the requirement that the model generate enough CP violation in $K^0\bar{K}^0$ mixing implies that $\mathrm{Im}C_5$ needs to be close to its upper bound. For our estimates we will use Eq. (3.3) as an equality in quoting numbers.

To calculate the effect of this interaction, we will utilize the bag-model calculations of Donoghue, Golowich, Holstein, and Ponce[20] (DGHP). The effect of the penguin operator on the S- and P-wave amplitude can be read off of Tables III and V of DGHP. The only complication arises in the separation of the Σ amplitudes according to the isospin of the final state, as is required by our usage above, Eqs. (2.18)–(2.20). For example, PCAC (partial conservation of axial-vector current) requires that the S-wave $\Delta I=\frac{1}{2}$ amplitude in $\Sigma^+\to n\pi^+$ vanish. This can only happen if the two amplitudes S_{13} and S_{11} cancel. Our procedure is to extract the amplitude from the data, assuming the data quoted in the Particle Data Group tables represent real amplitudes. The weak CP-violating phases are then added as calculated in the model. All amplitudes are quoted in units of 10^{-8}, and the strong-interaction phase shifts are not included.

The amplitudes which we find for Λ decay are

$$S(\Lambda_-^0)=-(\tfrac{2}{3})^{1/2}S_{11}e^{i\phi_1^S}+(\tfrac{1}{3})^{1/2}S_{33}e^{i\phi_3^S}$$
$$=32.8(1-0.42i\,\mathrm{Im}C_5)-0.3\ , \tag{3.4a}$$

$$P(\Lambda_-^0)=-(\tfrac{2}{3})^{1/2}P_{11}e^{i\phi_1^P}+(\tfrac{1}{3})^{1/2}P_{33}e^{i\phi_3^P}$$
$$=12.4(1-2.24i\,\mathrm{Im}C_5)-0.06\ . \tag{3.4b}$$

The numbers are quoted in the same order as the formulas above them. Note that only isospin-$\frac{1}{2}$ amplitudes have any weak phases. This is also true in the Weinberg-Higgs model.

In Ξ decay, our results have been given previously:

$$S(\Xi_-^-)=S_{12}e^{i\phi_{12}^S}+\tfrac{1}{2}S_{32}e^{i\phi_{32}^S}$$
$$=-46.2(1-0.29i\,\mathrm{Im}C_5)+1.1\ , \tag{3.5a}$$

$$P(\Xi_-^-)=P_{12}e^{i\phi_{12}^P}+\tfrac{1}{2}P_{32}e^{i\phi_{32}^P}$$
$$=10.2(1+0.92i\,\mathrm{Im}C_5)-0.1\ . \tag{3.5b}$$

Finally in the Σ system, we find

$$S(\Sigma_0^+)=\frac{\sqrt{2}}{3}S_{13}e^{i\phi_{13}^S}-\frac{4}{3\sqrt{5}}S_{33}e^{i\phi_{33}^S}+\frac{\sqrt{2}}{3}\bar{S}_1e^{i\bar{\phi}_1^S}$$
$$=-20.9(1-0.3i\,\mathrm{Im}C_5)-1.5$$
$$-10.3(1-0.3i\,\mathrm{Im}C_5)\ , \tag{3.6a}$$

$$P(\Sigma_0^+)=\frac{\sqrt{2}}{3}P_{13}e^{i\phi_{13}^P}-\frac{4}{3\sqrt{5}}P_{33}e^{i\phi_{33}^P}+\frac{\sqrt{2}}{3}\bar{P}_1e^{i\bar{\phi}_1^P}$$
$$=-0.3(1+20.0i\,\mathrm{Im}C_5)-1.9$$
$$+28.8(1-0.15i\,\mathrm{Im}C_5)\ . \tag{3.6b}$$

Even though these results are the best that can be done with present calculational methods, there can be considerable uncertainty. The S-wave amplitudes, when calculated in the quark model, generated by the full weak Hamiltonian match the experimental data roughly in amplitude, but have a somewhat different SU(3) structure. The baryon pole model for the P wave has a well-known factor of 2 defect in reproducing the magnitude,[21] although the signs are correctly given. Thus one should allow at least a factor of 2 uncertainty in the calculated phases.

