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|---|-----------------------|--|--|
| 1. REPORT NUMBER<br>AFIT/CI/NR 88-165   | 2. GOVT ACCESSION NO. | 3. REPORT'S CATALOG NUMBER<br>DTIC FILE COPY                   |  |
| 4. TITLE (and Subtitle)<br>HYPERVELOCITY ORBITAL INTERCEPT<br>GUIDANCE  |                       | 5. TYPE OF REPORT & PERIOD COVERED<br><del>THESIS</del>        |  |
|   |                       | 6. PERFORMING ORG. REPORT NUMBER                               |  |
| 7. AUTHOR(s)<br>SALVATORE ALFANO  |                       | 8. CONTRACT OR GRANT NUMBER(s)                                 |  |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>AFIT STUDENT AT: UNIVERSITY OF COLORADO  |                       | 10. PROGRAM ELEMENT, PROJECT, TASK<br>AREA & WORK UNIT NUMBERS |  |
| 11. CONTROLLING OFFICE NAME AND ADDRESS   |                       | 12. REPORT DATE<br>1988  |  |
|   |                       | 13. NUMBER OF PAGES<br>184                                     |  |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)<br>AFIT/NR<br>Wright-Patterson AFB OH 45433-6583  |                       | 15. SECURITY CLASS. (of this report)<br>UNCLASSIFIED           |  |
|   |                       | 15a. DECLASSIFICATION/DOWNGRADING<br>SCHEDULE                  |  |
| 16. DISTRIBUTION STATEMENT (of this Report)<br>DISTRIBUTED UNLIMITED: APPROVED FOR PUBLIC RELEASE   |                       |  |  |
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| 18. SUPPLEMENTARY NOTES<br>Approved for Public Release: IAW AFR 190-1<br>LYNN E. WOLAVER <i>Lynn Wolaver</i><br>Dean for Research and Professional Development<br>Air Force Institute of Technology<br>Wright-Patterson AFB OH 45433-6583 <i>8 Aug 88</i> |                       |  |  |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  |                       |  |  |
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HYPERVELOCITY ORBITAL INTERCEPT GUIDANCE

by

SALVATORE ALFANO

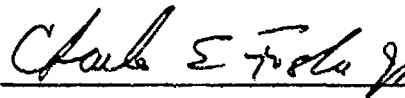
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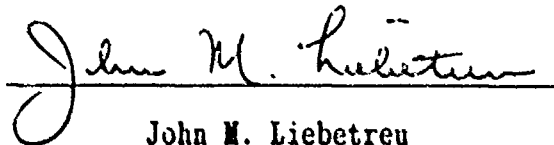
A thesis submitted to the  
Faculty of the Graduate School of the  
University of Colorado in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
Department of Electrical Engineering

1988

This thesis for the Doctor of Philosophy degree by  
Salvatore Alfano  
has been approved for the  
Department of  
Electrical Engineering  
by



Charles E. Fosha, Jr.



John M. Liebetreu

Date 14 APRIL 1988

To my wife, Michele, for her unfailing love and support

## ACKNOWLEDGEMENTS

First and foremost, I would like to thank God for giving man the ability, limited though it may be, to see and understand the workings of His wisdom and His hand in all of creation. My thanks go to Dr. Charles E. Fosha, Jr., my thesis advisor, and Dr. Robert B. Asher, for their guidance on this project. I would also like to thank Dr. John M. Liebetreu for his careful review of this manuscript as a second reader. In addition, I wish to thank the other members of my examining committee, Drs. Saber Elaydi, Ronald M. Sega, Renjeng Su, Mark A. Wickert, and Rodger E. Ziemer for their helpful suggestions.

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Alfano, Salvatore (Ph.D., Electrical Engineering)

Hypervelocity Orbital Intercept Guidance

Thesis directed by Associate Professor Charles E. Fosha, Jr.

Terminal guidance of a hypervelocity exo-atmospheric orbital interceptor with free end-time is examined. The pursuer is constrained to lateral thrusting with the evader modeled as an ICBM in its final boost phase. Proportional navigation, optimal control using certainty equivalence, dual control, and control with optimum thrust spacing are all examined. Also, a new approach called certainty control is developed for this problem. This algorithm constrains the final state to a function of projected estimate error to reduce control energy expenditure. All methods model the trajectories using splines and employ eight state Extended Kalman Filters with line-of-sight and range updates. The relative effectiveness of these control strategies is illustrated by applying them to various intercept problems. - (1)

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## CHAPTER I

### INTRODUCTION

Orbital interceptor performance can be enhanced by using a terminal guidance law that incorporates the orbital dynamics of the pursuer and evader plus the error knowledge of their estimates. The purpose of this research is to develop a guidance scheme for a hypervelocity, exo-atmospheric orbital vehicle in the final thirty seconds of flight that minimizes lateral thrusting while attempting to intercept a boosting missile. Much work has been done on the most common form of intercept guidance, proportional navigation, and its variations. This type of navigation assumes that the force of gravity acts equally on the pursuer and evader and can therefore be ignored in the relative dynamics. For orbital intercepts with large initial ranges the force of gravity will affect the relative trajectory and should be included in the equations of motion. To date, analytic solutions for such intercepts exist only when the pursuer's impact conditions are prespecified.

The general guidance schemes studied in this research attempt to minimize lateral velocity changes by varying the impact conditions through the use of splines. The pursuer is modeled as

a satellite with lateral thrusting capability using two-body orbital dynamics. The evader is modeled as an Intercontinental Ballistic Missile (ICBM) in its final boost phase, prior to burnout. The relative trajectory is propagated numerically to predicted impact time and then approximated by splines, eliminating the need to repeatedly propagate new trajectories when present conditions are varied. A search is then made for a new impact time and point that will minimize present interceptor velocity changes and final miss distance.

Six different variations of the general scheme are derived. The first scheme, presented in Chapter VI, uses a variable weighting factor and the principle of certainty equivalence to reduce velocity changes at the expense of final accuracy. The second scheme, also presented in Chapter VI, is a specialized version of the first, determining the velocity changes for zero miss. The third guidance algorithm in Chapter VI ignores the effects of gravity on the relative trajectory while attempting a zero miss solution. The fourth scheme, presented in Chapter VII, optimizes thrust spacing for a zero miss solution. The fifth scheme employs dual control techniques to reduce estimation error and is also presented in Chapter VII. The last algorithm is a new control approach that constrains the predicted miss distance to a function of final estimator error and is presented in Chapter VIII. Chapter IX summarizes the control strategies.

Target tracking is accomplished with a ranging device and line-of-sight sensors for in-plane and out-of-plane measurements. Noise corrupted data is processed through an eight state extended Kalman Filter with serial updates occurring every tenth of a second. The Kalman Filter equations are contained in Chapter X.

## CHAPTER II

## REVIEW OF LITERATURE

Much work has been done in the area of air-to-air intercept guidance. Guelman has derived a closed form solution for pure proportional navigation [1],[2] which is implemented in Chapter VI. Perturbation methods have been employed by Sridhar and Gupta [3]. Design procedures using optimal and stochastic control techniques abound [4]-[14] with variations of these techniques implemented in Chapters VI and VII. In the works cited above, the force of gravity is assumed to act equally on the pursuer and evader and is ignored in the relative dynamics. This 'flat earth' assumption is adequate for air-to-air encounters, but not for space-to-space. For orbital intercepts with large initial ranges the force of gravity will affect the relative trajectory and should be included in the equations of motion.

The literature for space-to-space guidance reveals many numerical approaches for determining present velocity for future rendezvous [15]-[21]. To date, analytic solutions for such intercepts exist only when the pursuer's impact conditions are pre-specified [19]. These works do not address hypervelocity intercept involving seconds, but are concerned with a much slower



rendezvous process involving hours or even days. Also, most of the literature reviewed assumed a passive, non-thrusting target. The literature that addressed thrusting targets was concerned with evasive maneuvering or 'gaming', the most recent being the paper by Menon and Calise [21]. A Defense Technology Information Center literature search revealed that the few papers addressing this problem are classified and therefore unavailable to the public.

The guidance schemes presented here attempt to minimize lateral velocity changes by varying the impact conditions through the use of splines. Splines were used by Johnson [16] in presenting a possible Earth-Mars transfer guidance algorithm. Dickmanns and Wells have used third order polynomials for general trajectory optimization [22], as well as Hargraves and Paris [23]. The splines eliminate the need to repeatedly propagate new trajectories when conditions are varied, resulting in faster searches. This feature makes them attractive for a hypervelocity orbital intercept where a fast and reasonably accurate numerical search is needed. Spline approximations are presented in Chapter V.

## CHAPTER III

## SYSTEM MODELING

In this chapter, the equations of motion for the evader and pursuer are developed, along with the necessary coordinate transformation for pursuer thrusting. Atmospheric drag will not be considered in the dynamics because the interceptor is assumed exo-atmospheric. Also, due to the pursuer's lateral thrusting limitation, the longitudinal axis will be assumed parallel to the pursuer's initial velocity vector.

It is convenient to transform the present coordinate frame to align the  $x$  axis with the pursuer's initial velocity vector. This is done by first rotating about the  $y$  axis until the  $z$  component of velocity is eliminated,

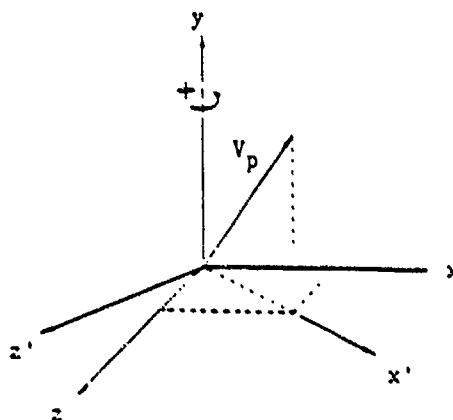


Figure 3-1. Rotation of coordinate frame about the  $y$  axis.

resulting in the following orthogonal transformation matrix:

$$[T_1] = \begin{bmatrix} \frac{\dot{x}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}} & 0 & \frac{\dot{z}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}} \\ 0 & 1 & 0 \\ \frac{-\dot{z}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}} & 0 & \frac{\dot{x}_p}{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}} \end{bmatrix} \quad (3-1)$$

The second rotation is about the new z axis ( $z'$ ), eliminating the y component of velocity.

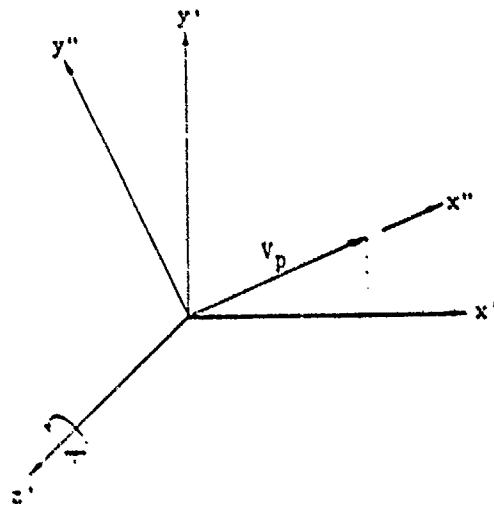


Figure 3-2. Rotation of the coordinate frame about the z axis.

This rotation yields the transformation matrix:

$$V_p = \sqrt{\dot{x}_p^2 + \dot{y}_p^2 + \dot{z}_p^2} \quad (3-2)$$

$$[T_2] = \begin{bmatrix} \frac{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}}{V_p} & \frac{\dot{y}_p}{V_p} & 0 \\ \frac{-\dot{y}_p}{V_p} & \frac{\sqrt{\dot{x}_p^2 + \dot{z}_p^2}}{V_p} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3-3)$$

Multiplying the two matrices in the proper order produces the overall transformation matrix:

$$[T] = [T_2][T_1] \quad (3-4)$$

The pursuer is modeled as a satellite traveling in excess of twelve kilometers per second with lateral thrusting capability using two-body orbital dynamics. Thrusting is prohibited along the longitudinal (x) axis to prevent sensor contamination and to satisfy the structural constraints of having forward sensors and a large aft booster to achieve hypervelocity speed. The equations of motion are:

$$\ddot{x}_p = \frac{-\mu x_p}{(x_p^2 + y_p^2 + z_p^2)^{3/2}} \quad (3-5)$$

$$\ddot{y}_p = \frac{-\mu y_p}{(x_p^2 + y_p^2 + z_p^2)^{3/2}} + a_y \quad (3-6)$$

$$\ddot{z}_p = \frac{-\mu z_p}{(x_p^2 + y_p^2 + z_p^2)^{3/2}} + a_z \quad (3-7)$$

where  $a_y$  and  $a_z$  are the lateral thrust accelerations,  $\mu$  is the earth's gravitational constant, and the double dots denote the second derivative with respect to time.

The evader is modeled as an Intercontinental Ballistic Missile (ICBM) in its final boost phase using two-body orbital dynamics. For tracking purposes the intercept must occur prior to burnout. Acceleration due to thrusting is computed in the direction of the booster's velocity vector. The equations of motion are:

$$A = \frac{A_0}{1 - \dot{m}_0 t} \quad (3-8)$$

$$\ddot{x}_E = \frac{-\mu x_E}{(x_E^2 + y_E^2 + z_E^2)^{3/2}} + \frac{A \dot{x}_E}{(\dot{x}_E^2 + \dot{y}_E^2 + \dot{z}_E^2)^{1/2}} \quad (3-9)$$

$$\ddot{y}_E = \frac{-\mu y_E}{(x_E^2 + y_E^2 + z_E^2)^{3/2}} + \frac{A \dot{y}_E}{(\dot{x}_E^2 + \dot{y}_E^2 + \dot{z}_E^2)^{1/2}} \quad (3-10)$$

$$\ddot{z}_E = \frac{-\mu z_E}{(x_E^2 + y_E^2 + z_E^2)^{3/2}} + \frac{A \dot{z}_E}{(\dot{x}_E^2 + \dot{y}_E^2 + \dot{z}_E^2)^{1/2}} \quad (3-11)$$

where  $A$  is the present acceleration,  $A_0$  the initial acceleration,  $\dot{m}_0$  the initial mass flow rate divided by mass, and  $t$  the time since ignition. The single dot denotes the first derivative with respect to time.

## CHAPTER IV

## PROBLEM STATEMENT AND TRUTH MODEL

Time-to-go and pursuer velocity changes are the control parameters that must be varied to minimize miss distance and fuel expended (i.e. velocity changes). This can be done by establishing a time remaining until intercept (time-to-go), propagating the equations of motion forward, and determining the miss distance. An iterative process can then be used to find the pursuer velocity needed to bring the miss distance to zero. The difference between current velocity and that needed for intercept, known as velocity-to-go, must be minimized. To accomplish this, the time-to-go is varied and the above procedure repeated until a minimum velocity-to-go is found.

The computation of needed velocity is time consuming because the equations of motion are nonlinear and do not lend themselves to closed form solution. These equations must be propagated numerically to intercept time whenever the initial velocity is varied. The above method will serve as the basis (truth) model for this control problem using the numerical techniques found in Maron [24].

A Newton-Raphson method for solving nonlinear systems is

employed to determine the proper values of the control parameters.

Let

$$\bar{u} = \begin{bmatrix} \overline{\Delta V}_y \\ \overline{\Delta V}_z \\ \overline{t}_{go} \end{bmatrix} \quad (4-1)$$

be a solution of the nonlinear system

$$\begin{bmatrix} f_1(\bar{u}) \\ f_2(\bar{u}) \\ f_3(\bar{u}) \end{bmatrix} = \begin{bmatrix} x_E(\overline{t}_{go}) - x_P(\overline{t}_{go}) \\ y_E(\overline{t}_{go}) - y_P(\overline{t}_{go}) - \overline{\Delta V}_y \overline{t}_{go} \\ z_E(\overline{t}_{go}) - z_P(\overline{t}_{go}) - \overline{\Delta V}_z \overline{t}_{go} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (4-2)$$

where  $\overline{t}_{go}$  is the time-to-go and the pursuer's velocity changes are  $\overline{\Delta V}_y$  and  $\overline{\Delta V}_z$ . The effect of small velocity changes in (4-2) can be considered linear because the pursuer is assumed to travel at hypervelocity, resulting in a near straight-line trajectory. Any error caused by this assumption will be accounted for in the succeeding iteration when the proposed velocity change is incorporated in the nonlinear dynamics.

The initial control values must be incrementally changed to satisfy (4-2). A linear approximation of the  $f$  vector for



changes in  $\underline{u}$  will yield approximate increments of the control parameters.

The linearized system becomes

$$[J] \begin{bmatrix} d\Delta V_y \\ d\Delta V_z \\ dt_{go} \end{bmatrix} = - \begin{bmatrix} f_1(\underline{u}) \\ f_2(\underline{u}) \\ f_3(\underline{u}) \end{bmatrix} \quad (4-3)$$

where  $J$  is the Jacobian matrix of the  $\underline{f}$  vector evaluated at  $\underline{u}$ :

$$[J] = \begin{bmatrix} \frac{\partial f_1(\underline{u})}{\partial \Delta V_y} & \frac{\partial f_1(\underline{u})}{\partial \Delta V_z} & \frac{\partial f_1(\underline{u})}{\partial t_{go}} \\ \frac{\partial f_2(\underline{u})}{\partial \Delta V_y} & \frac{\partial f_2(\underline{u})}{\partial \Delta V_z} & \frac{\partial f_2(\underline{u})}{\partial t_{go}} \\ \frac{\partial f_3(\underline{u})}{\partial \Delta V_y} & \frac{\partial f_3(\underline{u})}{\partial \Delta V_z} & \frac{\partial f_3(\underline{u})}{\partial t_{go}} \end{bmatrix} \quad (4-4)$$

Computing the partial derivatives yields

$$[J] = \begin{bmatrix} 0 & 0 & \{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\} \\ -t_{go} & 0 & \{\dot{y}_E(t_{go}) - \dot{y}_p(t_{go}) - \Delta V_y\} \\ 0 & -t_{go} & \{\dot{z}_E(t_{go}) - \dot{z}_p(t_{go}) - \Delta V_z\} \end{bmatrix} \quad (4-5)$$

To determine changes in the  $\underline{u}$  vector,  $\underline{f}$  is multiplied by the negative inverse of J

$$\begin{bmatrix} d\Delta V_y \\ d\Delta V_z \\ dt_{go} \end{bmatrix} = -[J]^{-1} \begin{bmatrix} f_1(u) \\ f_2(u) \\ f_3(u) \end{bmatrix} \quad (4-6)$$

$$[J]^{-1} = \begin{bmatrix} \frac{\dot{y}_E(t_{go}) - \dot{y}_p(t_{go}) - \Delta V_y}{\{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\}t_{go}} & -1/t_{go} & 0 \\ \frac{\dot{z}_E(t_{go}) - \dot{z}_p(t_{go}) - \Delta V_z}{\{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\}t_{go}} & 0 & -1/t_{go} \\ \frac{1}{\{\dot{x}_E(t_{go}) - \dot{x}_p(t_{go})\}} & 0 & 0 \end{bmatrix} \quad (4-7)$$

To find the control parameters the following procedure should be used. First, establish a time-to-go with zero velocity changes, a good choice being the time-to-go that yields the point of closest approach. This time-to-go is determined by propagating the orbits forward until a minimum relative distance is reached. Because the evader is assumed to be in its final boost phase throughout the intercept, this time-to-go will be less than or equal to time until ICBM thrust termination. Second, propagate the dynamic equations (3-5 thru 3-11) forward to the intercept time and determine the  $\underline{f}$  vector from (4-2). Changes to the control parameters are then obtained from (4-6). The velocity changes are applied to the pursuer's initial conditions and the procedure is repeated with the updated time-to-go until convergence occurs. The resulting control parameters will drive the miss distance to zero with minimum velocity changes. The difference between needed and present velocity are sufficient to determine the pursuer's thrust profile.

## CHAPTER V

## SPLINE APPROXIMATIONS

As discussed in Chapter IV, numerical propagation of the dynamic equations is very time consuming. It would be convenient to approximate the relative trajectory by a polynomial, eliminating the need for repeated propagation. Cubic splines lend themselves well to this application [16], [22], [23]. The current and final states can be used to generate cubic splines along each axis of the form

$$x(t) = At_{go}^3 + Bt_{go}^2 + Ct_{go} + D \quad (5-1)$$

By setting the current time to zero, D and C become the current position and velocity respectively, with time-to-go being the intercept time. Changes in velocity will be reflected only in the C coefficient and the final state can be easily determined for any intercept time. With this formulation, the determination of the spline coefficients is relatively simple. The current state gives D and C with no computations:

$$D = x(0) \quad (5-2)$$

$$C = \dot{x}(0) \quad (5-3)$$

The A and B coefficients can be computed using the final states and (5-1) as follows:

$$x(t_{go}) = At_{go}^3 + Bt_{go}^2 + Ct_{go} + D \quad (5-4)$$

$$\dot{x}(t_{go}) = 3At_{go}^2 + 2Bt_{go} + C \quad (5-5)$$

Because there are only two unknowns in the above two equations, algebraic manipulation yields:

$$A = \frac{2[x(0) - x(t_{go})]}{t_{go}^3} + \frac{[\dot{x}(0) + \dot{x}(t_{go})]}{t_{go}^2} \quad (5-6)$$

$$B = \frac{3[x(t_{go}) - x(0)]}{t_{go}^2} + \frac{[2\dot{x}(0) + \dot{x}(t_{go})]}{t_{go}} \quad (5-7)$$

Figures depicting distance errors associated with these approximations are provided in Appendix A.

There is an added versatility in using splines. Should the system model be changed, only the spline coefficients need be changed. The search algorithms based on the splines will remain the same, operating with the new coefficients. This is very beneficial as it is far simpler to recompute the coefficients than

to alter the algorithms.

To ensure accuracy, new spline coefficients are computed every cycle time. To accomplish this, the truth model is propagated forward to predicted impact time to obtain the needed final states. By using these updated final states every iteration, propagated roundoff error is eliminated in the spline coefficient computations.

## CHAPTER VI

## OPTIMAL CONTROL FORMULATION USING CERTAINTY EQUIVALENCE

Changes in pursuer lateral velocity will affect final position, velocity and time. These effects can be easily computed with the relative trajectory modeled by splines in the coordinate system discussed in Chapter II. The optimal control problem is to find the intercept time that minimizes changes in pursuer velocity while ensuring a hit. Techniques to solve such problems are addressed by Bryson and Ho [25] and summarized in the following paragraphs.

## PLAN A

To solve this problem, a relative spline equation is formed for each axis and a cost function is established. The cost function (L) incorporates velocity changes and miss distance multiplied by a weighting factor (K) and is represented as

$$L = \frac{K(x_1^2 + x_2^2 + x_3^2)}{2} + \frac{(\Delta V_y^2 + \Delta V_z^2)}{2} \quad (6-1)$$

where the final relative state vector is determined from the spline equations:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x(t_{go}) \\ y(t_{go}) \\ z(t_{go}) \end{bmatrix} = \begin{bmatrix} A_x t_{go}^3 + B_x t_{go}^2 + C_x t_{go} + D_x \\ A_y t_{go}^3 + B_y t_{go}^2 + (C_y - \Delta V_y) t_{go} + D_y \\ A_z t_{go}^3 + B_z t_{go}^2 + (C_z - \Delta V_z) t_{go} + D_z \end{bmatrix} \quad (6-2)$$

The cost function must now be minimized with respect to the control vector  $\underline{u}$ :

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} t_{go} \\ \Delta V_y \\ \Delta V_z \end{bmatrix} \quad (6-3)$$

As stated in Bryson and Ho [25], it should be possible to find a set of controls such that

$$\frac{\partial L}{\partial \underline{u}} = \underline{0} \quad (6-4)$$



Three equations arise from (6-4) with three unknowns, expressed here in vector form as

$$\underline{h} = \begin{bmatrix} h_1 \\ h_2 \\ h_2 \end{bmatrix} = \begin{bmatrix} K(x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3) \\ \Delta V_y - Kx_2t_{go} \\ \Delta V_z - Kx_3t_{go} \end{bmatrix} = 0 \quad (6-5)$$

with  $\dot{x}$  being

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 3A_x t_{go}^2 + 2B_x t_{go} + C_x \\ 3A_y t_{go}^2 + 2B_y t_{go} + C_y - \Delta V_y \\ 3A_z t_{go}^2 + 2B_z t_{go} + C_z - \Delta V_z \end{bmatrix} \quad (6-6)$$

As in the truth model, a Newton-Raphson method from Maron [24] is used to solve (6-5). It is important to note that this formulation differs from the truth model in two areas. First, a weighting factor has been introduced that allows a trade-off between miss distance and velocity changes. Zero miss distance is associated with infinite  $K$ , while zero  $K$  produces no velocity change. Second, the splines eliminate the need for repeated trajectory propagation, significantly reducing control parameter search time.

The Jacobian for (6-5) is

$$[J] = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & 0 \\ J_{31} & 0 & J_{33} \end{bmatrix} \quad (6-7)$$

where

$$J_{11} = K(x_1[6A_x t_{go} + 2B_x] + x_2[6A_y t_{go} + 2B_y] + x_3[6A_z t_{go} + 2B_z] + \dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2) \quad (6-8)$$

$$J_{12} = J_{21} = -K(x_2 + \dot{x}_2 t_{go}) \quad (6-9)$$

$$J_{13} = J_{31} = -K(x_3 + \dot{x}_3 t_{go}) \quad (6-10)$$

$$J_{22} = J_{33} = 1 + Kt_{go}^2 \quad (6-11)$$

Changes in the control vector are determined by

$$d\underline{u} = \begin{bmatrix} dt_{go} \\ d\Delta V_y \\ d\Delta V_z \end{bmatrix} = -[J]^{-1} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \quad (6-12)$$

$$[J]^{-1} = \frac{\begin{bmatrix} J_{22} & -J_{12} & -J_{13} \\ -J_{12} & (J_{11} - J_{13}^2/J_{22}) & (J_{12}J_{13}/J_{22}) \\ -J_{13} & (J_{12}J_{13}/J_{22}) & (J_{11} - J_{12}^2/J_{22}) \end{bmatrix}}{(J_{11}J_{22} - J_{13}^2 - J_{12}^2)} \quad (6-13)$$

To execute this procedure, initialize time-to-go (preferably to the point of closest approach) and determine the spline coefficients for this initial trajectory. Compute the  $x$ ,  $\dot{x}$ , and  $h$  vectors, in that order. Update the  $u$  vector using (6-12) and test for convergence. If convergence is not achieved recompute the above vectors and test again.

#### PLAN B

The formulation in this plan uses splines to determine the control parameters for a zero miss solution. This is a specialized version of Plan A where the weighting factor is set to infinity ( $K=\infty$ ). Because the only control for miss in the longitudinal direction is time-to-go,  $x_1$  in (6-2) is set equal to zero,

$$x_1 = A_x t_{go}^3 + B_x t_{go}^2 + C_x t_{go} + D_x = 0 \quad (6-14)$$

and  $t_{go}$  solved using numerical techniques. With time-to-go established, (6-2) is again used with  $x_2$  and  $x_3$  equal to zero, yielding equations for the velocity changes:

$$\Delta V_y = A_y t_{go}^2 + B_y t_{go} + C_y + D_y/t_{go} \quad (6-15)$$

$$\Delta V_z = A_z t_{go}^2 + B_z t_{go} + C_z + D_z/t_{go} \quad (6-16)$$

This plan is computationally less burdensome than Plan A because the complexity of the search is reduced.

#### PLAN C

Within a few seconds of intercept the acceleration due to gravity will be nearly identical for the pursuer and evader. Also, the booster, still thrusting in the final boost phase, will travel in a near straight line along its velocity vector. Ignoring gravity terms in the relative dynamics leads to a simpler and faster solution, reducing the guidance to proportional navigation [1]. The relative trajectories are expressed as

$$D_A = \frac{A_0}{\dot{m}_0^2} \sum_{i=2}^{\infty} \frac{(\dot{m}_0 t_{go})^i}{i(i-1)} \quad (6-17)$$

$$\begin{aligned}
 x(t_{go}) = & \left\{ [x_E(0) - x_p(0)] + [\dot{x}_E(0) - \dot{x}_p(0)]t_{go} \right. \\
 & \left. + \frac{D_A \dot{x}_E(0)}{\sqrt{\dot{x}_E^2(0) + \dot{y}_E^2(0) + \dot{z}_E^2(0)}} \right\} \quad (6-18)
 \end{aligned}$$

$$\begin{aligned}
 y(t_{go}) = & \left\{ [y_E(0) - y_p(0)] + [\dot{y}_E(0) - \dot{y}_p(0) - \Delta V_y]t_{go} \right. \\
 & \left. + \frac{D_A \dot{y}_E(0)}{\sqrt{\dot{x}_E^2(0) + \dot{y}_E^2(0) + \dot{z}_E^2(0)}} \right\} \quad (6-19)
 \end{aligned}$$

$$\begin{aligned}
 z(t_{go}) = & \left\{ [z_E(0) - z_p(0)] + [\dot{z}_E(0) - \dot{z}_p(0) - \Delta V_z]t_{go} \right. \\
 & \left. + \frac{D_A \dot{z}_E(0)}{\sqrt{\dot{x}_E^2(0) + \dot{y}_E^2(0) + \dot{z}_E^2(0)}} \right\} \quad (6-20)
 \end{aligned}$$

where  $D_A$  is the distance associated with thruster acceleration in the direction of booster velocity. As in Plan B, time-to-go is computed for zero miss on the x axis using (6-18), and then the velocity changes can be found from (6-19) and (6-20).

## CERTAINTY EQUIVALENCE

It should be noted that all these techniques use the principle of certainty equivalence, where expected values from a state estimator are substituted for random variables [26]. The pursuer's states are assumed known, but because the evader's states must be estimated, the resulting system is stochastic. Optimal control formulation is based on a system that is deterministic. In applying the certainty equivalence principle, the stochastic system is replaced by a deterministic one, using the expected values of the random variables from the estimator.

There is a drawback to this technique in the sense that imperfect knowledge of the present state produces needless thrusting. Any errors in the present state estimate cause errors in the predicted final state. This results in the computation of velocity changes based on the incorrect final state. Future iterations produce similar results requiring the pursuer to thrust excessively.

This excessive thrusting can be reduced using stochastic control techniques. Three formulations are examined in the following chapters. The first determines the optimum spacing of corrective thrusts for Plan B. The second uses dual control methods based on predicted error knowledge, such as filter covariance. The third constrains the miss distance to a function of predicted error knowledge, at the expense of accuracy.

## CHAPTER VII

## STOCHASTIC CONTROL

## OPTIMUM SPACING OF CORRECTIVE THRUSTS

Corrective thrusting in the presence of state estimate errors can be optimally spaced to reduce fuel [27]. A control effectiveness ratio ( $\rho$ ) is established to determine the spacing between thrusts. This ratio directly yields thrust times when control effectiveness is a linear function of time.

For this formulation, the number of corrective thrusts ( $N$ ) must be chosen to minimize the sum of thrusts ( $S_N$ ), which is total  $\Delta V$ . The behavior of  $\Delta V$  and miss distance as a function of  $\rho$  can be produced through digital computation and is done as part of the simulation to determine the best value of  $\rho$  for Plan B.

To enhance understanding this technique, assume the control effectiveness ratio is two ( $\rho=2$ ). This implies that corrective thrusting should take place when the control has half ( $1/\rho$ ) the effect of the previous corrective thrust. If control effectiveness is a near-linear function of time, as is the case for a hypervelocity vehicle, then it will be halved at about half the time to impact since the last thrust. Thrusting will take

place at the start of the intercept, at one-half time-to-go, one fourth time-to-go, one-eighth time-to-go and so on. With  $\rho=3$  the optimum thrust timing always occurs at a third of the time-to-go since the last correction. When the spacing is less than the estimator's cycle time, impact is imminent and thrust is terminated.

### DUAL CONTROL FORMULATION

Optimal control solutions require perfect knowledge of the states, but in reality the information provided to the controller is only an estimate. As stated by Aoki [28], a theory of control should take into account the 'imperfectness' of information. This explains the need to incorporate statistical decision theory in control formulation. A solution that uses imperfect information will be sub-optimal, but it is desirable for such a solution to have the intrinsic characteristics of optimality [5]. Recognition that the control affects not only the state but also its uncertainty leads to a form of stochastic control known as dual control. This method not only drives the system to some final state, but attempts to improve state uncertainty along the way. The result is often greater accuracy and/or reduced fuel consumption.

A dual control method for controlling stochastic nonlinear systems with free end-time was developed by Tse and



Bar-Shalom [5]. This method differs from the optimal control formulation presented in Chapter VI. Instead of minimizing the cost function  $L$  of (6-1), the expected value of the cost function ( $E\{L\}$ ) is minimized. To accomplish this, the final states and their covariances must be computed. This can be done by running the Extended Kalman Filter forward to predicted intercept time, as suggested by Tse, Bar-Shalom and Meier [4].

The solution involves establishing an expected cost function consisting of miss distance and covariance of each axis, along with the control. The cost function  $L$  from (6-1) is repeated here for convenience:

$$L = K \frac{(x_f^2 + y_f^2 + z_f^2)}{2} + \frac{(\Delta V_y^2 + \Delta V_z^2)}{2} \quad (7-1)$$

Assuming the estimates of the filter are Gaussian, the expected value is:

$$E\{L\} = K \left[ \frac{\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2}{2} + \frac{\hat{x}_f^2 + \hat{y}_f^2 + \hat{z}_f^2}{2} \right] + E \left\{ \frac{\Delta V_y^2 + \Delta V_z^2}{2} \right\} \quad (7-2)$$

The expected cost of (7-2) is conditioned on the controls. Two cases must be examined: the cost associated with the certainty equivalence (CE) solution (Plan A of Chapter VI) and the cost of deviating from that solution to improve the estimate. In this manner, the approximate best cost-to-go includes both estimation and control performance.

The expected cost of the CE solution is easily computed by determining the controls from Plan A and then running the Extended Kalman Filter forward to predicted impact time assuming measurement updates. The final filter data is then inserted into (7-2) to find the expected cost.

Finding the expected cost of deviating from the CE solution is computationally burdensome. The thrust direction that yields the greatest estimate improvement must first be determined. Thrusting in this direction will cause the expected miss distance to grow due to departure from the nominal (CE) path. It is therefore necessary to determine a new nominal path based on the deviation and include the control energy required for this path in the deviation cost estimate. Failure to do so may result in large expected miss distances that erroneously inflate the cost associated with deviation, causing the CE control of Plan A to always be chosen.

## CHAPTER VIII

## CERTAINTY CONTROL

As stated earlier, if the estimate is near perfect then optimal control should be used. For a less accurate estimate, dual control attempts to improve the measurement certainty, and thus the estimate, by expending control energy. This has been shown to work well if the certainty is a function of the control parameters [5]. Because range is included as a measurement, lateral deviations should not noticeably improve the estimate. For this reason, dual control techniques are not expected to work better than certainty equivalence formulations.

If the controls associated with cost do not affect state estimate certainty, fuel may be conserved by using that certainty to reduce the controls. By linking the controls to the certainty of the estimate, a near perfect estimate would yield the optimal control, with reduced control resulting from a poor estimate. To accomplish this, the predicted final states are constrained by a function of their variances at the final time. This form of control will be called certainty control and is implemented by establishing the cost function

$$L = \frac{\Delta V_y^2 + \Delta V_z^2}{2} \quad (8-1)$$

subject to the constraint:

$$f = \frac{x_f^2 + y_f^2 + z_f^2 - K[\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2]}{2} \leq 0 \quad (8-2)$$

where K is a weighting factor. The final state estimates ( $x_f$ ,  $y_f$ ,  $z_f$ ) and their deviations ( $\sigma_{xf}$ ,  $\sigma_{yf}$ ,  $\sigma_{zf}$ ) are determined by running the filter forward to predicted impact time without updates and then representing their time history with splines:

$$x_s = A_x t_{go}^3 + B_x t_{go}^2 + C_x t_{go} + D_x \quad (8-3)$$

$$y_s = A_y t_{go}^3 + B_y t_{go}^2 + C_y t_{go} + D_y \quad (8-4)$$

$$z_s = A_z t_{go}^3 + B_z t_{go}^2 + C_z t_{go} + D_z \quad (8-5)$$

$$x_f = x_s \quad (8-6)$$

$$y_f = y_s - \Delta V_y t_{go} \quad (8-7)$$

$$z_f = z_s - \Delta V_z t_{go} \quad (8-8)$$

$$\sigma_{xf} = A_{\sigma x} t_{go}^3 + B_{\sigma x} t_{go}^2 + C_{\sigma x} t_{go} + D_{\sigma x} \quad (8-9)$$

$$\sigma_{yf} = A_{\sigma y} t_{go}^3 + B_{\sigma y} t_{go}^2 + C_{\sigma y} t_{go} + D_{\sigma y} \quad (8-10)$$

$$\sigma_{zf} = A_{\sigma z} t_{go}^3 + B_{\sigma z} t_{go}^2 + C_{\sigma z} t_{go} + D_{\sigma z} \quad (8-11)$$

Conceptually, the constraint produces a deviation sphere about the predicted impact point. If the predicted miss is inside or touching the sphere, thrusting is not necessary. If the predicted miss is outside the sphere, minimum thrusting is determined to bring the miss to the surface of the sphere. As the estimates improve, the constraint tightens and the sphere shrinks. The spline representations allow this stochastic problem to be solved deterministically. The constraint is adjoined to the cost function to form the Hamiltonian [29]:

$$H = L + \lambda f \quad (8-12)$$

The partials of  $H$  with respect to the controls must equal zero:

$$\frac{\partial H}{\partial \Delta V_y} = \Delta V_y - \lambda y_f t_{go} = 0 \quad (8-13)$$

$$\frac{\partial H}{\partial \Delta V_z} = \Delta V_z - \lambda z_f t_{go} = 0 \quad (8-14)$$

$$\frac{\partial H}{\partial t_{go}} = \lambda(x_f \dot{x}_f + y_f \dot{y}_f + z_f \dot{z}_f - K[\sigma_{xf} \dot{\sigma}_{xf} + \sigma_{yf} \dot{\sigma}_{yf} + \sigma_{zf} \dot{\sigma}_{zf}]) = 0 \quad (8-15)$$

with the dot term expansions computed in Appendix B.

Equations 8-2, 8-13, 8-14, and 8-15 constitute four equations with four unknowns, which can be reduced two equations and two unknowns using (8-7) and (8-8). Substituting (8-7) into (8-13) yields

$$\Delta V_y = \frac{\lambda y_s t_{go}}{1 + \lambda t_{go}^2} \quad (8-16)$$

$$y_f = \frac{y_s}{1 + \lambda t_{go}^2} \quad (8-17)$$

In a similar manner, substituting (8-8) into (8-14) yields

$$\Delta V_z = \frac{\lambda z_s t_{go}}{1 + \lambda t_{go}^2} \quad (8-18)$$

$$z_f = \frac{z_s}{1 + \lambda t_{go}^2} \quad (8-19)$$

Equations 8-2 and 8-15 can now be solved in terms of  $\lambda$  and  $t_{go}$ . Once known,  $\Delta V_y$  and  $\Delta V_z$  can be determined from (8-16) and (8-18). The parameters  $\lambda$  and  $t_{go}$  can be found by numerical techniques using the Jacobian:

$$[J] \begin{bmatrix} dt_{go} \\ d\lambda \end{bmatrix} = \begin{bmatrix} -f_1 \\ -f_2 \end{bmatrix} \quad (8-20)$$

$$f_1 = \frac{x_f^2 + y_f^2 + z_f^2 - K[\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2]}{2} \quad (8-21)$$

$$f_2 = x_f \dot{x}_f + y_f \dot{y}_f + z_f \dot{z}_f - K[\sigma_{xf} \dot{\sigma}_{xf} + \sigma_{yf} \dot{\sigma}_{yf} + \sigma_{zf} \dot{\sigma}_{zf}] \quad (8-22)$$

with the elements of the Jacobian matrix computed in Appendix B.

Should the states be perfectly known, the  $\sigma$  terms will be zero. In this case, the equations for certainty control reduce to the optimal control formulation for Plan B. Should the estimate be poor, the  $\sigma$  terms will be large and the inequality constraint of (8-2) is met with very little (if any) change in velocity. This demonstrates the principle of certainty control, where the certainty of the estimate affects control energy expenditure.

## CHAPTER IX

## SUMMARY OF CONTROL STRATEGIES

In this chapter a brief summary of all the control strategies is presented. It is intended to give the reader a basis for quick comparison. The cost function of each algorithm is given, along with the requirements for computation.

Plan A is an optimal control, certainty equivalence formulation that minimizes the cost function:

$$L = \frac{K(x_f^2 + y_f^2 + z_f^2)}{2} + \frac{(\Delta V_y^2 + \Delta V_z^2)}{2}$$

This algorithm requires an estimate of the final relative states.

Plan B is a certainty equivalence formulation that minimizes the cost function:

$$L = \frac{(\Delta V_y^2 + \Delta V_z^2)}{2}$$



subject to the constraint

$$f = x_f^2 + y_f^2 + z_f^2 = 0 \quad .$$

This algorithm requires an estimate of the final relative states.

Plan C is a certainty equivalence formulation with the same cost function and requirements as Plan B. The difference between these strategies is that gravity is ignored in the dynamic equations used to estimate the final relative states.

The optimal spacing of corrective thrusts also uses the same cost function and requirements as Plan B. In this strategy, however, the pursuer is not permitted to thrust every cycle time. Thrust timing is controlled by selecting a control effectiveness ratio to minimize control energy expenditure.

Dual Control is a stochastic control formulation that attempts to improve the estimate, and thus accuracy, by minimizing the cost function:

$$E\{L\} = K \left[ \frac{\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2}{2} + \frac{\hat{x}_f^2 + \hat{y}_f^2 + \hat{z}_f^2}{2} \right] \\ + E \left[ \frac{\Delta V_y^2 + \Delta V_z^2}{2} \right]$$

This algorithm requires estimates of the final relative states, their filter variances, and the relationship between control and variance.

Certainty Control is a new stochastic control formulation that reduces the control based on the certainty of the estimate by minimizing the cost function

$$L = \frac{(\Delta V_y^2 + \Delta V_z^2)}{2}$$

subject to the constraint

$$\frac{\hat{x}_f^2 + \hat{y}_f^2 + \hat{z}_f^2 - K[\sigma_{xf}^2 + \sigma_{yf}^2 + \sigma_{zf}^2]}{2} \leq 0 .$$

This algorithm requires estimates of the final relative states and their filter variances.

## CHAPTER X

## EXTENDED KALMAN FILTERING

Optimal estimates of the pursuer and evader are needed for the search algorithms to converge properly. Due to the nature of the dynamics and sensors, the relative position and velocity must be estimated from sampled nonlinear measurements. The estimation problem for a nonlinear system having continuous dynamics and discrete-time measurements is addressed by Gelb [30]. The Extended Kalman Filter (EKF) was chosen over other estimation methods because the optimal estimate is determinate. That is, the dynamics and observations of the pursuer and evader can be well predicted in the presence of Gaussian noise.

A summary of the continuous-discrete EKF algorithm from Gelb [30] follows. The equations for the state dynamics and measurements, as well as the computations of the partial derivatives, can be found in Appendix C. The system model is a continuous model of the state dynamics with white Gaussian noise  $\{w(t)\}$  added.

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), t) + \underline{w}(t) \quad (10-1)$$

$$\underline{w}(t) \sim N(\underline{0}, Q(t)) \quad (10-2)$$

where  $\underline{w}$  is a Gaussian (normal) random vector with mean  $\underline{0}$  and covariance matrix  $Q$ . The measurement model is discrete and corrupted by white Gaussian noise  $\underline{v}_k$ :

$$\underline{z}_k = \underline{h}_k(\underline{x}(t_k)) + \underline{v}_k \quad (10-3)$$

$$\underline{v}_k \sim N(\underline{0}, R_k) \quad (10-4)$$

In either case, the noise is assumed uncorrelated for all  $t(k)$ .

The state estimate, denoted by a hat, is propagated from (10-1) with

$$\dot{\hat{\underline{x}}} = \underline{f}(\hat{\underline{x}}(t), t) \quad (10-5)$$

and the error covariance  $P(t)$  is propagated by

$$\dot{P}(t) = F(\hat{\underline{x}}(t), t)P(t) + P(t)F^T(\hat{\underline{x}}(t), t) + Q(t) \quad (10-6)$$

$$F(\hat{\underline{x}}(t), t) = \left. \frac{\partial \underline{f}(\underline{x}(t), t)}{\partial \underline{x}(t)} \right|_{\underline{x}(t) = \hat{\underline{x}}(t)} \quad (10-7)$$

The measurements determine the gain matrix  $K_k$  through the equations

$$H_k(\hat{x}_k(-)) = \frac{\partial h_k(x(t_k))}{\partial x(t_k)} \quad \left| \quad \begin{array}{l} x(t_k) = \hat{x}_k(-) \end{array} \right. \quad (10-8)$$

$$K_k = P_k(-) H_k^T(\hat{x}_k(-)) [H_k(\hat{x}_k(-)) P_k(-) H_k^T(\hat{x}_k(-)) + R_k]^{-1} \quad (10-9)$$

where the (-) symbolizes prior to update and the (+) after update.

With the gain matrix computed, the state estimate and error covariance can be updated by the following equations:

$$\hat{x}_k(+) = \hat{x}_k(-) + K_k [z_k - h_k(\hat{x}_k(-))] \quad (10-10)$$

$$P_k(+) = [I - K_k H_k(\hat{x}_k(-))] P_k(-) \quad (10-11)$$

It is advantageous to process measurements one at a time. This method, called serial updating [31], eliminates the requirement to compute a matrix inverse, thereby reducing computer load and avoiding the computational problems associated with inverting an ill-conditioned matrix. Also, measurements may be skipped without reformulating the filter equations, allowing greater flexibility in examining various tracking schemes. The simultaneous measurement components of the vector  $z_k$  can be considered serially over a very short time span.

## CHAPTER XI

## COMPUTER SIMULATION

A menu-driven program that simulates all the algorithms was written in FORTRAN 77 and run on a VAX 8600. The code for this program can be found in Appendix D. Six cases are examined to determine the accuracy and efficiency of each algorithm. In all cases the propagation ( $w$ ) and measurement ( $v$ ) noise properties associated with the filter are:

$$w_{x,y,z}(t) \sim N(0, 2.21516 \times 10^{-18} \frac{m^2}{s})$$

$$\dot{w}_{\dot{x},\dot{y},\dot{z}}(t) \sim N(0, 5.52049 \times 10^{-20} \frac{m^4}{s^3})$$

$$w_A(t) \sim N(0, 4.29831 \times 10^{-12} \frac{m^2}{s^5})$$

$$w_{\dot{m}}(t) \sim N(0, 2.493241 \times 10^{-7} \frac{1}{s^3})$$

$$v_{\theta,\gamma}(k) \sim N(0, 1.0 \times 10^{-8})$$

$$V_R(k) = N(0, 1.0 \times 10^{-8} \times R^2 \text{ m}^2)$$

where  $\theta$  is the out-of-plane line-of-sight angle,  $\gamma$  the in-plane line-of-sight angle and  $R$  is range.

The startup variances are:

$$\sigma_{xx}^2 = \sigma_{yy}^2 = \sigma_{zz}^2 = 100 \text{ m}^2$$

$$\sigma_{\dot{xx}}^2 = \sigma_{\dot{yy}}^2 = \sigma_{\dot{zz}}^2 = 10 \frac{\text{m}^2}{\text{s}^2}$$

$$\sigma_{\dot{\lambda}\dot{\lambda}}^2 = .1 \frac{\text{m}^2}{\text{s}^4}$$

$$\sigma_{\dot{mm}}^2 = 2.493241 \times 10^{-6} \frac{1}{\text{s}^2}$$

The pursuer's initial conditions for all cases are

$$x_p = -359899.441 \quad \text{m}$$

$$\dot{x}_p = 11991.950 \quad \frac{\text{m}}{\text{s}}$$

$$y_p = 6727335.870 \quad \text{m}$$

$$\dot{y}_p = 158.764 \quad \frac{m}{s}$$

$$z_p = 0.0 \quad m$$

$$\dot{z}_p = 0.0 \quad \frac{m}{s}$$

with a lateral acceleration range of 3-60  $m/s^2$  in each axis.

The booster's characteristics are modeled as

$$A_o = 3.15788 \quad \frac{m}{s^2}$$

$$\dot{m}_o = .01579 \quad \frac{1}{s}$$

with time-to-go equaling 30 seconds.

A time lag of one tenth second is used for all algorithms when computing velocity changes. It is unrealistic to assume the filter can process measurements, the controller determine thrust commands, and the thrusters respond to those commands all instantaneously. One cycle time is chosen to allow the velocity changes computed in the previous cycle to be implemented in the present cycle. The controller routines are built to take this lag into account. Also, thrusting is not permitted during the first three seconds of an intercept to account for target acquisition.



The following page shows the evader initial condition for six cases. Case I represents a head on, in-plane intercept. Case II represents a head on,  $10^\circ$  out-of-plane intercept. Case III represents a head on,  $20^\circ$  out-of-plane intercept. Case IV represents an in-plane tail chase. Case V represents a  $10^\circ$  out-of-plane tail chase. Case VI represents a  $20^\circ$  out-of-plane tail chase.

Table 11-1. Evader Initial Conditions

|                   | CASE I      | CASE II     | CASE III    | CASE IV     | CASE V      | CASE VI     |
|-------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $x_E$ (m)         | 205720.173  | 202600.41   | 193342.193  | -205720.173 | -202600.401 | -193342.193 |
| $\dot{x}_E$ (m/s) | -6799.072   | -6695.912   | -6390.106   | 6805.072    | 6701.912    | 6396.106    |
| $y_E$ (m)         | 6638526.297 | 6639834.472 | 6639829.206 | 6638526.297 | 6639834.427 | 6639829.206 |
| $\dot{y}_E$ (m/s) | 3101.494    | 3058.185    | 3058.501    | 3101.494    | 3058.185    | 3058.501    |
| $z_E$ (m)         | 0.0         | -38683.931  | -76213.728  | 0.0         | -38683.931  | -76213.728  |
| $\dot{z}_E$ (m/s) | 3.0         | 1282.071    | 2523.104    | 3.0         | 1282.71     | 2523.104    |

## CHAPTER XII

## RESULTS

A history of the miss and velocity changes with respect to multipliers is generated for those algorithms requiring multipliers (see Figures 12-1 through 12-24). A specific multiplier is then chosen for each plan for inclusion in the tables that follow.

One hundred Monte Carlo runs are generated per plan per case. The mean of each set of runs is adequate for judging relative performance. This performance is recorded in the six tables that follow the figures.

Appendix E contains the in-plane thrust profiles for Cases I and V of all plans. In the appendix, each profile uses the same random seed for startup to show the effect of estimate uncertainty on the various control strategies.

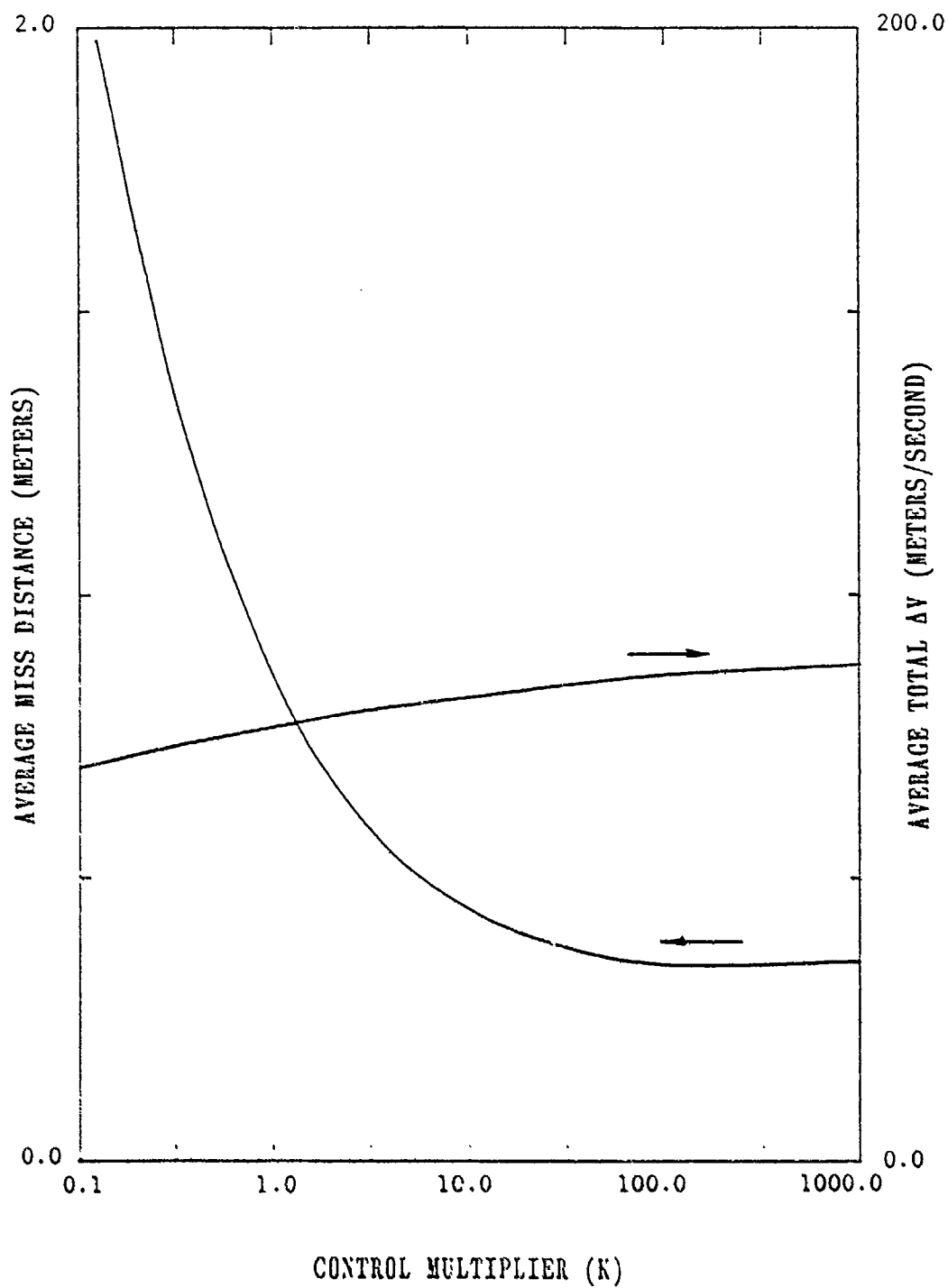


Figure 12-1. Performance of Plan A for Case I.

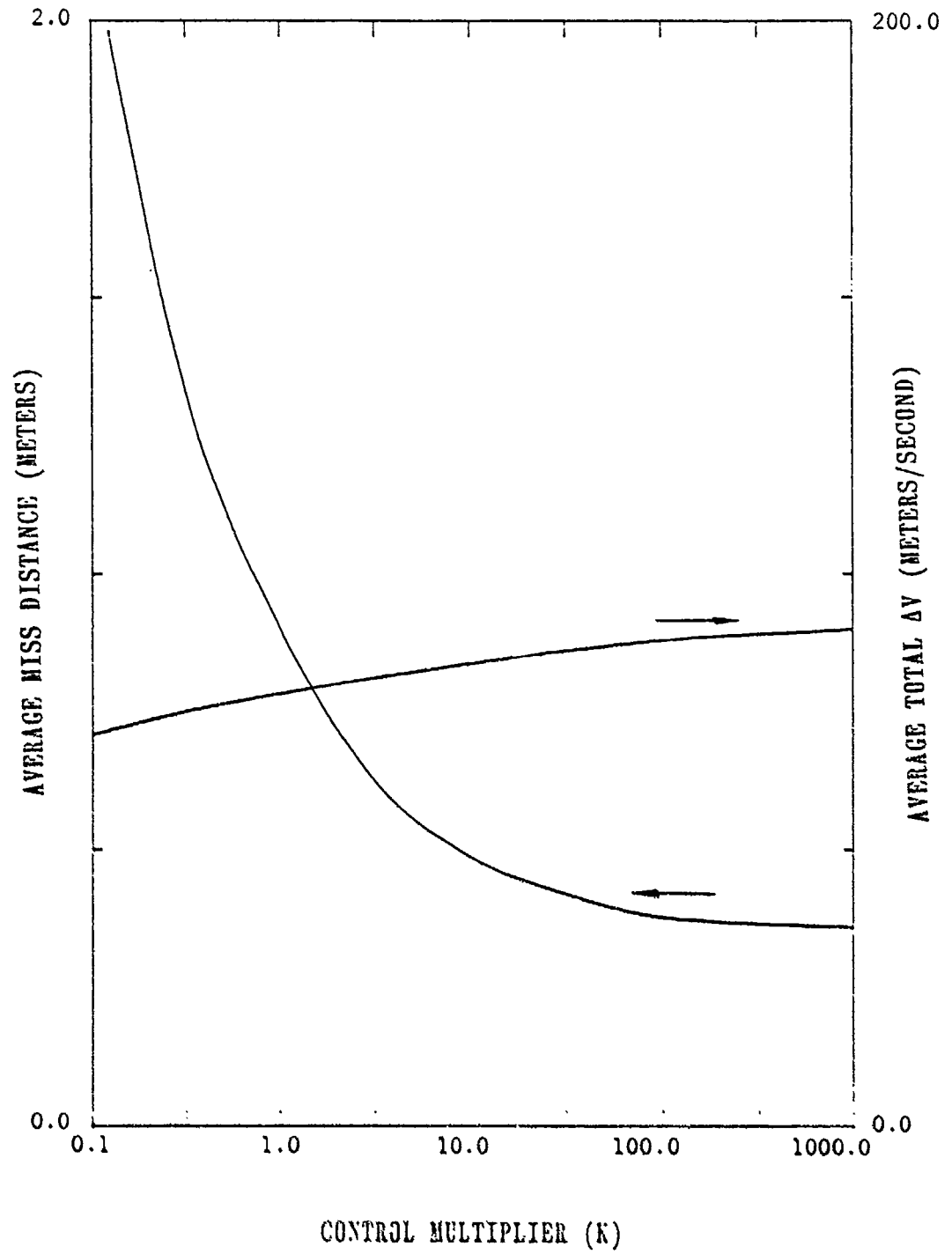


Figure 12-2. Performance of Plan A for Case II.

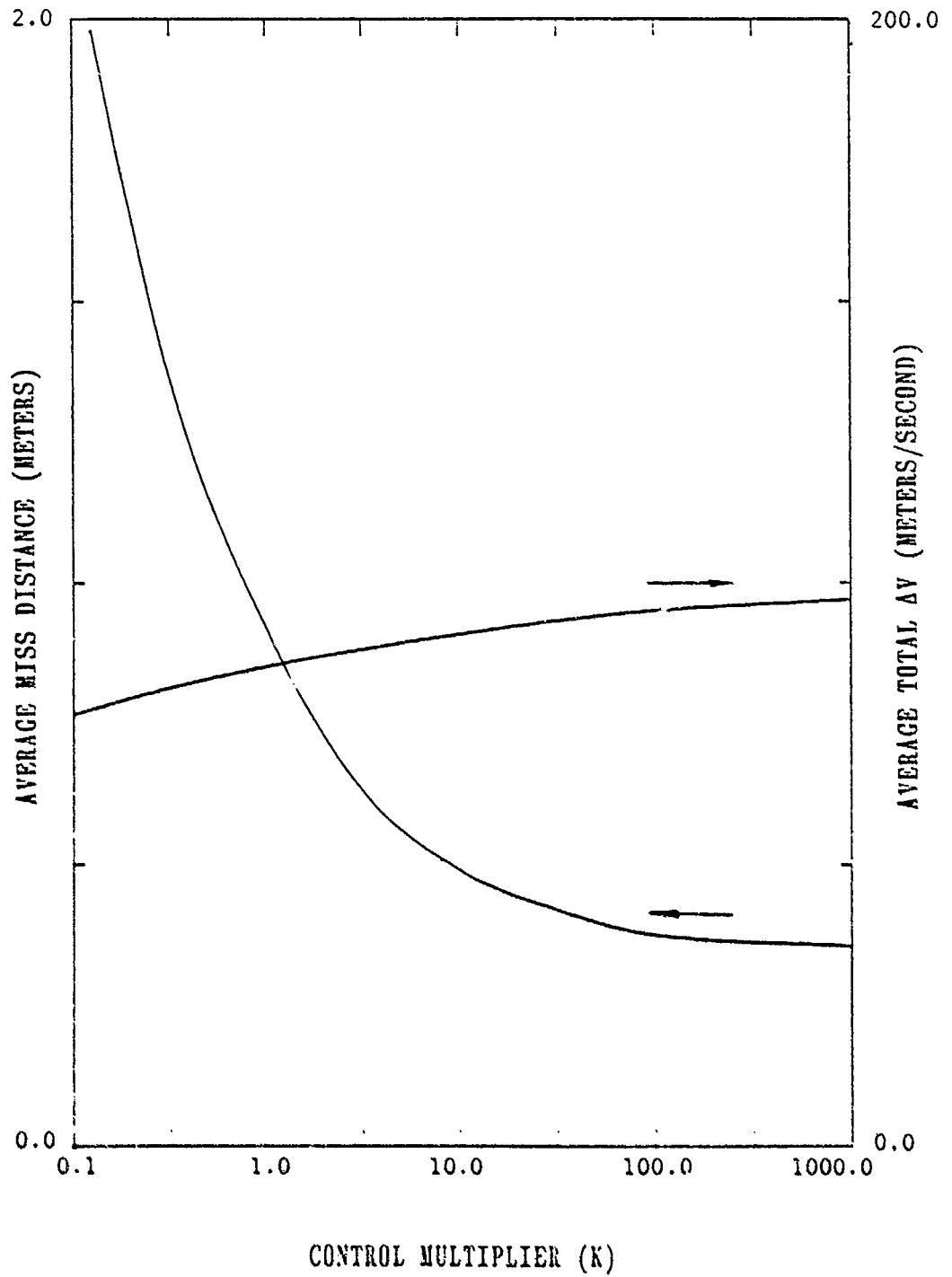


Figure 12-3. Performance of Plan A for Case III.

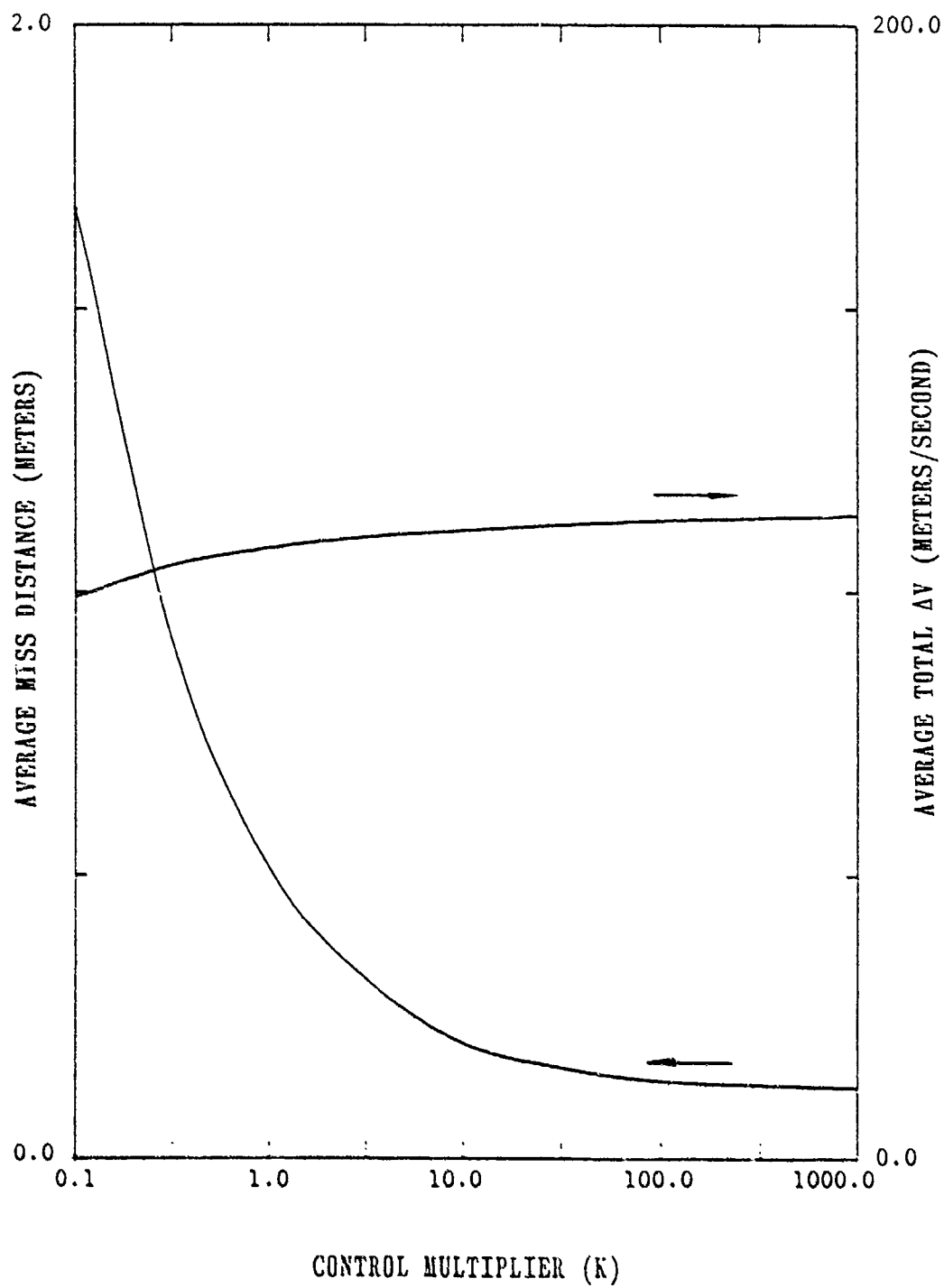


Figure 12-4. Performance of Plan A for Case IV.

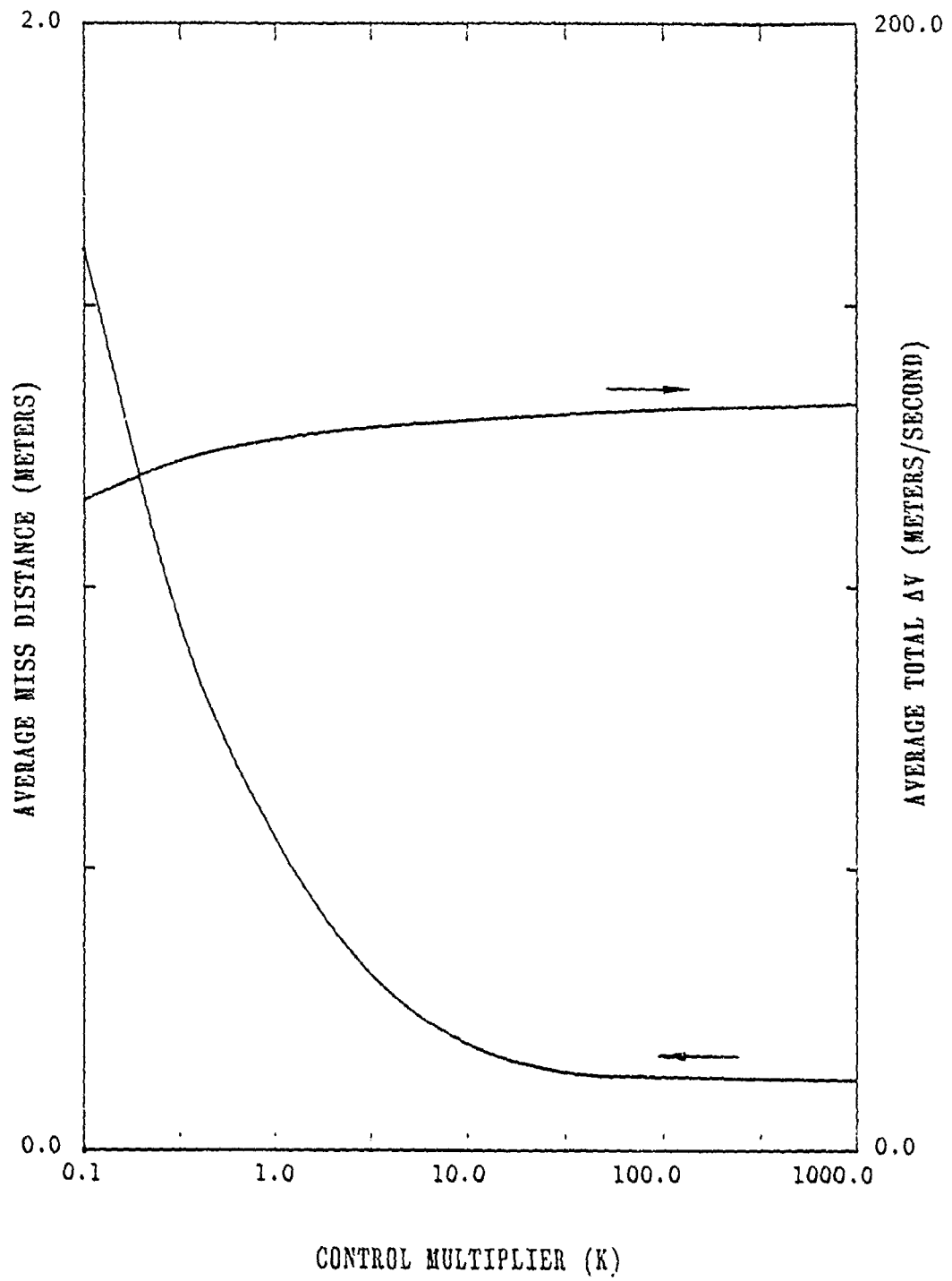


Figure 12-5. Performance of Plan A for Case V.



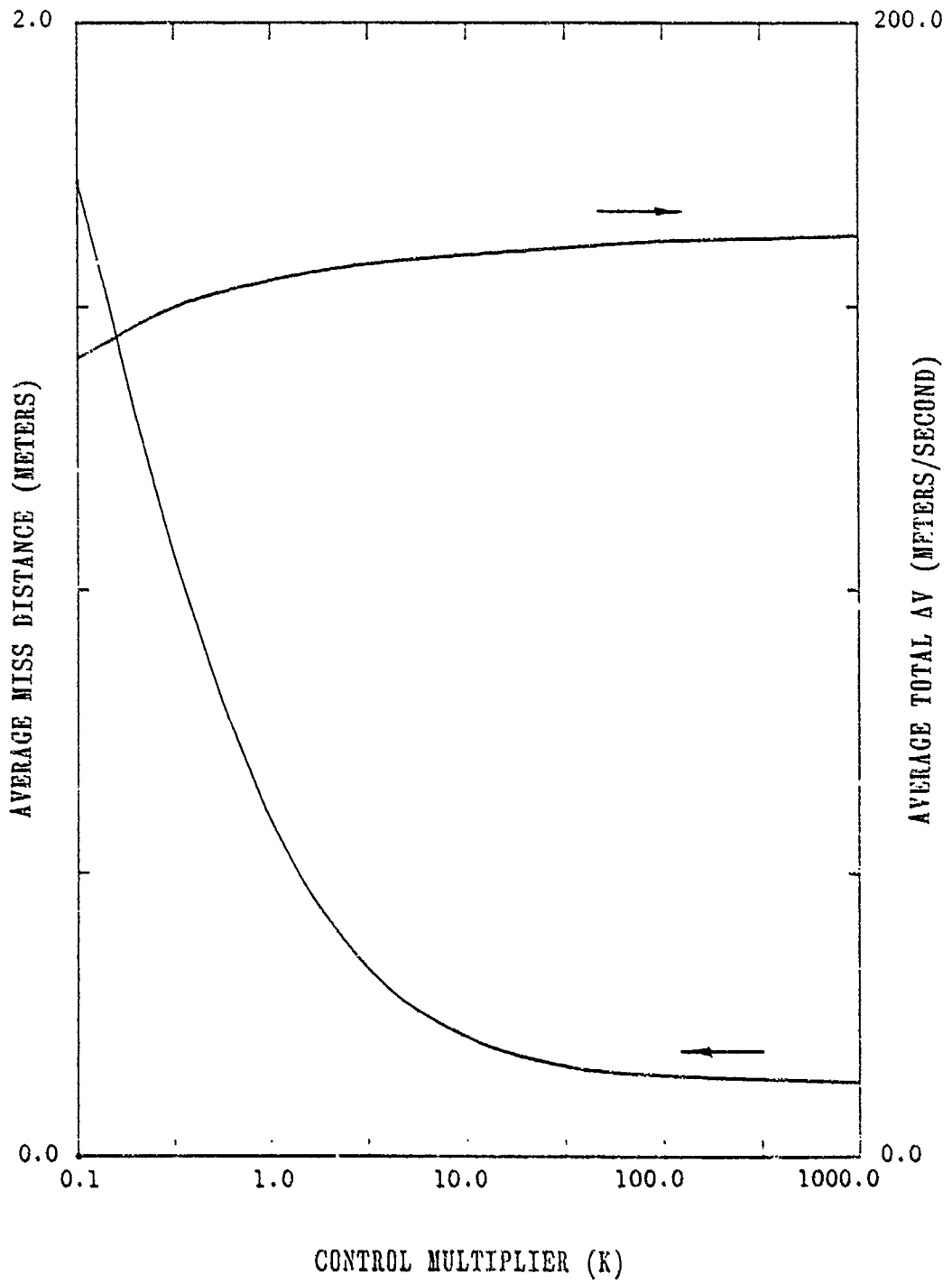


Figure 12-6. Performance of Plan A for Case VI.

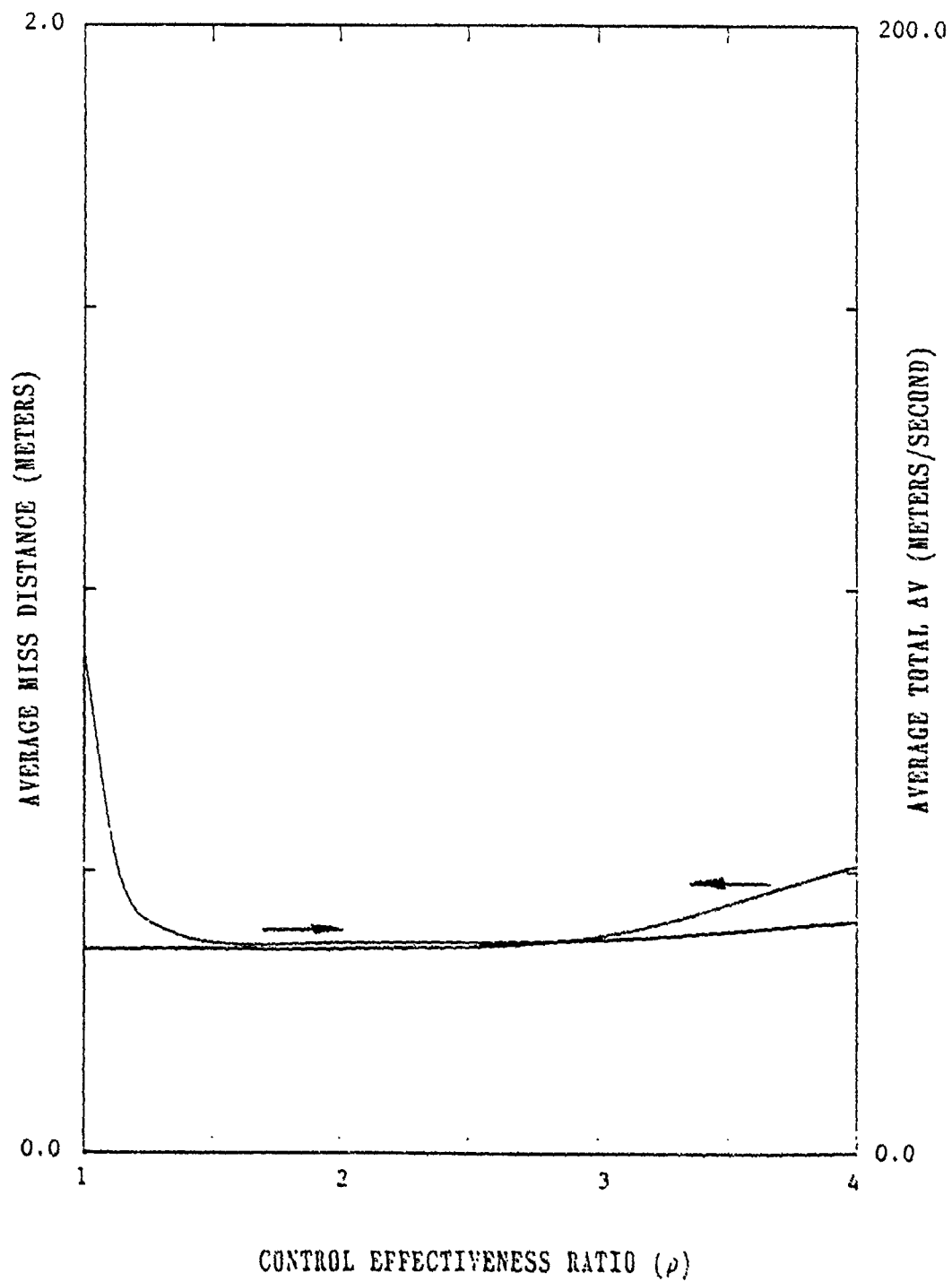


Figure 12-7. Performance of Optimum Thrust Spacing for Case I.

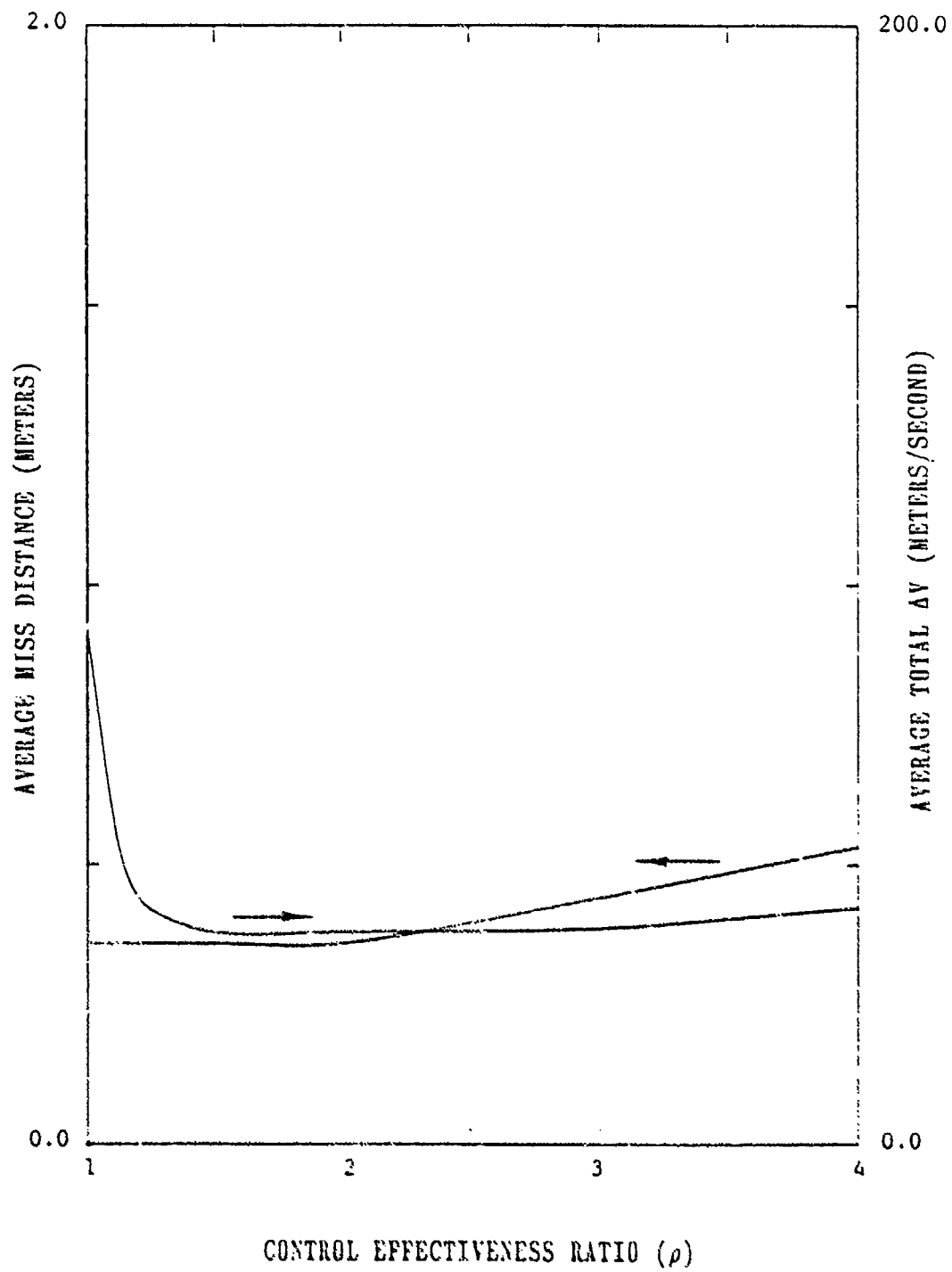


Figure 12-8. Performance of Optimum Thrust Spacing for Case II.

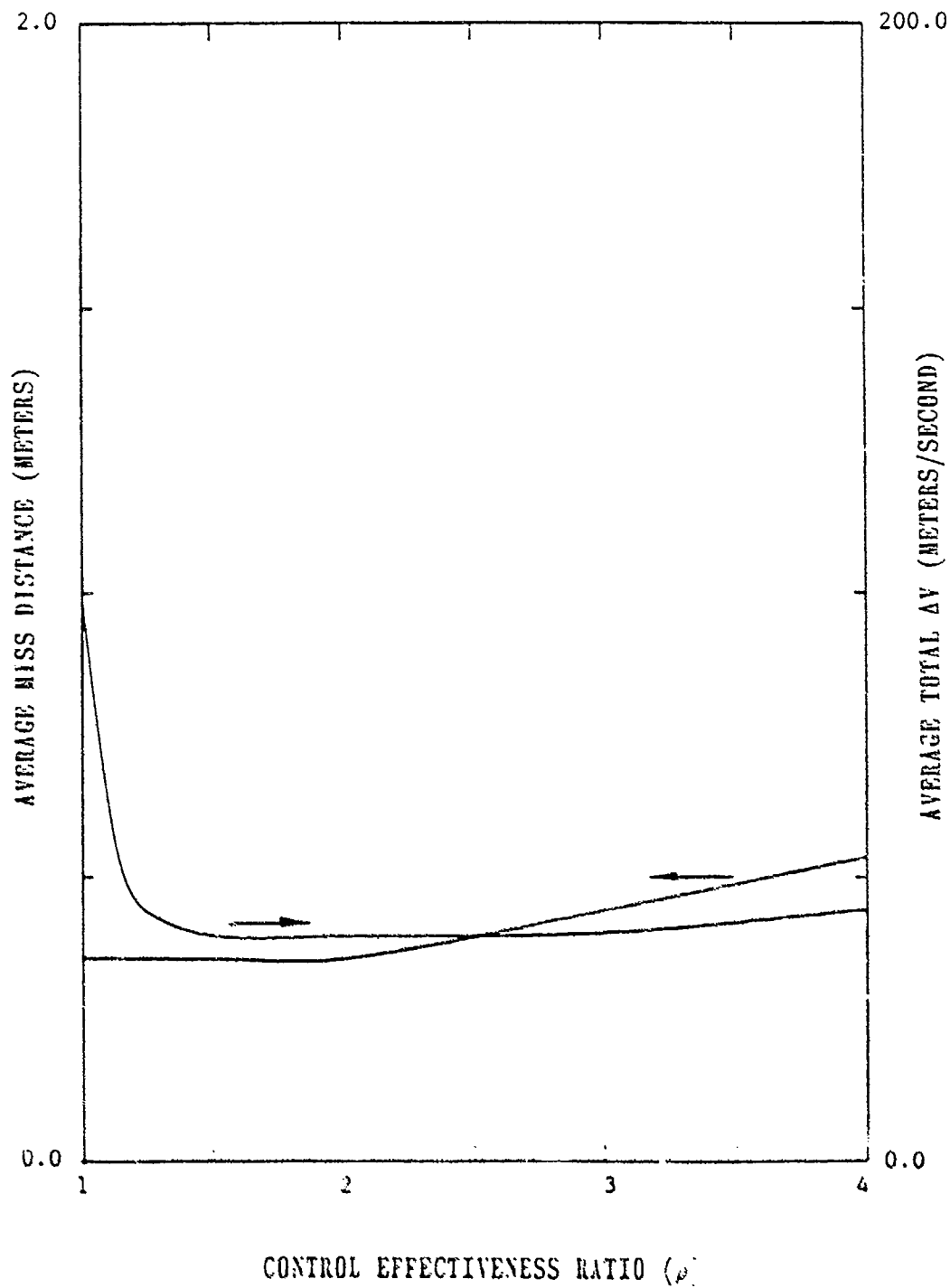


Figure 12-9. Performance of Optimum Thrust Spacing for Case III.

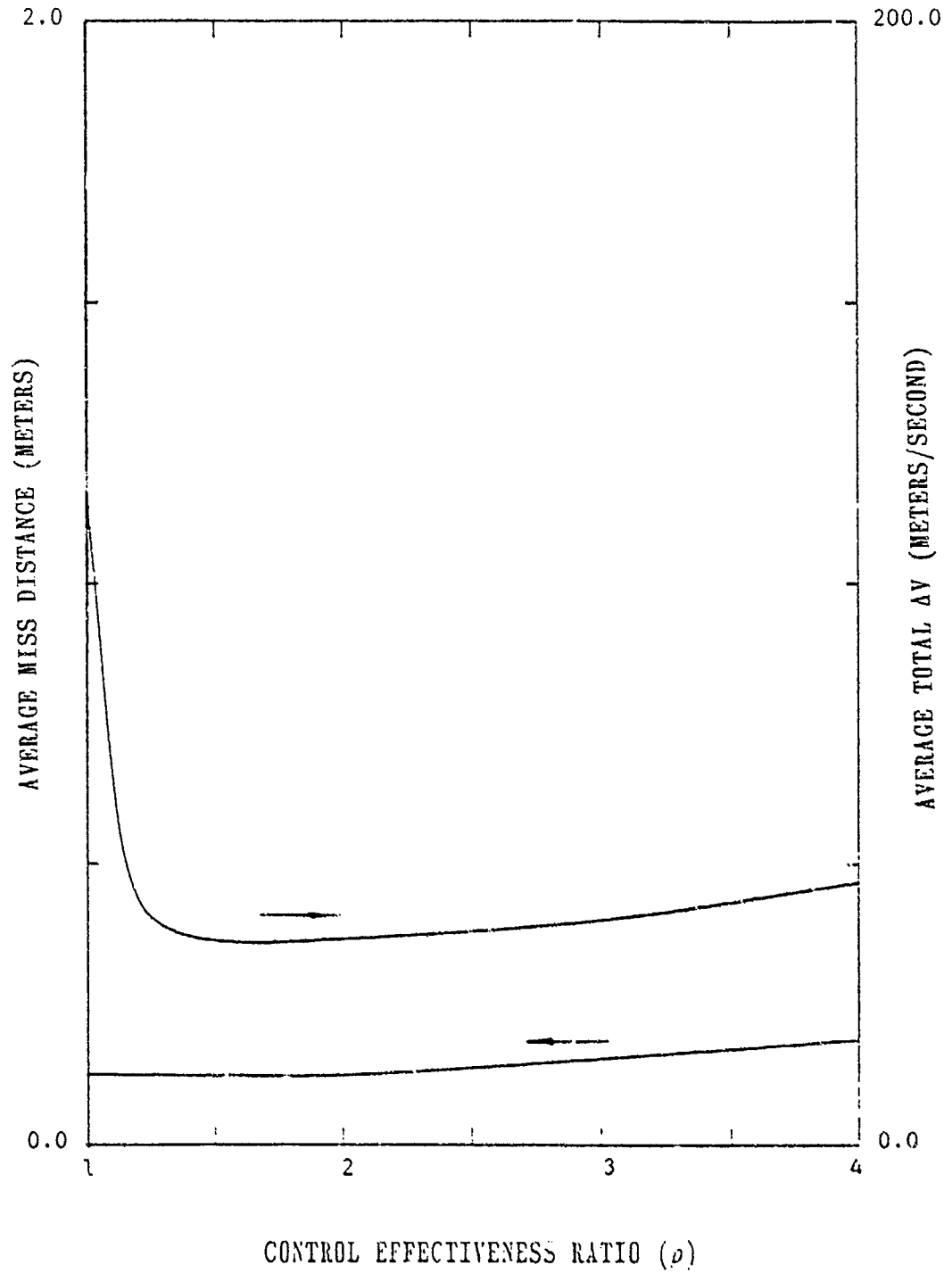


Figure 12-10. Performance of Optimum Thrust Spacing for Case IV.

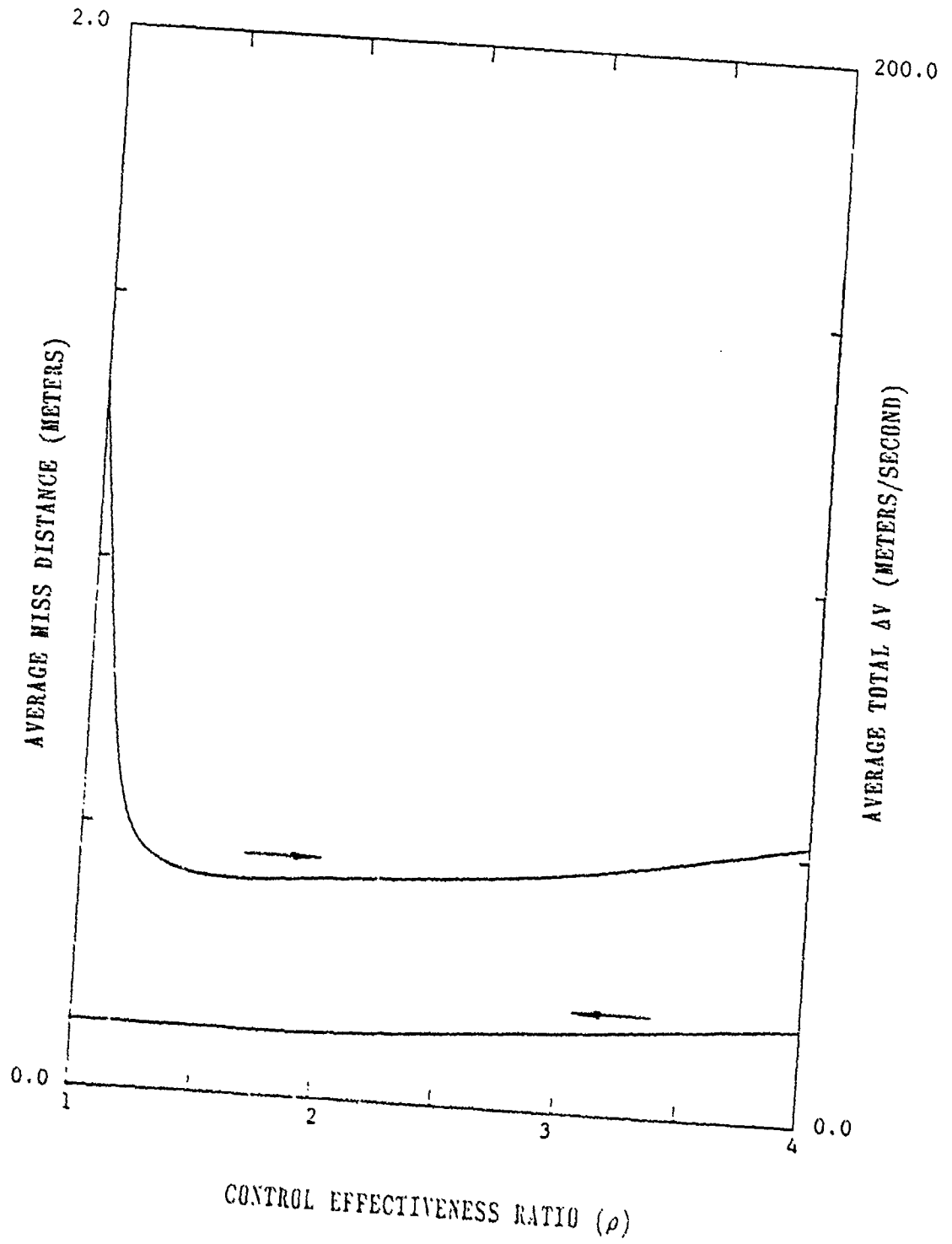


Figure 12-11. Performance of Optimum Thrust Spacing for Case V.