We will defer a complete discussion of the observables until the conclusion. However, we note that phases are typically of order

$$\phi\sim\mathrm{Im}C_5\sim 10^{-4}\ .$$

IV. WEINBERG-HIGGS MODEL

In the Weinberg-Higgs model, the CP violation is generated by the exchange of charged-Higgs bosons. The $\Delta S=2$ box diagram is small compared to the effect of $\Delta S=1$ CP violation. The model can be viable if dispersive effects in K^0-$\bar{K}^0$ mixing provide the major contribution in the kaon sector. That this may be the case has been shown in Ref. 22. The most important operator in the model involves the gluon field strength tensor $F_{\mu\nu}^A$:

$$\mathscr{L}_{CPV}=i\tilde{f}\,\bar{d}t^A\sigma^{\mu\nu}(1-\gamma_5)sF^A_{\mu\nu}\ . \quad (4.1)$$

The chiral-symmetry behavior of this operator, i.e., its transformation as $(\bar{3}_L,3_R)$ under chiral SU(3), played an important role in the analysis in the kaon sector. It will also be important below.

To normalize the strength of CP violation in this model, we use Ref. 22 to estimate the dispersive contribution to the K^0-$\bar{K}^0$ transition and to relate it to ϵ. We find

$$2m_K \mathrm{Im}M_{12}=2m_K\sqrt{2}\,|\epsilon|\,\Delta m \approx 2\times 10^{-7}\langle \pi^0|\mathscr{L}_{CPV}|K^0\rangle\ , \quad (4.2)$$

which yields

$$\langle \pi^0|\mathscr{L}_{CPV}|K^0\rangle = 5.8\times 10^{-11}\ \mathrm{GeV}^2\ . \quad (4.3)$$

The parameter ρ of Ref. 22 has been set equal to unity. If $\rho<1$, the strength of CP-odd signals in hyperon decay would be enhanced by a factor $1/\rho$.

The effect of the operator in $\mathscr{L}_{CPV}$ has been calculated in the bag model in Ref. 23 (DGHP). However, it has recently been discovered[24] that the PCAC analysis of hyperon decays is modified for operators with a $(\bar{3},3)$ transformation property. We include this modification in our analysis. The most important effect is the addition of a new diagram to the analysis of the S-wave amplitude. The new diagram is that of a $K\rightarrow$vacuum tadpole, as shown in Fig. 1. In the standard model, the $(8_L,1_R)$ chiral transformation property of the weak Hamiltonian forces this diagram to vanish. However $(\bar{3}_L,3_R)$ operators, such as the one that occurs in the Higgs-boson model, have a nonvanishing $K\rightarrow$vacuum matrix element. This can be most easily seen using effective chiral Lagrangian, where the $(\bar{3}_L,3_R)$ character is implied in the Lagrangian

$$\mathscr{L}_{\mathrm{eff}}=g\,\mathrm{Tr}(\lambda_6 M) \quad (4.4)$$

with

$$M=\exp\left[-i\frac{\lambda^A\phi^A}{f_\pi}\right] \quad (4.5)$$

and ϕ^A being the eight pseudoscalar fields. This Lagrangian has the expansion

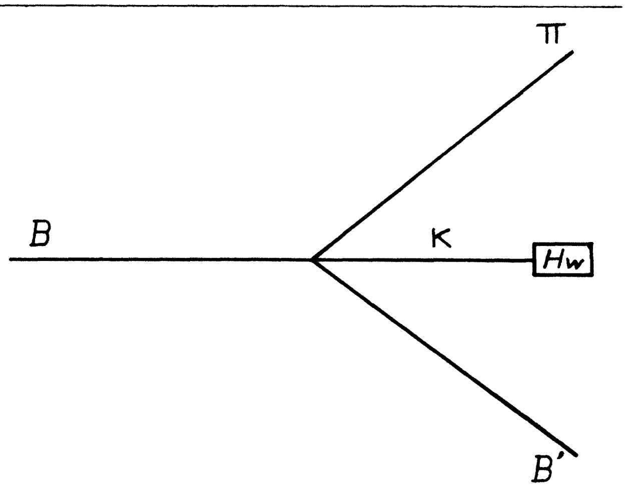

FIG. 1. Tadpole diagram for hyperon decay.

$$\mathscr{L}=\frac{2g}{F_\pi}\left[-i\phi_6-d_{6AB}\frac{\phi_A\phi_B}{2F_\pi}\right]+\cdots\ , \quad (4.6)$$

so that

$$\langle 0|\mathscr{L}_{CPV}|K^0\rangle = i2F_\pi\langle\pi^0|\mathscr{L}_{CPV}|K^0\rangle\ .$$