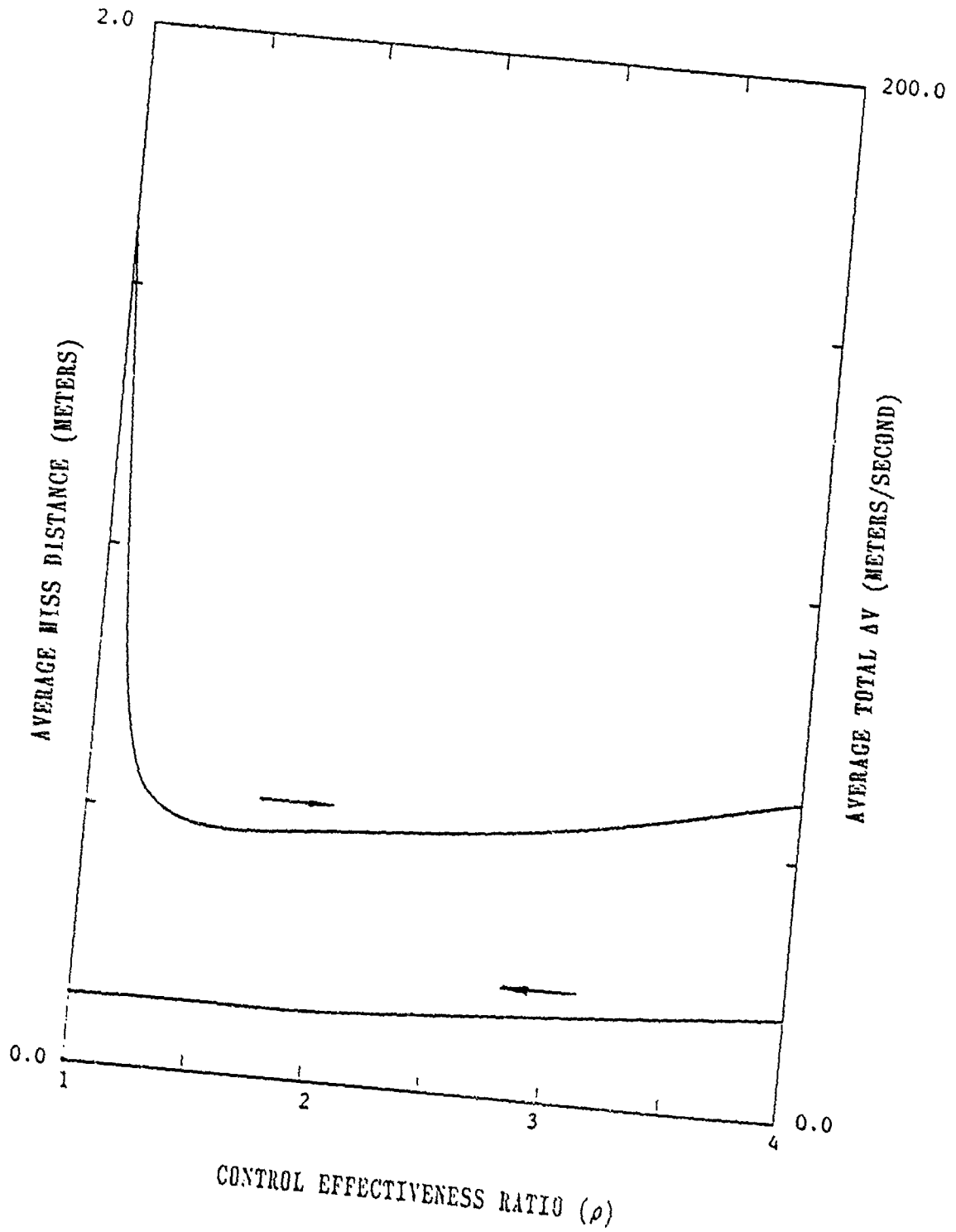


Figure 12-12. Performance of Optimum Thrust Spacing for Case VI.

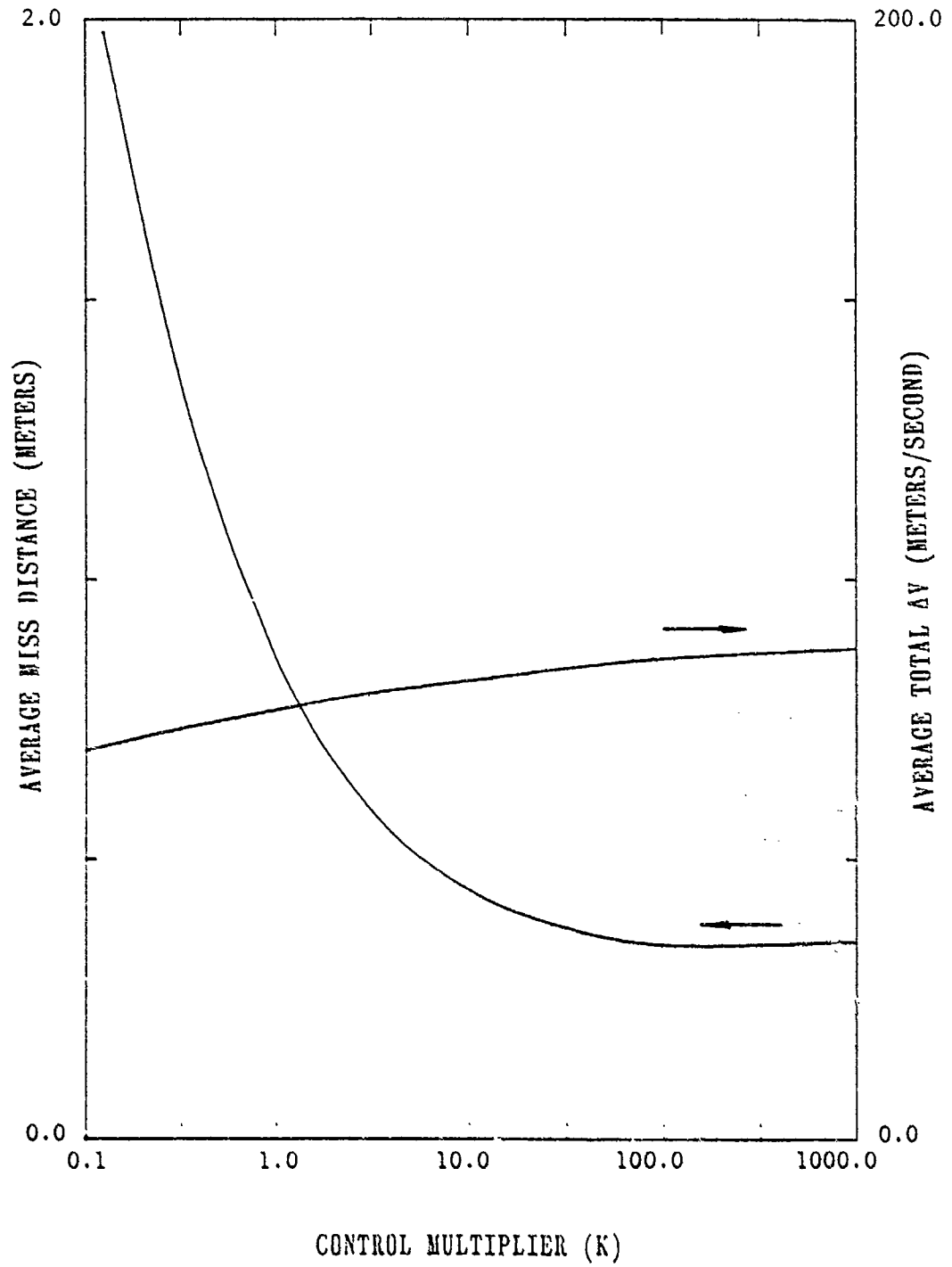


Figure 12-13. Performance of Dual Control for Case I.



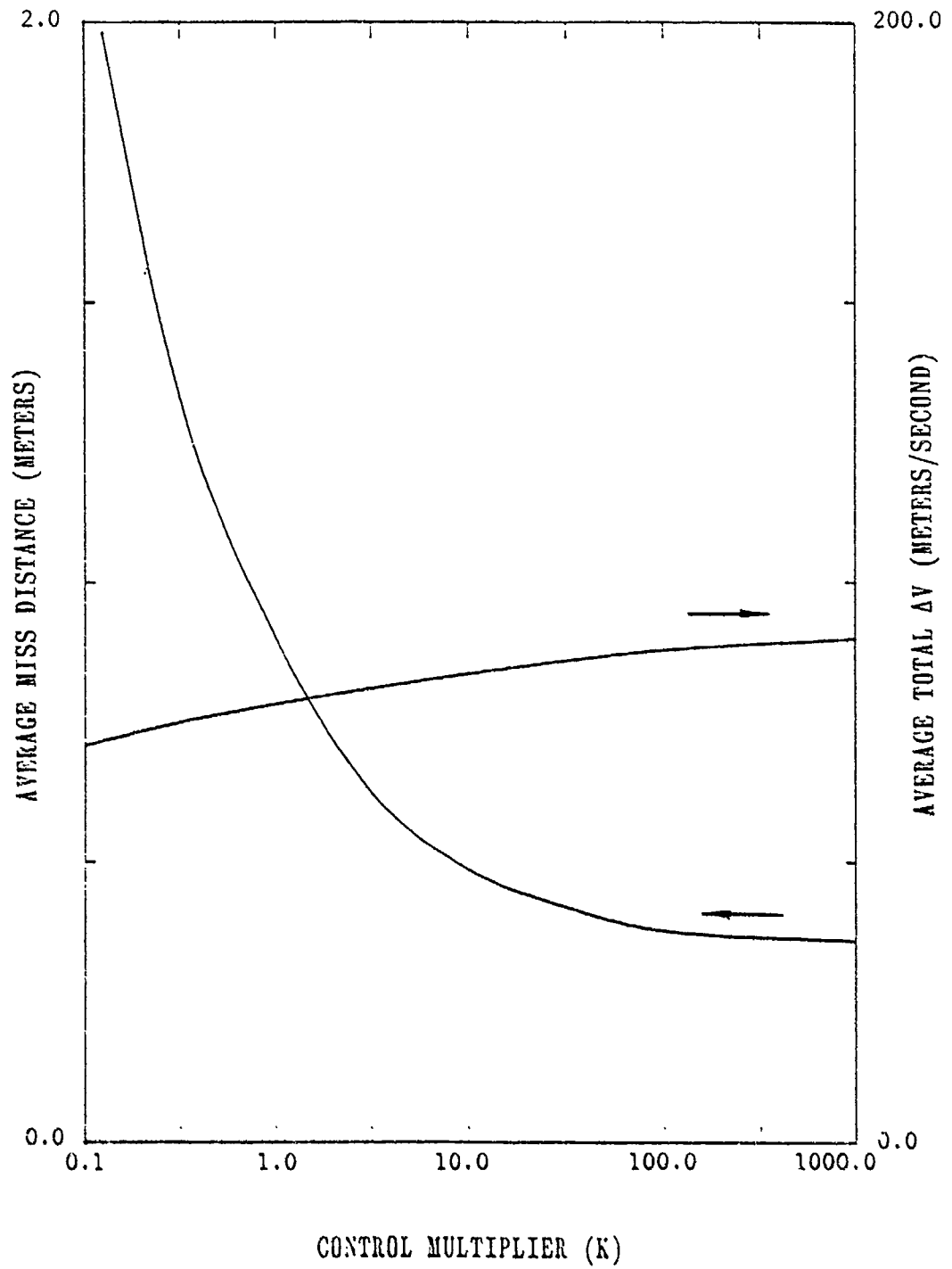


Figure 12-14. Performance of Dual Control for Case II.

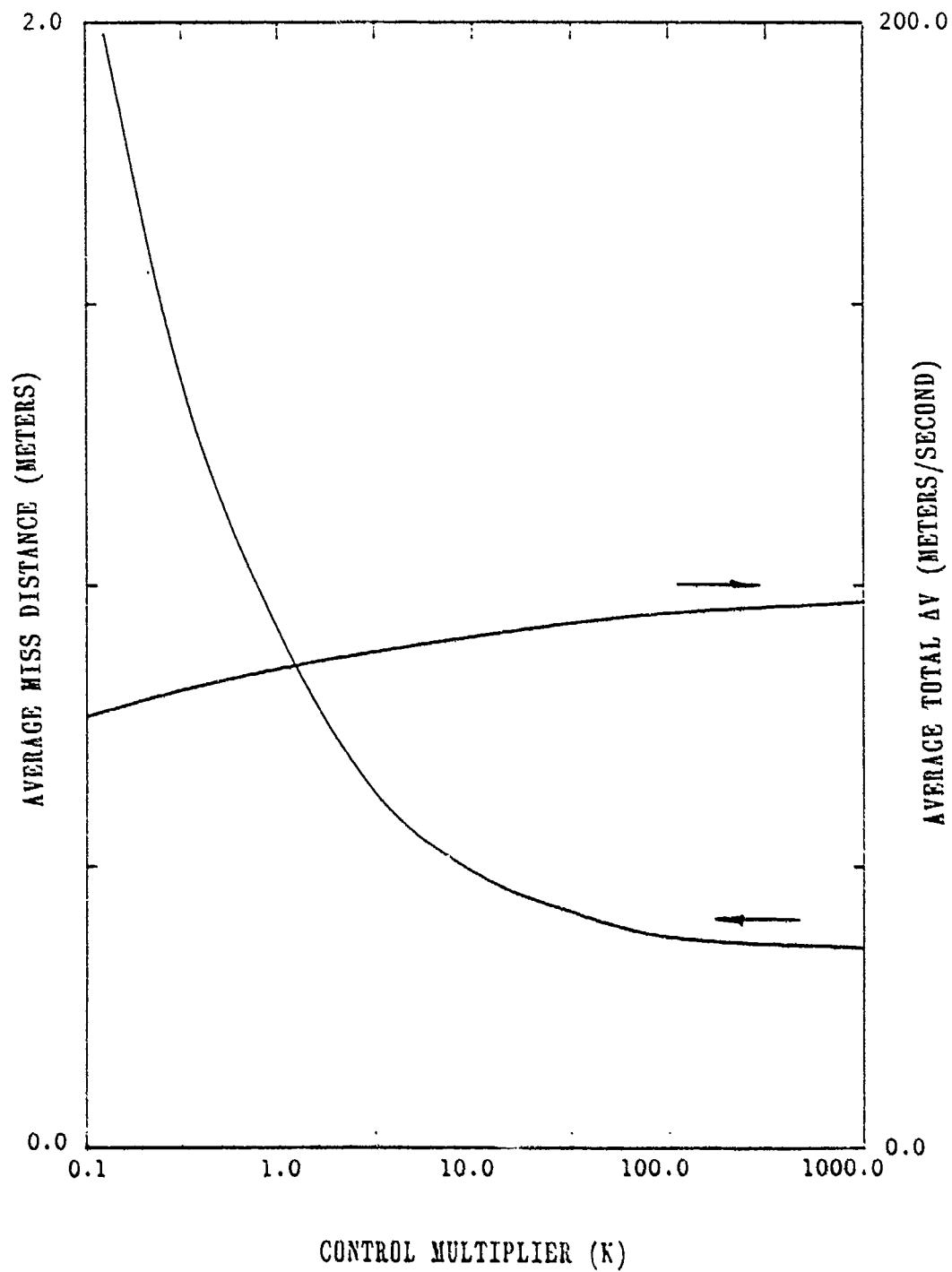


Figure 12-15. Performance of Dual Control for Case III.

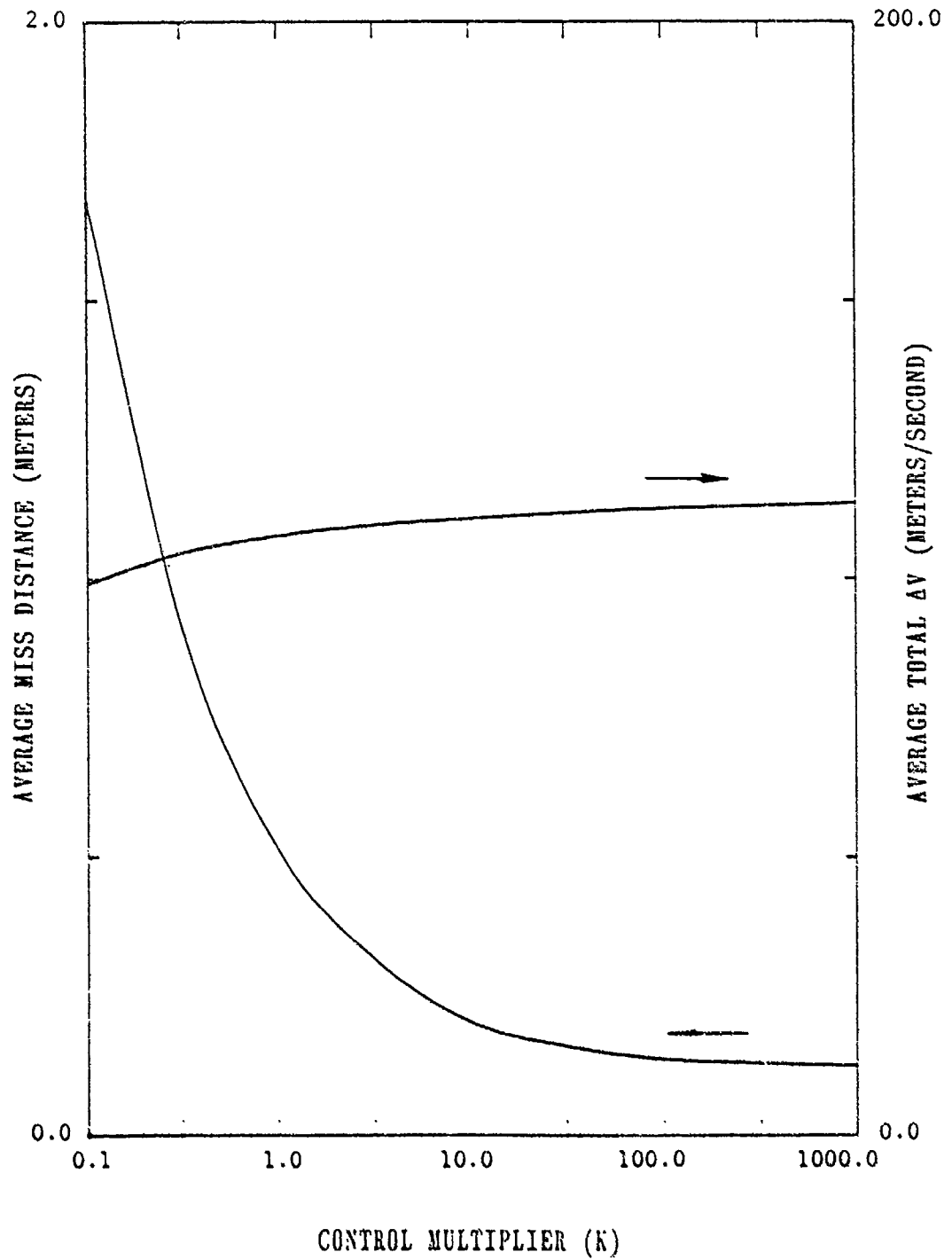


Figure 12-16. Performance of Dual Control for Case IV.

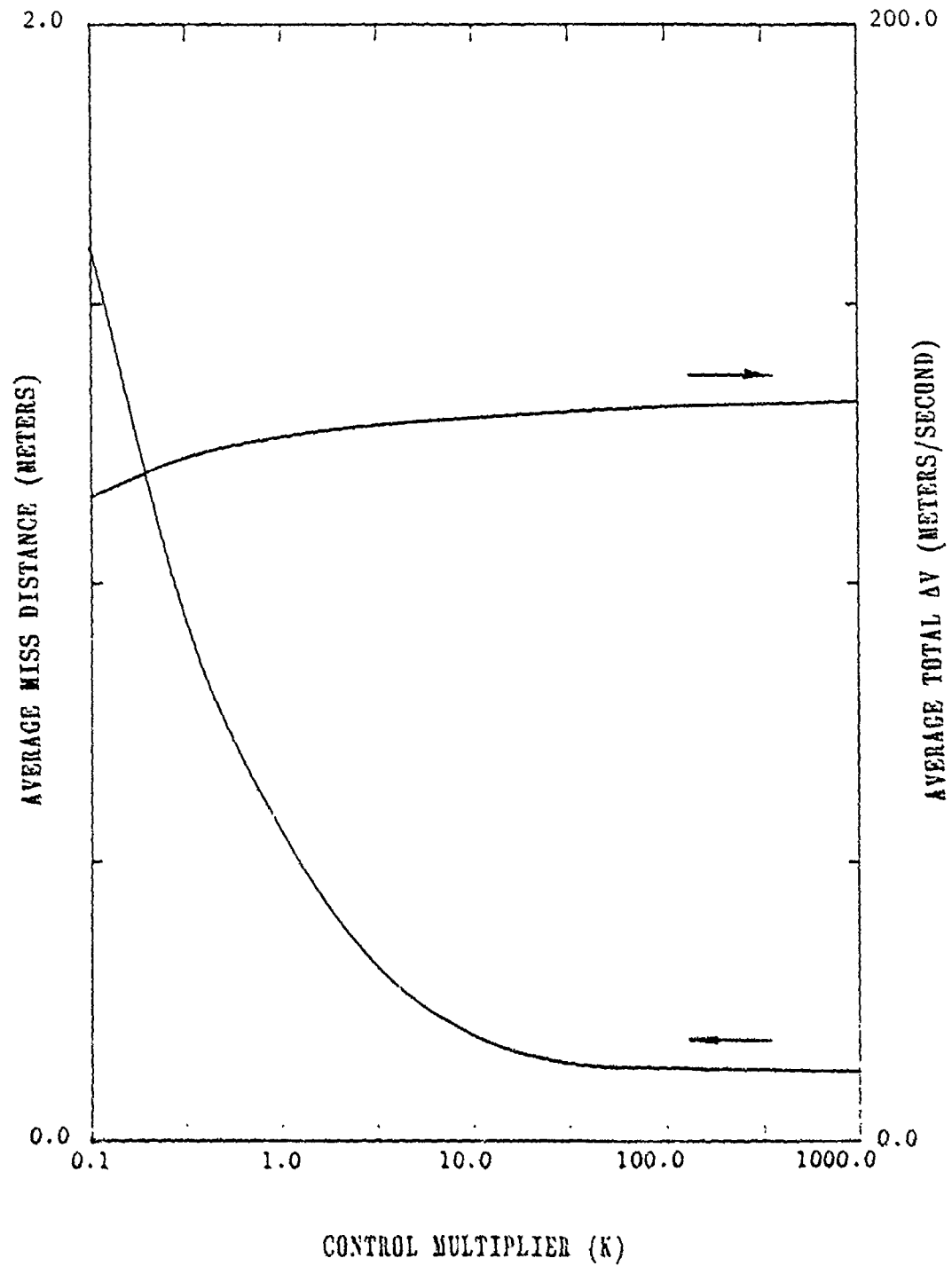


Figure 12-17. Performance of Dual Control for Case V.

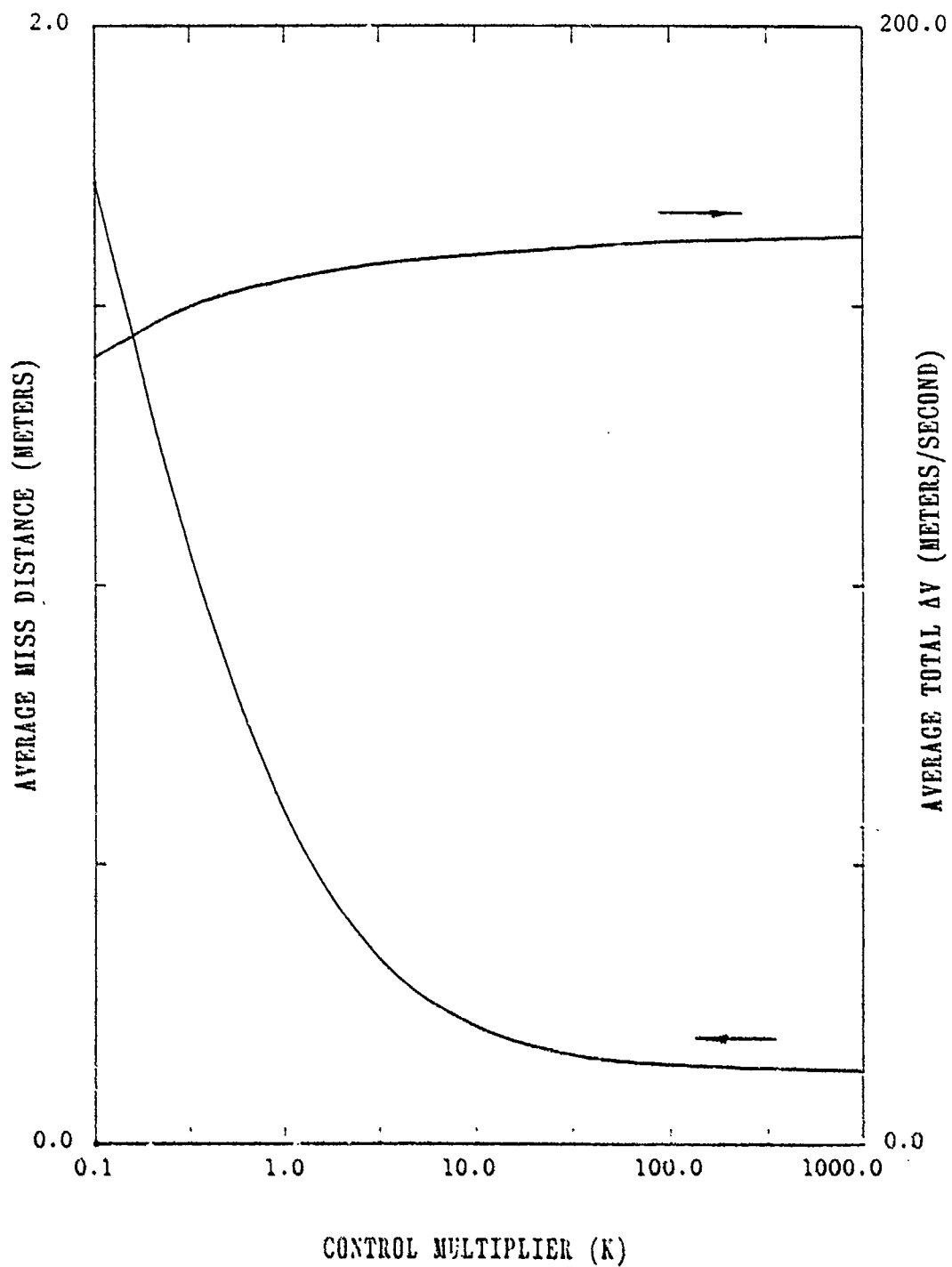


Figure 12-18. Performance of Dual Control for Case VI.

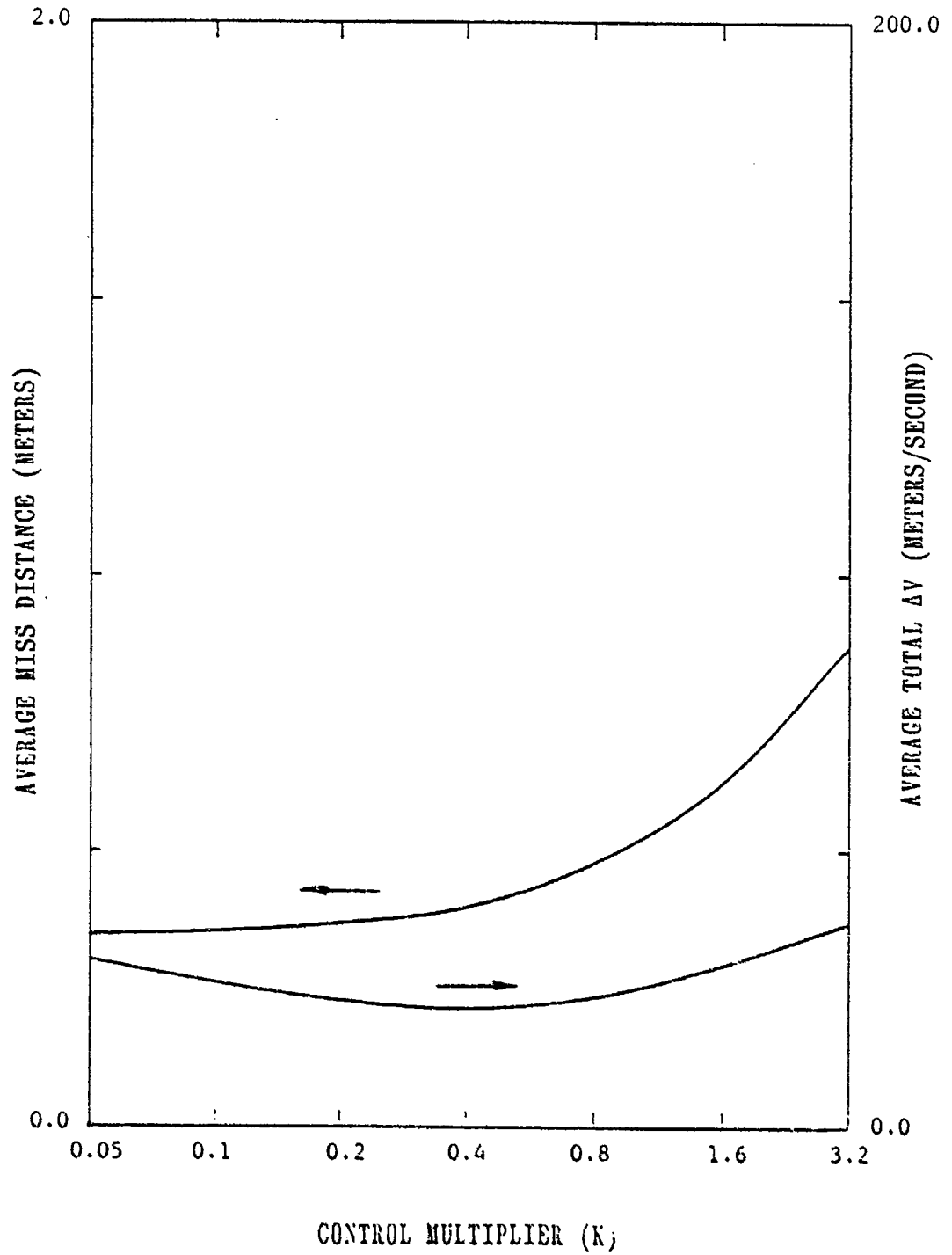


Figure 12-19. Performance of Certainty Control for Case I.

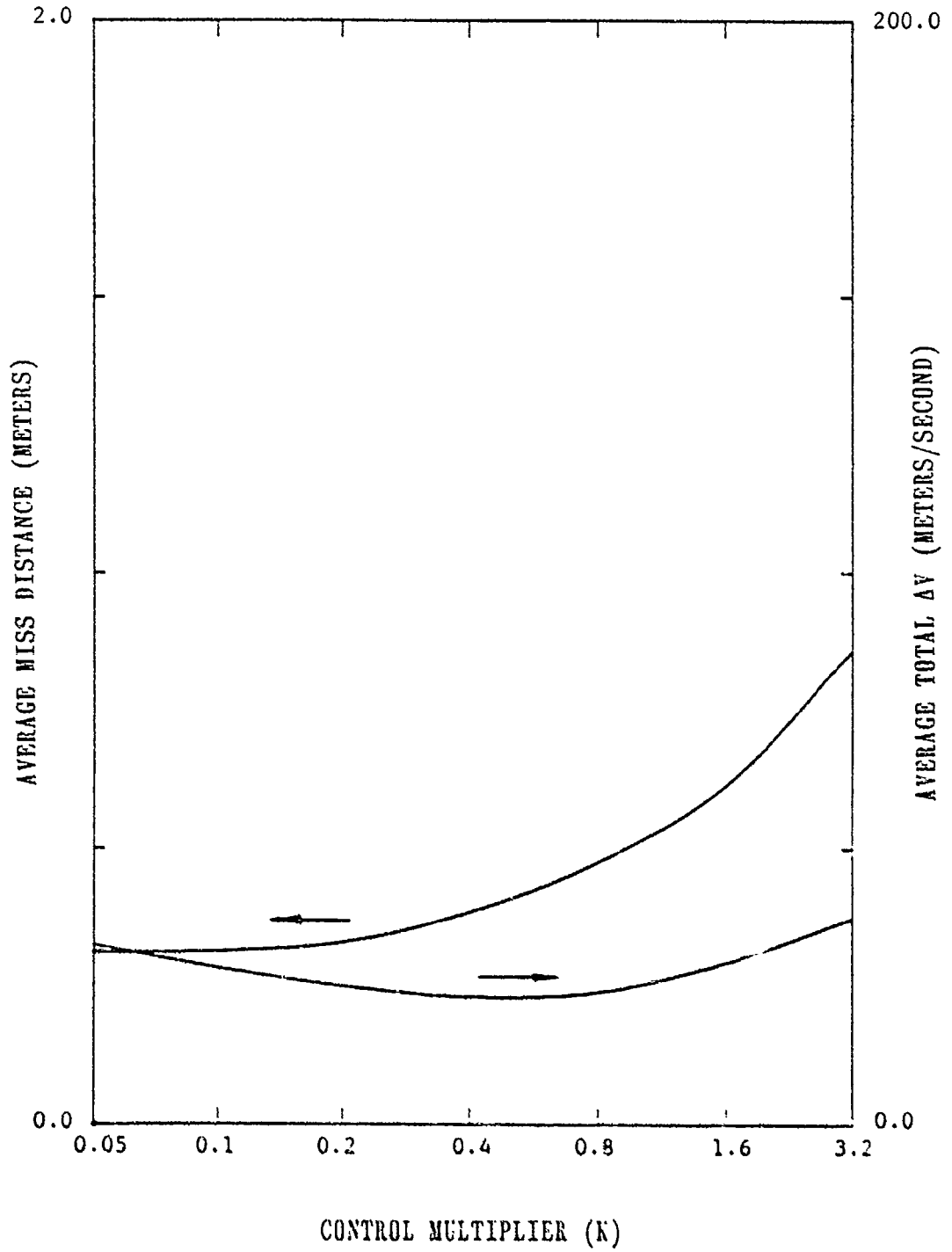


Figure 12-20. Performance of Certainty Control for Case II.

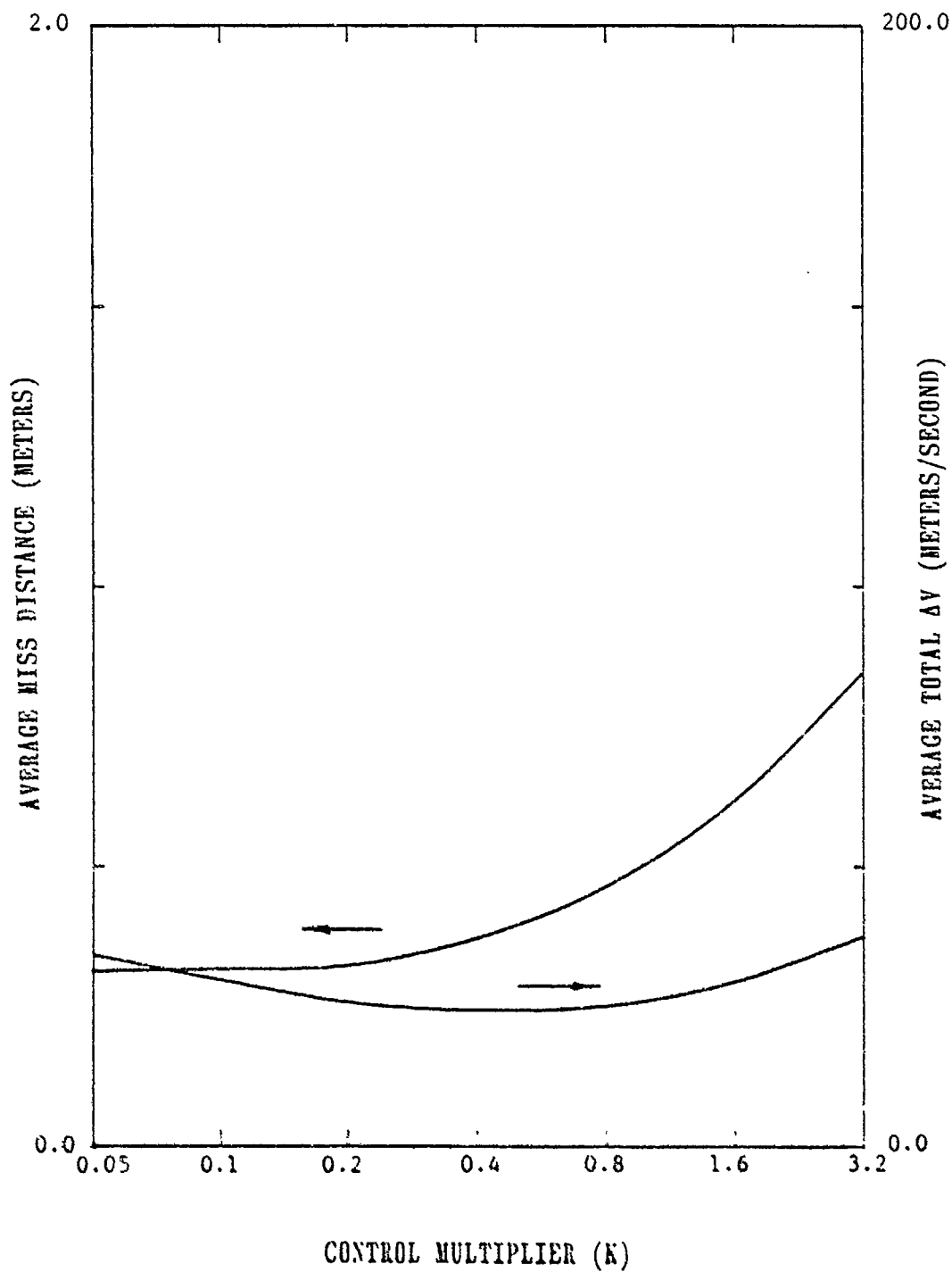


Figure 12-21. Performance of Certainty Control for Case III.



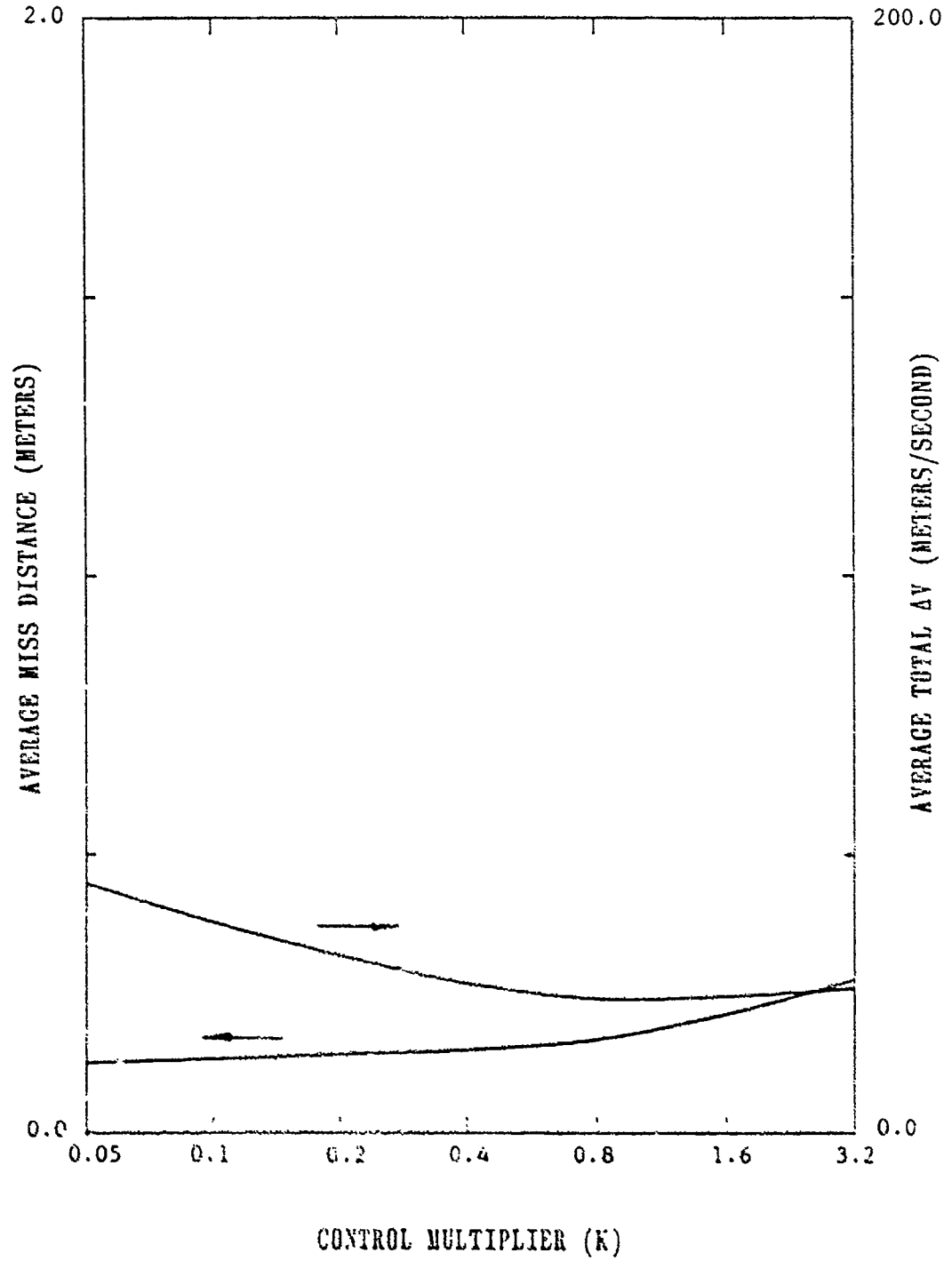


Figure 12-22. Performance of Certainty Control for Case IV.

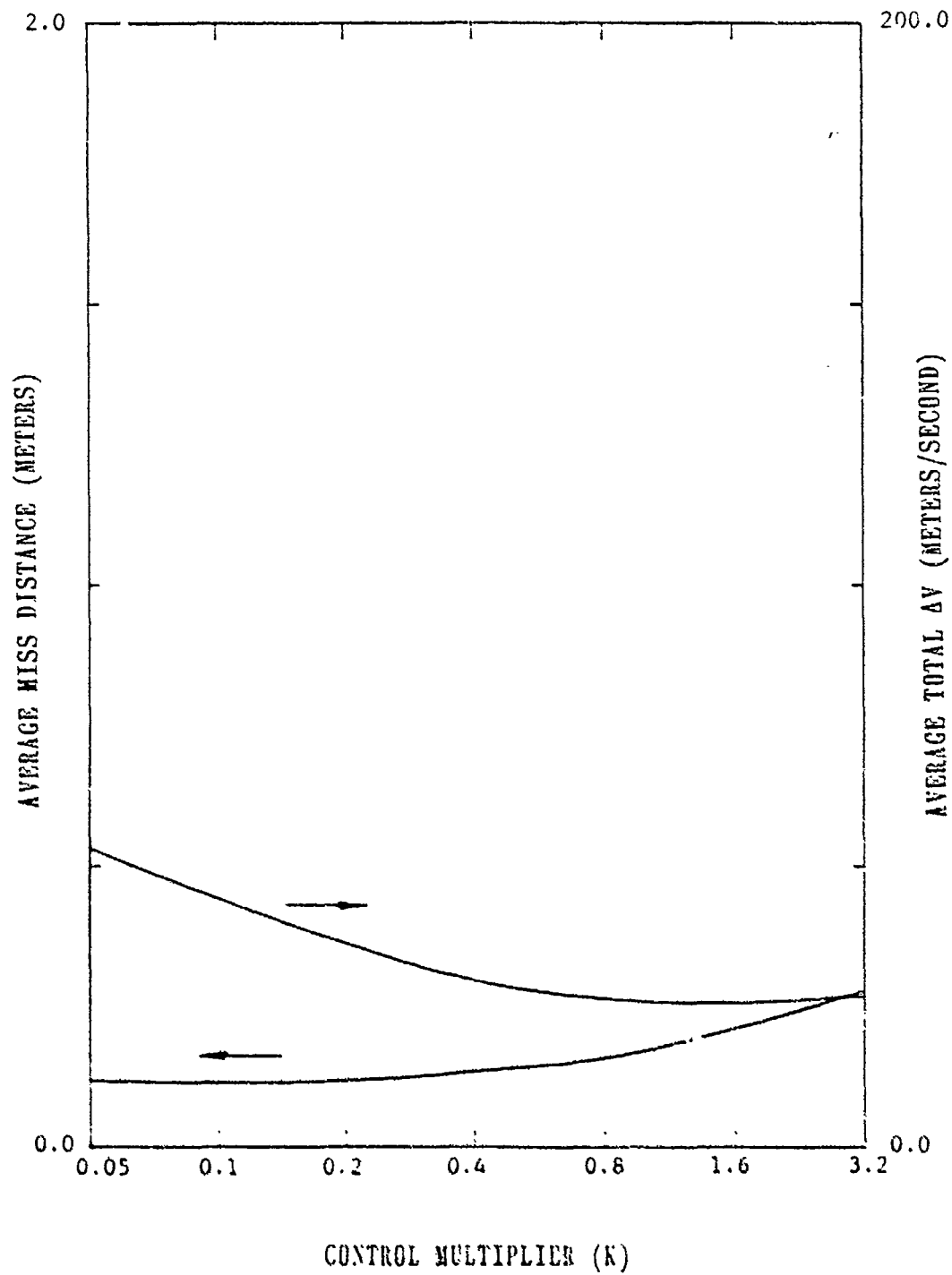


Figure 12-23. Performance of Certainty Control for Case V.

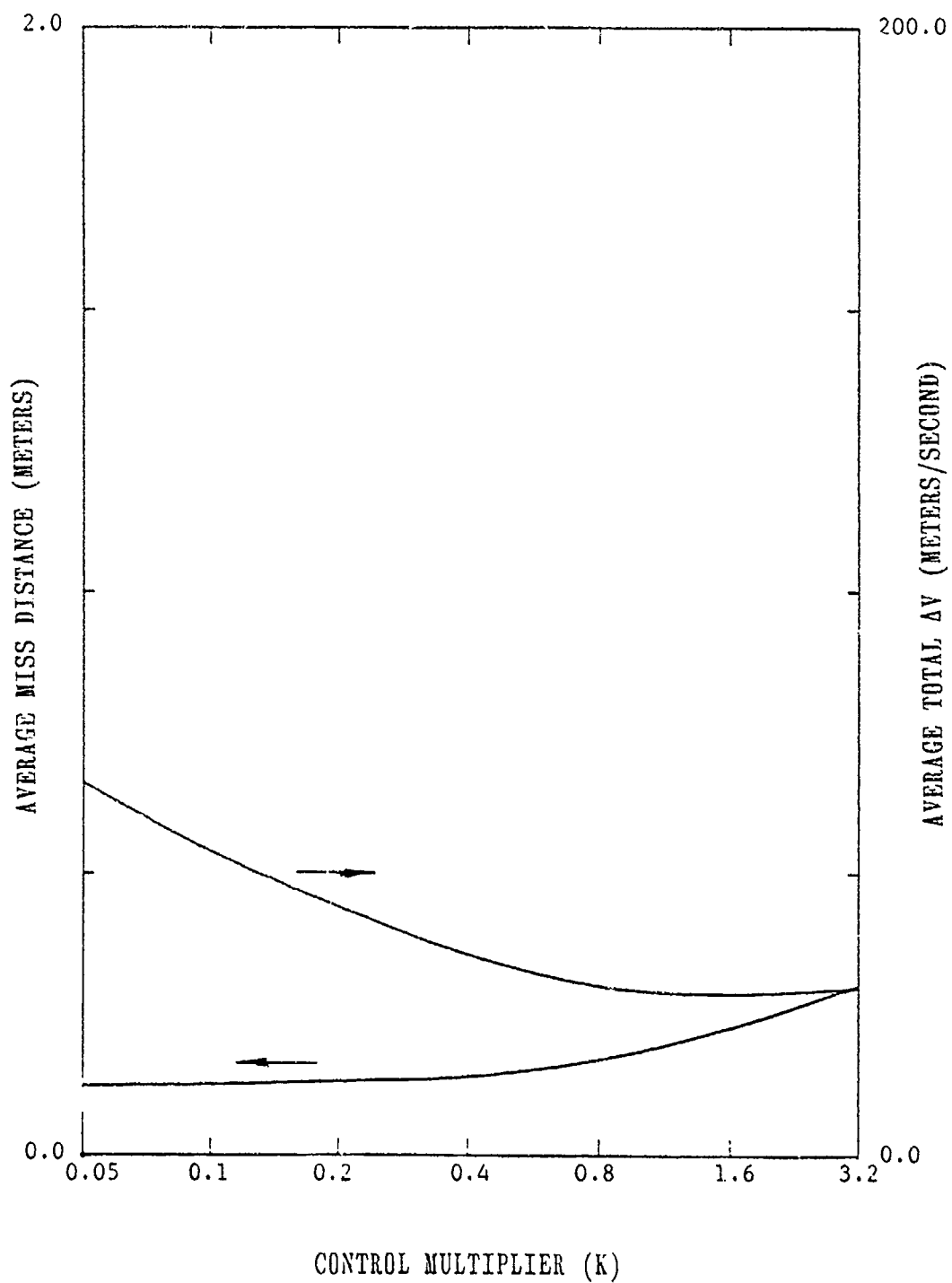


Figure 12-24. Performance of Certainty Control for Case VI.

Table 12-1. Case I Performance.  
(Head On, In-plane Intercept)

|   | MEAN<br>MISS<br>(METERS) | STANDARD<br>DEVIATION<br>(METERS) | MEAN<br>$\Delta V$<br>(M/S) | STANDARD<br>DEVIATION<br>(M/S) |
|---|--------------------------|-----------------------------------|-----------------------------|--------------------------------|
| PLAN A<br>(K=10)                                | .465                     | .230                              | 81.93                       | 5.64                           |
| PLAN B  | .362                     | .173                              | 88.29                       | 7.00                           |
| PLAN C  | .362                     | .174                              | 90.76                       | 6.86                           |
| OPTIMUM<br>THRUST<br>SPACING<br>( $\rho=1.75$ ) | .363                     | .175                              | 36.63                       | 8.14                           |
| DUAL<br>CONTROL<br>(K=10)                       | .465                     | .230                              | 81.93                       | 5.64                           |
| CERTAINTY<br>CONTROL<br>(K=.4)                  | .399                     | .215                              | 21.61                       | 3.79                           |
| TRUTH<br>WITH<br>NOISE                          | .527                     | .264                              | 81.44                       | 6.76                           |
| TRUTH<br>WITHOUT<br>NOISE                       | 0                        | NA                                | 7.31                        | NA                             |

Table 12-2. Case II Performance.  
 (Head On,  $10^\circ$  Out-of-Plane Intercept)

|   | MEAN<br>MISS<br>(METERS) | STANDARD<br>DEVIATION<br>(METERS) | MEAN<br>$\Delta V$<br>(M/S) | STANDARD<br>DEVIATION<br>(M/S) |
|---|--------------------------|-----------------------------------|-----------------------------|--------------------------------|
| PLAN A<br>(K=10)                                | .502                     | .224                              | 83.82                       | 6.99                           |
| PLAN B  | .360                     | .171                              | 90.39                       | 7.24                           |
| PLAN C  | .360                     | .171                              | 93.07                       | 7.39                           |
| OPTIMUM<br>THRUST<br>SPACING<br>( $\rho=1.75$ ) | .361                     | .171                              | 37.19                       | 8.50                           |
| DUAL<br>CONTROL<br>(K=10)                       | .502                     | .224                              | 83.82                       | 6.99                           |
| CERTAINTY<br>CONTROL<br>(K=.4)                  | .386                     | .191                              | 23.21                       | 4.18                           |
| TRUTH<br>WITH<br>NOISE                          | .545                     | .264                              | 83.69                       | 7.18                           |
| TRUTH<br>WITHOUT<br>NOISE                       | 0                        | NA                                | 7.54                        | NA                             |

Tabel 12-3. Case III Performance.  
 (Head On, 20° Out-of-Plane Intercept)

|   | MEAN<br>MISS<br>(METERS) | STANDARD<br>DEVIATION<br>(METERS) | MEAN<br>$\Delta V$<br>(M/S) | STANDARD<br>DEVIATION<br>(M/S) |
|---|--------------------------|-----------------------------------|-----------------------------|--------------------------------|
| PLAN A<br>(K=10)                                | .506                     | .225                              | 90.92                       | 7.51                           |
| PLAN B  | .358                     | .168                              | 97.53                       | 7.93                           |
| PLAN C  | .358                     | .168                              | 99.97                       | 7.89                           |
| OPTIMUM<br>THRUST<br>SPACING<br>( $\rho=1.75$ ) | .358                     | .168                              | 39.87                       | 8.63                           |
| DUAL<br>CONTROL<br>(K=10)                       | .506                     | .225                              | 90.92                       | 7.51                           |
| CERTAINTY<br>CONTROL<br>(K=.4)                  | .372                     | .185                              | 24.45                       | 4.50                           |
| TRUTH<br>WITH<br>NOISE                          | .534                     | .293                              | 90.77                       | 7.55                           |
| TRUTH<br>WITHOUT<br>NOISE                       | 0                        | NA                                | 7.78                        | NA                             |

Table 12-4. Case IV Performance.  
(In-Plane Tail Chase)

|   | MEAN<br>MISS<br>(METERS) | STANDARD<br>DEVIATION<br>(METERS) | MEAN<br>$\Delta V$<br>(M/S) | STANDARD<br>DEVIATION<br>(M/S) |
|---|--------------------------|-----------------------------------|-----------------------------|--------------------------------|
| PLAN A<br>(K=10)                                | .204                     | .105                              | 110.87                      | 9.89                           |
| PLAN B  | .126                     | .058                              | 113.72                      | 9.81                           |
| PLAN C  | .126                     | .057                              | 111.82                      | 9.25                           |
| OPTIMUM<br>THRUST<br>SPACING<br>( $\rho=1.75$ ) | .124                     | .057                              | 35.06                       | 10.34                          |
| DUAL<br>CONTROL<br>(K=10)                       | .204                     | .105                              | 110.87                      | 9.89                           |
| CERTAINTY<br>CONTROL<br>(K=.4)                  | .150                     | .081                              | 26.94                       | 6.78                           |
| TRUTH<br>WITH<br>NOISE                          | .376                     | .221                              | 105.71                      | 8.93                           |
| TRUTH<br>WITHOUT<br>NOISE                       | 0                        | NA                                | 8.75                        | NA                             |

Table 12-5. Case V Performance.  
 ( $10^\circ$  Out-of-Plane Tail Chase)

|   | MEAN<br>MISS<br>(METERS) | STANDARD<br>DEVIATION<br>(METERS) | MEAN<br>$\Delta V$<br>(M/S) | STANDARD<br>DEVIATION<br>(M/S) |
|---|--------------------------|-----------------------------------|-----------------------------|--------------------------------|
| PLAN A<br>(K=10)                                | .190                     | .100                              | 129.61                      | 13.29                          |
| PLAN B  | .126                     | .061                              | 132.65                      | 13.32                          |
| PLAN C  | .126                     | .061                              | 129.47                      | 12.26                          |
| OPTIMUM<br>THRUST<br>SPACING<br>( $\rho=1.75$ ) | .126                     | .059                              | 39.96                       | 12.85                          |
| DUAL<br>CONTROL<br>(K=10)                       | .190                     | .100                              | 129.61                      | 13.29                          |
| CERTAINTY<br>CONTROL<br>(K=.4)                  | .136                     | .076                              | 29.74                       | 9.10                           |
| TRUTH<br>WITH<br>NOISE                          | .379                     | .204                              | 123.57                      | 11.61                          |
| TRUTH<br>WITHOUT<br>NOISE                       | 0                        | NA                                | 9.52                        | NA                             |



Table 12-6. Case VI Performance.

(20° Out-of-Plane Tail Chase)

|   | MEAN<br>MISS<br>(METERS) | STANDARD<br>DEVIATION<br>(METERS) | MEAN<br>$\Delta V$<br>(M/S) | STANDARD<br>DEVIATION<br>(M/S) |
|---|--------------------------|-----------------------------------|-----------------------------|--------------------------------|
| PLAN A<br>(K=10)                                | .161                     | .079                              | 160.42                      | 14.89                          |
| PLAN B  | .135                     | .064                              | 162.73                      | 14.85                          |
| PLAN C  | .136                     | .064                              | 157.91                      | 13.50                          |
| OPTIMUM<br>THRUST<br>SPACING<br>( $\rho=1.75$ ) | .135                     | .065                              | 46.67                       | 15.90                          |
| DUAL<br>CONTROL<br>(K=10)                       | .161                     | .079                              | 160.42                      | 14.89                          |
| CERTAINTY<br>CONTROL<br>(K=.8)                  | .171                     | .087                              | 30.11                       | 10.40                          |
| TRUTH<br>WITH<br>NOISE                          | .396                     | .233                              | 151.56                      | 13.37                          |
| TRUTH<br>WITHOUT<br>NOISE                       | 0                        | NA                                | 10.13                       | NA                             |

As predicted, the dual control's performance is no better than the certainty equivalence formulation of Plan A. This is due to the fact that range is included as a measurement, causing the control to have virtually no effect on improving filter variance. Plan B is more accurate than Plan A, but more costly in energy. Again, this result is expected because the formulation of Plan B is based on infinite miss penalty ( $K=\infty$ ) for Plan A. By optimally spacing the thrusts of Plan B, energy expenditure is considerably reduced with little or no sacrifice in accuracy.

Plan C is just as accurate as Plan B, with slightly greater cost resulting from large initial intercept range. This extra cost is attributed to the negligible gravity assumption used in the formulation of Plan C. For the smaller ranges associated with a tail chase, Plan C was actually less costly than Plan B.

In every case, certainty control yields the least energy expenditure. This result is not surprising, as the formulation of certainty control is based on reducing control energy in the presence of poor estimates. This form of control works best because filter variance is range dependent. As range decreases, the control constraint tightens, and accuracy increases. Therefore, less fuel is used when range is great, with refinements made as impact nears.

The last two entries (truth with and without noise) do not use splines. Trajectory changes are computed as outlined in

Chapter IV. This data is included as a baseline reference of performance.

Because ranging is an active and costly process, various tracking schemes were examined to determine if ranging is needed. This was easily done in the simulation because of the serial updating discussed in Chapter X. Using only line-of-sight measurement angles, all algorithms are less accurate and/or require more velocity changes. It is of interest to note that the dual control guidance scheme, true to its nature, did expend control energy to improve the estimate. The improvement was very slight because of the pursuer's speed and lateral thrusting limits. Allowing one range update at midcourse also proved costly for all guidance schemes.

An attempt was made to reduce the order of the filter in the hopes of reducing processing time. The result was a serious degradation of performance for all algorithms. The system model is very sensitive to the evader booster characteristics,  $A$  and  $\dot{m}$ , which are estimated by the eight state filter. Failure of the filtering process to refine initial booster estimates allows greater acceleration errors to be passed on to the evader estimates, significantly reducing end-game accuracy.

## CHAPTER XIII

## CONCLUSIONS AND AREAS OF FURTHER RESEARCH

In this research, six guidance schemes were examined to determine their capability to minimize lateral velocity changes of a hypervelocity orbital intercept vehicle. Proportional navigation, optimal control using certainty equivalence, dual control, control with optimum thrust spacing, and certainty control were all examined. Certainty control was shown to be the most energy efficient.

Certainty control constrains the final condition to a function of final estimator accuracy in the absence of updates. This general approach is not limited to hypervelocity vehicles, and would suggest other applications of this form to stochastically control intercepts.

This control requires a measure of final estimator accuracy which was achieved by running the Extended Kalman Filter forward to intercept time without updates. This time consuming process could be eliminated if filter variances could be estimated by some function (polynomial or otherwise). Also, the constraint multiplier was assumed constant for this formulation. Perhaps a multiplier that was range or time dependent would further reduce

interceptor thrusting.

In summary, the approach identified by this research not only improves the efficiency of hypervelocity intercept, but can be applied to a broad range of stochastic problems where control energy does not improve filter accuracy. It is also possible to combine the effects of dual and certainty control in certain cases by initially using dual control to improve estimator accuracy and then switching to certainty control. End-game accuracy may be improved by switching from certainty control to a certainty equivalence formulation just prior to impact.

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## APPENDIX A

## SPLINE APPROXIMATION ERRORS

Because the splines discussed in Chapter V are only approximations, there will be a small difference from the true trajectories modeled by them. To examine these errors, six figures are generated from a worst case scenario. Case I is considered the worst because of the high relative velocities. From this case profiles are generated for no velocity change, a velocity change of one meter per second, maximum  $\Delta V_y$ , and maximum  $\Delta V_z$ .

Figures A-1 and A-2 show no error at predicted impact time. This is expected because the splines are constrained to match final position and velocity. Figure A-3 reflects the error caused by a one meter per second in-plane velocity change decreasing as time-to-go decreases. Figure A-4 shows a similar effect for an out-of-plane velocity change. Figure A-5 shows the effect of maximum  $\Delta V_y$  on trajectory error in the region of impact. Figure A-6 shows a similar effect for maximum out-of-plane thrusting.

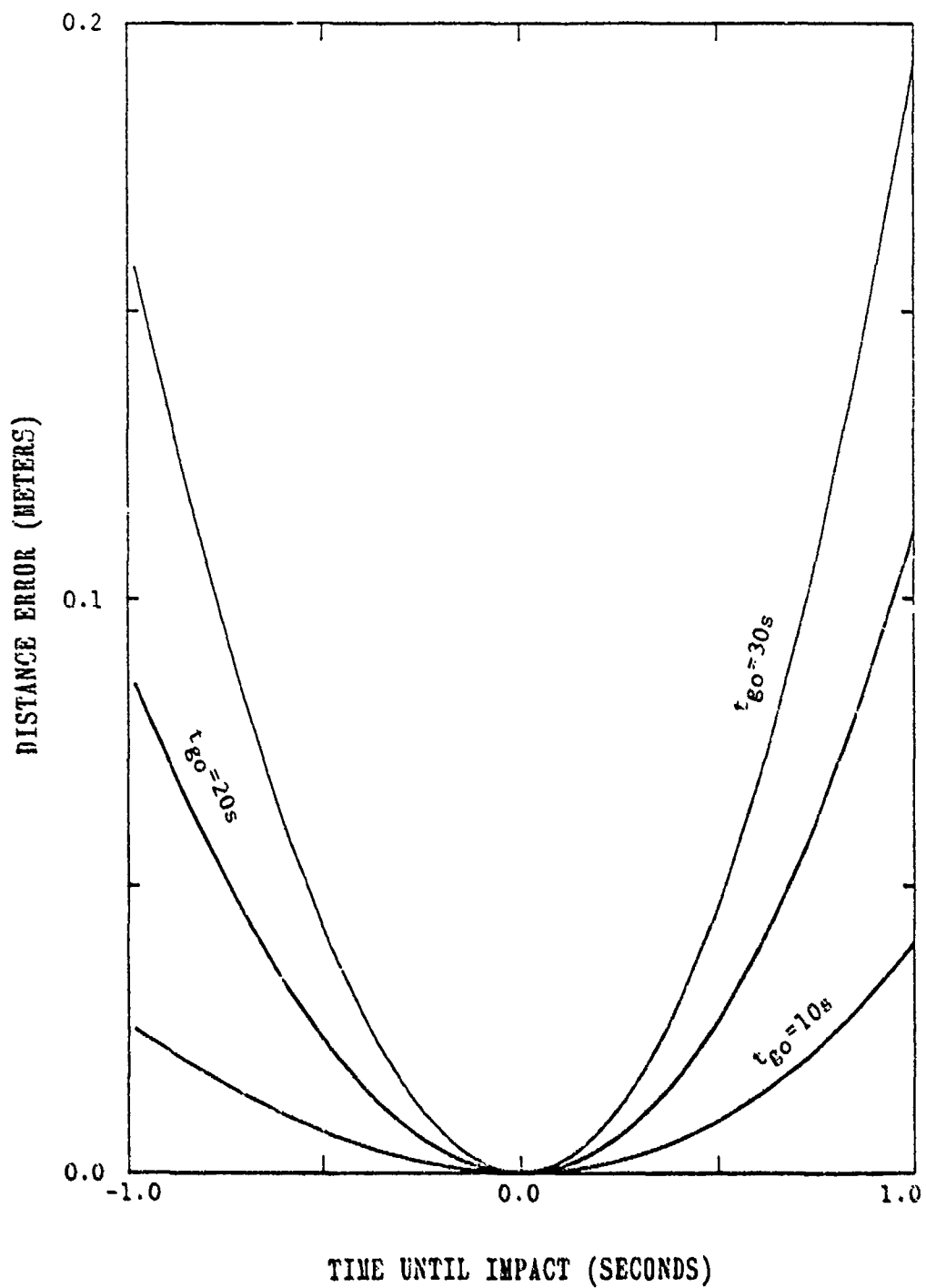


Figure A-1. Distance error of spline trajectory vs. time for zero velocity change.

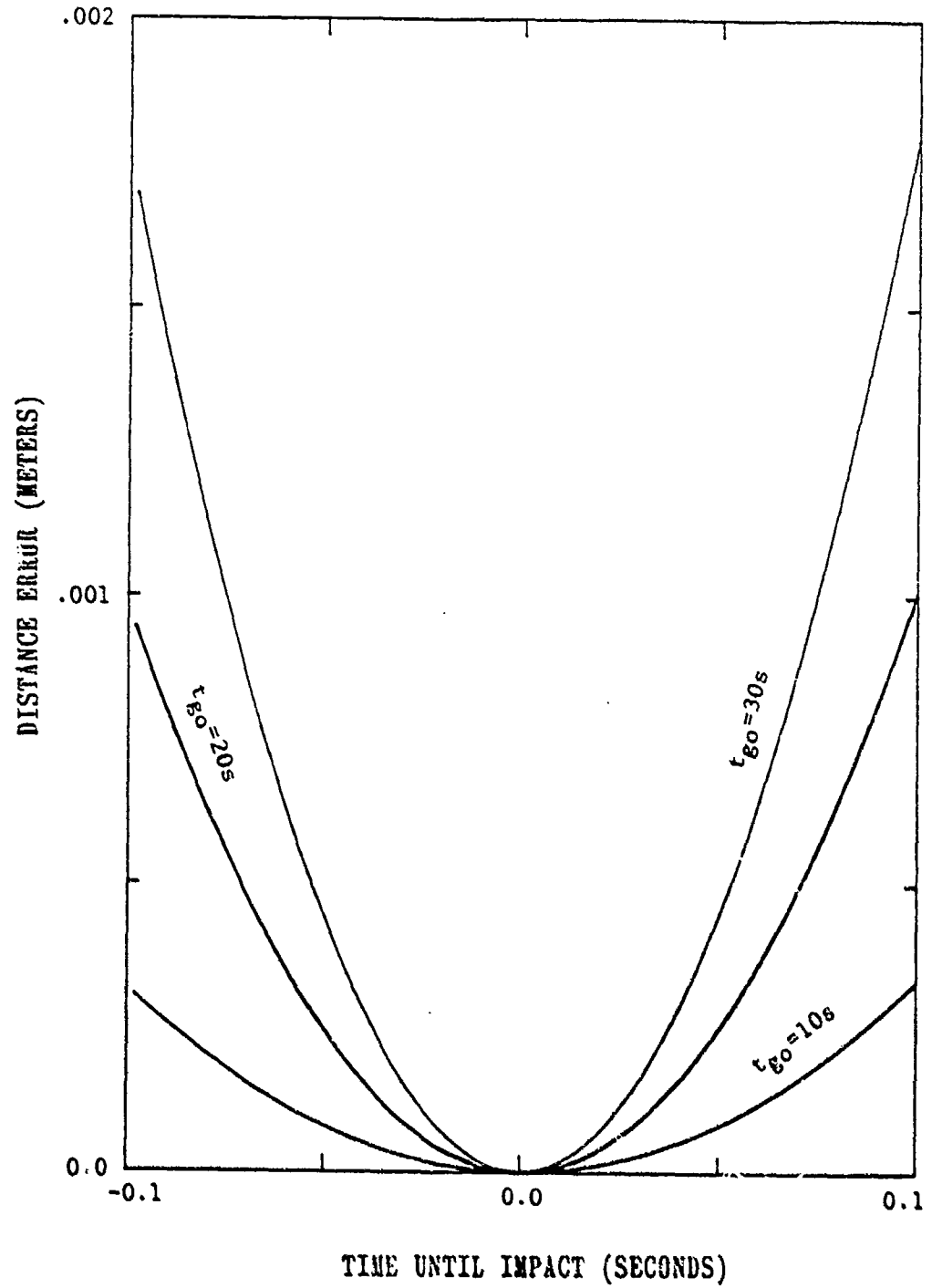


Figure A-2. Distance error of spline trajectory vs. time for zero velocity change.

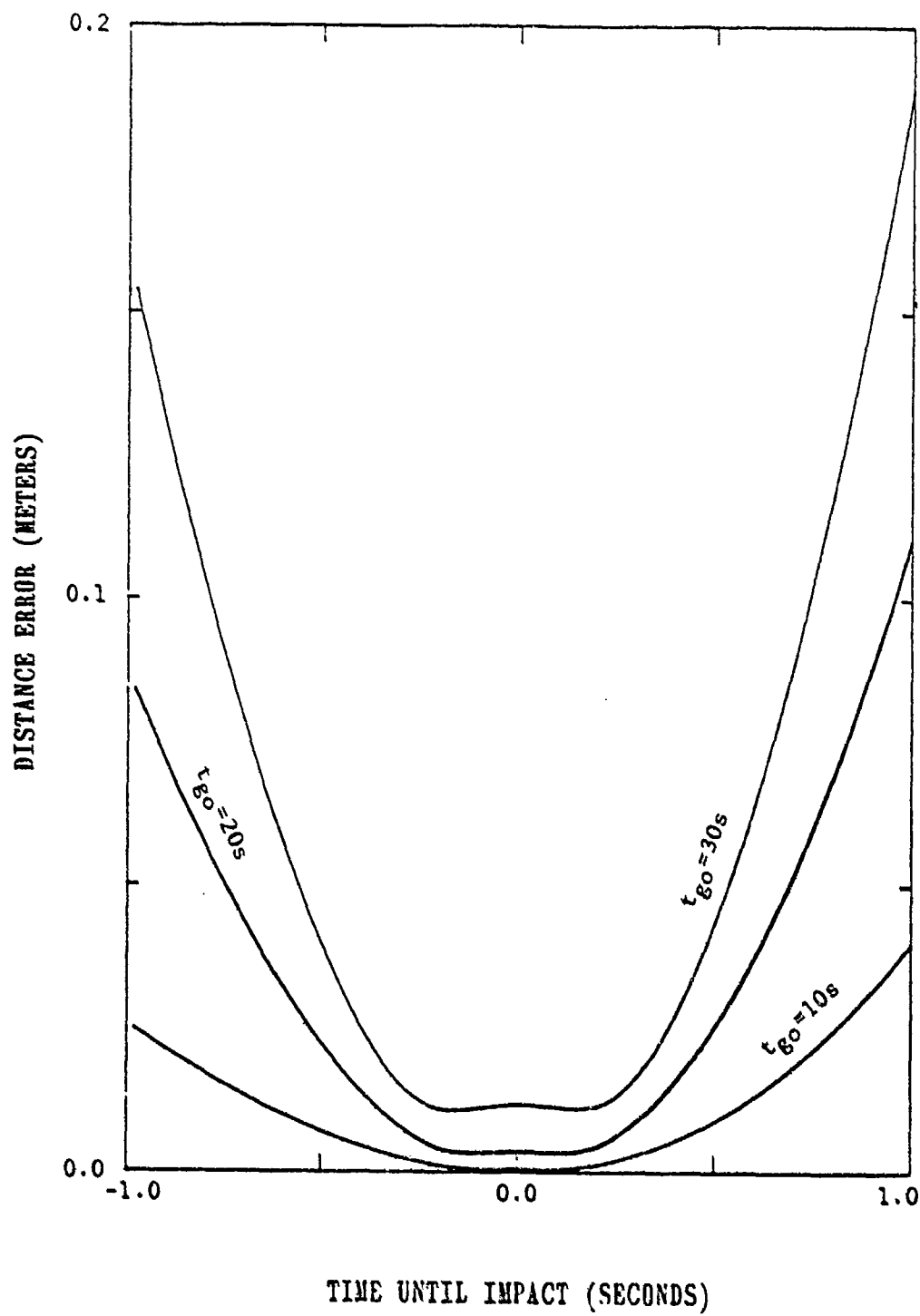


Figure A-3. Distance error of spline trajectory vs. time for  $\Delta V_y = 1$  m/s.

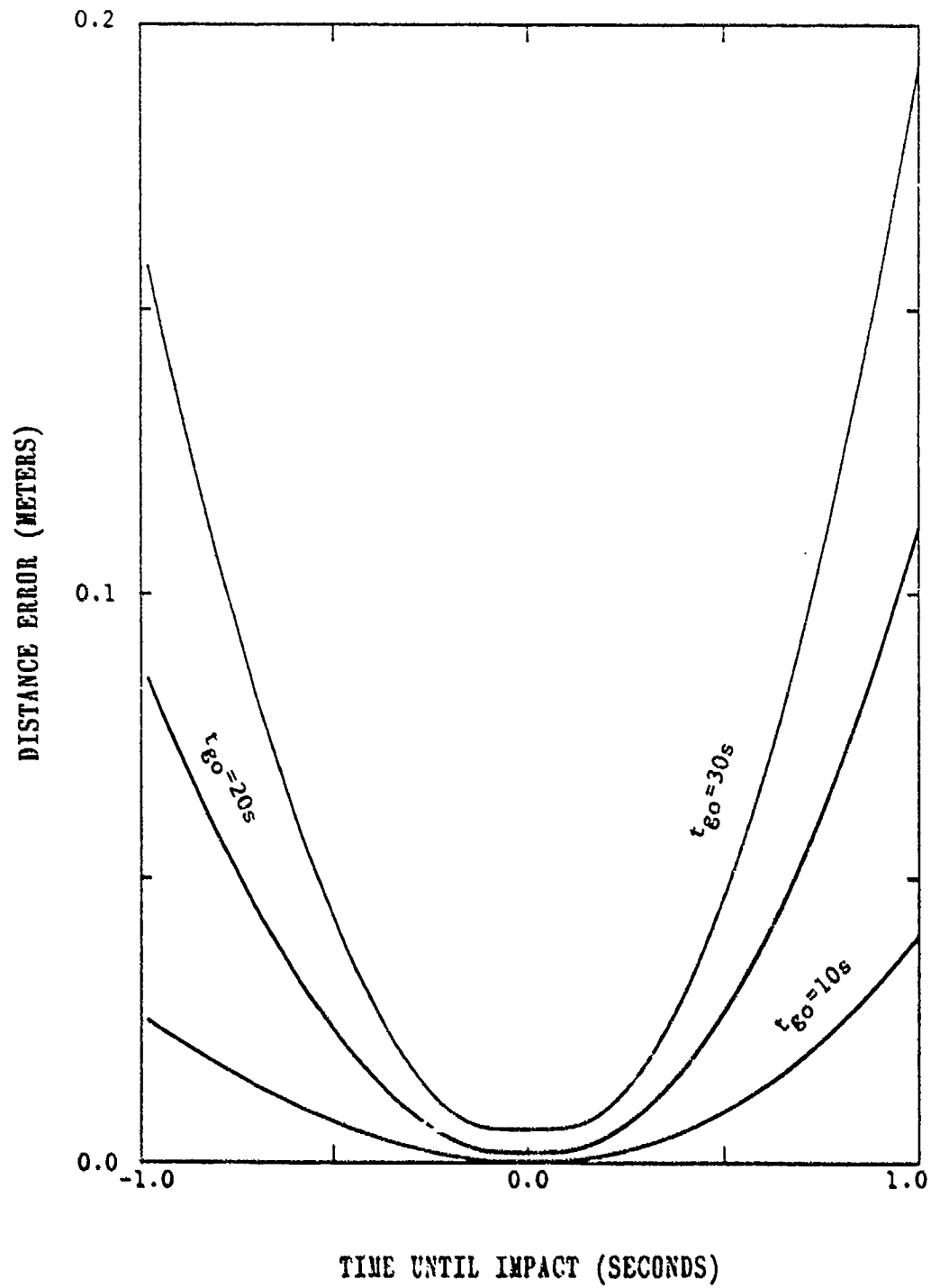


Figure A-4. Distance error of spline trajectory vs. time for  $\Delta V_z = 1$  m/s.

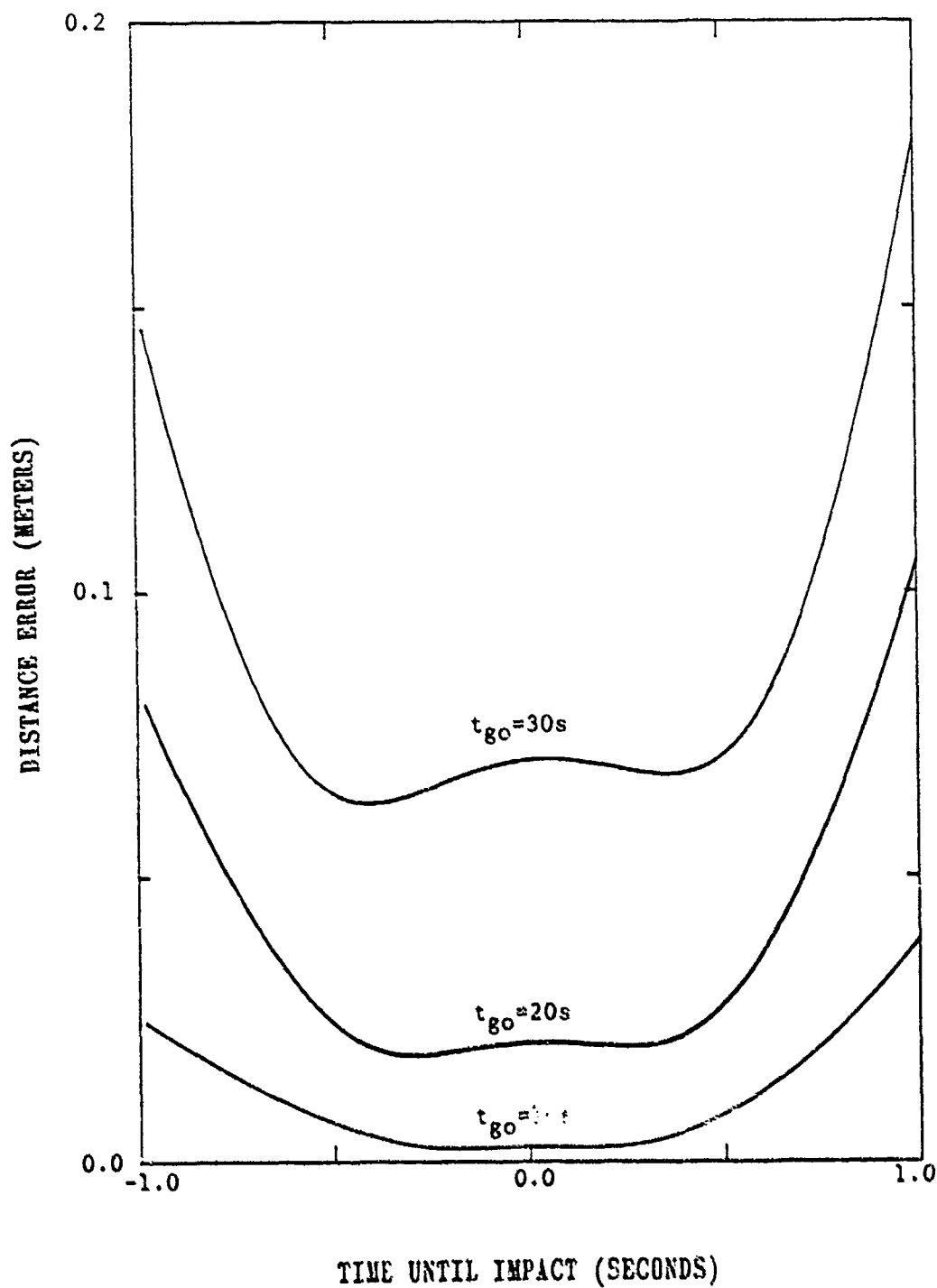


Figure A-5. Distance error of spline trajectory vs. time for maximum  $\Delta V_y$  ( $\Delta V_y = 6$  m/s).

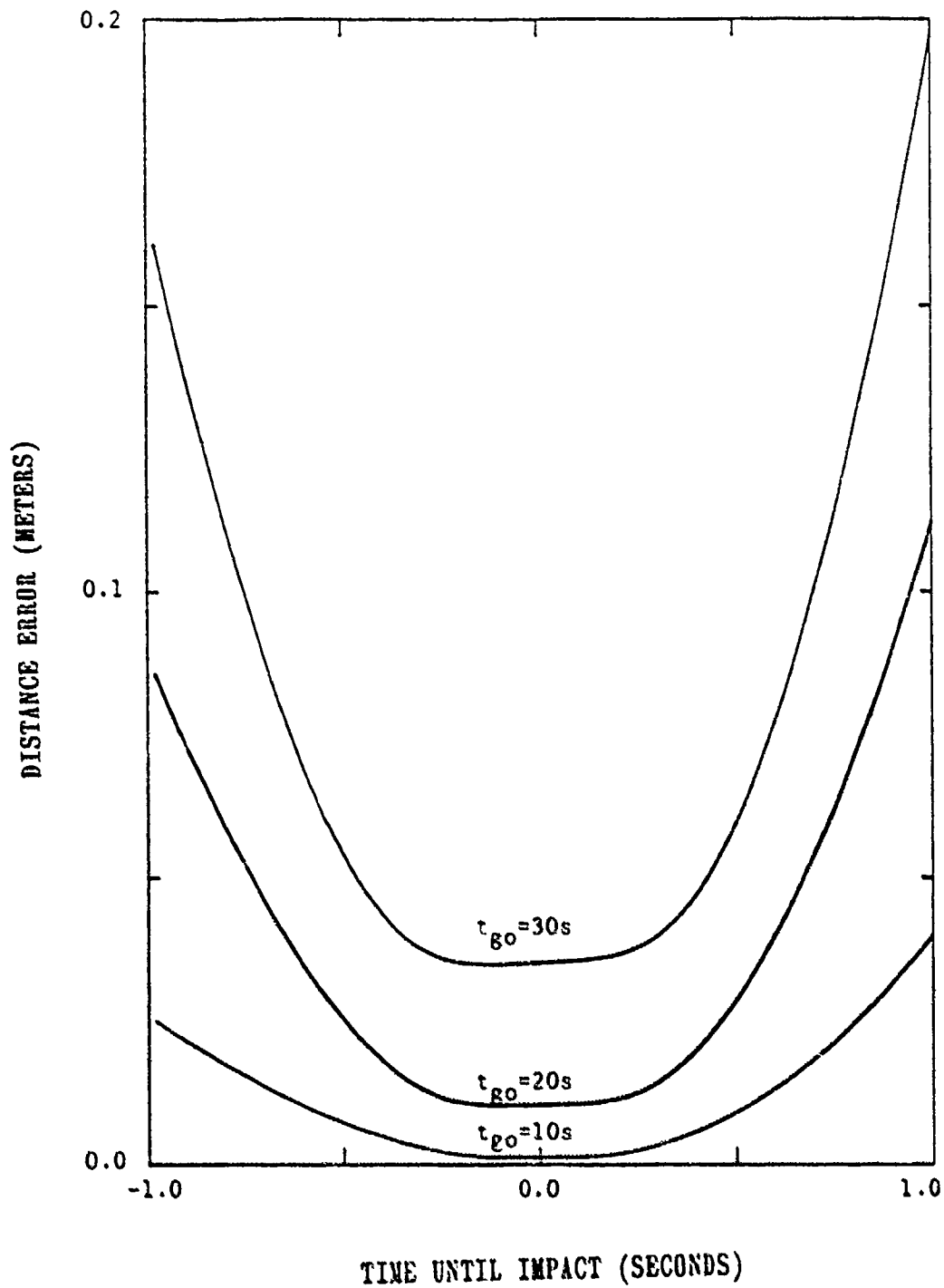


Figure A-6. Distance error of spline trajectory vs. time for maximum  $\Delta V_z$  ( $\Delta V_z = 6$  m/s).

## APPENDIX B

## DERIVATION OF CERTAINTY CONTROL EQUATIONS

The dot terms for (8-15) are computed as follows:

$$\dot{x}_f = 3A_x t_{go}^2 + 2B_x t_{go} + C_x \quad (B-1)$$

$$\dot{y}_f = 3A_y t_{go}^2 + 2B_y t_{go} + C_y - \Delta V_y \quad (B-2)$$

$$\dot{z}_f = 3A_z t_{go}^2 + 2B_z t_{go} + C_z - \Delta V_z \quad (B-3)$$

$$\dot{\sigma}_{xf} = 3A_{\sigma x} t_{go}^2 + 2B_{\sigma x} t_{go} + C_{\sigma x} \quad (B-4)$$

$$\dot{\sigma}_{yf} = 3A_{\sigma y} t_{go}^2 + 2B_{\sigma y} t_{go} + C_{\sigma y} \quad (B-5)$$

$$\dot{\sigma}_{zf} = 3A_{\sigma z} t_{go}^2 + 2B_{\sigma z} t_{go} + C_{\sigma z} \quad (B-6)$$

The Jacobian matrix elements for (8-20) are:

$$J_{11} = \frac{\partial f_1}{\partial t_{go}} = f_2 \quad (B-7)$$



$$J_{12} = \frac{\partial f_1}{\partial \lambda} = y_f \frac{\partial y_f}{\partial \lambda} + z_f \frac{\partial z_f}{\partial \lambda} \quad (\text{B-8})$$

$$\frac{\partial y_f}{\partial \lambda} = \frac{-y_s t_{go}^2}{(1 + \lambda t_{go}^2)^2} \quad (\text{B-9})$$

$$\frac{\partial z_f}{\partial \lambda} = \frac{-z_s t_{go}^2}{(1 + \lambda t_{go}^2)^2} \quad (\text{B-10})$$

$$J_{21} = \frac{\partial f_2}{\partial t_{go}} = x_f \ddot{x}_f + \dot{x}_f^2 + y_f \ddot{y}_f + \dot{y}_f^2 + z_f \ddot{z}_f + \dot{z}_f^2 - K[\sigma_{xf} \ddot{\sigma}_{xf} + \dot{\sigma}_{xf}^2 + \sigma_{yf} \ddot{\sigma}_{yf} + \dot{\sigma}_{yf}^2 + \sigma_{zf} \ddot{\sigma}_{zf} + \dot{\sigma}_{zf}^2] \quad (\text{B-11})$$

$$\ddot{x}_f = 6A_{\sigma x} t_{go} + 2B_x \quad (\text{B-12})$$

$$\ddot{y}_f = 6A_y t_{go} + 2B_y - \lambda(\dot{y}_f t_{go} + y_f) \quad (\text{B-13})$$

$$\ddot{z}_f = 6A_z t_{go} + 2B_z - \lambda(\dot{z}_f t_{go} + z_f) \quad (\text{B-14})$$

$$\ddot{\sigma}_{xf} = 6A_{\sigma x} t_{go} + 2B_{\sigma x} \quad (\text{B-15})$$

$$\ddot{\sigma}_{yf} = 6A_{\sigma y} t_{go} + 2B_{\sigma y} \quad (\text{B-16})$$

$$\ddot{\sigma}_{zf} = 6A_{\sigma z} t_{go} + 2B_{\sigma z} \quad (\text{B-17})$$

$$J_{22} = \frac{\partial f_2}{\partial \lambda} = \frac{\partial y_f}{\partial \lambda} \dot{y}_f + y_f \frac{\partial \dot{y}_f}{\partial \lambda} + \frac{\partial z_f}{\partial \lambda} \dot{z}_f + z_f \frac{\partial \dot{z}_f}{\partial \lambda} \quad (\text{B-18})$$

$$\frac{\partial \dot{y}_f}{\partial \lambda} = - \frac{y_s t_{g0}}{(1 + \lambda t_{g0}^2)^2} \quad (\text{B-19})$$

$$\frac{\partial \dot{z}_f}{\partial \lambda} = - \frac{z_s t_{g0}}{(1 + \lambda t_{g0}^2)^2} \quad (\text{B-20})$$

## APPENDIX C

## EXTENDED KALMAN FILTER EQUATIONS

The EKF states are defined from (3-5) through (3-11) as follows:

$$x_1 = x_E - x_p \quad (C-1)$$

$$x_2 = \dot{x}_E - \dot{x}_p \quad (C-2)$$

$$x_3 = y_E - y_p \quad (C-3)$$

$$x_4 = \dot{y}_E - \dot{y}_p \quad (C-4)$$

$$x_5 = z_E - z_p \quad (C-5)$$

$$x_6 = \dot{z}_E - \dot{z}_p \quad (C-6)$$

$$x_7 = \Lambda \quad (C-7)$$

$$x_8 = \dot{m} \quad (C-8)$$

Determining the F matrix components from (10-7) yields

$$E = \sqrt{x_E^2 + y_E^2 + z_E^2} \quad (C-9)$$

$$\dot{E} = \sqrt{\dot{x}_E^2 + \dot{y}_E^2 + \dot{z}_E^2} \quad (C-10)$$

$$F_{12} = F_{34} = F_{56} = 1 \quad (C-11)$$

$$F_{21} = \frac{-\mu}{E^3} + \frac{3x_E^2 \mu}{E^5} \quad (C-12)$$

$$F_{22} = \frac{-x_E^2 A}{E^3} + \frac{A}{E} \quad (C-13)$$

$$F_{23} = F_{41} = \frac{3x_E y_E \mu}{E^5} \quad (C-14)$$

$$F_{24} = F_{42} = \frac{-\dot{x}_E \dot{y}_E A}{E^3} \quad (C-15)$$

$$F_{25} = F_{61} = \frac{3x_E z_E \mu}{E^5} \quad (C-16)$$

$$F_{26} = F_{62} = \frac{-\dot{x}_E \dot{z}_E A}{E^3} \quad (C-17)$$

$$F_{27} = \frac{\dot{x}_E}{E} \quad (C-18)$$

$$F_{43} = \frac{-\mu}{E^3} + \frac{3y_E^2 \mu}{E^5} \quad (C-19)$$

$$F_{44} = \frac{-\dot{y}_E^2 A}{E^3} + \frac{A}{E} \quad (C-20)$$

$$F_{45} = F_{63} = \frac{3y_E z_E^2 \mu}{E^5} \quad (C-21)$$

$$F_{46} = F_{64} = \frac{-\dot{y}_E \dot{z}_E A}{E^3} \quad (C-22)$$

$$F_{47} = \frac{\dot{y}_E}{E} \quad (C-23)$$

$$F_{65} = \frac{-\mu}{E^3} + \frac{3z_E^2 \mu}{E^5} \quad (C-24)$$

$$F_{66} = \frac{-\dot{z}_E^2 A}{E^3} + \frac{A}{E} \quad (C-25)$$

$$F_{67} = \frac{\dot{z}_E}{E} \quad (C-26)$$

$$F_{77} = \dot{m} \quad (C-27)$$

$$F_{78} = A \quad (C-28)$$

$$F_{88} = 2\dot{m} \quad . \quad (C-29)$$

All other elements are zero.

The measurements of range and line-of sight angles are:

$$z_{1k} = \sqrt{x_1^2 + x_3^2 + x_5^2} + V_{Rk} \quad (C-30)$$

$$z_{2k} = \text{TAN}^{-1}(x_5/x_1) + V_{\theta k} \quad (C-31)$$

$$z_{3k} = \text{TAN}^{-1}(x_3/x_1) + V_{\gamma k} \quad (C-32)$$

The  $H_k$  vectors for serial update from (10-8) are:

$$R = \sqrt{x_1^2 + x_3^2 + x_5^2} \quad (C-33)$$

$$H_{1k1} = x_1/R \quad (C-34)$$

$$H_{1k3} = x_3/R \quad (C-35)$$

$$H_{1k5} = x_5/R \quad (C-36)$$

$$H_{2k1} = \frac{-x_5}{(x_1^2 + x_5^2)} \quad (C-37)$$

$$H_{2k5} = \frac{x_1}{(x_1^2 + x_5^2)} \quad (C-38)$$

$$H_{3k1} = \frac{-x_3}{(x_1^2 + x_3^2)} \quad (C-39)$$

$$H_{3k3} = \frac{x_1}{(x_1^2 + x_3^2)} \quad (C-40)$$

All other elements are zero.

## APPENDIX D

## COMPUTER SIMULATION PROGRAM

Contained here are the routines used to simulate the guidance algorithms of Chapters IV through IX. The program (main code) is separate from the supporting routines (subroutines) with the following labeling:

1. KEVSIM - This is the main program for the hyper-velocity orbital intercept.
2. TOOL 1 - Contained here are the subroutines needed for orbit propagation.
3. TOOL 2 - The coordinate transformation matrix subroutines are found here.
4. TOOL 3 - The Extended Kalman Filter subroutines are kept here.
5. TOOL 4 - The subroutines for all the guidance algorithms plus the truth model are here.

The source code for the above can be found on the following pages. All the code is written in FORTRAN 77.



## C PROGRAM KEWSIM

C THIS PROGRAM IS A HYPERVELOCITY ORBITAL INTERCEPT  
 C SIMULATION THAT FINDS THE VELOCITY CHANGES WITH AN  
 C EIGHT, SIX OR THREE STATE EXTENDED KALMAN FILTER  
 C AND THREE MEASUREMENTS.