The S-wave amplitudes are modified by this term in the following way:

$$S(\Lambda^0_0)=-\frac{i}{2F_\pi}\langle N|H_w|\Lambda\rangle+\frac{i}{2F_\pi}\left[\frac{3}{2}\right]^{1/2}\frac{M_\Lambda-M_N}{M_s-M_d}\times\left[\frac{M_s+M_d}{iF_K M_K{}^2}\langle 0|H_2|K\rangle\right]\ , \quad (4.7a)$$

$$S(\Sigma^+_0)=-\frac{i}{2F_\pi}\langle P|H_w|\Sigma^+\rangle+\frac{i}{2F_\pi}\frac{M_\Sigma-M_P}{M_s-M_d}\left[\frac{M_s+M_d}{iF_K M_K{}^2}\langle 0|H_w|K\rangle\right]\ , \quad (4.7b)$$

$$S(\Xi^0_0)=-\frac{i}{2F_\pi}\langle\Lambda|H_w|\Xi^0\rangle-\frac{i}{2F_\pi}\left[\frac{3}{2}\right]^{1/2}\frac{M-M_\Lambda}{M_s-M_d}\times\left[\frac{M_s+M_d}{iF_K M_K{}^2}\langle 0|H_w|K\rangle\right]\ . \quad (4.7c)$$

The relative sizes and signs of these terms may be checked using the Feinberg-Kabir-Weinberg theorem,[25] which requires that all the amplitudes vanish if the Hamiltonian is $\bar{d}(1-\gamma_5)s$. The contribution of the tadpole term turns out to be important because the phases generated by them are larger than those of the baryon terms. However there is substantial cancellation, between the K tadpole terms in the S wave and the $K\rightarrow\pi$ pole terms in the P wave, when comparing $\phi_S-\phi_P$. In the P waves we use the standard baryon-plus-kaon pole model. Again we recall that the kaon poles are of order $M_\pi{}^2/M_K{}^2$ (and hence negligible) in the KM model, due to the $(8_L,1_R)$ chiral property, but are important for a $(\bar{3},3)$ Lagrangian.

Our results in this model are given below. In Λ decay

$$S(\Lambda^0_-)=-(\tfrac{3}{2})^{1/2}S_{11}e^{i\phi^S_1}+(\tfrac{1}{2})^{1/2}S_{33}e^{i\phi^S_3} = 32.8(1-1.2i\times 10^{-3})-0.3\ , \quad (4.8a)$$

$$P(\Lambda^0_-)=-(\tfrac{2}{3})^{1/2}P_{11}e^{i\phi^P_1}+(\tfrac{1}{3})^{1/2}P_{33}e^{i\phi^P_3} = 12.4(1-1.4i\times 10^{-3})-0.06\ , \quad (4.8b)$$

while in Ξ decay

$$S(\Xi^-_-)=S_{12}e^{i\phi^S_{12}}+\tfrac{1}{2}S_{32}e^{i\phi^S_{32}} = -4.62(1-0.7i\times 10^{-3})+1.1\ , \quad (4.9a)$$

TABLE II. The *CP*-violating observables for the KM model.

	Δ	A	B	C	$(\beta/\alpha)_{CPC}$
$\Lambda^0 \to p\pi^-$	-5.4×10^{-7}	-0.5×10^{-4}	3.0×10^{-3}	-3.6×10^{-4}	0.124
$\Xi^- \to \Lambda^0\pi^-$	0	-0.7×10^{-4}	8.4×10^{-4}	2.6×10^{-4}	0.287
$\Sigma^- \to n\pi^-$	0	1.6×10^{-4}	-1.2×10^{-2}	-1.4×10^{-3}	0.116
$\Sigma^+ \to p\pi^0$	-6.2×10^{-7}	-3.2×10^{-7}	-4.2×10^{-4}	-1.3×10^{-5}	0.033
$\Sigma^+ \to n\pi^+$	6.0×10^{-7}	-1.6×10^{-4}	-8.4×10^{-7}	5.7×10^{-4}	3.5

$$P(\Xi_-^-)=P_{12}e^{i\phi_{12}^P}r\tfrac{1}{2}P_{32}e^{i\phi_{32}^P}$$
$$=10.2(1+0.4i\times10^{-3})-0.1\ . \qquad (4.9b)$$