C LINK AS FOLLOWS:

C LINK KEWSIM, TOOL1, TOOL2, TOOL3, TOOL4

## C PROGRAM DICTIONARY

|   |        |   |
|---|--------|---|
| C | A      | EVADER ACCELERATION DUE TO THRUSTING    |
| C | AD     | DUMMY ACCELERATION                      |
| C | ASIGX  | SIGMA X AXIS SPLINE COEFFICIENT OF T**3 |
| C | ASIGY  | SIGMA Y AXIS SPLINE COEFFICIENT OF T**3 |
| C | ASIGZ  | SIGMA Z AXIS SPLINE COEFFICIENT OF T**3 |
| C | AT     | DUMMY ACCELERATION                      |
| C | AX     | X AXIS SPLINE COEFFICIENT OF T**3       |
| C | AY     | Y AXIS SPLINE COEFFICIENT OF T**3       |
| C | AZ     | Z AXIS SPLINE COEFFICIENT OF T**3       |
| C | BSIGX  | SIGMA X AXIS SPLINE COEFFICIENT OF T**2 |
| C | BSIGY  | SIGMA Y AXIS SPLINE COEFFICIENT OF T**2 |
| C | BSIGZ  | SIGMA Z AXIS SPLINE COEFFICIENT OF T**2 |
| C | BX     | X AXIS SPLINE COEFFICIENT OF T**2       |
| C | BY     | Y AXIS SPLINE COEFFICIENT OF T**2       |
| C | BZ     | Z AXIS SPLINE COEFFICIENT OF T**2       |
| C | COUNT  | ITERATION FINAL COUNT                   |
| C | COV    | COVARIANCE MATRIX                       |
| C | CVD    | COVARIANCE MATRIX TRACE ELEMENTS        |
| C | CVDD   | DUMMY COVARIANCE MATRIX                 |
| C | CVDUAL | DUMMY COVARIANCE MATRIX                 |
| C | CSIGX  | SIGMA X AXIS SPLINE COEFFICIENT OF T    |
| C | CSIGY  | SIGMA Y AXIS SPLINE COEFFICIENT OF T    |
| C | CSIGZ  | SIGMA Z AXIS SPLINE COEFFICIENT OF T    |
| C | CX     | X AXIS SPLINE COEFFICIENT OF T          |
| C | CY     | Y AXIS SPLINE COEFFICIENT OF T          |
| C | CZ     | Z AXIS SPLINE COEFFICIENT OF T          |
| C | DDX    | CHANGE IN X VELOCITY (INERTIAL FRAME)   |
| C | DDY    | CHANGE IN Y VELOCITY (INERTIAL FRAME)   |
| C | DDZ    | CHANGE IN Z VELOCITY (INERTIAL FRAME)   |
| C | DELTAY | CHANGE IN Y VELOCITY (BODY FRAME)       |
| C | DELTAZ | CHANGE IN Z VELOCITY (BODY FRAME)       |
| C | DH     | DUMMY STEP SIZE                         |
| C | DSIGX  | SIGMA X AXIS SPLINE COEFFICIENT         |
| C | DSIGY  | SIGMA Y AXIS SPLINE COEFFICIENT         |
| C | DSIGZ  | SIGMA Z AXIS SPLINE COEFFICIENT         |
| C | DTGO   | DUMMY TIME-TO-GO                        |
| C | DUM    | DUMMY VARIABLE                          |
| C | DV     | INCREMENTAL VELOCITY CHANGE             |
| C | DX     | X AXIS SPLINE COEFFICIENT               |
| C | DY     | Y AXIS SPLINE COEFFICIENT               |
| C | DZ     | Z AXIS SPLINE COEFFICIENT               |
| C | FILTER | NUMBER OF FILTER STATES                 |

|   |        |  |
|---|--------|--|
| C | GAMMA  | OBSERVED LINE-OF-SIGHT ANGLE (IN PLANE)  |
| C | G2     | GAUSSIAN POINT                           |
| C | H      | STEP SIZE                                |
| C | I      | ITERATION COUNTER                        |
| C | J      | ITERATION COUNTER                        |
| C | JUP    | UPPER LIMIT ON 'J' ITERATION COUNTER     |
| C | K      | CONSTRAINT/COST FUNCTION MULTIPLIER      |
| C | KDEVF  | CONSTRAINT BASED ON FINAL COVARIANCE     |
| C | KFLAG  | KALMAN GAIN CONVERGENCE FLAG             |
| C | MAXDV  | MAXIMUM INCREMENTAL VELOCITY CHANGE      |
| C | MAXG   | MAXIMUM THRUST ACCELERATION (G FORCES)   |
| C | MDOT   | UNITIZED MASS FLOW RATE OF EVADER        |
| C | MDOTD  | DUMMY MASS FLOW RATE OF EVADER           |
| C | MDOTT  | DUMMY MASS FLOW RATE OF EVADER           |
| C | MEAN   | MEAN OF MEASUREMENT RESIDUALS            |
| C | MINDV  | MINIMUM INCREMENTAL VELOCITY CHANGE      |
| C | MING   | MINIMUM THRUST ACCELERATION (G FORCES)   |
| C | MISS2  | ESTIMATED MISS DISTANCE SQUARED          |
| C | OPT    | OPTION                                   |
| C |        | 1 - WITHOUT KALMAN FILTER                |
| C |        | 2 - WITH KALMAN FILTER                   |
| C |        | 3 - WITH KALMAN FILTER + PRINTOUT        |
| C | PLAN   | PLAN OPTION                              |
| C |        | 1 - PLAN A                               |
| C |        | 2 - PLAN B                               |
| C |        | 3 - PLAN C                               |
| C |        | 4 - DUAL CONTROL                         |
| C |        | 5 - CERTAINTY CONTROL                    |
| C |        | 6 - TRUTH MODEL                          |
| C | Q      | PROPAGATION NOISE VARIANCE               |
| C | RANGE  | RANGE MEASUREMENT OF EVADER FROM PURSUER |
| C | RES    | MEASUREMENT RESIDUALS                    |
| C | RHO    | CONTROL EFFECTIVENESS RATIO              |
| C | R3     | MEASUREMENT NOISE VARIANCE               |
| C | SEED   | RANDOM NUMBER SEED                       |
| C | SFILTR | SIMULATION FILTER                        |
| C | SFLAG  | SEARCH CONVERGENCE FLAG                  |
| C | SIGMAM | STANDARD DEVIATION OF MEASUREMENTS       |
| C | SIGT0  | INITIAL X,Y,Z DEVIATIONS AND THEIR RATES |
| C | SIGTF  | FINAL X,Y,Z DEVIATIONS AND THEIR RATES   |
| C | SIMCNT | SIMULATION COUNTER                       |
| C | SKFLAG | SIMULATION KALMAN GIAN CONVERGENCE FLAG  |
| C | SNUM   | SIMULATION NUMBER                        |
| C | SPLAN  | SIMULATION PLAN                          |
| C | SRANGE | SIMULATION RANGE                         |
| C | SSFLAG | SIMULATION SEARCH CONVERGENCE FLAG       |
| C | SVTOT  | SIMULATION TOTAL VELOCITY CHANGE         |
| C | SW     | INTEGER SWITCH FOR FUNCTION 'GAUSS'      |
| C | T      | TIME                                     |
| C | TD     | DUMMY TIME                               |
| C | TGO    | TIME-TO-GO (UNTIL IMPACT)                |
| C | THETA  | OBSERVED LOS ANGLE (OUT OF PLANE)        |
| C | TMAT   | TRANSFORMATION MATRIX                    |
| C | TOL    | RANGE TOLERANCE FOR SEARCH ROUTINE       |

```

C      TSTART      START TIME FOR CONTROL
C      UPDATE      UPDATE FLAG FOR EKF
C                  0 - NO UPDATE
C                  1 - UPDATE
C                  2 - UPDATE WITH RESIDUALS SET TO ZERO
C      VAR          VARIANCE OF MEASUREMENT RESIDUALS
C      VEL          EVADER VELOCITY
C      VTOT        TOTAL VELOCITY CHANGE
C      XDUAL       DUMMY XHAT VECTOR
C      XDUALD      DUMMY XHAT VECTOR
C      XE          STATE VECTOR OF EVADER
C      XED         DUMMY STATE VECTOR OF EVADER
C      XEDD        DUMMY STATE VECTOR OF EVADER
C      XEEST       ESTIMATED STATE VECTOR OF EVADER
C      XET         TRANSFORMED STATE VECTOR OF EVADER
C      XHAT        ESTIMATED STATES
C                  1 - RELATIVE X POSITION
C                  2 - RELATIVE X VELOCITY
C                  3 - RELATIVE Y POSITION
C                  4 - RELATIVE Y VELOCITY
C                  5 - RELATIVE Z POSITION
C                  6 - RELATIVE Z VELOCITY
C                  7 - A
C                  8 - MDOT
C      XP          STATE VECTOR OF PURSUER
C      XPD         DUMMY STATE VECTOR OF PURSUER
C      XPDD        DUMMY STATE VECTOR OF PURSUER
C      XPEST       ESTIMATED STATE VECTOR OF PURSUER
C      XPT         TRANSFORMED STATE VECTOR OF PURSUER
C      XR          RELATIVE EVADER STATE VECTOR

```

C DECLARE VARIABLES

```

REAL*8 A,MDOT,T,TIME,H,XE(6),XP(6),XED(6),XPD(6)
REAL*8 TGO,MING,RES(3),VAR(3),MEAN(3),XHAT(8)
REAL*8 DELTAY,DELTAZ,DV,K,DH,TD,TMAT(3,3),MAXG
REAL*8 AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ,G2
REAL*8 DDW,DDY,DDZ,XET(6),XPT(6),SIGMAM(3),MISS2
REAL*8 COV(8,8),Q(8,8),R3(3,3),XEEST(6),XPEST(6)
REAL*8 MAXDV,MINDV,VTOT,XR(6),RANGE,THETA,GAMMA
REAL*8 MDOTD,MDOTT,SIGT0(6),SIGTF(6)
REAL*8 GAUSS,AD,AT,TOL,DUM,TSTART,KDEVF,RHO
REAL*8 SRANGE,SVTOT,ASIGZ,BSIGZ,CSIGZ,DSIGZ
REAL*8 ASIGX,BSIGX,CSIGX,DSIGX,ASIGY,BSIGY,CSIGY
REAL*8 DSIGY,CVD(6),XPDD(6),CVDD(8,8),XEDD(6)
REAL*8 XDUAL(8),XDUALD(8),CVDUAL(8,8),DTGO
INTEGER I,J,JUP,COUNT,SIMCNT,SEED,OPT,SW,PLAN
INTEGER SNUM,UPDATE,SSFLAG
INTEGER KFLAG,SFLAG,FILTER,SPLAN,SFILTR,SKFLAG

```

```

C *****
C * INITIALIZATIONS *
C *****

C READ IN INITIAL CONDITIONS FOR DYNAMICS
1   FORMAT(2X,3F14.3)
6   FORMAT(2X,F8.2)
8   FORMAT(2X,F9.5)

      OPEN(UNIT=2,NAME=' [ENGR.THESIS.SALFANO]INIT.DAT',
+        TYPE='OLD',READONLY)
      READ(2,1)XE(1),XE(3),XE(5)
      READ(2,1)XE(2),XE(4),XE(6)
      READ(2,1)XP(1),XP(3),XP(5)
      READ(2,1)XP(2),XP(4),XP(6)
      READ(2,6)TGO
      READ(2,8)A
      READ(2,8)MDOT
      CLOSE(2)
      PRINT *, ' '
      PRINT *, ' ENTER TIME STEP VALUE'
      READ *, H
      PRINT *, ' '
      PRINT *, ' ENTER CONTROL/CONSTRAINT MULTIPLIER'
      READ *, K
      PRINT *, ' '
      PRINT *, ' ENTER CONTROL EFFECTIVENESS RATIO'
      READ *, RHO
      IF (RHO .LT. 1.0) RHO=1.0

C READ IN FILTER MEASUREMENT STANDARD DEVIATIONS
C AND ASSIGN COVARIANCES TO R MATRIX DIAGONAL
7   FORMAT(F14.10)
      OPEN(UNIT=3,NAME=' [ENGR.THESIS.SALFANO]
+        FILTER8.REL',TYPE='OLD',READONLY)
      READ(3,7)SIGMAM(1)
      READ(3,7)SIGMAM(2)
      READ(3,7)SIGMAM(3)
      CLOSE(3)
      XR(1)=XE(1)-XP(1)
      XR(3)=XE(3)-XP(3)
      XR(5)=XE(5)-XP(5)
      RANGE=SQRT(XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5))
      R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
      R3(2,2)=SIGMAM(2)*SIGMAM(2)
      R3(3,3)=SIGMAM(3)*SIGMAM(3)

C READ IN THE NEXT SEED
      OPEN(UNIT=4,NAME=' SIM.STATS',TYPE='OLD',READONLY)
9   FORMAT(2X,F9.5,2X,F9.3,2(2X,I2),2(2X,I5))
10  FORMAT(2X,I3,2X,I14)
      READ(4,10)SNUM,SEED

```

```

PRINT *, ' '
PRINT *, ' WHAT FILTER DO YOU CHOOSE ? '
PRINT *, ' 8 - EIGHT STATE EKF '
PRINT *, ' 6 - SIX STATE EKF '
PRINT *, ' 60 - SIX STATE EKF WITHOUT GRAVITY '
READ *, FILTER

C ZERO OUT OFF DIAGONAL FILTER MATRIX COMPONENTS
DO 40 I=2,8
  JUP=I-1
  DO 40 J=1,JUP
    COV(I,J)=0.0
    COV(J,I)=0.0
    Q(I,J)=0.0
    Q(J,I)=0.0
40 CONTINUE

C ESTABLISH ACCELERATION AND
C MDOT PROPAGATION VARIANCES
DUM=.1*MDOT*H
Q(8,8)=DUM*DUM/H
Q(7,7)=A*A*DUM*DUM*H

C COMPUTE AND INITIALIZE PROPAGATION VARIANCES
C COMPUTE TRANSFORMATION MATRIX
CALL COMPTV(XP, TMAT)

C TRANSFORM ESTIMATED STATE VARIABLES
CALL TRANSFWD(XP(1),XP(3),XP(5),
+ TMAT,XPT(1),XPT(3),XPT(5))
CALL TRANSFWD(XP(2),XP(4),XP(6),
+ TMAT,XPT(2),XPT(4),XPT(6))
CALL TRANSFWD(XE(1),XE(3),XE(5),
+ TMAT,XET(1),XET(3),XET(5))
CALL TRANSFWD(XE(2),XE(4),XE(6),
+ TMAT,XET(2),XET(4),XET(6))

C INITIALIZE STATE VECTORS IN NEW FRAME
DO 50 I=1,6
  XP(I)=XPT(I)
  XE(I)=XET(I)
  XED(I)=XET(I)
50 CONTINUE

C ESTABLISH DUMMY TIME STEP
DH=H/256

C PROPAGATE DUMMY VARIABLES FORWARD ONE STEP
TD=0.0
AD=A
MDOTD=MDOT
DO 60 I=1,256
  CALL RK4SYSE(TD,XED,DH,AD,MDOTD)
  TD=TD+DH
60 CONTINUE

```

```

C   PROPAGATE TRANSFORMED VARIABLES FORWARD ONE STEP
T=0.0
IF (FILTER .EQ. 8) THEN
  AT=A
  MDOTT=MDOT
  CALL RK4SYSE(T,XET,H,AT,MDOTT)
ENDIF
IF (FILTER .EQ. 6) THEN
  AT=A+SQRT(H*Q(7,7))
  MDOTT=MDOT+SQRT(H*Q(8,8))
  CALL RK4SYSE(T,XET,H,AT,MDOTT)
ENDIF
IF (FILTER .EQ. 60) THEN
  XET(1)=XET(1)+H*XET(2)
  XET(3)=XET(3)+H*XET(4)
  XET(5)=XET(5)+H*XET(6)
  AT=A+SQRT(H*Q(7,7))
  DUM=1.0+H*AT/SQRT(XET(2)**2+XET(4)**2+
+                   XET(6)**2)
  XET(2)=XET(2)*DUM
  XET(4)=XET(4)*DUM
  XET(6)=XET(6)*DUM
ENDIF

C   COMPUTE REMAINING Q DIAGONAL COMPONENTS
Q(1,1)=((XET(1)-XED(1))**2+(XET(3)-XED(3))**2+
+       (XET(5)-XED(5))**2)/3.0/H
Q(3,3)=Q(1,1)
Q(5,5)=Q(1,1)
Q(2,2)=((XET(2)-XED(2))**2+(XET(4)-XED(4))**2+
+       (XET(6)-XED(6))**2)/3.0/H
Q(4,4)=Q(2,2)
Q(6,6)=Q(2,2)
Q(7,7)=(AT-AD)*(AT-AD)/H

C   ASSIGN STARTUP COVARIANCES
COV(1,1)=100.0
COV(2,2)=SQRT(COV(1,1))
COV(3,3)=COV(1,1)
COV(4,4)=COV(2,2)
COV(5,5)=COV(1,1)
COV(6,6)=COV(2,2)
COV(7,7)=(0.1*A)**2
COV(8,8)=(0.1*MDOT)**2

C   INITIALIZE VARIABLES
TOL=0.0001
TSTART=3.0
SFLAG=0
KFLAG=0
SW=0
DELTAY=0.0
DELTAZ=0.0
VTOT=0.0

```

```

MAXG=6.0
MING=0.05*MAXG
MAXDV=MAXG*10.0*H
MINDV=MING*10.0*H
COUNT=100

```

C ASK USER TO CHOOSE CONTROL METHOD

```

PRINT *, ' '
PRINT *, ' WHAT CONTROL METHOD DO YOU CHOOSE ? '
PRINT *, ' 1 - PLAN A '
PRINT *, ' 2 - PLAN B '
PRINT *, ' 3 - PLAN C '
PRINT *, ' 4 - DUAL CONTROL '
PRINT *, ' 5 - CERTAINTY CONTROL '
PRINT *, ' 6 - TRUTH MODEL '
READ *, PLAN
UPDATE=1
IF (PLAN .EQ. 4) UPDATE=2
IF (PLAN .EQ. 5) UPDATE=0

```

C ASK USER FOR NOISE OPTION

```

PRINT *, ' '
PRINT *, ' CHOOSE YOUR OPTION '
PRINT *, ' 1 - NO NOISE '
PRINT *, ' 2 - NOISE '
PRINT *, ' 3 - NOISE + SCREEN PRINTOUT '
PRINT *, ' 4 - NOISE + DATAFILE PRINTOUT '
READ *, OPT

```

C INITIALIZE ESTIMATED VARIABLES

```

DO 100 I=1,6
  XPEST(I)=XP(I)
  IF (OPT .EQ. 1) THEN
    XEEST(I)=XE(I)
  ELSE
    XEEST(I)=XE(I)+SQRT(COV(I,I))*
+      GAUSS(SEED,SW,G2)
  ENDIF
  KHAT(I)=XEEST(I)-XPEST(I)
100 CONTINUE
IF (OPT .EQ. 1) THEN
  KHAT(7)=A
  KHAT(8)=MDOT
ELSE
  KHAT(7)=A+SQRT(COV(7,7))*GAUSS(SEED,SW,G2)
  KHAT(8)=MDOT+SQRT(COV(8,8))*GAUSS(SEED,SW,G2)
ENDIF

```

C OPEN UNITS FOR WRITING OUTPUT DATA

```

5 FORMAT(F13.7,2X,F13.7)
IF (OPT .EQ. 4) THEN
  OPEN(UNIT=11,FILE='RES1.DAT',STATUS='NEW',
+    IOSTAT=ISTAT)

```

```
OPEN(UNIT=12, FILE='RES2.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=13, FILE='RES3.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=14, FILE='RES4.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=15, FILE='RES5.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=16, FILE='RES6.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=17, FILE='RES7.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=18, FILE='RES8.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=19, FILE='DELTAY.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=20, FILE='DELTAZ.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=21, FILE='COV1.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=22, FILE='COV2.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=23, FILE='COV3.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=24, FILE='COV4.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=25, FILE='COV5.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=26, FILE='COV6.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=27, FILE='COV7.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=28, FILE='COV8.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=29, FILE='MISS.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=30, FILE='TOL.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=31, FILE='COV1M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=32, FILE='COV2M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=33, FILE='COV3M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=34, FILE='COV4M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=35, FILE='COV5M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=36, FILE='COV6M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=37, FILE='COV7M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
OPEN(UNIT=38, FILE='COV8M.DAT', STATUS='NEW',
+   IOSTAT=ISTAT)
ENDIF
```



```

C *****
C * BEGIN SIMULATION LOOP *
C *****

      DO 990 SIMCNT=1,50000

C PRINT RESIDUALS, VELOCITY CHANGES,
C AND COVARIANCES TO DATAFILES
      IF (OPT .EQ. 4) THEN
        WRITE(11,5),T,XEEST(1)-XE(1)
        WRITE(12,5),T,XEEST(2)-XE(2)
        WRITE(13,5),T,XEEST(3)-XE(3)
        WRITE(14,5),T,XEEST(4)-XE(4)
        WRITE(15,5),T,XEEST(5)-XE(5)
        WRITE(16,5),T,XEEST(6)-XE(6)
        WRITE(17,5),T,XHAT(7)-A
        WRITE(18,5),T,XHAT(8)-MDOT
        IF ((DELTAY .NE. 0.0) .OR. (SIMCNT .EQ. 1))
+       WRITE(19,5),T,DELTAY
        IF ((DELTAZ .NE. 0.0) .OR. (SIMCNT .EQ. 1))
+       WRITE(20,5),T,DELTAZ
        WRITE(21,5),T,SQRT(COV(1,1))
        WRITE(22,5),T,SQRT(COV(2,2))
        WRITE(23,5),T,SQRT(COV(3,3))
        WRITE(24,5),T,SQRT(COV(4,4))
        WRITE(25,5),T,SQRT(COV(5,5))
        WRITE(26,5),T,SQRT(COV(6,6))
        WRITE(27,5),T,SQRT(COV(7,7))
        WRITE(28,5),T,SQRT(COV(8,8))
        WRITE(29,5),T,SQRT(MISS2)
        IF (KDEVF .GT. 0.0) WRITE(30,5),T,KDEVF
        WRITE(31,5),T,-SQRT(COV(1,1))
        WRITE(32,5),T,-SQRT(COV(2,2))
        WRITE(33,5),T,-SQRT(COV(3,3))
        WRITE(34,5),T,-SQRT(COV(4,4))
        WRITE(35,5),T,-SQRT(COV(5,5))
        WRITE(36,5),T,-SQRT(COV(6,6))
        WRITE(37,5),T,-SQRT(COV(7,7))
        WRITE(38,5),T,-SQRT(COV(8,8))
      ENDIF

C TEST TO SEE IF TIME IS UP
      IF (TGO .LE. H) GOTO 995

C REASSIGN DUMMY VARIABLES
      DO 105 I=1,6
        XPD(I)=XPEST(I)
        XED(I)=XEEST(I)
105    CONTINUE

```

```

IF ((UPDATE .NE. 1) .AND. (T .GE. TSTART)) THEN
  DO 106 I=1,8
    XDUAL(I)=XHAT(I)
    DO 106 J=1,8
      CVDUAL(I,J)=COV(I,J)
106   CONTINUE
ENDIF

C PROPAGATE DUMMY VARIABLES FORWARD ONE STEP
TD=T
IF ((UPDATE .NE. 1) .AND. (T .GE. TSTART)) THEN
  RANGE=SQRT(XDUAL(1)*XDUAL(1)+XDUAL(3)*XDUAL(3)+
+ XDUAL(5)*XDUAL(5))
  R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
  IF (FILTER .EQ. 8) CALL EKF8(XDUAL,XED,XPD,TD,
+ H,CVDUAL,Q,R3,0.0,0.0,0.0,KFLAG,RES,UPDATE)
  IF (FILTER .EQ. 6) THEN
    Q(2,2)=1.0+XDUAL(7)*H*Q(8,8)
    Q(4,4)=Q(2,2)
    Q(6,6)=Q(2,2)
    CALL EKF6(XDUAL,XED,XPD,TD,H,CVDUAL,Q,
+ R3,0.0,0.0,0.0,KFLAG,RES,UPDATE)
  ENDIF
  AD=XDUAL(7)
  MDOTD=XDUAL(8)
ELSE
  AD=XHAT(7)
  MDOTD=XHAT(8)
  CALL RK4SYSP(TD,XPD,H)
  CALL RK4SYSE(TD,XED,H,AD,MDOTD)
ENDIF
TD=TD+H
TGO=TGO-H

C INITIALIZE TRANSFORMED VARIABLES TO DUMMY VARIABLES
AT=AD
MDOTT=MDOTD
DO 110 I=1,6
  XET(I)=XED(I)
  XPT(I)=XPD(I)
110 CONTINUE
IF (PLAN .EQ. 4) THEN
  DO 111 I=1,8
    XDUALD(I)=XDUAL(I)
    DO 111 J=1,8
      CVDD(I,J)=CVDUAL(I,J)
111 CONTINUE
  DO 112 I=1,6
    XEDD(I)=XED(I)
    XPDD(I)=XPD(I)
112 CONTINUE
ENDIF

```

```

C STORE INITIAL VALUES OF DEVIATIONS FOR
C SPLINE COMPUTATION
  IF ((PLAN .EQ. 5) .AND.
    +   (T .GE. TSTART)) THEN
    SIGT0(1)=SQRT(CVDUAL(1,1))
    SIGT0(2)=(SIGT0(1)-SQRT(COV(1,1)))/H
    SIGT0(3)=SQRT(CVDUAL(3,3))
    SIGT0(4)=(SIGT0(3)-SQRT(COV(3,3)))/H
    SIGT0(5)=SQRT(CVDUAL(5,5))
    SIGT0(6)=(SIGT0(5)-SQRT(COV(5,5)))/H
  ENDIF

C ESTABLISH SPLINE TIME STEP
  IF (TGO/COUNT .LT. H) COUNT=COUNT-1
  IF (COUNT .LT. 1) COUNT=1
  DH=TGO/COUNT

C PROPAGATE DUMMY VARIABLES FORWARD
C TO PREDICTED IMPACT TIME
  IF ((PLAN .EQ. 5) .AND.
    +   (T .GE. TSTART)) THEN
    DO 113 I=1,COUNT
      IF (I .EQ. COUNT) THEN
        CVD(1)=CVDUAL(1,1)
        CVD(3)=CVDUAL(3,3)
        CVD(5)=CVDUAL(5,5)
      ENDIF
      RANGE=SQRT(XDUAL(1)*XDUAL(1)+XDUAL(3)*
    +         XDUAL(3)+XDUAL(5)*XDUAL(5))
      R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
      IF (FILTER .EQ. 8) THEN
    +         CALL EKF8(XDUAL,XED,XPD,TD,DH,CVDUAL,Q,R3,
    +           0.0,0.0,0.0,KFLAG,RES,UPDATE)
      ENDIF
      IF (FILTER .EQ. 6) THEN
        Q(2,2)=1.0+XDUAL(7)*DH*Q(8,8)
        Q(4,4)=Q(2,2)
        Q(6,6)=Q(2,2)
    +         CALL EKF6(XDUAL,XED,XPD,TD,DH,CVDUAL,Q,
    +           R3,0.0,0.0,0.0,KFLAG,RES,UPDATE)
      ENDIF
      TD=TD+DH
113 CONTINUE
    SIGTF(1)=SQRT(CVDUAL(1,1))
    SIGTF(2)=(SIGTF(1)-SQRT(CVD(1)))/DH
    SIGTF(3)=SQRT(CVDUAL(3,3))
    SIGTF(4)=(SIGTF(3)-SQRT(CVD(3)))/DH
    SIGTF(5)=SQRT(CVDUAL(5,5))
    SIGTF(6)=(SIGTF(5)-SQRT(CVD(5)))/DH
    KDEVF=SQRT(K*(CVDUAL(1,1)+CVDUAL(3,3)+
    +   CVDUAL(5,5)))
  ELSE

```

```

DO 115 I=1,COUNT
  CALL RK4SYSP(TD,XPD,DH)
  CALL RK4SYSE(TD,XED,DH,AD,MDOTD)
  TD=TD+DH
115  CONTINUE
  ENDIF

C COMPUTE FINAL ESTIMATED RELATIVE STATES
DO 120 I=1,6
  XR(I)=XED(I)-XPD(I)
120  CONTINUE

C COMPUTE ESTIMATED MISS DISTANCE SQUARED
MISS2=XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5)

C UPDATE DUMMY TIME-TO-GO
DTGO=TD-T-H

C COMPUTE VELOCITY CHANGES
IF ((TGO .GT. H) .AND.
+   (T .GE. TSTART)) THEN

C COMPUTE RELATIVE SPLINE COEFFICIENTS
CALL SPLINE(XET(1)-XPT(1),XET(2)-XPT(2),XR(1),
+   XR(2),DTGO,AX,BX,CX,DX)
CALL SPLINE(XET(3)-XPT(3),XET(4)-XPT(4),XR(3),
+   XR(4),DTGO,AY,BY,CY,DY)
CALL SPLINE(XET(5)-XPT(5),XET(6)-XPT(6),XR(5),
+   XR(6),DTGO,AZ,BZ,CZ,DZ)

IF (PLAN .EQ. 5) THEN
  CALL SPLINE(SIGT0(1),SIGT0(2),SIGTF(1),
+   SIGTF(2),DTGO,ASIGX,BSIGX,CSIGX,DSIGX)
  CALL SPLINE(SIGT0(3),SIGT0(4),SIGTF(3),
+   SIGTF(4),DTGO,ASIGY,BSIGY,CSIGY,DSIGY)
  CALL SPLINE(SIGT0(5),SIGT0(6),SIGTF(5),
+   SIGTF(6),DTGO,ASIGZ,BSIGZ,CSIGZ,DSIGZ)
ENDIF

C COMPUTE OPTIMAL CHANGES IN VELOCITY AND
C IMPACT TIME
IF (PLAN .EQ. 1) THEN
  CALL SEARCHA(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,
+   CZ,DZ,K,TGO,DELTAY,DELTAZ,TOL,SFLAG)
ENDIF

IF (PLAN .EQ. 2) THEN
  CALL SEARCHB(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,
+   CZ,DZ,TGO,DELTAY,DELTAZ,TOL,SFLAG)
ENDIF

```

```

IF (PLAN .EQ. 3) THEN
  DO 135 I=1,6
    XR(I)=XET(I)-XPT(I)
135  CONTINUE
    CALL SEARCHC(XR,XET(2),XET(4),XET(6),AT,
+      MDOT,TGO,DELTAY,DELTAZ,TOL,SFLAG)
  ENDIF

IF (PLAN .EQ. 4) THEN
  CALL SEARCHD(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,
+      CZ,DZ,K,TGO,DELTAY,DELTAZ,TOL,SFLAG,
+      XD'JALD,XEDD,XPDD,CVDD,MAXDV,COUNT,
+      Q,R3,SIGMAM,H)
  ENDIF

IF (PLAN .EQ. 5) THEN
  CALL SEARCHCC(AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,
+      CZ,DZ,ASIGX,BSIGX,CSIGX,DSIGX,ASIGY,BSIGY,
+      CSIGY,DSIGY,ASIGZ,BSIGZ,CSIGZ,DSIGZ,K,TGO,
+      DELTAY,DELTAZ,TOL,SFLAG)
  ENDIF

IF (PLAN .EQ. 6) THEN
  CALL SEARCHT(XPT,XET,AT,MDOTT,H,T,TGO,
+      DELTAY,DELTAZ,TOL,SFLAG,COUNT)
  ENDIF

C  BOUND VELOCITY CHANGES
  IF (ABS(DELTAY) .LT. MINDV) THEN
    DELTAY=0.0
  ELSE
    IF (DELTAY .GT. MAXDV) DELTAY=MAXDV
    IF (DELTAY .LT. -MAXDV) DELTAY=-MAXDV
  ENDIF
  IF (ABS(DELTAZ) .LT. MINDV) THEN
    DELTAZ=0.0
  ELSE
    IF (DELTAZ .GT. MAXDV) DELTAZ=MAXDV
    IF (DELTAZ .LT. -MAXDV) DELTAZ=-MAXDV
  ENDIF

  DV=ABS(DELTAY)+ABS(DELTAZ)
  IF (DV .GE. MAXDV) THEN
    TSTART=T
  ELSE
    TSTART=T+(RHC-1.0)*TGO/RHO
  ENDIF

ELSE

  DELTAY=0.0
  DELTAZ=0.0
  DV=0.0

```

```

ENDIF

IF (DV .GE. MINDV) WRITE(*,1)DELTAY,DELTAZ,TGO

C PROPAGATE REAL VARIABLES AND PURSUER ESTIMATE
C FORWARD ONE STEP (I.M.U. ASSUMED PERFECT)
CALL RK4SYSP(T,XP,H)
CALL RK4SYSE(T,XE,H,A,MDOT)
DO 210 I=1,6
XPEST(I)=XP(I)
210 CONTINUE

C COMPUTE SENSOR MEASUREMENTS (PLUS NOISE)
XR(1)=XE(1)-XP(1)
XR(3)=XE(3)-XP(3)
XR(5)=XE(5)-XP(5)
RANGE=SQRT(XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5))
R3(1,1)=RANGE*SIGMAM(1)*RANGE*SIGMAM(1)
THETA=ATAN(XR(5)/XR(1))
GAMMA=ATAN(XR(3)/XR(1))
IF (OPT .NE. 1) THEN
RANGE=RANGE*(1.0+GAUSS(SEED,SW,G2))*SIGMAM(1)
THETA=THETA+GAUSS(SEED,SW,G2)*SIGMAM(2)
GAMMA=GAMMA+GAUSS(SEED,SW,G2)*SIGMAM(3)
ENDIF

C GET FILTER ESTIMATES
IF (FILTER .EQ. 8) CALL EKF8(XHAT,XEEST,XPEST,
+ T,H,COV,Q,R3,RANGE,THETA,GAMMA,KFLAG,RES,1)
IF (FILTER .EQ. 6) THEN
Q(2,2)=1.0+XHAT(7)*H*Q(8,8)
Q(4,4)=Q(2,2)
Q(6,6)=Q(2,2)
CALL EKF6(XHAT,XEEST,XPEST,T,H,
+ COV,Q,R3,RANGE,THETA,GAMMA,KFLAG,RES,1)
ENDIF

IF (FILTER .EQ. 60) CALL EKF60(XHAT,XEEST,
+ XPEST,T,H,COV,Q,R3,RANGE,THETA,GAMMA,KFLAG,RES)

C UPDATE EVADER ESTIMATE USING RELATIVE ESTIMATE
DO 200 I=1,6
XEEST(I)=XPEST(I)+XHAT(I)
200 CONTINUE

C RECURSIVELY COMPUTE MEAN AND VARIANCE OF
C MEASUREMENT RESIDUALS
IF (SIMCNT .EQ. 1) THEN
VAR(1)=RES(1)*RES(1)
VAR(2)=RES(2)*RES(2)
VAR(3)=RES(3)*RES(3)
ELSE
VAR(1)=VAR(1)*(SIMCNT-2)/(SIMCNT-1)+
+ (MEAN(1)-RES(1))*(MEAN(1)-RES(1))/SIMCNT

```

```

      VAR(2)=VAR(2)*(SIMCNT-2)/(SIMCNT-1)+
+      (MEAN(2)-RES(2))*(MEAN(2)-RES(2))/SIMCNT
      VAR(3)=VAR(3)*(SIMCNT-2)/(SIMCNT-1)+
+      (MEAN(3)-RES(3))*(MEAN(3)-RES(3))/SIMCNT
      ENDIF
      MEAN(1)=(MEAN(1)*(SIMCNT-1)+RES(1))/SIMCNT
      MEAN(2)=(MEAN(2)*(SIMCNT-1)+RES(2))/SIMCNT
      MEAN(3)=(MEAN(3)*(SIMCNT-1)+RES(3))/SIMCNT

C   PRINT ESTIMATED AND TRUE STATES AND COVARIANCES
      IF (OPT .EQ. 3) THEN
          PRINT *, ' '
          PRINT *, ' ESTIMATED STATE, TRUE STATE, ERROR'
          PRINT *, ' AND COVARIANCE'
          DO 220 I=1,6
              PRINT *,XHAT(I),XE(I)-XP(I),
+              XHAT(I)-XE(I)+XP(I)
          PRINT *, ' ',COV(I,I)
220      CONTINUE
          ENDIF

C   APPLY VELOCITY CHANGES
      XP(4)=XP(4)+DELTAY
      XP(6)=XP(6)+DELTAZ
      XPEST(4)=XP(4)
      XPEST(6)=XP(6)
      XHAT(4)=XHAT(4)-DELTAY
      XHAT(6)=XHAT(6)-DELTAZ
      VTOT=VTOT+DV

C   UPDATE TIME
      T=T+H

990  CONTINUE

C   *****
C   * END SIMULATION LOOP *
C   *****

995  CONTINUE

C   PRINT SQUARE ROOT OF COVARIANCE DIAGONAL
      PRINT *, ' '
      PRINT *, ' DEVIATIONS, ERROR'
      DO 800 I=1,6
          PRINT *,SQRT(COV(I,I)),XEEST(I)-XE(I)
800  CONTINUE
      PRINT *,SQRT(COV(7,7)),XHAT(7)-A
      PRINT *,SQRT(COV(8,8)),XHAT(8)-MDOT

```

```

C PROPAGATE REAL DATA TO FINAL PREDICTED IMPACT TIME
  DH=TGO
  CALL RK4SYSP(T,XP,DH)
  CALL RK4SYSE(T,XE,DH,A,MDOT)
  T=T+DH

C ITERATE TO FIND POINT OF CLOSEST APPROACH
  DH=H
  DO 310 J=1,25
    DO 300 I=1,6
      XR(I)=XE(I)-XP(I)
300   CONTINUE
      IF (ABS(DH) .LE. 0.0000001) GOTO 320
      DH=XR(1)*XR(2)+XR(3)*XR(4)+XR(5)*XR(6)
      DH=-DH/(XR(2)*XR(2)+XR(4)*XR(4)+XR(6)*XR(6))
      CALL RK4SYSP(T,XP,DH)
      CALL RK4SYSE(T,XE,DH,A,MDOT)
      T=T+DH
310   CONTINUE

320   CONTINUE

C PRINT CONVERGENCE MESSAGE
  PRINT *, ' '
  PRINT *, ' SEARCH NON-CONVERGENCE =', SFLAG
  PRINT *, ' GAIN NON-CONVERGENCE =', KFLAG
  PRINT *, ' '

C PRINT TIME AND MISS OF CLOSEST APPROACH
  PRINT *, ' IMPACT TIME : ', T
  RANGE=SQRT(XR(1)*XR(1)+XR(3)*XR(3)+XR(5)*XR(5))
  PRINT *, ' TOTAL VELOCITY CHANGE : ', VTOT
  PRINT *, ' MISS DISTANCE : ', RANGE
  PRINT *, XR(1), XR(3), XR(5)

C CLOSE OUTPUT DATA FILES
  IF (OPT .EQ. 4) THEN
    DO 340 I=11,38
      CLOSE(I)
340   CONTINUE
  ENDIF

C READ IN PREVIOUS SIMULATION DATA, SORT ON RANGE,
C AND WRITE TO NEW FILE

  OPEN(UNIT=5, FILE='SIM.STATS', STATUS='NEW',
+      IOSTAT=ISTAT)

  WRITE(5,10)(SNUM+1), SEED
  SRANGE=0.0
C   RANGE=SIGN(RANGE, XR(3))
  J=0

```



```
DO 350 I=1, SNUM
  READ(4,9)SRANGE, SVTOT, SPLAN, SFILTR, SSFLAG, SKFLAG
  IF ((J .EQ. 0) .AND. (RANGE .LE. SRANGE)) THEN
    WRITE(5,9)RANGE, VTOT, PLAN, FILTER, SFLAG, KFLAG
    J=1
  ENDIF
  WRITE(5,9)SRANGE, SVTOT, SPLAN, SFILTR, SSFLAG, SKFLAG
350 CONTINUE
  IF (J .EQ. 0)
+   WRITE(5,9)RANGE, VTOT, PLAN, FILTER, SFLAG, KFLAG

  CLOSE(4)
  CLOSE(5)

  END
```

```

C      TOOL1

C      THIS IS A COLLECTION OF SUBROUTINES NEEDED FOR
C      ORBIT PROPAGATION
C      IN THE HYPERVELOCITY ORBITAL INTERCEPT PROGRAM

C      SUBROUTINE DICTIONARY

C      A          EVADER ACCELERATION DUE TO THRUSTING
C      AD         DUMMY ACCELERATION
C      COUNT      ITERATION FINAL COUNT
C      MDOT       UNITIZED MASS FLOW RATE OF EVADER
C      MDOTD      DUMMY MASS FLOW RATE OF EVADER
C      H          STEP SIZE
C      I          ITERATION COUNTER
C      T          TIME
C      TFINAL     FINAL TIME
C      XE         STATE VECTOR OF EVADER
C      XP         STATE VECTOR OF PURSUER

```

```

SUBROUTINE XPSYSP(X,F)

```

```

C      THIS SUBROUTINE EVALUATES THE FUNCTIONS FOR RK4SYSP
C      (ORBITAL DYNAMICS FOR TWO BODY PROBLEM FOR PURSUER)
C      (X VECTOR = [X XDOT Y YDOT Z ZDOT])
C      (F VECTOR = [XDOT XDOUBLEDOT YDOT YDOUBLEDOT
C                  ZDOT ZDOUBLEDOT])

```

```

REAL*8 X(6),F(6),RSQRD,CONST

```

```

RSQRD=X(1)*X(1)+X(3)*X(3)+X(5)*X(5)
CONST=-3.986012E14/RSQRD/SQRT(RSQRD)
F(1)=X(2)
F(2)=-CONST*X(1)
F(3)=X(4)
F(4)=-CONST*X(3)
F(5)=X(6)
F(6)=-CONST*X(5)

```

```

END

```

```

SUBROUTINE RK4SYSP(T,X,H)

```

```

C      THIS SUBROUTINE IS A ONE STEP RUNGE-KUTTA
C      4TH ORDER INTEGRATOR FOR THE PURSUER DYNAMICS

```

```

REAL*8 T,X(6),H,F1(6),F2(6),F3(6)
REAL*8 F4(6),H2,DUMX(6)
INTEGER I

H2=0.5*H

C FIND F1
  CALL XPSYSP(X,F1)

C FIND F2
  DO 100 I=1,6
    DUMX(I)=X(I)+H2*F1(I)
100  CONTINUE
    CALL XPSYSP(DUMX,F2)

C FIND F3
  DO 200 I=1,6
    DUMX(I)=X(I)+H2*F2(I)
200  CONTINUE
    CALL XPSYSP(DUMX,F3)

C FIND F4
  DO 300 I=1,6
    DUMX(I)=X(I)+H*F3(I)
300  CONTINUE
    CALL XPSYSP(DUMX,F4)

C UPDATE THE STATE
  DO 400 I=1,6
    X(I)=X(I)+H*(F1(I)+F2(I)+F2(I)+F3(I)+
+          F3(I)+F4(I))/6.0
400  CONTINUE

  END

```

SUBROUTINE EULERP(T,X,H)

```

C THIS SUBROUTINE IS A ONE STEP EULER INTEGRATOR
C FOR THE PURSUER DYNAMICS

```

```

REAL*8 T,X(6),F1(6),H
INTEGER I

C FIND F1
  CALL XPSYSP(X,F1)

C UPDATE THE STATE
  DO 400 I=1,6
    X(I)=X(I)+H*F1(I)
400  CONTINUE

  END

```

## SUBROUTINE XPSYSE(X,F,A,MDOT,T)

```

C THIS SUBROUTINE EVALUATES THE FUNCTIONS FOR RK4SYSE
C (ORBITAL DYNAMICS FOR TWO BODY PROBLEM FOR EVADER
C USING THE ROCKET EQUATION:
C  $A=A_0/(1-MDOT*T)$ 
C (X VECTOR = [X XDOT Y YDOT Z ZDOT])
C (F VECTOR = [XDOT XDOUBLEDOT YDOT YDOUBLEDOT ZDOT
C ZDOUBLEDOT ADOT])

```

```

REAL*8 X(6),F(8),RSQRD,CONST,MDOT,T,A,V

```

```

RSQRD=X(1)*X(1)+X(3)*X(3)+X(5)*X(5)
CONST=-3.986012E14/RSQRD/SQRT(RSQRD)
V=SQRT(X(2)*X(2)+X(4)*X(4)+X(6)*X(6))

```

```

F(1)=X(2)
F(2)=CONST*X(1)+A*X(2)/V
F(3)=X(4)
F(4)=CONST*X(3)+A*X(4)/V
F(5)=X(6)
F(6)=CONST*X(5)+A*X(6)/V
F(7)=A*MDOT
F(8)=MDOT*MDOT

```

```

END

```

## SUBROUTINE RK4SYSE(T,X,H,A,MDOT)

```

C THIS SUBROUTINE IS A ONE STEP RUNGE-KUTTA
C 4TH ORDER INTEGRATOR FOR THE EVADER DYNAMICS

```

```

REAL*8 T,X(6),H,F1(8),F2(8),F3(8),F4(8)
REAL*8 H2,DUMX(6),A,MDOT,AD,MDOTD
INTEGER I

```

```

H2=0.5*H

```

```

C FIND F1
CALL XPSYSE(X,F1,A,MDOT,T)

```

```

C FIND F2
DO 100 I=1,6
DUMX(I)=X(I)+H2*F1(I)
100 CONTINUE
AD=A+H2*F1(7)
MDOTD=MDOT+H2*F1(8)
CALL XPSYSE(DUMX,F2,AD,MDOTD,T)

```

```

C  FIND F3
      DO 200 I=1,6
          DUMX(I)=X(I)+H2*F2(I)
200  CONTINUE
      AD=A+H2*F2(7)
      MDOTD=MDOT+H2*F2(8)
      CALL XPSYSE(DUMX,F3,AD,MDOTD,T)

C  FIND F4
      DO 300 I=1,6
          DUMX(I)=X(I)+H*F3(I)
300  CONTINUE
      AD=A+H*F3(7)
      MDOTD=MDOT+H2*F3(8)
      CALL XPSYSE(DUMX,F4,AD,MDOTD,T)

C  UPDATE THE STATE
      DO 400 I=1,6
          X(I)=X(I)+H*(F1(I)+F2(I)+F2(I)+F3(I)+
+              F3(I)+F4(I))/6.0
400  CONTINUE
      A=A+H*(F1(7)+F2(7)+F2(7)+F3(7)+
+          F3(7)+F4(7))/6.0
      MDOT=MDOT+H*(F1(8)+F2(8)+F2(8)+F3(8)+
+          F3(8)+F4(8))/6.0

      END

```

SUBROUTINE EULERE(T,X,H,A,MDOT)

C THIS SUBROUTINE IS A ONE STEP EULER INTEGRATOR  
C FOR THE EVADER DYNAMICS

```

      REAL*8 T,X(6),F1(8),H,A,MDOT
      INTEGER I

```

```

C  FIND F1
      CALL XPSYSE(X,F1,A,MDOT,T)

```

```

C  UPDATE THE STATE
      DO 400 I=1,6
          X(I)=X(I)+H*F1(I)
400  CONTINUE
      A=A+H*F1(7)
      MDOT=MDOT+H*F1(8)

```

END

```

C      TOOL2

C      THIS IS A COLLECTION OF SUBROUTINES NEEDED FOR
C      COORDINATE TRANSFORMATIONS
C      IN THE HYPERVELOCITY ORBITAL INTERCEPT PROGRAM

C      SUBROUTINE DICTIONARY

C      XHAT      ESTIMATED RELATIVE STATE VECTOR
C      XP        STATE VECTOR OF PURSUER
C      XPD       DUMMY STATE VECTOR OF PURSUER
C      TMAT      TRANSFORMATION MATRIX
C      TMATA     TRANSFORMATION MATRIX

      SUBROUTINE COMPTLOS(XHAT, TMAT)

C      THIS SUBROUTINE COMPUTES THE MATRIX (TMAT) THAT
C      TRANSFORMS THE REFERENCE FRAME TO THE LOS FRAME
C      WHERE THE X AXIS OF THE LOS FRAME LIES ALONG THE
C      RELATIVE POSTION VECTOR.

      REAL*8 XHAT(6), TMAT(3,3), A, R, AR

      R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
+          XHAT(5)*XHAT(5))
      A=SQRT(XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5))
      IF (A .LT. .00001) THEN A=.00001
      AR=A*R
      TMAT(1,1)=XHAT(1)/R
      TMAT(1,2)=XHAT(3)/R
      TMAT(1,3)=XHAT(5)/R
      TMAT(2,1)=-XHAT(1)*XHAT(3)/AR
      TMAT(2,2)=A/R
      TMAT(2,3)=-XHAT(3)*XHAT(5)/AR
      TMAT(3,1)=-XHAT(5)/A
      TMAT(3,2)=0.0
      TMAT(3,3)=XHAT(1)/A

      END

      SUBROUTINE COMPTV(XP, TMAT)

C      THIS SUBROUTINE COMPUTES THE MATRIX (TMAT) THAT
C      TRANSFORMS THE REFERENCE FRAME TO THE BODY FRAME
C      WHERE THE X AXIS OF THE BODY FRAME LIES ALONG
C      THE PURSUER'S VELOCITY VECTOR AND THE PURSUER'S
C      RADIUS VECTOR IS IN THE NEW XY PLANE.

```

```

REAL*8 XP(6),XPD(6),TMATA(3,3),TMAT(3,3)
REAL*8 A,B,V,AV

V=SQRT(XP(2)*XP(2)+XP(4)*XP(4)+XP(6)*XP(6))
A=SQRT(XP(2)*XP(2)+XP(6)*XP(6))
IF (A .LT. .00001) THEN A=.00001
AV=A*V
TMAT(1,1)=XP(2)/V
TMAT(1,2)=XP(4)/V
TMAT(1,3)=XP(6)/V
TMAT(2,1)=-XP(2)*XP(4)/AV
TMAT(2,2)=A/V
TMAT(2,3)=-XP(4)*XP(6)/AV
TMAT(3,1)=-XP(6)/A
TMAT(3,2)=0.0
TMAT(3,3)=XP(2)/A

CALL TRANSFWD(XP(1),XP(3),XP(5),TMAT,XPD(1),
+             XPD(3),XPD(5))
B=SQRT(XPD(3)*XPD(3)+XPD(5)*XPD(5))
IF (B .LT. .0001) RETURN
TMATA(2,1)=(TMAT(2,1)*XPD(3)+TMAT(3,1)*XPD(5))/B
TMATA(3,1)=(TMAT(3,1)*XPD(3)-TMAT(2,1)*XPD(5))/B
TMATA(2,2)=(TMAT(2,2)*XPD(3)+TMAT(3,2)*XPD(5))/B
TMATA(3,2)=(TMAT(3,2)*XPD(3)-TMAT(2,2)*XPD(5))/B
TMATA(2,3)=(TMAT(2,3)*XPD(3)+TMAT(3,3)*XPD(5))/B
TMATA(3,3)=(TMAT(3,3)*XPD(3)-TMAT(2,3)*XPD(5))/B
TMAT(2,1)=TMATA(2,1)
TMAT(3,1)=TMATA(3,1)
TMAT(2,2)=TMATA(2,2)
TMAT(3,2)=TMATA(3,2)
TMAT(2,3)=TMATA(2,3)
TMAT(3,3)=TMATA(3,3)

END

```

SUBROUTINE TRANSFWD(X,Y,Z,TMAT,XT,YT,ZT)

C THIS ROUTINE TAKES THE X,Y,Z VECTOR AND USES  
C THE TRANSFORMATION MATRIX TMAT TO FORM THE  
C VECTOR XT,YT,ZT

```

REAL*8 X,Y,Z,TMAT(3,3),XT,YT,ZT
INTEGER I

XT=TMAT(1,1)*X+TMAT(1,2)*Y+TMAT(1,3)*Z
YT=TMAT(2,1)*X+TMAT(2,2)*Y+TMAT(2,3)*Z
ZT=TMAT(3,1)*X+TMAT(3,2)*Y+TMAT(3,3)*Z

END

```

```
SUBROUTINE TRANSBKWD(X,Y,Z,TMAT,XT,YT,ZT)
C THIS ROUTINE TAKES THE X,Y,Z VECTOR AND USES THE
C INVERSE OF THE TRANSFORMATION MATRIX TMAT TO
C FORM THE VECTOR XT,YT,ZT
```

```
REAL*8 X,Y,Z,TMAT(3,3),XT,YT,ZT
INTEGER I
```

```
XT=TMAT(1,1)*X+TMAT(2,1)*Y+TMAT(3,1)*Z
YT=TMAT(1,2)*X+TMAT(2,2)*Y+TMAT(3,2)*Z
ZT=TMAT(1,3)*X+TMAT(2,3)*Y+TMAT(3,3)*Z
```

```
END
```



C        TOOL3

C        THIS IS A COLLECTION OF SUBROUTINES NEEDED FOR  
 C        EXTENDED KALMAN FILTERING  
 C        IN THE HYPERVELOCITY ORBITAL INTERCEPT PROGRAM

C        SUBROUTINE DICTIONARY

|   |         |  |
|---|---------|--|
| C | ACC DEN | DENOMINATOR OF EVADER ACCELERATION TERM  |
| C | ACCEL   | PRESENT EVADER THRUSTING ACCELERATION    |
| C | COV     | COVARIANCE MATRIX                        |
| C | DET     | THE DETERMINANT OF THE MATRIX 'MAT'      |
| C | DMAT    | DUMMY MATRIX                             |
| C | DCMAT   | DUMMY COLUMN MATRIX                      |
| C | DT      | TIME STEP                                |
| C | DUM     | DUMMY VARIABLE                           |
| C | E       | RADIUS OF EVADER                         |
| C | E2      | E**2                                     |
| C | E3      | E**3                                     |
| C | E5      | E**5                                     |
| C | EDOT    | VELOCITY OF EVADER                       |
| C | EDOT2   | EDOT**2                                  |
| C | EDOT3   | EDOT**3                                  |
| C | F       | MATRIX OF STATE PARTIAL DERIVATIVES      |
| C | GAIN    | GAIN MATRIX                              |
| C | GAMMA   | OBSERVED LINE-OF-SIGHT ANGLE (IN PLANE)  |
| C | H       | MATRIX OF MEASUREMENT PARTIALS           |
| C | HYP13   | XHAT(1)**2 + XHAT(3)**2                  |
| C | HYP15   | XHAT(1)**2 + XHAT(5)**2                  |
| C | I       | COUNTER                                  |
| C | J       | COUNTER                                  |
| C | JUP     | UPPER LIMIT ON 'J' COUNTER               |
| C | K       | COUNTER                                  |
| C | KFLAG   | GAIN CONVERGENCE FLAG                    |
| C | MEAN    | GAUSSIAN MEAN                            |
| C |         | 1 - WITHOUT FILTER                       |
| C |         | 2 - WITH FILTER                          |
| C |         | 3 - WITH FILTER + PRINTOUT               |
| C | PDOT    | TIME DERIVATIVE OF COVARIANCE MATRIX     |
| C | POLD    | PROPAGATED COVARIANCE MATRIX             |
| C | Q       | VARIANCE OF FILTER STATE NOISE           |
| C | R       | ESTIMATE OF RANGE OF EVADER FROM PURSUER |
| C | RES     | MEASUREMENT RESIDUALS                    |
| C | R3      | VARIANCE OF MEASUREMENT NOISE            |
| C | RANGE   | MEASUREMENT OF RELATIVE RANGE            |
| C | SEED    | SEED FOR RANDOM NUMBER GENERATOR         |
| C | SIGMA   | GAUSSIAN STANDARD DEVIATION              |
| C | SUM     | SUM OF UNIFORMLY DISTRIBUTED NUMBERS     |
| C | T       | TIME                                     |
| C | THETA   | OBSERVED OUT-OF-PLANE LOS ANGLE          |
| C | U       | GRAVITATIONAL CONSTANT                   |

```

C      UPDATE      FLAG FOR UPDATING EKF
C                  0  NO UPDATE
C                  1  UPDATE
C                  2  UPDATE WITH RESIDUALS EQUAL TO ZERO
C      V           ESTIMATE OF RELATIVE RANGE RATE
C      VAR         VARIANCE OF MEASUREMENT
C      XEEST       ESTIMATED EVADER VECTOR
C      XHAT        ESTIMATE OF X VECTOR (BODY FRAME)
C                  1  RELATIVE X POSITION
C                  2  RELATIVE X VELOCITY
C                  3  RELATIVE Y POSITION
C                  4  RELATIVE Y VELOCITY
C                  5  RELATIVE Z POSITION
C                  6  RELATIVE Z VELOCITY
C                  7  PRESENT ACCELERATION OF EVADER
C                  8  BOOSTER MASS FLOW RATE
C      XPEST       ESTIMATED PURSUER VECTOR

```

```

      FUNCTION GAUSS(SEED,SW,G2)
C      CREATES A POINT HAVING A GAUSSIAN DISTRIBUTION WITH
C      MEAN=0.0
C      SIGMA=1.0

```

```

      REAL MTH$RANDOM,A,B
      REAL*8 GAUSS,G2
      INTEGER SEED,SW,I

```

```

      IF (SW .GT. 0) THEN
        SW=0
        GAUSS=G2
      ELSE
        SW=1
        A=SQRT(-2.0*ALOG(MTH$RANDOM(SEED)))
        B=2.0*ACOS(-1.0)*(MTH$RANDOM(SEED))
        G2=A*COS(B)
        GAUSS=A*SIN(B)
      ENDIF

```

```

      END

```

```

      SUBROUTINE EKF8(XHAT,XEEST,XPEST,T,DT,COV,Q,R3,
+      RANGE,THETA,GAMMA,KFLAG,RES,UPDATE)

```

```

C      THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C      VELOCITY VECTORS AND ACCELERATION AND MASS FLOW
C      RATE OF THE EVADER USING AN EIGHT STATE EXTENDED

```

C KALMAN FILTER WITH SERIAL UPDATES OF RANGE AND  
 C TWO LINE-OF-SIGHT ANGLES.  
 C (GELB, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

```

REAL*8 XHAT(8),COV(8,8),RANGE,THETA,GAMMA,F(8,8)
REAL*8 E,E2,E3,E5,U,EDOT,EDOT2,Q(8,8),RES(3)
REAL*8 EDOT3,T,HYP15,HYP13,H(8)
REAL*8 XEEST(6),XPEST(6),DT,PDOT(8,8),R,R3(3,3)
REAL*8 POLD(8,8),DMAT,DCMAT(8),GAIN(8)
INTEGER I,J,JUP,K,KFLAG,UPDATE

```

C INITIALIZE DUMMY VARIABLES

```

U=3.986012E14
E2=XEEST(1)*XEEST(1)+XEEST(3)*XEEST(3)+
+ XEEST(5)*XEEST(5)
E=SQRT(E2)
E3=E2*E
E5=E3*E2
EDOT2=XEEST(2)*XEEST(2)+XEEST(4)*XEEST(4)+
+ XEEST(6)*XEEST(6)
EDOT=SQRT(EDOT2)
EDOT3=EDOT*EDOT2

```

C COMPUTE F MATRIX

```

DO 200 I=1,8
  F(1,I)=0.0
  F(3,I)=0.0
  F(5,I)=0.0
  F(7,I)=0.0
  F(8,I)=0.0

```

200 CONTINUE

```

F(1,2)=1.0
F(3,4)=1.0
F(5,6)=1.0

```

```

F(2,1)=(-1.0+3.0*XEEST(1)*XEEST(1)/E2)*U/E3
F(2,2)=(1.0-XEEST(2)*XEEST(2)/EDOT2)*XHAT(7)/EDOT
F(2,3)=3.0*U*XEEST(1)*XEEST(3)/E5
F(2,4)=-XEEST(2)*XHAT(7)*XEEST(4)/EDOT3
F(2,5)=3.0*U*XEEST(1)*XEEST(5)/E5
F(2,6)=-XEEST(2)*XHAT(7)*XEEST(6)/EDOT3
F(2,7)=XEEST(2)/EDOT
F(2,8)=0.0

```

```

F(4,1)=F(2,3)
F(4,2)=F(2,4)
F(4,3)=(-1.0+3.0*XEEST(3)*XEEST(3)/E2)*U/E3
F(4,4)=(1.0-XEEST(4)*XEEST(4)/EDOT2)*XHAT(7)/EDOT
F(4,5)=3.0*U*XEEST(3)*XEEST(5)/E5
F(4,6)=-XEEST(4)*XHAT(7)*XEEST(6)/EDOT3
F(4,7)=XEEST(4)/EDOT
F(4,8)=0.0

```

```

F(6,1)=F(2,5)
F(6,2)=F(2,6)
F(6,3)=F(4,5)
F(6,4)=F(4,6)
F(6,5)=(-1.0+3.0*XEEST(5)*XEEST(5)/E2)*U/E3
F(6,6)=(1.0-XEEST(6)*XEEST(6)/EDOT2)*XHAT(7)/EDOT
F(6,7)=XEEST(6)/EDOT
F(6,8)=0.0

```

```

F(7,7)=XHAT(8)
F(7,8)=XHAT(7)

```

```

F(8,8)=XHAT(8)+XHAT(8)

```

```

C PROPAGATE COVARIANCE MATRIX FORWARD (EULER'S METHOD)
C (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)

```

```

DO 300 I=1,8
  DO 300 J=1,I
    PDOT(I,J)=Q(I,J)
    DO 300 K=1,8
      PDOT(I,J)=PDOT(I,J)+F(I,K)*COV(K,J)+
+
300 + COV(I,K)*F(J,K)
    CONTINUE
  DO 310 I=1,8
    DO 310 J=1,I
      POLD(I,J)=COV(I,J)+(PDOT(I,J)*DT)
310 CONTINUE

```

```

C REASSIGN COV AND ZERO OUT H MATRIX

```

```

DO 320 I=1,8
  COV(I,I)=POLD(I,I)
  H(I)=0.0
  JUP=I-1
  DO 320 J=1,JUP
    COV(I,J)=POLD(I,J)
    COV(J,I)=POLD(I,J)
320 CONTINUE

```

```

C PROPAGATE STATE ESTIMATE FORWARD ONE STEP

```

```

CALL RK4SYSE(T,XPEST,DT,XHAT(7),XHAT(8))
IF (UPDATE .NE. 1) CALL RK4SYSP(T,XPEST,DT)
DO 330 I=1,6
  XHAT(I)=XEEST(I)-XPEST(I)
330 CONTINUE

```

```

IF (UPDATE .NE. 0) THEN

```

```

C PERFORM RANGE UPDATE

```

```

R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
+
XHAT(5)*XHAT(5))
H(1)=XHAT(1)/R
H(3)=XHAT(3)/R
H(5)=XHAT(5)/R

```

```

RES(1)=RANGE-R
IF (UPDATE .NE. 1) RES(1)=0.0
CALL UPDATE8(XHAT,H,COV,R3(1,1),RES(1),KFLAG)

```

```

C PERFORM THETA UPDATE
HYP15=XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5)
H(1)=-XHAT(5)/HYP15
H(3)=0.0
H(5)=XHAT(1)/HYP15
RES(2)=THETA-ATAN(XHAT(5)/XHAT(1))
IF (UPDATE .NE. 1) RES(2)=0.0
CALL UPDATE8(XHAT,H,COV,R3(2,2),RES(2),KFLAG)

```

```

C PERFORM GAMMA UPDATE
HYP13=XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)
H(1)=-XHAT(3)/HYP13
H(3)=XHAT(1)/HYP13
H(5)=0.0
RES(3)=GAMMA-ATAN(XHAT(3)/XHAT(1))
IF (UPDATE .NE. 1) RES(3)=0.0
CALL UPDATE8(XHAT,H,COV,R3(3,3),RES(3),KFLAG)

```

```

ENDIF

```

```

END

```

```

SUBROUTINE UPDATE8(XHAT,H,COV,VAR,RES,KFLAG)

```

```

C THIS SUBROUTINE DOES ONE SERIAL UPDATE FOR THE
C EIGHT STATE EXTENDED KALMAN FILTER

```

```

REAL*8 XHAT(8),H(8),COV(8,8),VAR,RES
REAL*8 DMAT,DCMAT(8),GAIN(8),POLD(8,8)
INTEGER I,J,JUP,KFLAG

```

```

C INITIALIZE COUNT
COUNT=0

```

```

C COMPUTE MATRIX (1X1) FOR INVERSION
DO 110 I=1,8
  DCMAT(I)=0.0
  DO 110 J=1,8
    DCMAT(I)=DCMAT(I)+COV(I,J)*H(J)
110 CONTINUE
  DMAT=VAR
  DO 120 I=1,8
    DMAT=DMAT+DCMAT(I)*H(I)
120 CONTINUE

```

```

C COMPUTE GAIN MATRIX
  DO 140 I=1,8
    GAIN(I)=0.0
    DO 130 J=1,8
      GAIN(I)=GAIN(I)+COV(I,J)*H(J)
130   CONTINUE
      GAIN(I)=GAIN(I)/DMAT
140   CONTINUE

C UPDATE COVARIANCE MATRIX AND LIMIT THE GAIN TO
C PREVENT NEGATIVE DIAGONAL COVARIANCES
  DO 150 I=1,8
    DCMAT(I)=0.0
    DO 150 J=1,8
      DCMAT(I)=DCMAT(I)+H(J)*COV(J,I)
150   CONTINUE

      DO 160 I=1,8
        POLD(I,I)=COV(I,I)-GAIN(I)*DCMAT(I)
        IF (POLD(I,I) .LT. 0.0) THEN
          IF (COUNT .LT. 10) THEN
            DO 155 J=1,8
              GAIN(J)=GAIN(J)/2.0
155          CONTINUE
              COUNT=COUNT+1
              KFLAG=KFLAG+1
              GOTO 150
            ELSE
              KFLAG=KFLAG+990
              RETURN
            ENDIF
          ENDIF
          JUP=I-1
          DO 160 J=1, JUP
            POLD(I,J)=COV(I,J)-GAIN(I)*DCMAT(J)
160          CONTINUE

C UPDATE ESTIMATE
  DO 170 I=1,8
    XHAT(I)=XHAT(I)+GAIN(I)*RES
170   CONTINUE

C UPDATE COV MATRIX
  DO 175 I=1,8
    COV(I,I)=POLD(I,I)
    JUP=I-1
    DO 175 J=1, JUP
      COV(I,J)=POLD(I,J)
      COV(J,I)=POLD(I,J)
175   CONTINUE

      END

```

```

SUBROUTINE EKF6(XHAT,XEEST,XPEST,T,DT,COV,Q,R3,
+ RANGE,THETA,GAMMA,KFLAG,RES,UPDATE)

C THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C VELOCITY VECTORS AND ACCELERATION OF THE EVADER
C USING A SIX STATE EXTENDED KALMAN FILTER WITH
C SERIAL UPDATES OF RANGE AND TWO LOS ANGLES.
C (GELB, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

REAL*8 XHAT(8),COV(8,8),RANGE,THETA,GAMMA,F(8,8)
REAL*8 E,E2,E3,E5,U,EDOT,EDOT2,Q(8,8),RES(3)
REAL*8 EDOT3,T,HYP15,HYP13,H(8)
REAL*8 XEEST(6),XPEST(6),DT,PDOT(8,8),R,R3(3,3)
REAL*8 POLD(8,8),DMAT,DCMAT(8),GAIN(8)
INTEGER I,J,JUP,K,KFLAG,UPDATE

C INITIALIZE DUMMY VARIABLES
U=3.986012E14
E2=XEEST(1)*XEEST(1)+XEEST(3)*XEEST(3)+
+ XEEST(5)*XEEST(5)
E=SQRT(E2)
E3=E2*E
E5=E3*E2
EDOT2=XEEST(2)*XEEST(2)+XEEST(4)*XEEST(4)+
+ XEEST(6)*XEEST(6)
EDOT=SQRT(EDOT2)
EDOT3=EDOT*EDOT2

C COMPUTE F MATRIX
DO 200 I=1,8
F(1,I)=0.0
F(3,I)=0.0
F(5,I)=0.0
F(7,I)=0.0
F(8,I)=0.0
200 CONTINUE

F(1,2)=1.0
F(3,4)=1.0
F(5,6)=1.0

F(2,1)=(-1.0+3.0*XEEST(1)*XEEST(1)/E2)*U/E3
F(2,2)=(1.0-XEEST(2)*XEEST(2)/EDOT2)*XHAT(7)/EDOT
F(2,3)=3.0*U*XEEST(1)*XEEST(3)/E5
F(2,4)=-XEEST(2)*XHAT(7)*XEEST(4)/EDOT3
F(2,5)=3.0*U*XEEST(1)*XEEST(5)/E5
F(2,6)=-XEEST(2)*XHAT(7)*XEEST(6)/EDOT3
F(2,7)=0.0
F(2,8)=0.0

F(4,1)=F(2,5)
F(4,2)=F(2,4)
F(4,3)=(-1.0+3.0*XEEST(3)*XEEST(3)/E2)*U/E3

```

```

F(4,4)=(1.0-XEEST(4)*XEEST(4)/EDOT2)*XHAT(7)/EDOT
F(4,5)=3.0*U*XEEST(3)*XEEST(5)/E5
F(4,6)=-XEEST(4)*XHAT(7)*XEEST(6)/EDOT3
F(4,7)=0.0
F(4,8)=0.0

```

```

F(6,1)=F(2,5)
F(6,2)=F(2,6)
F(6,3)=F(4,5)
F(6,4)=F(4,6)
F(6,5)=(-1.0+3.0*XEEST(5)*XEEST(5)/E2)*U/E3
F(6,6)=(1.0-XEEST(6)*XEEST(6)/EDOT2)*XHAT(7)/EDOT
F(6,7)=0.0
F(6,8)=0.0

```

```

C PROPAGATE COVARIANCE MATRIX FORWARD (EULER'S METHOD)
C (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)

```

```

DO 300 I=1,6
  DO 300 J=1,I
    PDOT(I,J)=Q(I,J)
    DO 300 K=1,6
      PDOT(I,J)=PDOT(I,J)+F(I,K)*COV(K,J)+
+
      COV(I,K)*F(J,K)
300 CONTINUE
DO 310 I=1,6
  DO 310 J=1,I
    POLD(I,J)=COV(I,J)+(PDOT(I,J)*DT)
310 CONTINUE

```

```

C REASSIGN COV AND ZERO OUT H MATRIX

```

```

DO 320 I=1,6
  COV(I,I)=POLD(I,I)
  H(I)=0.0
  JUP=I-1
  DO 320 J=1,JUP
    COV(I,J)=POLD(I,J)
    COV(J,I)=POLD(I,J)
320 CONTINUE

```

```

C PROPAGATE STATE ESTIMATE FORWARD ONE STEP

```

```

CALL RK4SYSE(T,XEEST,DT,XHAT(7),XHAT(8))
IF (UPDATE .NE. 1) CALL RK4SYSP(T,XPEST,DT)
DO 330 I=1,6
  XHAT(I)=XEEST(I)-XPEST(I)
330 CONTINUE

```

```

IF (UPDATE .NE. 0) THEN

```

```

C PERFORM RANGE UPDATE

```

```

R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
+
XHAT(5)*XHAT(5))
H(1)=XHAT(1)/R
H(3)=XHAT(3)/R
H(5)=XHAT(5)/R

```



```

RES(1)=RANGE-R
IF (UPDATE .NE. 1) RES(1)=0.0
CALL UPDATE6(XHAT,H,COV,R3(1,1),RES(1),KFLAG)

C PERFORM THETA UPDATE
HYP15=XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5)
H(1)=-XHAT(5)/HYP15
H(3)=0.0
H(5)=XHAT(1)/HYP15
RES(2)=THETA-ATAN(XHAT(5)/XHAT(1))
IF (UPDATE .NE. 1) RES(2)=0.0
CALL UPDATE6(XHAT,H,COV,R3(2,2),RES(2),KFLAG)

C PERFORM GAMMA UPDATE
HYP13=XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)
H(1)=-XHAT(3)/HYP13
H(3)=XHAT(1)/HYP13
H(5)=0.0
RES(3)=GAMMA-ATAN(XHAT(3)/XHAT(1))
IF (UPDATE .NE. 1) RES(3)=0.0
CALL UPDATE6(XHAT,H,COV,R3(3,3),RES(3),KFLAG)

ENDIF

END

SUBROUTINE EKF60(XHAT,XEEST,XPEST,T,DT,COV,Q,R3,
+ RANGE,THETA,GAMMA,KFLAG,RES)

C THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C VELOCITY VECTORS AND ACCELERATION OF THE EVADER
C USING A SIX STATE EXTENDED KALMAN FILTER WITH
C SERIAL UPDATES OF RANGE AND TWO LOS ANGLES.
C (GELB, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

REAL*8 XHAT(8),COV(8,8),RANGE,THETA,GAMMA,F(8,8)
REAL*8 E,E2,E3,E5,U,EDOT,EDOT2,Q(8,8),RES(3)
REAL*8 EDOT3,T,HYP15,HYP13,H(8)
REAL*8 XEEST(6),XPEST(6),DT,PDOT(8,8),R,R3(3,3)
REAL*8 POLD(8,8),DMAT,DCMAT(8),GAIN(8)
INTEGER I,J,JUP,K,KFLAG

C INITIALIZE DUMMY VARIABLES
U=3.986012E14
E2=XEEST(1)*XEEST(1)+XEEST(3)*XEEST(3)+
+ XEEST(5)*XEEST(5)
E=SQRT(E2)
E3=E2*E
E5=E3*E2
EDOT2=XEEST(2)*XEEST(2)+XEEST(4)*XEEST(4)+
+ XEEST(6)*XEEST(6)

```

```

      EDOT=SQRT(EDOT2)
      EDOT3=EDOT*EDOT2

C   COMPUTE F MATRIX
      DO 200 I=1,8
        DO 200 J=1,8
          F(I,J)=0.0
200   CONTINUE

      F(1,2)=1.0
      F(3,4)=1.0
      F(5,6)=1.0

      F(2,2)=(1.0-XEEST(2)*XEEST(2)/EDOT2)*XHAT(7)/EDOT
      F(2,4)=-XEEST(2)*XHAT(7)*XEEST(4)/EDOT3
      F(2,6)=-XEEST(2)*XHAT(7)*XEEST(6)/EDOT3

      F(4,2)=F(2,4)
      F(4,4)=(1.0-XEEST(4)*XEEST(4)/EDOT2)*XHAT(7)/EDOT
      F(4,6)=-XEEST(4)*XHAT(7)*XEEST(6)/EDOT3

      F(6,2)=F(2,6)
      F(6,4)=F(4,6)
      F(6,6)=(1.0-XEEST(6)*XEEST(6)/EDOT2)*XHAT(7)/EDOT

C   PROPAGATE COVARIANCE MATRIX FORWARD (EULER'S METHOD)
C   (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)
      DO 300 I=1,6
        DO 300 J=1,I
          PDOT(I,J)=Q(I,J)
          DO 300 K=1,6
            PDOT(I,J)=PDOT(I,J)+F(I,K)*COV(K,J)+
+           COV(I,K)*F(J,K)
300   CONTINUE
        DO 310 I=1,6
          DO 310 J=1,I
            POLD(I,J)=COV(I,J)+(PDOT(I,J)*DT)
310   CONTINUE

C   REASSIGN COV AND ZERO OUT H MATRIX
      DO 320 I=1,6
        COV(I,I)=POLD(I,I)
        H(I)=0.0
        JUP=I-1
        DO 320 J=1,JUP
          COV(I,J)=POLD(I,J)
          COV(J,I)=POLD(I,J)
320   CONTINUE

C   PROPAGATE STATE ESTIMATES FORWARD ONE STEP
      XEEST(1)=XEEST(1)+DT*XEEST(2)
      XEEST(3)=XEEST(3)+DT*XEEST(4)
      XEEST(5)=XEEST(5)+DT*XEEST(6)

```

```

XEEST(2)=XEEST(2)*(1.0+DT*XHAT(7)/EDOT)
XEEST(4)=XEEST(4)*(1.0+DT*XHAT(7)/EDOT)
XEEST(6)=XEEST(6)*(1.0+DT*XHAT(7)/EDOT)
XHAT(7)=XHAT(7)/(1.0-XHAT(8)*DT)
XHAT(8)=XHAT(8)/(1.0-XHAT(8)*DT)

```

```

DO 330 I=1,6
  XHAT(I)=XEEST(I)-XPEST(I)
330 CONTINUE

```

```

C PERFORM RANGE UPDATE
  R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
+ XHAT(5)*XHAT(5))
  H(1)=XHAT(1)/R
  H(3)=XHAT(3)/R
  H(5)=XHAT(5)/R
  RES(1)=RANGE-R
  CALL UPDATE6(XHAT,H,COV,R3(1,1),RES(1),KFLAG)

```

```

C PERFORM THETA UPDATE
  HYP15=XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5)
  H(1)=-XHAT(5)/HYP15
  H(3)=0.0
  H(5)=XHAT(1)/HYP15
  RES(2)=THETA-ATAN(XHAT(5)/XHAT(1))
  CALL UPDATE6(XHAT,H,COV,R3(2,2),RES(2),KFLAG)

```

```

C PERFORM GAMMA UPDATE
  HYP13=XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)
  H(1)=-XHAT(3)/HYP13
  H(3)=XHAT(1)/HYP13
  H(5)=0.0
  RES(3)=GAMMA-ATAN(XHAT(3)/XHAT(1))
  CALL UPDATE6(XHAT,H,COV,R3(3,3),RES(3),KFLAG)

```

END

SUBROUTINE UPDATE6(XHAT,H,COV,VAR,RES,KFLAG)

```

C THIS SUBROUTINE DOES ONE SERIAL UPDATE FOR THE
C SIX STATE EXTENDED KALMAN FILTER

```

```

REAL*8 XHAT(8),H(8),COV(8,8),VAR,RES
REAL*8 DMAT,DCMAT(8),GAIN(8),POLD(8,8)
INTEGER I,J,JUP,KFLAG

```

```

C INITIALIZE COUNT
  COUNT=0

```

```

C   COMPUTE MATRIX (1X1) FOR INVERSION
      DO 110 I=1,6
          DCMAT(I)=0.0
          DO 110 J=1,6
              DCMAT(I)=DCMAT(I)+COV(I,J)*H(J)
110   CONTINUE
          DMAT=VAR
          DO 120 I=1,6
              DMAT=DMAT+DCMAT(I)*H(I)
120   CONTINUE

C   COMPUTE GAIN MATRIX
      DO 140 I=1,6
          GAIN(I)=0.0
          DO 130 J=1,6
              GAIN(I)=GAIN(I)+COV(I,J)*H(J)
130   CONTINUE
          GAIN(I)=GAIN(I)/DMAT
140   CONTINUE

C   UPDATE COVARIANCE MATRIX AND LIMIT THE GAIN TO
C   PREVENT NEGATIVE DIAGONAL COVARIANCES
      DO 150 I=1,6
          DCMAT(I)=0.0
          DO 150 J=1,6
              DCMAT(I)=DCMAT(I)+H(J)*COV(J,I)
150   CONTINUE

          DO 160 I=1,6
              POLD(I,I)=COV(I,I)-GAIN(I)*DCMAT(I)
              IF (POLD(I,I) .LT. 0.0) THEN
                  IF (COUNT .LT. 10) THEN
                      DO 155 J=1,6
                          GAIN(J)=GAIN(J)/2.0
155   CONTINUE
                          COUNT=COUNT+1
                          KFLAG=KFLAG+1
                          GOTO 150
                      ELSE
                          KFLAG=KFLAG+990
                          RETURN
                      ENDIF
                  ENDIF
                  JUP=I-1
                  DO 160 J=1,JUP
                      POLD(I,J)=COV(I,J)-GAIN(I)*DCMAT(J)
160   CONTINUE

C   UPDATE ESTIMATE
      DO 170 I=1,6
          KHAT(I)=KHAT(I)+GAIN(I)*RES
170   CONTINUE

```

```

C  UPDATE COV MATRIX
  DO 175 I=1,6
    COV(I,I)=POLD(I,I)
    JUP=I-1
    DO 175 J=1,JUP
      COV(I,J)=POLD(I,J)
      COV(J,I)=POLD(I,J)
175  CONTINUE

      END

```

```

      SUBROUTINE EKF3(XHAT,XEEST,XPEST,T,DT,COV,Q,R3,
+                   RANGE,THETA,GAMMA,KFLAG,RES)

```

```

C  THIS SUBROUTINE ESTIMATES RELATIVE POSITION AND
C  VELOCITY VECTORS AND ACCELERATION OF THE EVADER
C  USING A THREE STATE EXTENDED KALMAN FILTER WITH
C  SERIAL UPDATES OF RANGE AND TWO LOS ANGLES.
C  (GELB, 'APPLIED OPTIMAL ESTIMATION', PP. 182-192)

```

```

      REAL*8 XHAT(8),COV(8,8),RANGE,THETA,GAMMA,F(8,3)
      REAL*8 E,E2,E3,E5,U,EDOT,EDOT2,Q(8,8),RES(3)
      REAL*8 EDOT3,T,HYP15,HYP13,H(8)
      REAL*8 XEEST(6),XPEST(6),DT,PDOT(8,8),R,R3(3,3)
      REAL*8 POLD(8,8),DMAT,DCMAT(8),GAIN(8)
      INTEGER I,J,JUP,K,KFLAG

```

```

C  INITIALIZE DUMMY VARIABLES
      U=3.986012E14
      E2=XEEST(1)*XEEST(1)+XEEST(3)*XEEST(3)+
+      XEEST(5)*XEEST(5)
      E=SQRT(E2)
      E3=E2*E
      E5=E3*E2
      EDOT2=XEEST(2)*XEEST(2)+XEEST(4)*XEEST(4)+
+      XEEST(6)*XEEST(6)
      EDOT=SQRT(EDOT2)
      EDOT3=EDOT*EDOT2

```

```

C  COMPUTE F MATRIX
      DO 200 I=1,8
        DO 200 J=1,8
          F(I,J)=0.0
200  CONTINUE

```

```

      F(1,1)=DT*(-1.0+3.0*XEEST(1)*XEEST(1)/E2)*U/E3
      F(1,3)=DT*3.0*U*XEEST(1)*XEEST(3)/E5
      F(1,5)=DT*3.0*U*XEEST(1)*XEEST(5)/E5

```

```

F(3,1)=F(1,3)
F(3,3)=DT*(-1.0+3.0*XEEST(3)*XEEST(3)/E2)*U/E3
F(3,5)=DT*3.0*U*XEEST(3)*XEEST(5)/E5

```

```

F(5,1)=F(1,5)
F(5,3)=F(3,5)
F(5,5)=DT*(-1.0+3.0*XEEST(5)*XEEST(5)/E2)*U/E3

```

```

C PROPAGATE COVARIANCE MATRIX FORWARD (EULER'S METHOD)
C (USING SYMMETRY, COMPUTE LOWER TRIANGULAR PDOT)

```

```

DO 300 I=1,5,2
  DO 300 J=1,I,2
    PDOT(I,J)=Q(I,J)
    DO 300 K=1,5,2
      PDOT(I,J)=PDOT(I,J)+F(I,K)*COV(K,J)+
+      COV(I,K)*F(J,K)
300 CONTINUE
DO 310 I=1,5,2
  DO 310 J=1,I,2
    POLD(I,J)=COV(I,J)+(PDOT(I,J)*DT)
310 CONTINUE

```

```

C REASSIGN COV AND ZERO OUT H MATRIX

```

```

DO 320 I=1,5,2
  COV(I,I)=POLD(I,I)
  H(I)=0.0
  H(I+1)=0.0
  JUP=I-2
  DO 320 J=1,JUP,2
    COV(I,J)=POLD(I,J)
    COV(J,I)=POLD(I,J)
320 CONTINUE

```

```

C PROPAGATE STATE ESTIMATES FORWARD ONE STEP

```

```

CALL RK4SYSE(T,XEEST,DT,XHAT(7),XHAT(8))
DO 330 I=1,6
  XHAT(I)=XEEST(I)-XPEST(I)
330 CONTINUE

```

```

C PERFORM RANGE UPDATE

```

```

R=SQRT(XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)+
+ XHAT(5)*XHAT(5))
H(1)=XHAT(1)/R
H(3)=XHAT(3)/R
H(5)=XHAT(5)/R
RES(1)=RANGE-R
CALL UPDATE3(XHAT,H,COV,R3(1,1),RES(1),KFLAG)

```

```

C PERFORM THETA UPDATE

```

```

HYP15=XHAT(1)*XHAT(1)+XHAT(5)*XHAT(5)
H(1)=-XHAT(5)/HYP15
H(3)=0.0
H(5)=XHAT(1)/HYP15

```

```
RES(2)=THETA-ATAN(XHAT(5)/XHAT(1))
CALL UPDATE3(XHAT,H,COV,R3(2,2),RES(2),KFLAG)
```

```
C PERFORM GAMMA UPDATE
HYP13=XHAT(1)*XHAT(1)+XHAT(3)*XHAT(3)
H(1)=-XHAT(3)/HYP13
H(3)=XHAT(1)/HYP13
H(5)=0.0
RES(3)=GAMMA-ATAN(XHAT(3)/XHAT(1))
CALL UPDATE3(XHAT,H,COV,R3(3,3),RES(3),KFLAG)
```

```
END
```

```
SUBROUTINE UPDATE3(XHAT,H,COV,VAR,RES,KFLAG)
```

```
C THIS SUBROUTINE DOES ONE SERIAL UPDATE FOR THE
C THREE STATE EXTENDED KALMAN FILTER
```

```
REAL*8 XHAT(8),H(8),COV(8,8),VAR,RES
REAL*8 DMAT,DCMAT(8),GAIN(8),POLD(8,8)
INTEGER I,J,JUP,KFLAG
```

```
C INITIALIZE COUNT
COUNT=0
```

```
C COMPUTE MATRIX (1X1) FOR INVERSION
DO 110 I=1,5,2
DCMAT(I)=0.0
DO 110 J=1,5,2
DCMAT(I)=DCMAT(I)+COV(I,J)*H(J)
110 CONTINUE
DMAT=VAR
DO 120 I=1,6
DMAT=DMAT+DCMAT(I)*H(I)
120 CONTINUE
```

```
C COMPUTE GAIN MATRIX
DO 140 I=1,5,2
GAIN(I)=0.0
GAIN(I+1)=0.0
DO 130 J=1,5,2
GAIN(I)=GAIN(I)+COV(I,J)*H(J)
130 CONTINUE
GAIN(I)=GAIN(I)/DMAT
140 CONTINUE
```

```

C  UPDATE COVARIANCE MATRIX AND LIMIT THE GAIN TO
C  PREVENT NEGATIVE DIAGONAL COVARIANCES
      DO 150 I=1,5,2
          DCMAT(I)=0.0
          DO 150 J=1,5,2
              DCMAT(I)=DCMAT(I)+H(J)*COV(J,I)
150    CONTINUE

      DO 160 I=1,5,2
          POLD(I,I)=COV(I,I)-GAIN(I)*DCMAT(I)
          IF (POLD(I,I) .LT. 0.0) THEN
              IF (COUNT .LT. 10) THEN
                  DO 155 J=1,5,2
                      GAIN(J)=GAIN(J)/2.0
155    CONTINUE
                      COUNT=COUNT+1
                      KFLAG=KFLAG+1
                      GOTO 150
                  ELSE
                      KFLAG=KFLAG+990
                      RETURN
                  ENDIF
              ENDIF
              JUP=I-2
              DO 160 J=1,JUP,2
                  POLD(I,J)=COV(I,J)-GAIN(I)*DCMAT(J)
160    CONTINUE

C  UPDATE ESTIMATE
          XHAT(1)=XHAT(1)+GAIN(1)*RES
          XHAT(3)=XHAT(3)+GAIN(3)*RES
          XHAT(5)=XHAT(5)+GAIN(5)*RES

C  UPDATE COV MATRIX
          DO 175 I=1,5,2
              COV(I,I)=POLD(I,I)
              JUP=I-2
              DO 175 J=1,JUP,2
                  COV(I,J)=POLD(I,J)
                  COV(J,I)=POLD(I,J)
175    CONTINUE

      END

```



## C        TOOL4

C        THIS IS A COLLECTION OF SUBROUTINES NEEDED FOR  
 C        SPLINE COMPUTATION AND NUMERICAL SEARCH OF  
 C        OPTIMAL SOLUTION FOR THE HYPERVELOCITY ORBITAL  
 C        INTERCEPT PROGRAM

## C        SUBROUTINE DICTIONARY

|   |        |  |
|---|--------|--|
| C | A      | PRESENT ACCELERATION OF EVADER         |
| C | ACOE   | A SPLINE COEFFICIENT                   |
| C | AD     | DUMMY ACCELERATION                     |
| C | ASIGX  | A SPLINE COEFFICIENT OF SIGMAX         |
| C | ASIGY  | A SPLINE COEFFICIENT OF SIGMAY         |
| C | ASIGZ  | A SPLINE COEFFICIENT OF SIGMAZ         |
| C | AX     | A SPLINE COEFFICIENT OF X              |
| C | AY     | A SPLINE COEFFICIENT OF Y              |
| C | AZ     | A SPLINE COEFFICIENT OF Z              |
| C | BCOE   | B SPLINE COEFFICIENT                   |
| C | BSIGX  | B SPLINE COEFFICIENT OF SIGMAX         |
| C | BSIGY  | B SPLINE COEFFICIENT OF SIGMAY         |
| C | BSIGZ  | B SPLINE COEFFICIENT OF SIGMAZ         |
| C | BX     | B SPLINE COEFFICIENT OF X              |
| C | BY     | B SPLINE COEFFICIENT OF Y              |
| C | BZ     | B SPLINE COEFFICIENT OF Z              |
| C | CCOE   | C SPLINE COEFFICIENT                   |
| C | CFLAG  | CONVERGENCE FLAG                       |
| C | COST1  | DUAL CONTROL COST                      |
| C | COST2  | DUAL CONTROL COST                      |
| C | CSIGX  | C SPLINE COEFFICIENT OF SIGMAX         |
| C | CSIGY  | C SPLINE COEFFICIENT OF SIGMAY         |
| C | CSIGZ  | C SPLINE COEFFICIENT OF SIGMAZ         |
| C | CVDD   | COVARIANCE MATRIX FOR EKF              |
| C | CVDR   | COVARIANCE MATRIX FOR EKF              |
| C | CVDY   | COVARIANCE MATRIX FOR EKF              |
| C | CVDZ   | COVARIANCE MATRIX FOR EKF              |
| C | CVD2   | COVARIANCE MATRIX FOR EKF              |
| C | CVDIFY | SELECTED COVARIANCE DIFFERENCE         |
| C | CVDIFZ | SELECTED COVARIANCE DIFFERENCE         |
| C | CVTOT  | SELECTED COVARIANCE TOTAL              |
| C | CX     | C SPLINE COEFFICIENT OF X              |
| C | CY     | C SPLINE COEFFICIENT OF Y              |
| C | CZ     | C SPLINE COEFFICIENT OF Z              |
| C | DA     | DISTANCE DUE TO BOOSTER ACCELERATION   |
| C | DCOE   | D SPLINE COEFFICIENT                   |
| C | DDY    | CHANGE IN Y VELOCITY                   |
| C | DDZ    | CHANGE IN Z VELOCITY                   |
| C | DELTAY | CHANGE IN Y VELOCITY                   |
| C | DELTAZ | CHANGE IN Z VELOCITY                   |
| C | DEN    | DENOMINATOR (FOR JACOBIAN DETERMINANT) |
| C | DSIGX  | D SPLINE COEFFICIENT OF SIGMAX         |
| C | DSIGY  | D SPLINE COEFFICIENT OF SIGMAY         |
| C | DSIGZ  | D SPLINE COEFFICIENT OF SIGMAZ         |

|   |        |                                       |
|---|--------|---------------------------------------|
| C | DTGO   | CHANGE IN TIME-TO-GO                  |
| C | DUMMY  | DUMMY VARIABLE                        |
| C | DVY    | Y VELOCITY CHANGE FOR DEVIATION       |
| C | DVYA   | Y VELOCITY CHANGE FOR DEVIATION       |
| C | DVZ    | Z VELOCITY CHANGE FOR DEVIATION       |
| C | DVZA   | Z VELOCITY CHANGE FOR DEVIATION       |
| C | DX     | D SPLINE COEFFICIENT OF X             |
| C | DY     | D SPLINE COEFFICIENT OF Y             |
| C | DZ     | D SPLINE COEFFICIENT OF Z             |
| C | FSIGX  | SIGMA X SPLINE COEFF FOR Y VEL CHANGE |
| C | FSIGY  | SIGMA Y SPLINE COEFF FOR Y VEL CHANGE |
| C | FSIGZ  | SIGMA Z SPLINE COEFF FOR Y VEL CHANGE |
| C | F1     | NONLINEAR SYSTEM EQUATION VALUE       |
| C | F2     | NONLINEAR SYSTEM EQUATION VALUE       |
| C | F3     | NONLINEAR SYSTEM EQUATION VALUE       |
| C | GSIGX  | SIGMA X SPLINE COEFF FOR Z VEL CHANGE |
| C | GSIGY  | SIGMA Y SPLINE COEFF FOR Z VEL CHANGE |
| C | GSIGZ  | SIGMA Z SPLINE COEFF FOR Z VEL CHANGE |
| C | H      | ITERATION TIME STEP SIZE              |
| C | I      | COUNTER                               |
| C | J      | COUNTER                               |
| C | J11    | JACOBIAN MATRIX 1,1 ELEMENT           |
| C | J12    | JACOBIAN MATRIX 1,2 ELEMENT           |
| C | J13    | JACOBIAN MATRIX 1,3 ELEMENT           |
| C | J22    | JACOBIAN MATRIX 2,2 ELEMENT           |
| C | LAMBDA | LAGRANGE MULTIPLIER                   |
| C | MAXDV  | MAXIMUM PERMISSIBLE VELOCITY CHANGE   |
| C | MISS   | MISS DISTANCE                         |
| C | MDOT   | UNITIZED MASS FLOW RATE OF EVADER     |
| C | OLDTGO | PREVIOUS TIME-TO-GO                   |
| C | RADICL | QUANTITY INSIDE RADICAL SIGN          |
| C | RANGE  | RELATIVE RANGE                        |
| C | SIG    | DEVIATION AT FINAL TIME               |
| C | SIGMAM | MEASUREMENT DEVIATIONS                |
| C | SIGX   | X DEVIATION AT FINAL TIME             |
| C | SIGY   | Y DEVIATION AT FINAL TIME             |
| C | SIGZ   | Z DEVIATION AT FINAL TIME             |
| C | SIGXDT | TIME DERIVATIVE OF SIGX               |
| C | SIGYDT | TIME DERIVATIVE OF SIGY               |
| C | SIGZDT | TIME DERIVATIVE OF SIGZ               |
| C | TD     | DUMMY TIME                            |
| C | TGO    | TIME-TO-GO                            |
| C | TGO3   | 3.0*TGO                               |
| C | TGO6   | 6.0*TGO                               |
| C | TLIM   | TIME STEP LIMIT                       |
| C | TOL    | MISS TOLERANCE                        |
| C | VA     | VELOCITY DUE TO BOOSTER ACCELERATION  |
| C | VE     | EVADER VELOCITY MAGNITUDE             |
| C | VX     | EVADER UNITIZED X VELOCITY            |
| C | VXE    | EVADER X VELOCITY                     |
| C | VY     | EVADER UNITIZED Y VELOCITY            |
| C | VYE    | EVADER Y VELOCITY                     |
| C | VZ     | EVADER UNITIZED Z VELOCITY            |

```

C      VZE      EVADER Z VELOCITY
C      XDUALD   EKF STATE ESTIMATE
C      XDUALR   EKF STATE ESTIMATE
C      XDUALY   EKF STATE ESTIMATE
C      XDUALZ   EKF STATE ESTIMATE
C      XDUAL2   EKF STATE ESTIMATE
C      XE       EVADER RELATIVE STATE VECTOR
C      XEDD     EVADER RELATIVE STATE VECTOR
C      XEDR     EVADER RELATIVE STATE VECTOR
C      XEDY     EVADER RELATIVE STATE VECTOR
C      XEDZ     EVADER RELATIVE STATE VECTOR
C      XED2     EVADER RELATIVE STATE VECTOR
C      XF       FINAL RELATIVE X POSITION
C      XFDOT    FINAL RELATIVE X VELOCITY
C      XP       PURSUER RELATIVE STATE VECTOR
C      XPDD     PURSUER RELATIVE STATE VECTOR
C      XPDR     PURSUER RELATIVE STATE VECTOR
C      XPDY     PURSUER RELATIVE STATE VECTOR
C      XPDZ     PURSUER RELATIVE STATE VECTOR
C      XPD2     PURSUER RELATIVE STATE VECTOR
C      XR       RELATIVE STATE VECTOR
C      X0       INITIAL RELATIVE X POSITION
C      X1       RELATIVE X POSITION AT ONE THIRD POINT
C      X2       RELATIVE X POSITION AT TWO THIRDS POINT
C      X3       RELATIVE X POSITION AT FINAL POINT
C      X0DOT    INITIAL RELATIVE X VELOCITY
C      YF       FINAL RELATIVE Y POSITION
C      YFDOT    FINAL RELATIVE Y VELOCITY
C      ZF       FINAL RELATIVE Z POSITION
C      ZFDOT    FINAL RELATIVE Z VELOCITY

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SUBROUTINE SPLINE4(X0,X1,X2,X3,TGO,
+                 ACOEF,BCOEF,CCOEF,DCOEF)

```

```

C THIS SUBROUTINE CREATES THE SPLINE COEFFICIENTS
C FOR THE EQUATION:
C      ACOEF*T**3 + BCOEF*T**2 + CCOEF*T + DCOEF
C GIVEN FOUR EQUISPACED POINTS AND TIME-TO-GO.
C PRESENT TIME IS EQUAL TO ZERO.