Finally, the Σ amplitudes are

$$S(\Sigma_0^+)=\frac{\sqrt{2}}{3}S_{13}e^{i\phi_{13}^S}-\frac{4}{3\sqrt{5}}S_{33}e^{i\phi_{33}^S}+\frac{\sqrt{2}}{3}\bar{S}_1e^{i\bar{\phi}_1^S}$$
$$=-20.9(1-1.0i\times10^{-3})-1.5$$
$$-10.3(1-1.0i\times10^{-3})\ , \qquad (4.10a)$$

$$P(\Sigma_0^+)=\frac{\sqrt{2}}{3}P_{13}e^{i\phi_{13}^P}-\frac{4}{3\sqrt{5}}P_{33}e^{i\phi_{33}^P}+\frac{\sqrt{2}}{3}\bar{P}_1e^{i\bar{\phi}_1^P}$$
$$=-0.3(1-25.0i\times10^{-3})-1.9$$
$$+28.8(1+0.1i\times10^{-3})\ . \qquad (4.10b)$$

We note here that the phases are typically of order 10^{-3}.

V. LEFT-RIGHT-SYMMETRIC MODEL

In this section, we estimate weak phases ϕ_i's in the left-right-symmetric model. There are several versions of left-right-symmetric models of *CP* nonconservation, but the most appealing is that with the "isoconjugate structure" which generates sizable $\Delta S=1$ *CP*-odd interaction even though $\epsilon'/\epsilon=0$ in kaon decay (in the limit of no W_L-W_R mixing). The full $\Delta S=1$ Hamiltonian has the form

$$H_w=\frac{G_F}{\sqrt{2}}\sin\theta_1\cos\theta_1(O_{LL}+\eta e^{i\beta}O_{RR})\ , \qquad (5.1)$$

where $\eta=M_{W_L}{}^2/M_{W_R}{}^2$; O_{LL} and O_{RR} are identical operators, except that O_{LL} is a product of two left-handed currents whereas O_{RR} has two right-handed currents. Because of this structure one can easily see that all $\Delta S=1$ parity-conserving processes have an identical phase factor $1+i\eta\beta$, while all parity-nonconserving ones have phase $1-i\eta\beta$. We have

$$\phi_i^S=\eta\beta\ ,$$
$$\phi_i^P=-\eta\beta\ , \qquad (5.2)$$

for all decays. $\eta\beta$ can be obtained from *CP* nonconservation in kaon mixing. In this model, the box diagrams[10] yield

$$\epsilon=\frac{G_F{}^2}{12\sqrt{2}\pi^2}\frac{s_1{}^2f_K{}^2M_KM_C{}^2}{\Delta M}B\left[60C_{LR}\frac{\mathcal{M}_{LR}}{\mathcal{M}_{LL}}\eta\beta\right]\ , \qquad (5.3)$$

where $C_{LR}\sim3$ is the quantum-chromodynamic correction factor and

$$\frac{\mathcal{M}_{LR}}{\mathcal{M}_{LL}}=\frac{\langle K^0|\bar{s}_Rd_L\bar{s}_Ld_R|\bar{K}^0\rangle}{\langle K^0|\bar{s}_L\gamma_\mu d_L\bar{s}_L\gamma^\mu d_L|\bar{K}^0\rangle}\ . \qquad (5.4)$$

$\mathcal{M}_{LR}/\mathcal{M}_{LL}$ is in the range 3–10. Note the large factor $60C_{LR}M_{LR}/M_{LL}$ which enhances the left-right box diagram, but decreases the size of the signals in the hyperon sector. The value for ϵ implies

$$\eta\beta\simeq\frac{\mathcal{M}_{LL}}{\mathcal{M}_{LR}}\times2.2\times10^{-4}\ . \qquad (5.5)$$

In our later estimates we will use $\mathcal{M}_{LL}/\mathcal{M}_{LR}\simeq\frac{1}{5}$, that is $\eta\beta=4.4\times10^{-5}$.