```

```

REAL*8 X0,X1,X2,X3,TGO,ACOEF,BCOEF,CCOEF,DCOEF

```

```

ACOEF=(-4.5*X0 +13.5*X1 -13.5*X2 +4.5*X3)/
+      (TGO*TGO*TGO)
BCOEF=(+9.0*X0 -22.5*X1 +18.0*X2 -4.5*X3)/
+      (TGO*TGO)
CCOEF=(-5.5*X0 +9.0*X1 -4.5*X2 +1.0*X3)/TGO
DCOEF=X0

```

```

END

```

```

SUBROUTINE SPLINE(X0,X0DOT,XF,XFDOT,TGO,
+               ACOEF,BCOEF,CCOEF,DCOEF)

```

```

C THIS SUBROUTINE CREATES THE SPLINE COEFFICIENTS
C FOR THE EQUATION:
C   ACOEF*T**3 + BCOEF*T**2 + CCOEF*T + DCOEF
C GIVEN INITIAL POSITION AND VELOCITY AND
C FINAL POSITION, VELOCITY, AND TIME-TO-GO.
C PRESENT TIME IS EQUAL TO ZERO.

```

```

REAL*8 X0,X0DOT,XF,XFDOT,TGO
REAL*8 ACOEF,BCOEF,CCOEF,DCOEF

```

```

DCOEF=X0
CCOEF=X0DOT
ACOEF=(2.0*(X0-XF)/TGO+X0DOT+XFDOT)/TGO/TGO
BCOEF=(3.0*(XF-X0)/TGO-2.0*X0DOT-XFDOT)/TGO

```

```

END

```

```

SUBROUTINE SEARCHA(AX,BX,CX,DX,AY,BY,CY,DY,
+ AZ,BZ,CZ,DZ,K,TGO,DELTAY,DELTAZ,TOL,CFLAG)

```

```

C THIS SUBROUTINE NUMERICALLY MINIMIZES THE
C COST FUNCTION:
C   L = K*(XF**2+XY**2+XZ**2)/2 +
C       (DELTAX**2 + DELTAY**2)/2
C BY VARYING THE TIME-TO-GO (TGO) AND THE VELOCITY
C CHANGES (DELTAY AND DELTAZ) TO BRING THE MISS
C DISTANCE WITHIN SOME TOLERANCE.
C XF,YF AND ZF ARE COMPUTED FROM THE SPLINE
C COEFFICIENTS AX,BX,...,DZ AT THE FINAL TIME.
C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON
C SEARCH SCHEME FOR NON-LINEAR SYSTEMS.
C (PP. 176-179 OF MARON, 'NUMERICAL ANALYSIS,
C A PRACTICAL APPROACH')

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REAL*8 AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ,DDY
REAL*8 K,TGO,DELTAY,DELTAZ,XF,YF,ZF,F1,F2,F3
REAL*8 XFDOT,YFDOT,ZFDOT,J11,J12,J13,J22,TGO3
REAL*8 MISS,TOL,TGO6,OLDTGO,SCALE,DEN,DTGO,DDZ
INTEGER I,CFLAG

```

```

C INITIALIZE VELOCITY CHANGES AND OLD FINAL TIME
DELTAY=0.0
DELTAZ=0.0
OLDTGO=TGO

```

```

C BEGIN SEARCH LOOP
DO 10 I=1,25

```

```

C COMPUTE FINAL RELATIVE POSITIONS
  XF=((AX*TGO+BX)*TGO+CX)*TGO+DX
  YF=((AY*TGO+BY)*TGO+CY-DELTAY)*TGO+DY
  ZF=((AZ*TGO+BZ)*TGO+CZ-DELTAZ)*TGO+DZ
  MISS=SQRT(XF*XF+YF*YF+ZF*ZF)

C TEST FOR NEARNESS (THIS IS RANGE RELATIVE)
  IF (MISS .LE. TOL) RETURN

C COMPUTE FINAL RELATIVE VELOCITIES
  TGO3=TGO+TGO+TGO
  XFDOT=(AX*TGO3+BX+BX)*TGO+CX
  YFDOT=(AY*TGO3+BY+BY)*TGO+CY-DELTAY
  ZFDOT=(AZ*TGO3+BZ+BZ)*TGO+CZ-DELTAZ

C COMPUTE NONLINEAR SYSTEM EQUATIONS
  F1=K*(XF*XFDOT+YF*YFDOT+ZF*ZFDOT)
  F2=DELTAY-K*YF*TGO
  F3=DELTAZ-K*ZF*TGO

C COMPUTE NECESSARY ELEMENTS OF JACOBIAN MATRIX
  TGO6=TGO3+TGO3
  J11=K*(XF*(AX*TGO6+BX+BX)+YF*(AY*TGO6+BY+BY)+
+      ZF*(AZ*TGO6+BZ+BZ)+XFDOT*XFDOT+
+      YFDOT*YFDOT+ZFDOT*ZFDOT)
  J12=-K*(YF+YFDOT*TGO)
  J13=-K*(ZF+ZFDOT*TGO)
  J22=1.0+K*TGO*TGO

C COMPUTE CHANGES IN CONTROL PARAMETERS
  DEN=(J11*J22-J13*J13-J12*J12)
  DTGO=- (F1*J22-J12*F2-J13*F3)/DEN
  DDY=- (-J12*F1+(J11-J13*J13/J22)*F2+
+      F3*J12*J13/J22)/DEN
  DDZ=- (-J13*F1+F2*J12*J13/J22+
+      (J11-J12*J12/J22)*F3)/DEN

C UPDATE CONTROL PARAMETERS
  TGO=TGO+DTGO
  DELTAY=DELTAY+DDY
  DELTAZ=DELTAZ+DDZ

C TEST FOR CONVERGENCE OF FINAL TIME
  IF (ABS(DTGO) .LT. (.000001*TGO)) RETURN
10 CONTINUE

C INCREMENT CONVERGENCE FLAG
  CFLAG=CFLAG+1

C ZERO OUT VELOCITY CHANGES AND RESET TIME
  DELTAY=0.0
  DELTAZ=0.0
  TGO=OLDTGO
  END

```

```

SUBROUTINE SEARCHB(AX,BX,CX,DX,AY,BY,CY,DY,
+   AZ,BZ,CZ,DZ,TGO,DELTAY,DELTAZ,TOL,CFLAG)

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```

C THIS SUBROUTINE FINDS THE TIME-TO-GO AND VELOCITY
C CHANGES WITH MISS DISTANCE WITHIN SOME EXTERNAL
C TOLERANCE.
C THIS IS DONE BY VARYING THE TIME-TO-GO (TGO) TO
C VARY THE X MISS DISTANCE.
C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON
C SEARCH SCHEME (PP. 48-53 OF MARON, 'NUMERICAL
C ANALYSIS, A PRACTICAL APPROACH')
C XF,AND XFDOT ARE COMPUTED FROM THE SPLINE
C COEFFICIENTS AX,BX,CX,DX AT THE FINAL TIME.
C ONCE THE FINAL TIME IS KNOWN THE VELOCITY CHANGES
C (DELTAY AND DELTAZ) ARE COMPUTED.

```

```

REAL*8 AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ,DTGO
REAL*8 TGO,DELTAY,DELTAZ,XFDOT,SCALE,XF,YF,ZF
REAL*8 MISS,TOL,DDY,DDZ,OLDTGO
INTEGER I,CFLAG

```

```

C INITIALIZE VARIABLES
  DELTAY=0.0
  DELTAZ=0.0
  OLDTGO=TGO

C BEGIN SEARCH LOOP
  DO 10 I=1,10

C COMPUTE FINAL POSITIONS AND MISS DISTANCE
  XF=((AX*TGO+BX)*TGO+CX)*TGO+DX
  YF=((AY*TGO+BY)*TGO+CY-DELTAY)*TGO+DY
  ZF=((AZ*TGO+BZ)*TGO+CZ-DELTAZ)*TGO+DZ
  MISS=SQRT(XF*XF+YF*YF+ZF*ZF)

C TEST FOR NEARNESS
  IF (MISS .LE. TOL) RETURN

C COMPUTE CHANGE IN TIME-TO-GO
  XFDOT=(AX*(TGO+TGO+TGO)+BX+BX)*TGO+CX
  DTGO=-XF/XFDOT
  TGO=TGO+DTGO

C COMPUTE CHANGES IN VELOCITIES
  YF=((AY*TGO+BY)*TGO+CY-DELTAY)*TGO+DY
  ZF=((AZ*TGO+BZ)*TGO+CZ-DELTAZ)*TGO+DZ
  DDY=(AY*TGO+BY)*TGO+CY-DELTAY+DY/TGO
  DELTAY=DELTAY+DDY
  DDZ=(AZ*TGO+BZ)*TGO+CZ-DELTAZ+DZ/TGO
  DELTAZ=DELTAZ+DDZ

C TEST FOR CONVERGENCE OF FINAL TIME
  IF (ABS(DTGO) .LT. (.000001*TGO)) RETURN

```

```

10    CONTINUE

C    INCREMENT CONVERGENCE FLAG
      CFLAG=CFLAG+1

C    ZERO OUT VELOCITY CHANGES AND RESET TIME
      DELTAY=0.0
      DELTAZ=0.0
      TGO=OLDTGO

      END

      SUBROUTINE SEARCHC(XR,VXE,VYE,VZE,A,MDOT,
+                       TGO,DELTAY,DELTAZ,TOL,CFLAG)

C    THIS SUBROUTINE DETERMINES TIME-TO-GO AND VELOCITY
C    CHANGES FOR PLAN C WITH MISS DISTANCE WITHIN SOME
C    TOLERANCE.
C    THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON
C    SEARCH SCHEME (PP. 48-53 OF MARON, 'NUMERICAL
C    ANALYSIS, A PRACTICAL APPROACH')
C    ONCE THE TIME-TO-GO IS KNOWN THE VELOCITY CHANGES
C    (DELTAY AND DELTAZ) ARE COMPUTED.

      REAL*8 DTGO,XR(6),VXE,VYE,VZE,A,MDOT,DA,VE
      REAL*8 TF,DELTAY,DELTAZ,XF,YF,ZF,XFDOT,SCALE
      REAL*8 OLDTGO,VX,VY,VZ,MISS,TGO,TOL,MT
      INTEGER I,J,CFLAG

C    COMPUTE EVADER UNITIZED VELOCITY VECTOR COMPONENTS
      VE=SQRT(VXE*VXE+VYE*VYE+VZE*VZE)
      VX=VXE/VE
      VY=VYE/VE
      VZ=VZE/VE

C    INITIALIZE VARIABLES
      DELTAY=0.0
      DELTAZ=0.0
      OLDTGO=TGO

C    COMPUTE DISTANCE DUE TO BOOSTER ACCELERATION
      DA=0.0
      MT=MDOT*TGO
      DO 15 J=2,50
         DA=DA+MT**J/(J*(J-1))
15    CONTINUE
      DA=DA*A/(MDOT*MDOT)

C    BEGIN SEARCH LOOP
      DO 30 I=1,10

```

```

C COMPUTE FINAL RELATIVE POSITION AND MISS DISTANCE
  XF=XR(1)+XR(2)*TGO+DA*VX
  YF=XR(3)+(XR(4)-DELTAY)*TGO+DA*VY
  ZF=XR(5)+(XR(6)-DELTAZ)*TGO+DA*VZ
  MISS=SQRT(XF*XF+YF*YF+ZF*ZF)

C TEST FOR NEARNESS
  IF (MISS .LE. TOL) RETURN

C COMPUTE CHANGE IN TIME-TO-GO
  XFDOT=XR(2)+DA*VX/TGO
  DTGO=-XF/XFDOT
  TGO=TGO+DTGO

C COMPUTE CHANGES IN VELOCITIES
  DA=0.0
  MT=MDOT*TGO
  DO 25 J=2,50
    DA=DA+MT**J/(J*(J-1))
25  CONTINUE
    DA=DA*A/(MDOT*MDOT)
    DELTAY=(DA*VY+XR(3))/TGO+XR(4)
    DELTAZ=(DA*VZ+XR(5))/TGO+XR(6)

C TEST FOR CONVERGENCE OF TIME-TO-GO
  IF (ABS(DTGO) .LT. (.000001*TGO)) RETURN

30  CONTINUE

C INCREMENT CONVERGENCE FLAG
  CFLAG=CFLAG+1

C ZERO OUT VELOCITY CHANGES AND RESET TIME
  DELTAY=0.0
  DELTAZ=0.0
  TGO=OLDTGO

  END

```

```

SUBROUTINE SEARCHCC(AX,BX,CX,DX,AY,BY,CY,DY,
+ AZ,BZ,CZ,DZ,ASIGX,BSIGX,CSIGX,DSIGX,ASIGY,
+ BSIGY,CSIGY,DSIGY,ASIGZ,BSIGZ,CSIGZ,DSIGZ,
+ K,TGO,DELTAY,DELTAZ,TOL,CFLAG)

```

```

C THIS SUBROUTINE NUMERICALLY MINIMIZES THE
C COST FUNCTION:
C   L = (DELTAX**2 + DELTAY**2)/2
C SUBJECT TO THE CONSTRAINT :
C   XF*XF+YF*YF+ZF*ZF <= K*(SIGMAF*SIGMAF)

```



C BY VARYING THE TIME-TO-GO (TGO) AND THE VELOCITY  
 C CHANGES (DELTAY AND DELTAZ).  
 C XF,YF AND ZF ARE COMPUTED FROM THE SPLINE  
 C COEFFICIENTS AX,BX,...,DZ AT THE FINAL TIME,  
 C AS ARE THE FINAL SIGMAS.  
 C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON  
 C SEARCH SCHEME FOR NON-LINEAR SYSTEMS.  
 C (PP. 176-179 OF MARON, 'NUMERICAL ANALYSIS,  
 C A PRACTICAL APPROACH')

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REAL*8 AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ
REAL*8 DTGO,TLIM,SCALE,DEN,OLDTGO,KSIG
REAL*8 K,TGO,DELTAY,DELTAZ,XF,YF,ZF,F1,F2,F3
REAL*8 XFDOT,YFDOT,ZFDOT,J11,J12,J21,J22,TGO3
REAL*8 ASIGX,BSIGX,CSIGX,DSIGX,ASIGY,BSIGY
REAL*8 CSIGY,DSIGY,DLAM,PERT,MISS,F1T,F2T,F1L,F2L
REAL*8 TOL,ASIGZ,BSIGZ,CSIGZ,DSIGZ,LAMBDA
REAL*8 SIGX,SIGY,SIGZ,SIGXDT,SIGYDT,SIGZDT
REAL*8 DYDL,DZDL,DYDSDL,DZDSDL,TGO6,YS,ZS
REAL*8 SIGXDD,SIGYDD,SIGZDD,XFDD,YFDD,ZFDD
INTEGER I,J,CFLAG

```

C INITIALIZE PARAMETERS

```

PERT=0.0000001
OLDTGO=TGO
TLIM=0.01*TGO
DELTAY=0.0
DELTAZ=0.0

```

C DETERMINE IF CONSTRAINT CAN BE SATISFIED

C WITHOUT THRUSTING  
 DO 5 I=1,10

C COMPUTE FINAL RELATIVE POSITIONS AND DEVIATIONS

```

XF=((AX*TGO+BX)*TGO+CX)*TGO+DX
YF=((AY*TGO+BY)*TGO+CY)*TGO+DY
ZF=((AZ*TGO+BZ)*TGO+CZ)*TGO+DZ
SIGX=((ASIGX*TGO+BSIGX)*TGO+CSIGX)*TGO+DSIGX
SIGY=((ASIGY*TGO+BSIGY)*TGO+CSIGY)*TGO+DSIGY
SIGZ=((ASIGZ*TGO+BSIGZ)*TGO+CSIGZ)*TGO+DSIGZ

```

C COMPUTE FINAL RELATIVE VELOCITIES

C AND DEVIATION DERIVATIVES

```

TGO3=TGO+TGO+TGO
XFDOT=(AX*TGO3+BX+BX)*TGO+CX
YFDOT=(AY*TGO3+BY+BY)*TGO+CY
ZFDOT=(AZ*TGO3+BZ+BZ)*TGO+CZ
SIGXDT=(ASIGX*TGO3+BSIGX+BSIGX)*TGO+CSIGX
SIGYDT=(ASIGY*TGO3+BSIGY+BSIGY)*TGO+CSIGY
SIGZDT=(ASIGZ*TGO3+BSIGZ+BSIGZ)*TGO+CSIGZ

```

```

C COMPUTE NONLINEAR SYSTEM EQUATIONS
  F1=(XF*XF+YF*YF+ZF*ZF-K*(SIGX*SIGX+
+   SIGY*SIGY+SIGZ*SIGZ))/2.0
C DETERMINE IF CONSTRAINT IS SATISFIED
  IF (F1 .LT. TOL) RETURN
  F2=XF*XFDOT+YF*YFDOT+ZF*ZFDOT-K*(SIGX*SIGXDT+
+   SIGY*SIGYDT+SIGZ*SIGZDT)
  F3=XFDOT*XFDOT+YFDOT*YFDOT+ZFDOT*ZFDOT-K*
+   (SIGXDT*SIGXDT+SIGYDT*SIGYDT+SIGZDT*SIGZDT)

C COMPUTE NEW TGO
  DTGO=-F2/F3
  TGO=TGO+DTGO

C TEST FOR CONVERGENCE OF FINAL TIME
  IF (ABS(DTGO) .LT. (TOL*TLIM)) GOTO 6

5 CONTINUE
6 CONTINUE

C INITIALIZE LAMBDA
  MISS=SQRT(XF*XF+YF*YF+ZF*ZF)
  KSIG=SQRT(K*(SIGX*SIGX+SIGY*SIGY+SIGZ*SIGZ))
  LAMBDA=(1.0-KSIG/MISS)/TGO/TGO

C BEGIN SEARCH LOOP
  DO 15 I=1,20

C COMPUTE VELOCITY CHANGES
  YS=((AY*TGO+BY)*TGO+CY)*TGO+DY
  ZS=((AZ*TGO+BZ)*TGO+CZ)*TGO+DZ
  DEN=1.0+LAMBDA*TGO*TGO
  DELTAY=YS*TGO*LAMBDA/DEN
  DELTAZ=ZS*TGO*LAMBDA/DEN

C COMPUTE FINAL RELATIVE POSITIONS AND DEVIATIONS
  XF=((AX*TGO+BX)*TGO+CX)*TGO+DX
  YF=YS-DELTAY*TGO
  ZF=ZS-DELTAZ*TGO
  SIGX=((ASIGX*TGO+BSIGX)*TGO+CSIGX)*TGO+DSIGX
  SIGY=((ASIGY*TGO+BSIGY)*TGO+CSIGY)*TGO+DSIGY
  SIGZ=((ASIGZ*TGO+BSIGZ)*TGO+CSIGZ)*TGO+DSIGZ

C COMPUTE FINAL RELATIVE VELOCITIES
AND DEVIATION DERIVATIVES
  TGO3=TGO+TGO+TGO
  XFDOT=(AX*TGO3+BX+BX)*TGO+CX
  YFDOT=(AY*TGO3+BY+BY)*TGO+CY-DELTAY
  ZFDOT=(AZ*TGO3+BZ+BZ)*TGO+CZ-DELTAZ
  SIGXDT=(ASIGX*TGO3+BSIGX+BSIGX)*TGO+CSIGX
  SIGYDT=(ASIGY*TGO3+BSIGY+BSIGY)*TGO+CSIGY
  SIGZDT=(ASIGZ*TGO3+BSIGZ+BSIGZ)*TGO+CSIGZ

```

```

C  COMPUTE NONLINEAR SYSTEM EQUATIONS
      F1=(XF*XF+YF*YF+ZF*ZF-K*(SIGX*SIGX+
+       SIGY*SIGY+SIGZ*SIGZ))/2.0
      F2=XF*XFDOT+YF*YFDOT+ZF*ZFDOT-K*
+       (SIGX*SIGXDT+SIGY*SIGYDT+SIGZ*SIGZDT)

C  TEST FOR CONVERGENCE OF CONSTRAINTS
      IF ((ABS(F1) .LT. TOL) .AND.
+       (ABS(F2) .LT. TOL)) RETURN

C  COMPUTE NECESSARY PARTIALS
      DYDTDL=-YF*TGO/DEN
      DZDTDL=-ZF*TGO/DEN
      DYDL=DYDTDL*TGO
      DZDL=DZDTDL*TGO
      TGO6=TGO3+TGO3
      XFDD=AX*TGO6+BX+BX
      YFDD=AY*TGO6+BY+BY-LAMBDA*(YFDOT*TGO+YF)
      ZFDD=AZ*TGO6+BZ+BZ-LAMBDA*(ZFDOT*TGO+ZF)
      SIGXDD=ASIGX*TGO6+BSIGX+BSIGX
      SIGYDD=ASIGY*TGO6+BSIGY+BSIGY
      SIGZDD=ASIGZ*TGO6+BSIGZ+BSIGZ

C  COMPUTE NECESSARY ELEMENTS OF JACOBIAN MATRIX
      J11=F2
      J12=YF*DYDL+ZF*DZDL
      J21=XF*XFDD+XFDOT*XFDOT+YF*YFDD+YFDOT*YFDOT+
+       ZF*ZFDD+ZFDOT*ZFDOT
      J21=J21-K*(SIGX*SIGXDD+SIGXDT*SIGXDT+
+       SIGY*SIGYDD+SIGYDT*SIGYDT+SIGZ*SIGZDD+
+       SIGZDT*SIGZDT)
      J22=DYDL*YFDOT+YF*DYDTDL+DZDL*ZFDOT+ZF*DZDTDL

C  COMPUTE CHANGES IN CONTROL PARAMETERS
      DEN=(J11*J22-J12*J21)
      DTGO=(-F1*J22+F2*J12)/DEN
      DLAM=(F1*J21-F2*J11)/DEN
      SCALE=ABS(DTGO/TLIM)
      IF (SCALE .GT. 1.0) THEN
          DTGO=DTGO/SCALE
          DLAM=DLAM/SCALE
      ENDIF

C  UPDATE CONTROL PARAMETERS
      TGO=TGO+DTGO
      LAMBDA=LAMBDA+DLAM

15  CONTINUE

C  INCREMENT CONVERGENCE FLAG
      CFLAG=CFLAG+1

```

```

C ZERO OUT VELOCITY CHANGES AND RESET TIME
  DELTAY=0.0
  DELTAZ=0.0
  TGO=OLDTGO

  END

```

```

SUBROUTINE SEARCHD(AX,BX,CX,DX,AY,BY,CY,DY,AZ,
+ BZ,CZ,DZ,K,TGO,DELTAY,DELTAZ,TOL,CFLAG,XDUALD,
+ XEDD,XPDD,CVDD,MAXDV,COUNT,Q,R3,SIGMAM,H)

```

```

C THIS SUBROUTINE NUMERICALLY MINIMIZES THE
C COST FUNCTION:
C 
$$L = E\{K*(XF**2+XY**2+XZ**2)/2 +$$

C 
$$(DELTAX**2 + DELTAY**2)/2\}$$

C BY VARYING THE TIME-TO-GO (TGO) AND THE VELOCITY
C CHANGES (DELTAY AND DELTAZ) TO BRING THE MISS
C DISTANCE WITHIN SOME TOLERANCE.
C XF,YF AND ZF ARE COMPUTED FROM THE SPLINE
C COEFFICIENTS AX,BX,...,DZ AT THE FINAL TIME.

```

```

REAL*8 AX,BX,CX,DX,AY,BY,CY,DY,AZ,BZ,CZ,DZ
REAL*8 K,TGO,DELTAY,DELTAZ,DH,Q(8,8),RES(3)
REAL*8 DVY,DVZ,XDUALD(8),CVDD(8,8),XPDD(6)
REAL*8 MISS,TOL,XEDD(6),R3(3,3),TD,RANGE,DVYA
REAL*8 SIGMAM(3),COST1,COST2,CVTOT,DVZA
REAL*8 XDUAL2(8),CVD2(8,8),XED2(6),XPD2(6)
REAL*8 CVDY(8,8),CVDZ(8,8),XEDY(6),XEDZ(6)
REAL*8 XPDY(6),XPDZ(6),XDUALY(8),XDUALZ(8)
REAL*8 XF,YF,ZF,XFDOT,YFDOT,ZFDOT,MAXDV,DTGO
REAL*8 CVDR(8,8),XEDR(6),XPDR(6),XDUALR(8)
REAL*8 DEN,CVDIFY,CVDIFZ,RADICL,RTGO
INTEGER I,J,CFLAG,COUNT,KFLAG

```

```

C COMPUTE CERTAINTY EQUIVALENCE SOLUTION
  CALL SEARCHA(AX,BX,CX,DX,AY,BY,CY,DY,
+ AZ,BZ,CZ,DZ,K,TGO,DELTAY,DELTAZ,TOL,CFLAG)

```

```

C ASSIGN DUMMY FILTER VARIABLES
  DO 100 I=1,8
    XDUAL2(I)=XDUALD(I)
    XDUALY(I)=XDUALD(I)
    XDUALZ(I)=XDUALD(I)
    XDUALR(I)=XDUALD(I)
  DO 100 J=1,8
    CVD2(I,J)=CVDD(I,J)
    CVDY(I,J)=CVDD(I,J)
    CVDZ(I,J)=CVDD(I,J)
    CVDR(I,J)=CVDD(I,J)
100 CONTINUE

```

```

DO 110 I=1,6
  XED2(I)=XEDD(I)
  XEDY(I)=XEDD(I)
  XEDZ(I)=XEDD(I)
  XEDR(I)=XEDD(I)
  XPD2(I)=XPDD(I)
  XPDY(I)=XPDD(I)
  XPDZ(I)=XPDD(I)
  XPDR(I)=XPDD(I)
110  CONTINUE

C  DETERMINE BEST DIRECTION TO IMPROVE THE ESTIMATE
  DVY=MAXDV
  DVZ=MAXDV
  XPDY(4)=XPDY(4)+DVY
  XPDZ(6)=XPDZ(6)+DVZ
  XDUALY(4)=XDUALY(4)-DVY
  XDUALZ(6)=XDUALZ(6)-DVZ

  DH=TGO/COUNT
  DO 300 I=1,COUNT
    RANGE=SQRT(XDUALR(1)*XDUALR(1)+
+     XDUALR(3)*XDUALR(3)+XDUALR(5)*XDUALR(5))
    R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
    CALL EKF8(XDUALR,XEDR,XPDR,TD,DH,CVDR,Q,R3,
+     0.0,0.0,0.0,KFLAG,RES,2)
    RANGE=SQRT(XDUALY(1)*XDUALY(1)+
+     XDUALY(3)*XDUALY(3)+XDUALY(5)*XDUALY(5))
    R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
    CALL EKF8(XDUALY,XEDY,XPDY,TD,DH,CVDY,Q,R3,
+     0.0,0.0,0.0,KFLAG,RES,2)
    RANGE=SQRT(XDUALZ(1)*XDUALZ(1)+
+     XDUALZ(3)*XDUALZ(3)+XDUALZ(5)*XDUALZ(5))
    R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
    CALL EKF8(XDUALZ,XEDZ,XPDZ,TD,DH,CVDZ,Q,R3,
+     0.0,0.0,0.0,KFLAG,RES,2)
300  CONTINUE

  CVTOT=CVDR(1,1)+CVDR(3,3)+CVDR(5,5)
  CVDIFY=CVTOT-CVDY(1,1)-CVDY(3,3)-CVDY(5,5)
  CVDIFZ=CVTOT-CVDZ(1,1)-CVDZ(3,3)-CVDZ(5,5)
  DEN=CVDIFY*CVDIFY+CVDIFZ*CVDIFZ
  IF (DEN .GT. 1.0E-20) THEN
    DEN=SQRT(DEN)
    DVY=DVY*CVDIFY/DEN
    DVZ=DVZ*CVDIFZ/DEN
  ELSE
    RETURN
  ENDIF

C  ENSURE C.E. SOLUTION IS WITHIN THRUST LIMITS
  RTGO=TGO-H
  RADICL=RTGO*RTGO-(DELTAY+DELTAY)*RTGO*H/MAXDV

```

```

IF (RADICL .LE. 0.0) THEN
  DELTAY=MAXDV*INT(RTGO/H)/2.0
  COST1=(DELTAY+DELTAY)**2
ELSE
  COST1=((RTGO-SQRT(RADICL))*MAXDV/H)**2
ENDIF
RADICL=RTGO*RTGO-(DELTAY+DELTAY)*RTGO*H/MAXDV
IF (RADICL .LE. 0.0) THEN
  DELTAZ=MAXDV*INT(RTGO/H)/2.0
  COST1=COST1+(DELTAY+DELTAY)**2
ELSE
  COST1=COST1+((RTGO-SQRT(RADICL))*MAXDV/H)**2
ENDIF

```

C COMPUTE COST OF CERTAINTY EQUIVALENCE SOLUTION

```

XDUALD(4)=XDUALD(4)-DELTAY
XDUALD(6)=XDUALD(6)-DELTAY
XPDD(4)=XPDD(4)+DELTAY
XPDD(6)=XPDD(6)+DELTAY
TD=0.0
DH=H
RANGE=SQRT(XDUALD(1)*XDUALD(1)+
+ XDUALD(3)*XDUALD(3)+XDUALD(5)*XDUALD(5))
R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
CALL EKF8(XDUALD,XEDD,XPDD,TD,DH,CVDD,Q,R3,
+ 0.0,0.0,0.0,KFLAG,RES,2)
DTGO=TGO-DH
DH=DTGO/COUNT
DO 200 I=1,COUNT
  RANGE=SQRT(XDUALD(1)*XDUALD(1)+
+ XDUALD(3)*XDUALD(3)+XDUALD(5)*XDUALD(5))
  R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
  CALL EKF8(XDUALD,XEDD,XPDD,TD,DH,CVDD,Q,R3,
+ 0.0,0.0,0.0,KFLAG,RES,2)
200 CONTINUE
CVTOT=CVDD(1,1)+CVDD(3,3)+CVDD(5,5)
COST1=COST1+K*(CVTOT+XDUALD(1)*XDUALD(1)+
+ XDUALD(3)*XDUALD(3)+XDUALD(5)*XDUALD(5))

```

C DETERMINE COST OF IMPROVING THE ESTIMATE

```

XPD2(4)=XPD2(4)+DVY
XPD2(6)=XPD2(6)+DVZ
XDUAL2(4)=XDUAL2(4)-DVY
XDUAL2(6)=XDUAL2(6)-DVZ

```

C MOVE FORWARD ONE STEP

```

DH=H
RANGE=SQRT(XDUAL2(1)*XDUAL2(1)+
+ XDUAL2(3)*XDUAL2(3)+XDUAL2(5)*XDUAL2(5))
R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
CALL EKF8(XDUAL2,XED2,XPD2,TD,DH,CVD2,Q,R3,
+ 0.0,0.0,0.0,KFLAG,RES,2)

```

```

C      COMPUTE NEW SPLINES
      CY=CY-DVY
      CZ=CZ-DVZ
      XF=((AX*TGO+BX)*TGO+CX)*TGO+DX
      YF=((AY*TGO+BY)*TGO+CY)*TGO+DY
      ZF=((AZ*TGO+BZ)*TGO+CZ)*TGO+DZ
      XFDOT=(3.0*AX*TGO+BX+BX)*TGO+CX
      YFDOT=(3.0*AY*TGO+BY+BY)*TGO+CY
      ZFDOT=(3.0*AZ*TGO+BZ+BZ)*TGO+CZ
      DTGO=TGO-H
      CALL SPLINE(XDUAL2(1),XDUAL2(2),XF,XFDOT,
+              DTGO,AX,BX,CX,DX)
      CALL SPLINE(XDUAL2(3),XDUAL2(4),YF,YFDOT,
+              DTGO,AY,BY,CY,DY)
      CALL SPLINE(XDUAL2(5),XDUAL2(6),ZF,ZFDOT,
+              DTGO,AZ,BZ,CZ,DZ)

C      COMPUTE NEW CERTAINTY EQUIVALENCE SOLUTION
      CALL SEARCHA(AX,BX,CX,DX,AY,BY,CY,DY,
+      AZ,BZ,CZ,DZ,K,DTGO,DVYA,DVZA,TOL,CFLAG)

C      ENSURE NEW C.E. SOLUTION IS WITHIN THRUST LIMITS
      RTGO=DTGO-H
      RADICL=RTGO*RTGO-(DVYA+DVYA)*RTGO*H/MAXDV
      IF (RADICL.LE. 0.0) THEN
          DVYA=MAXDV*INT(RTGO/H)/2.0
          COST2=(DVYA+DVYA)**2
      ELSE
          COST2=((RTGO-SQRT(RADICL))*MAXDV/H)**2
      ENDIF
      RADICL=RTGO*RTGO-(DVZA+DVZA)*RTGO*H/MAXDV
      IF (RADICL.LE. 0.0) THEN
          DVZA=MAXDV*INT(RTGO/H)/2.0
          COST2=COST2+(DVZA+DVZA)**2
      ELSE
          COST2=COST2+((RTGO-SQRT(RADICL))*MAXDV/H)**2
      ENDIF

C      COMPUTE NEXT ITERATION COST
      XDUAL2(4)=XDUAL2(4)-DVYA
      XDUAL2(6)=XDUAL2(6)-DVZA
      XPD2(4)=XPD2(4)+DVYA
      XPD2(6)=XPD2(6)+DVZA
      DH=DTGO/COUNT
      DO 400 I=1,COUNT
          RANGE=SQRT(XDUAL2(1)*XDUAL2(1)+
+              XDUAL2(3)*XDUAL2(3)+XDUAL2(5)*XDUAL2(5))
          R3(1,1)=SIGMAM(1)*RANGE*SIGMAM(1)*RANGE
          CALL EKF8(XDUAL2,XED2,XPD2,TD,DH,CVD2,Q,R3,
+              0.0,0.0,0.0,KFLAG,RES,2)
400  CONTINUE
      CVTOT=CVD2(1,1)+CVD2(3,3)+CVD2(5,5)

```

```

COST2=COST2+K*(CVTOT+XDUAL2(1)*XDUAL2(1)+
+ XDUAL2(3)*XDUAL2(3)+XDUAL2(5)*XDUAL2(5))+
+ DVY*DVY+DVZ*DVZ

```

```

C CHOOSE THE LEAST COST OPTION
  IF (COST1 .GT. COST2) THEN
    PRINT *, 'DEVIATING FROM NOMINAL PATH'
    DELTAY=DVY
    DELTAZ=DVZ
    TGO=DTGO+H
  ENDIF

  END

```

```

SUBROUTINE SEARCHT(XP,XE,A,MDOT,H,T,TGO,
+ DELTAY,DELTAZ,TOL,CFLAG,COUNT)

```

```

C THIS SUBROUTINE VARIES THE TIME-TO-GO (TGO) AND THE
C VELOCITY CHANGES (DELTAY AND DELTAZ) WHILE BRINGING
C MISS DISTANCE WITHIN A PRESPECIFIED TOLERANCE.
C THIS IS ACCOMPLISHED BY EMPLOYING A NEWTON-RAPHSON
C SEARCH SCHEME FOR NON-LINEAR SYSTEMS. (PP. 176-179
C OF MARON, 'NUMERICAL ANALYSIS, A PRACTICAL APPROACH')
C XP,YF,ZF,XFDOT,YFDOT, AND ZFDOT ARE COMPUTED
C NUMERICALLY.

```

```

REAL*8 XP(6),XE(6),XPD(6),XED(6),T,TGO,DELTAY
REAL*8 XF,YF,ZF,SCALE,XFDOT,YFDOT,ZFDOT,TD,DTGO
REAL*8 A,MDOT,TOL,AD,MDOTD,OLDTGO,DH,DELTAZ,H
REAL*8 DDY,DDZ
INTEGER I,J,COUNT,CFLAG

```

```

C INITIALIZE VELOCITY CHANGES
  DELTAY=0.0
  DELTAZ=0.0
  OLDTGO=TGO

```

```

C BEGIN SEARCH LOOP
  DO 10 J=1,10

```

```

C INITIALIZE DUMMY VARIABLES
  DO 110 I=1,6
    XED(I)=XE(I)
    XPD(I)=XP(I)
110 CONTINUE
    XPD(4)=XPD(4)+DELTAY
    XPD(6)=XPD(6)+DELTAZ
    AD=A
    MDOTD=MDOT

```



```
C  PROPAGATE DUMMY VARIABLES FORWARD TO IMPACT TIME
    TD=T
    DH=TGO/COUNT
    DO 115 I=1,COUNT
        CALL RK4SYSP(TD,XPD,DH)
        CALL RK4SYSE(TD,XED,DH,AD,MDOTD)
        TD=TD+H
115  CONTINUE

C  ASSIGN FINAL RELATIVE POSITIONS
    XF=XED(1)-XPD(1)
    YF=XED(3)-XPD(3)
    ZF=XED(5)-XPD(5)
    MISS=SQRT(XF*XF+YF*YF+ZF*ZF)

C  TEST FOR NEARNESS
    IF (MISS .LE. TOL) RETURN

C  ASSIGN FINAL RELATIVE VELOCITIES
    XFDOT=XED(2)-XPD(2)
    YFDOT=XED(4)-XPD(4)
    ZFDOT=XED(6)-XPD(6)

C  COMPUTE CHANGES IN CONTROL PARAMETERS AND UPDATE
    DTGO=-XF/XFDOT
    DDY=(DTGO*YFDOT+YF)/TGO
    DDZ=(DTGO*ZFDOT+ZF)/TGO
    TGO=TGO+DTGO
    DELTAY=DELTAY+DDY
    DELTAZ=DELTAZ+DDZ

C  TEST FOR CONVERGENCE OF TIME-TO-GO
    IF (ABS(DTGO) .LT. (.000001*TGO)) RETURN

10  CONTINUE

C  INCREMENT CONVERGENCE FLAG
    CFLAG=CFLAG+1

C  ZERO OUT VELOCITY CHANGES AND RESET TIME
    DELTAY=0.0
    DELTAZ=0.0
    TGO=OLDTGO

    END
```

## APPENDIX E

## IN-PLANE THRUST PROFILES

This appendix contains the in-plane thrust profiles for Cases I and V, showing the effect of estimate uncertainty on the various control strategies. Each profile was started with the same random seed to form a basis for comparison.

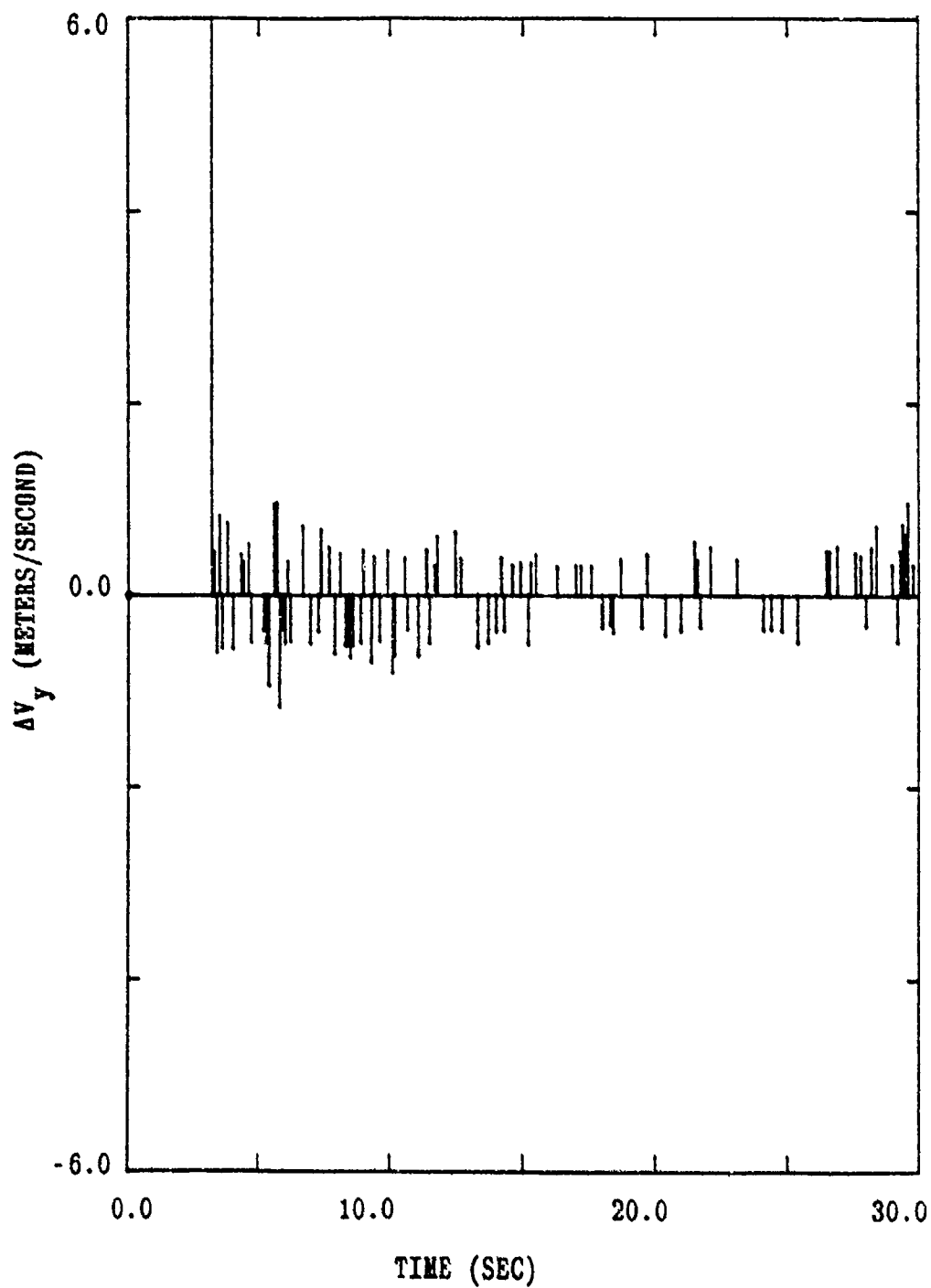


Figure E-1. In-plane thrust profile of Plan A for Case I.

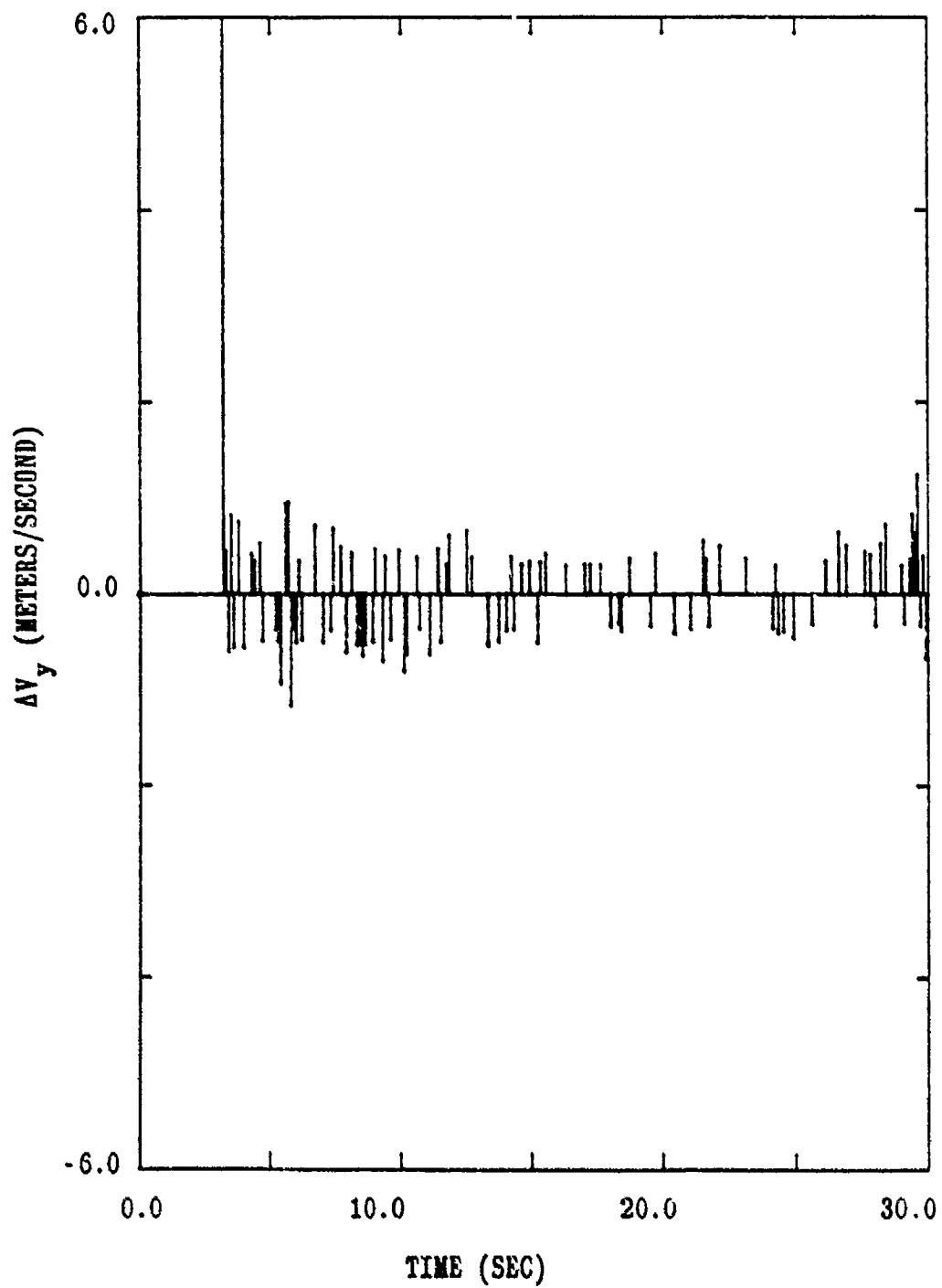


Figure E-2. In-plane thrust profile of Plan B for Case I.

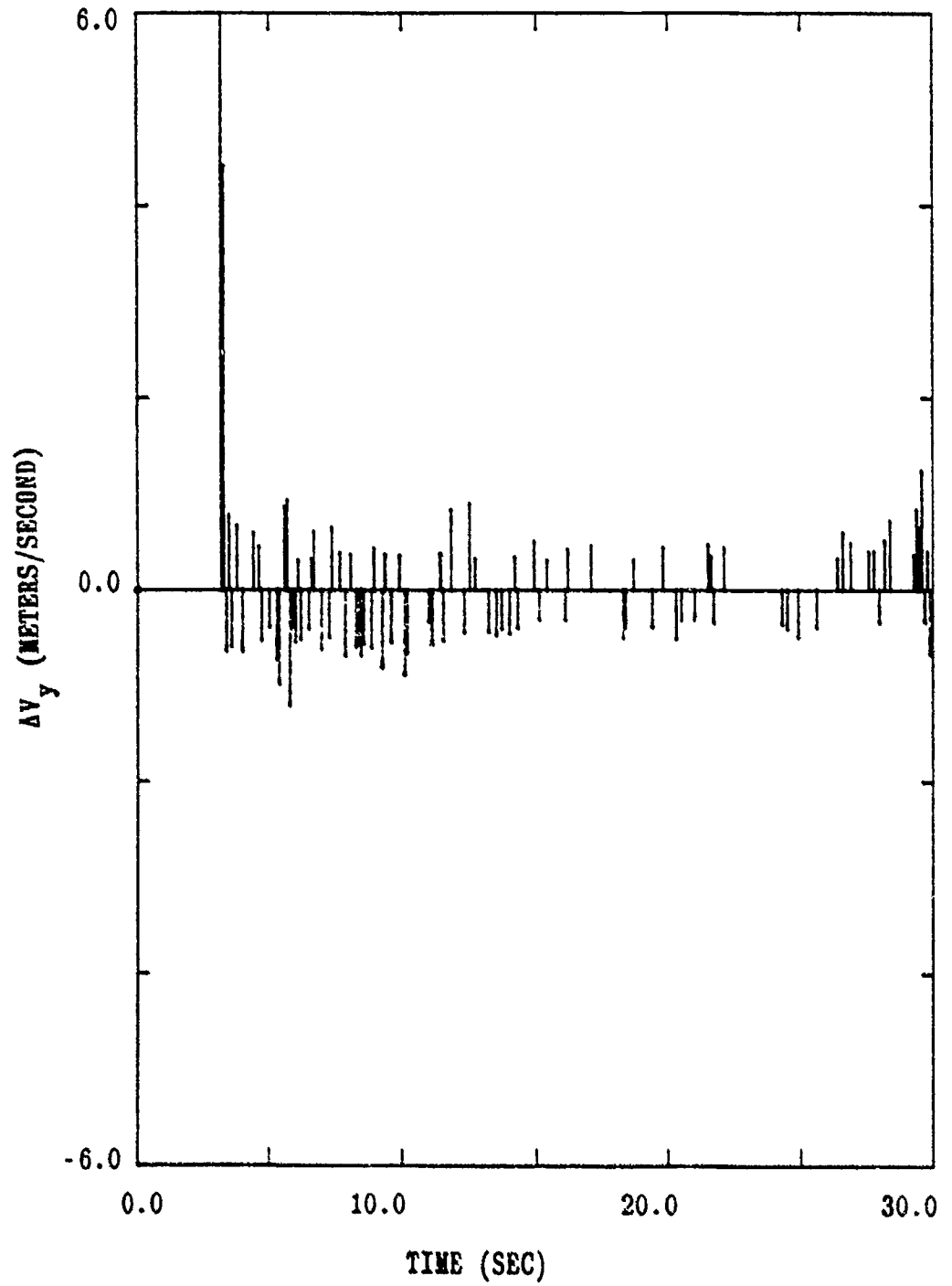


Figure E-3. In-plane thrust profile of Plan C for Case I.

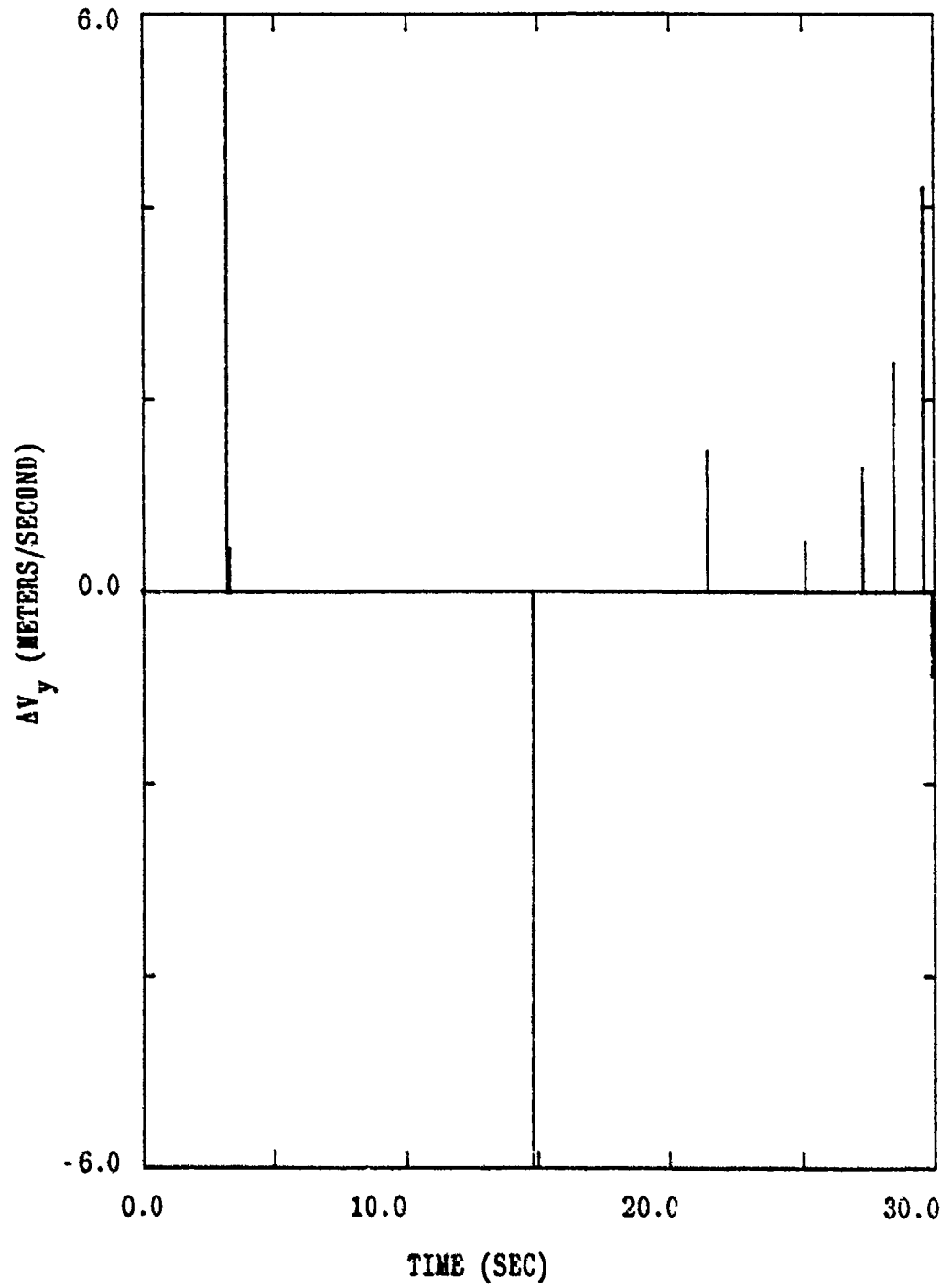


Figure E-4. In-plane Optimum Thrust Spacing profile for Case I.

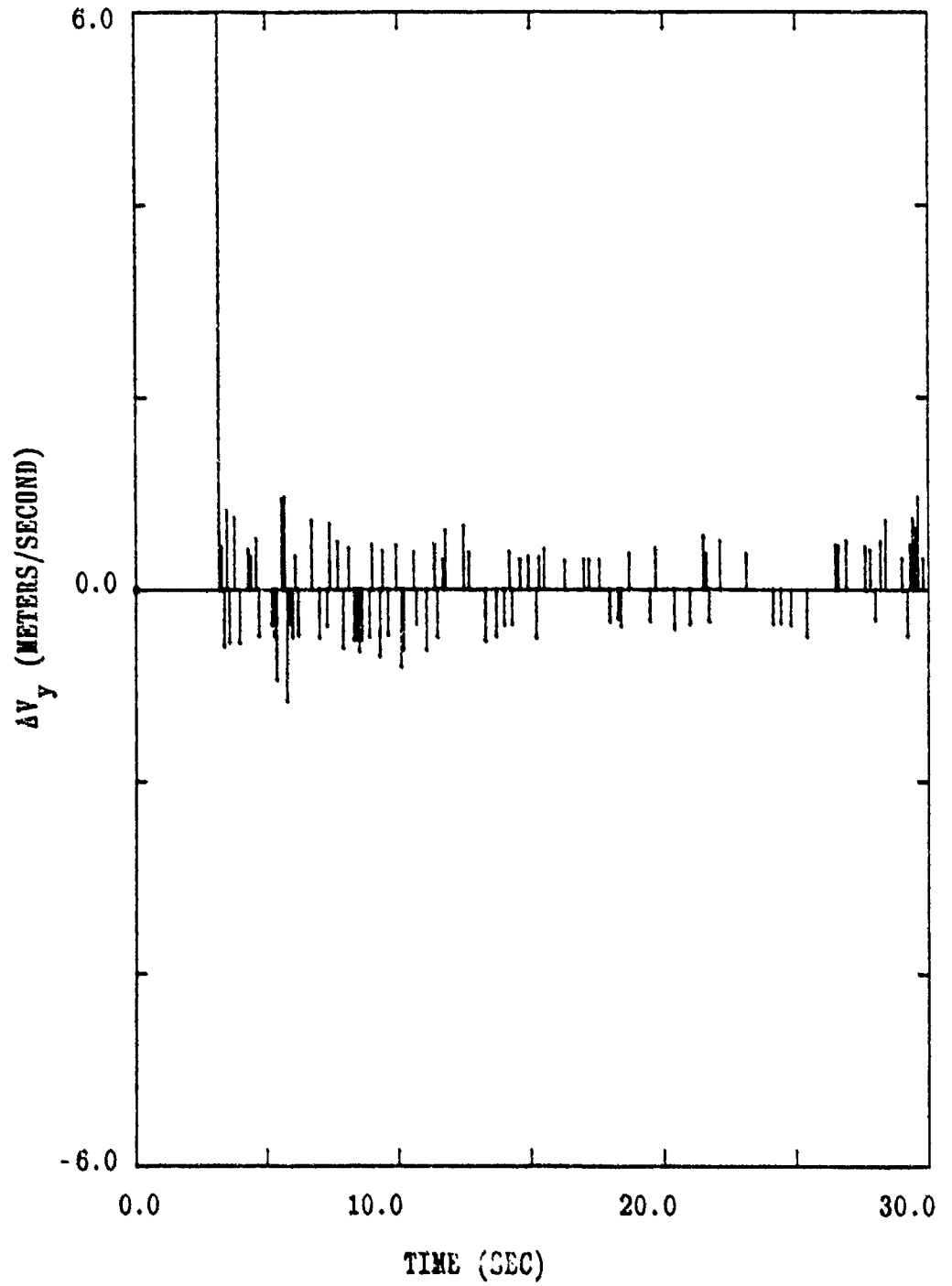


Figure E-5. In-plane thrust profile of Dual Control for Case I.

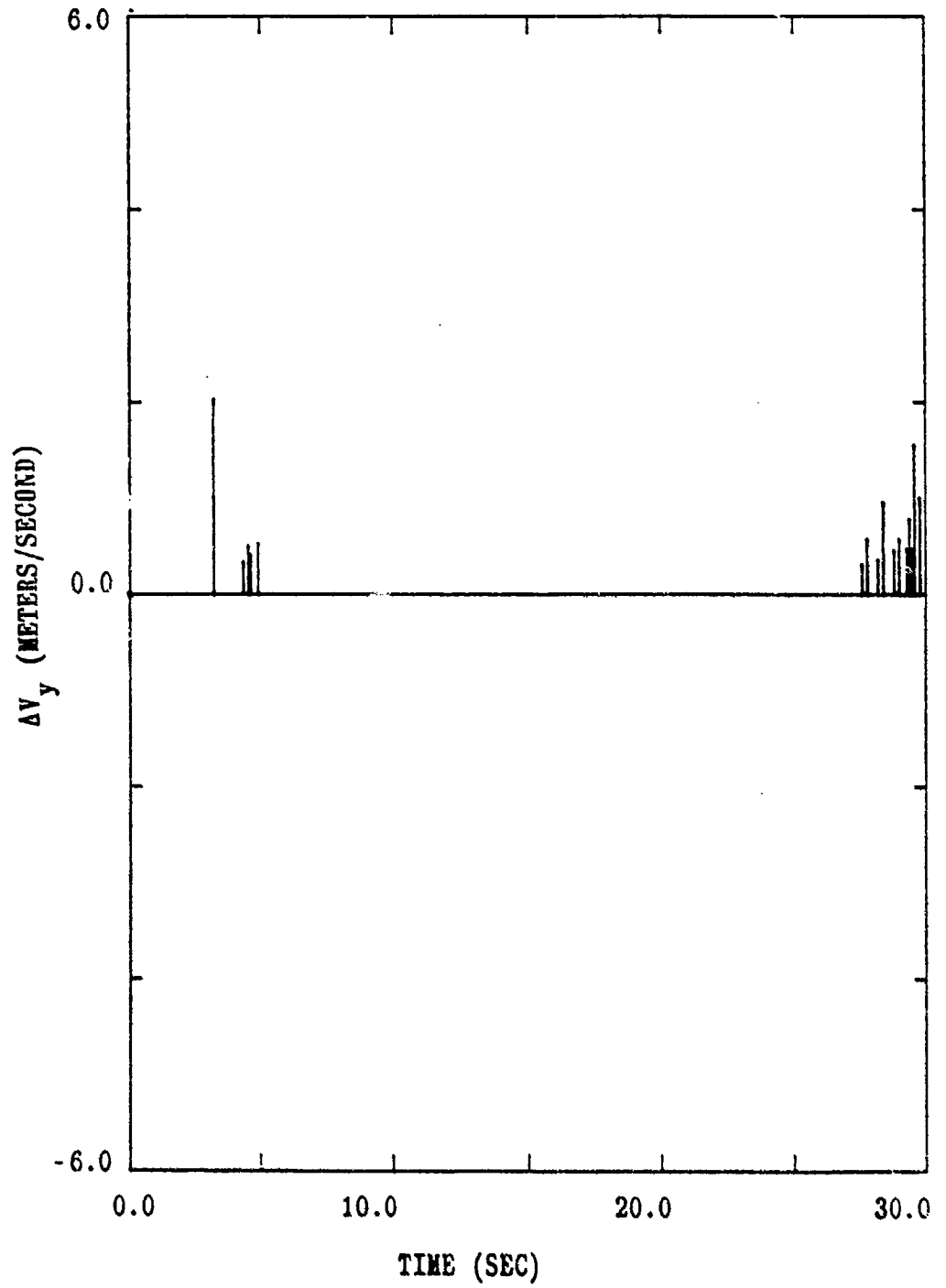


Figure E-6. In-plane thrust profile of Certainty Control for Case I.



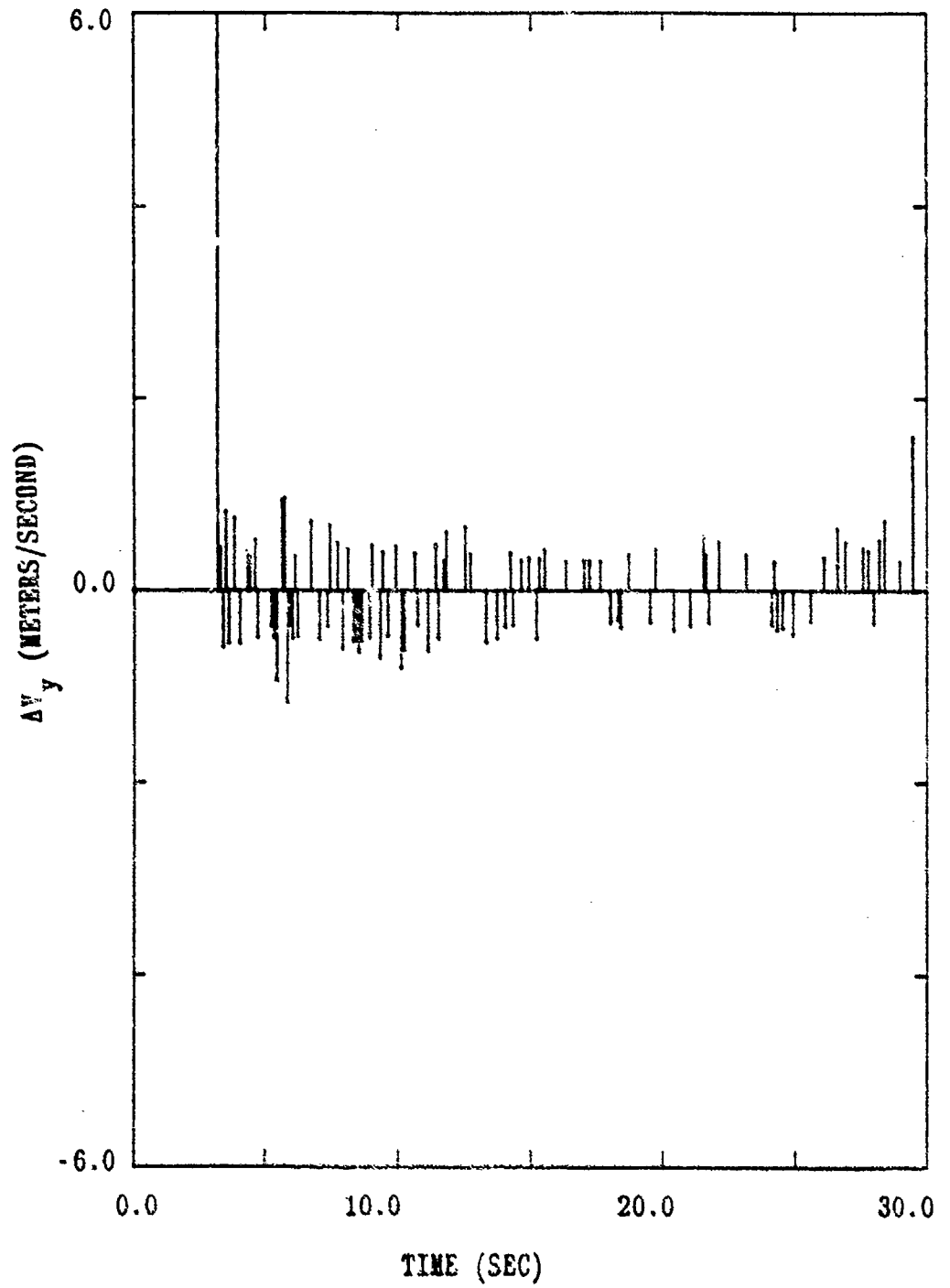


Figure E-7. In-plane thrust profile of Truth Model for Case I.

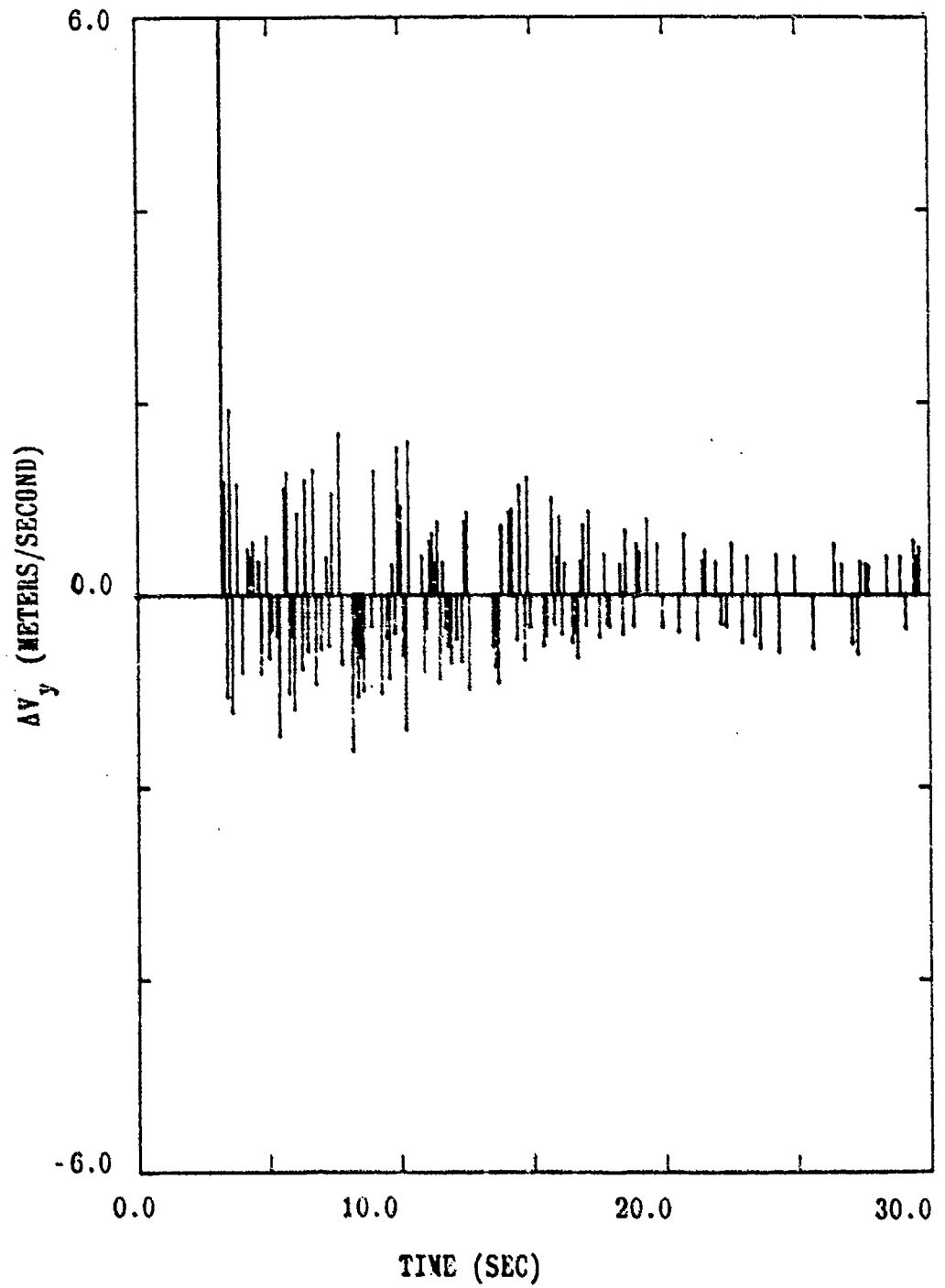


Figure E-8. In-plane thrust profile of Plan A for Case V.

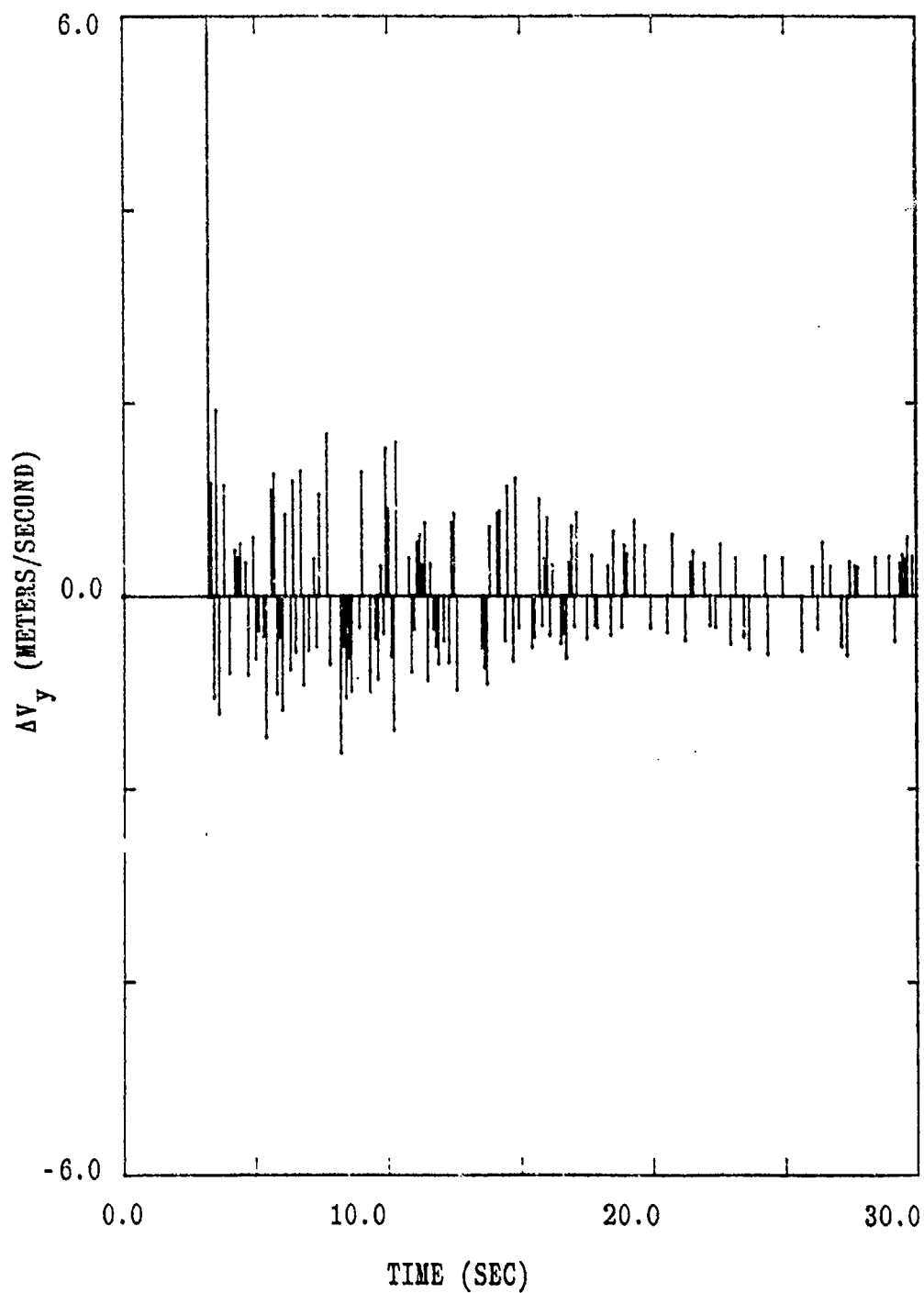


Figure E-9. In-plane thrust profile of Plan B for Case V.

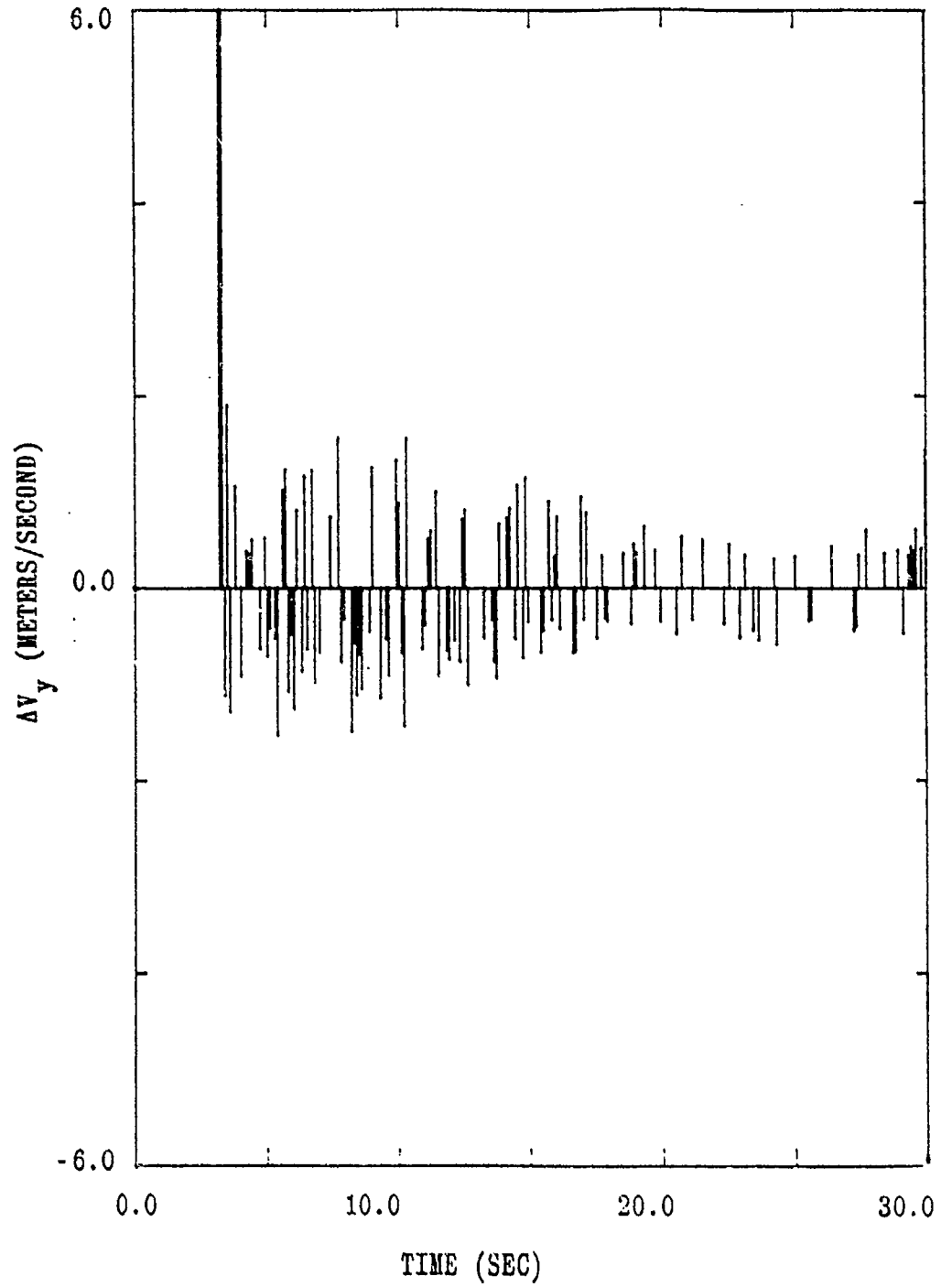


Figure E-10. In-plane thrust profile of Plan C for Case V.

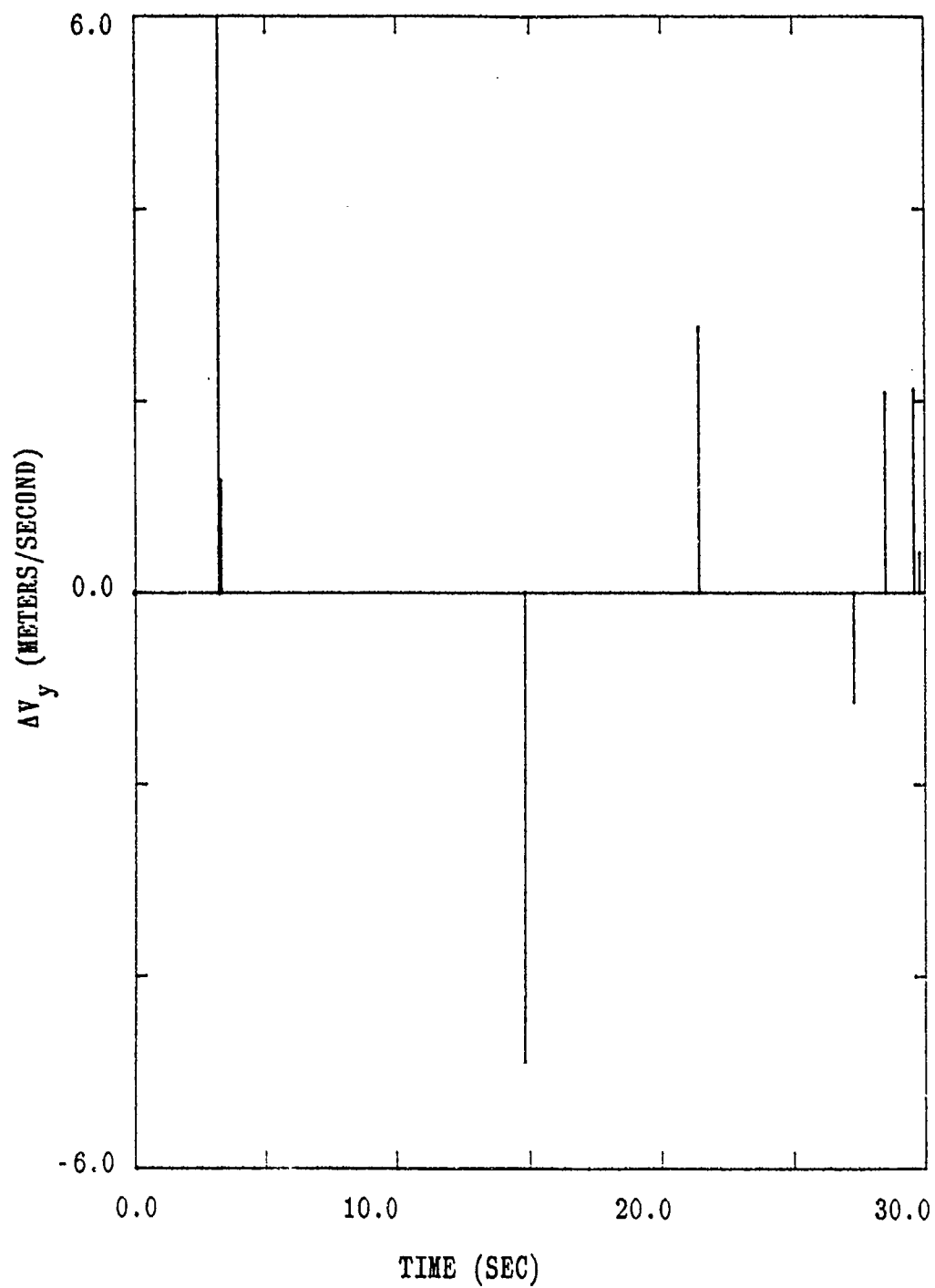


Figure E-11. In-plane Optimum Thrust Spacing profile for Case V.

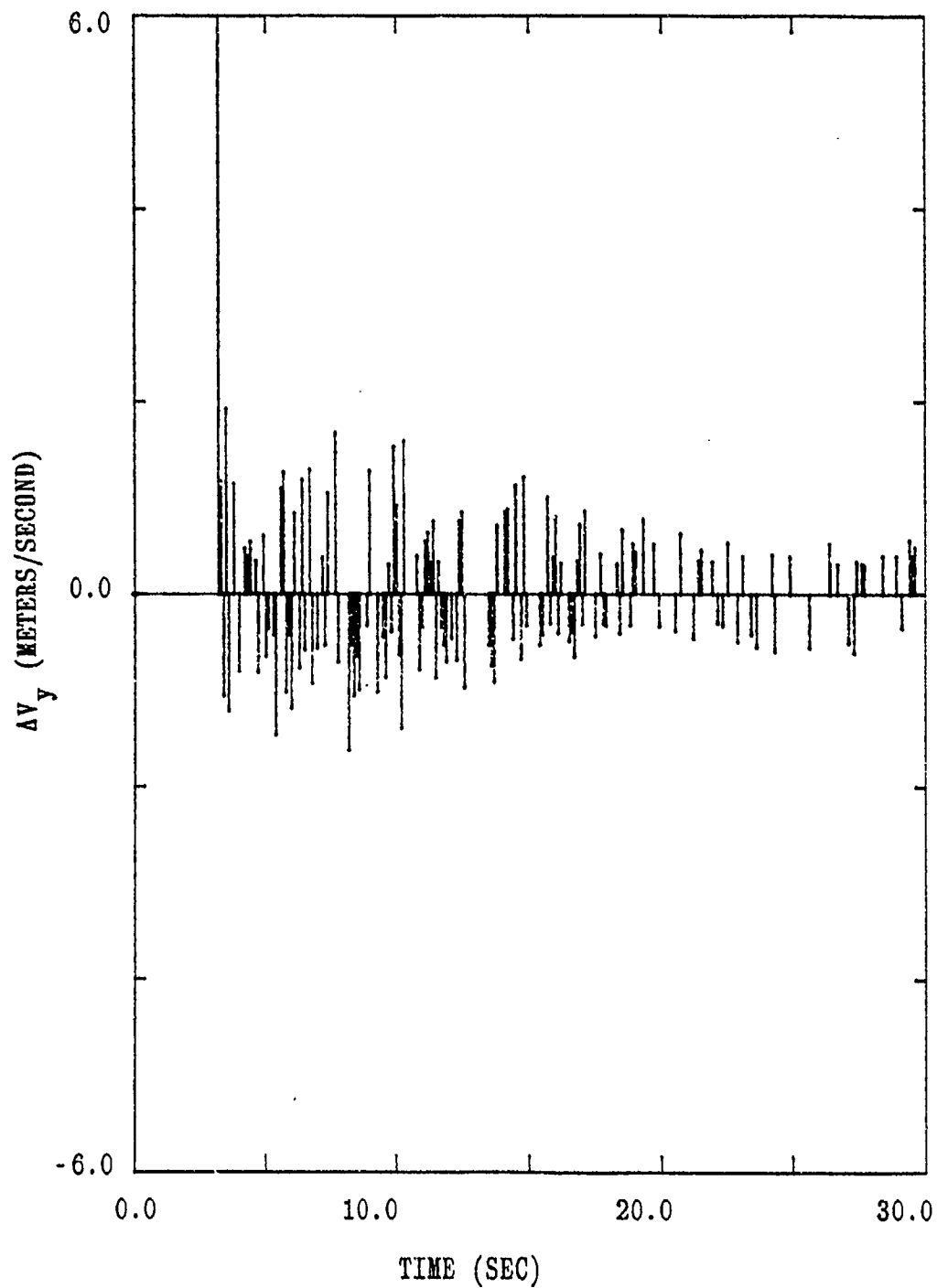


Figure E-12. In-plane thrust profile of Dual Control for Case V.

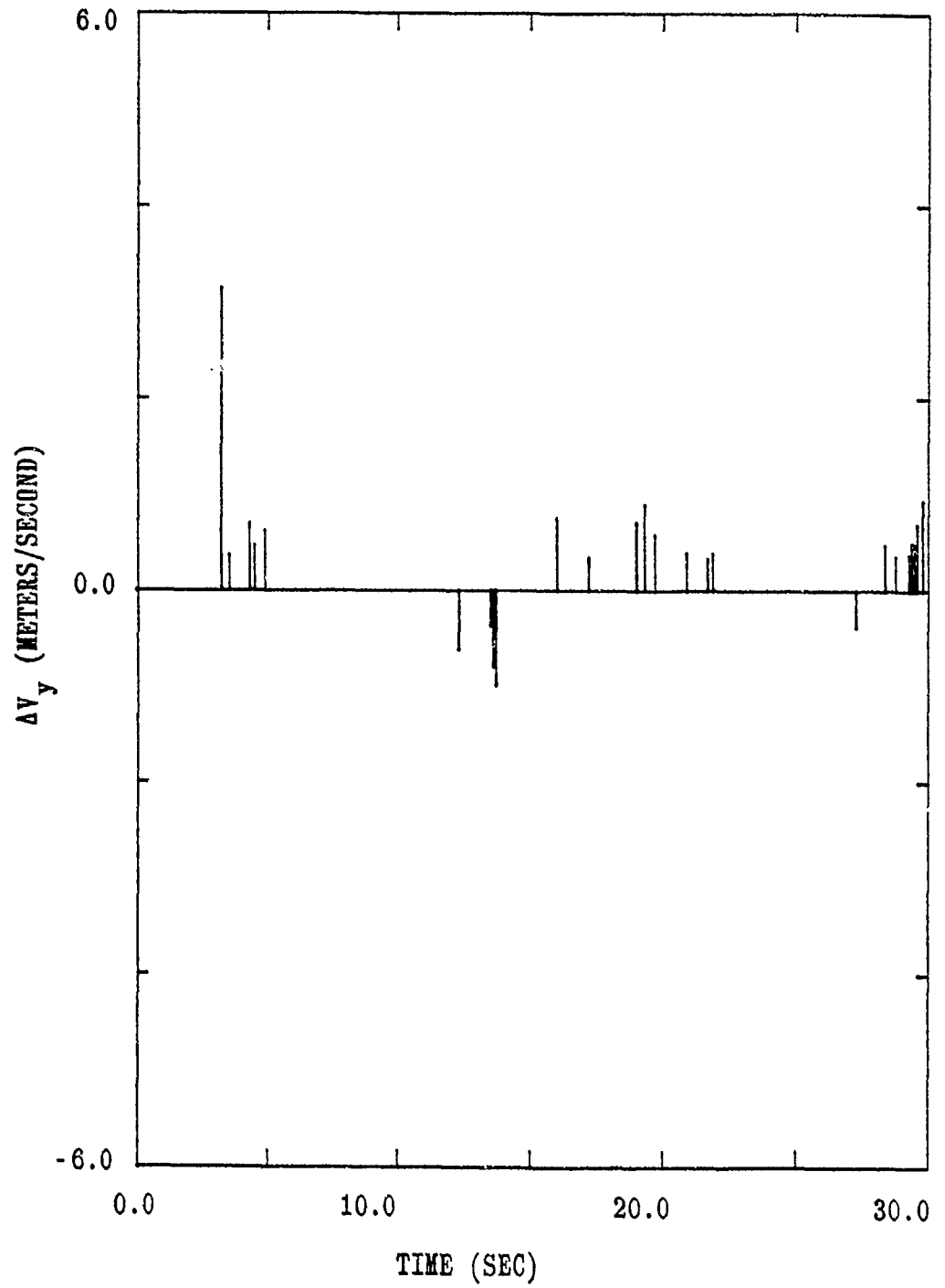


Figure E-13. In-plane thrust profile of Certainty Control for Case V.

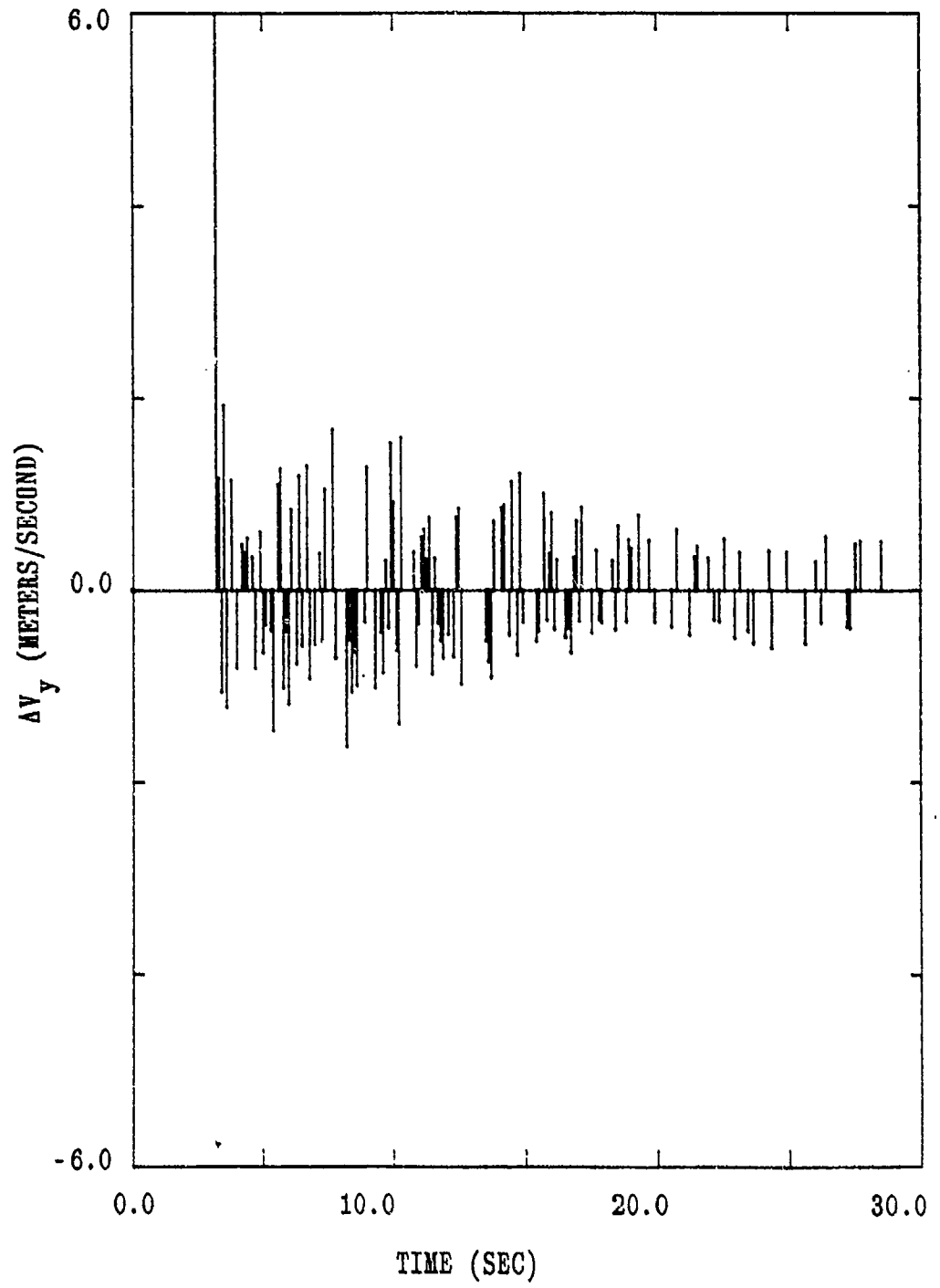


Figure E-14. In-plane thrust profile of Truth Model for Case V.



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ABSTRACT: Terminal guidance of a hypervelocity exo-atmospheric orbital interceptor with free end-time is examined. The pursuer is constrained to lateral thrusting with the evader modeled as an ICBM in its final boost phase. Proportional navigation, optimal control using certainty equivalence, dual control, and control with optimum thrust spacing are all examined. Also, a new approach called certainty control is developed for this problem. This algorithm constrains the final state to a function of projected estimate error to reduce control energy expenditure. All methods model the trajectories using splines and employ eight state Extended Kalman Filters with line-of-sight and range updates. The relative effectiveness of these control strategies is illustrated by applying them to various intercept problems.