Because all *S* and *P* weak phases are equal, the calculation for Δ, A, B, and C is much simplified. We have

$$\Delta=0\ , \qquad (5.6a)$$

$$A=-\left[\frac{\beta}{\alpha}\right]_{CPC}\sin(\phi^P-\phi^S)\ , \qquad (5.6b)$$

$$B=\left[\frac{\alpha}{\beta}\right]_{CPC}\sin(\phi^P-\phi^S)\ , \qquad (5.6c)$$

$$C=\left[1+\left[\frac{\beta}{\alpha}\right]_{CPC}\right]\sin(\phi^P-\phi^S)\ , \qquad (5.6d)$$

TABLE III. The *CP*-violating observables for the Weinberg-Higgs model.

	Δ	A	B	C
$\Lambda^0 \to p\pi^-$	-7.8×10^{-6}	-2.5×10^{-5}	1.6×10^{-3}	-0.2×10^{-3}
$\Xi^- \to \Lambda^0\pi^-$	0	-3.2×10^{-4}	3.8×10^{-3}	1.2×10^{-3}
$\Sigma^- \to n\pi^-$	0	-1.1×10^{-3}	8.6×10^{-2}	1.0×10^{-2}
$\Sigma^+ \to p\pi^0$	1.4×10^{-5}	-3.2×10^{-5}	3.9×10^{-2}	1.3×10^{-3}
$\Sigma^+ \to n\pi^+$	-1.3×10^{-5}	-3.4×10^{-3}	2.6×10^{-5}	1.2×10^{-2}

TABLE IV. The CP-violating observables for the left-right-symmetric model.

	Δ	A	B	C
$\Lambda^0 \to p\pi^-$	0	-1.1×10^{-5}	7.0×10^{-4}	-8.8×10^{-5}
$\Xi^- \to \Lambda^0\pi^-$	0	2.5×10^{-5}	-3.1×10^{-4}	-0.9×10^{-4}
$\Sigma^- \to n\pi^-$	0	1.0×10^{-5}	-7.6×10^{-4}	-0.89×10^{-4}
$\Sigma^+ \to p\pi^0$	0	-2.9×10^{-5}	-2.7×10^{-3}	-8.9×10^{-5}
$\Sigma^+ \to n\pi^+$	0	3.1×10^{-4}	-2.5×10^{-5}	-1.0×10^{-3}

VI. CONCLUSIONS

In the preceding sections we have defined all of the observables and provided calculations of the weak phases in several models. Here we combine these to obtain predictions for the various quantities.

First in Table II we consider the KM model. Here we use the upper bound on $\mathrm{Im}\,C_5$ as an estimate of its value. Tables III and IV provide the same estimate in the Weinberg-Higgs-boson and left-right-symmetric models, respectively. Not included in the table is the (β/α) charge asymmetry in Ξ decays. For this we find

$$\left[\frac{\beta}{\alpha}\right]_{\Xi^-}-\left[\frac{\beta}{\alpha}\right]_{\Xi^0}=\begin{cases}6.2\times10^{-6}\ , & \text{KM}\ ,\\ 6.3\times10^{-5}\ , & \text{Weinberg-Higgs}\ ,\\ 0\ , & \text{left-right}\ .\end{cases}$$

As expected the Weinberg-Higgs-boson model provides the largest signal in practically all cases. In Table II we also show the expected values of $(\beta/\alpha)_{CPC}$. These are uncertain by about 10% due to the $\pm1^\circ$ uncertainty in the phase shifts. Unless these can be reduced to 0.1°, the parameter C seems difficult to use.

The rough size of the signal in the various asymmetries can be readily understood by inspection of the original definitions. Both the strong interaction phase shifts and $\Delta I=\frac{3}{2}$ effect are small and can suppress the signal even if the weak CP-violating phase is large. We normalize[26] all of the models ultimately to the only known measure of CP nonconservation, i.e., ϵ. If we characterize each model by the amount of $\Delta S=1$ CP violation using a parameter X, our earlier estimates would indicate the following pattern.

Model	X (approximately)
Superweak	0
Heavy neutral Higgs boson	0
Kobayashi-Maskawa	$20\epsilon'$
Charged-Higgs-boson (Weinberg)	ϵ
Left-right-symmetric	$(\mathcal{M}_{LL}/\mathcal{M}_{RL})\epsilon$

Using this, and counting the factors of $\sin\delta$ or A_3/A_1 in the observables, one can understand the strengths of the signals

$$(\Gamma-\bar{\Gamma})/(\Gamma+\bar{\Gamma})\approx\sin(\delta_3-\delta_1)(A_3/A_1)X\approx10^{-5}(X/\epsilon)\ ,$$

$$(\alpha+\bar{\alpha})/(\alpha-\bar{\alpha})\approx\tan(\delta_S-\delta_P)X\approx10^{-4}(X/\epsilon)\ ,$$

$$(\beta+\bar{\beta})/(\beta-\bar{\beta})\approx X/\tan(\delta_S-\delta_P)\approx10^{-2}(X/\epsilon)\ ,$$

$$\beta_{\Xi^0}/\alpha_{\Xi^0}-\beta_{\Xi^-}/\alpha_{\Xi^-}\approx(A_3/A_1)X\sim10^{-4}(X/\epsilon)\ .$$

As pointed out previously,[1] the ratio involving the β parameter is the one where any CP violation would be most evident. We would hope that these tests could be carried out and provide new evidence on the nature of CP nonconservation.

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[1] J. F. Donoghue and S. Pakvasa, Phys. Rev. Lett. **55**, 162 (1985).

[2] For an overview of weak interaction calculations including CP violation, see J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rep. **131**, 319 (1986). Up-to-date reviews of CP violation include L. Wolfenstein (unpublished) and B. Winstein (unpublished).

[3] L. Wolfenstein, Phys. Lett. **13**, 562 (1984).

[4] J. Pati, R. N. Mohapatra, and L. Wolfenstein, Phys. Rev. D **11**, 3362 (1975).

[5] T. Brown, N. Deshpande, S. Pakvasa, and H. Sugawara, Phys. Lett. **141B**, 95 (1984).

[6] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[7] M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. **B120**, 315 (1977); F. Gilman and M. Wise, Phys. Lett. **93B**, 129 (1980).

[8] S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976); T. D. Lee, Phys. Rev. D **8**, 1226 (1973); Phys. Rep. **96**, 143 (1974).

[9] R. N. Mohapatra, University of Maryland Report No. 85-124 (unpublished); R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); G. Branco, J. M. Frere, and J. M. Gerard, Nucl. Phys. **B221**, 317 (1983); H. Harari and M. Leurer, *ibid.* **B233**, 221 (1984).

[10] G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **47**, 552 (1981).

[11] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1645 (1957); R. Gatto, Nucl. Phys. **5**, 183 (1958).

[12] Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).

[13] T. Brown, S. Pakvasa, and S. F. Tuan, Phys. Rev. Lett. **51**,

1823 (1983); L. L. Chau and H.-Y. Cheng, Phys. Lett. **B131**, 302 (1983); J. Donoghue and S. Pakvasa, Phys. Rev. Lett. **55**, 162 (1985); L. Wolfenstein and D. Chang, in *Intense Medium Energy Sources of Strangeness,* proceedings of the Theoretical Symposium, Santa Cruz, 1983, edited by T. Goldman, H. E. Haber, and H. F. W. Hadrozinski (AIP Conf. Proc. No. 102) (AIP, New York, 1983), p. 73.

[14]O. E. Overseth and S. Pakvasa, Phys. Rev. **184**, 1663 (1969).

[15]L. Roper *et al.*, Phys. Rev. **138**, 190 (1965).

[16]R. Nath and B. R. Kumar, Nuovo Cimento **36**, 669 (1965).

[17]B. Martin, Phys. Rev. **138**, B1136 (1965).

[18]O. E. Overseth, Phys. Lett. **111B**, 286 (1982).

[19]F. Gilman and M. Wise, Phys. Rev. D **27**, 1128 (1983).

[20]J. F. Donoghue, E. Golowich, B. R. Holstein, and W. Ponce, Phys. Rev. D **21**, 186 (1980).

[21]R. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interaction of Elementary Particles* (Wiley, New York, 1969); see Overseth and Pakvasa in Ref. 14.

[22]J. F. Donoghue and B. R. Holstein, Phys. Rev. D **32**, 1152 (1985).

[23]J. F. Donoghue, E. Golowich, B. R. Holstein, and W. Ponce, Phys. Rev. D **23**, 1213 (1981).

[24]J. F. Donoghue, B. R. Holstein, and W. Ponce, Phys. Rev. D **23**, 1213 (1981).

[25]J. F. Donoghue and B. R. Holstein, Phys. Rev. D **33**, 2717 (1986). As will be seen, some of the results of Ref. 1 have been modified by this new analysis.

[26]G. Feinberg, P. K. Kabir, and S. Weinberg, Phys. Rev. Lett. **3**, 527 (1957).