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HYSTERESIS, IMPORT PENETRATION, AND EXCHANGE RATE PASS-THROUGH*

AVINASH DIXIT

A competitive industry has established home firms, and foreign firms with entry and exit costs. The real exchange rate follows a Brownian motion. Industry equilibrium is determined using methods of option pricing. Entry requires the operating profit to exceed the interest on the entry cost, and similarly for exit. The middle band of rates without entry or exit yields hysteresis; it is found to be very wide for plausible parameter values. The exchange rate pass-through to domestic prices is found to be close to one in the phases where foreign firms enter or exit, and near zero otherwise.

I. INTRODUCTION

As the dollar has fallen from its 1985 heights, the U. S. trade balance has been slow to improve, and foreign firms have been reluctant to raise their prices in the U. S. market. Among the economic explanations of these facts, there is a new and appealing concept of economic "hysteresis," an effect that persists after the cause that brought it about has been removed. The argument is that firms must incur sunk costs to enter new markets, and cannot recoup these costs if they exit. Thus, foreign firms that entered the U. S. market when the dollar was high do not abandon their sunk investments when the dollar falls.

This idea was put forward by Baldwin [1986], and its best known exposition is in Baldwin and Krugman [1986]. But these formal models failed to do it justice. Baldwin assumes perfect foresight, while Baldwin and Krugman assume that the *levels* of the

*I am very grateful to Andrew Caplin for valuable discussions, to Alan Blinder and two referees for comments on earlier versions, and to the National Science Foundation for financial support under grant no. SES-8509536. I have also benefited from reading Krugman's [1988] exposition of the model.

real exchange rate at successive instants of time are independently and identically distributed. While theory needs simplifying assumptions, this is far too unrealistic. Krugman and Baldwin [1987] conduct a statistical test of hysteresis by introducing a dummy variable for the post-1984:2 and post-1985:2 periods in the trade volume equations, and find the hysteretic effect quantitatively unimportant. But this is an ad hoc test leading to a provisional finding that awaits better theory, and empirical work better grounded in the theory.

In the context of job search and unemployment under demand uncertainty, Lucas and Prescott [1974] allow a general Markov process for demand and establish the existence of zones of inaction, producing what we would now call hysteresis. However, the model is too general to yield any specific insights about the nature and the magnitude of hysteresis.

In this paper I take an intermediate approach. The real exchange rate is assumed to be a random walk, or more accurately in continuous time, a Brownian motion. As a first approximation, this has considerable empirical support; see Frankel and Meese [1987]. Also, we shall see that the reversion to purchasing power parity that can be detected from long time series of real exchange rates only strengthens the hysteretic effect.

The most important benefit of the Brownian motion specification is that we can think of firms' entry and exit choices as options, and bring to bear the techniques and the intuition available from financial economics. Greater exchange rate volatility makes the entry and exit options more valuable and therefore less readily exercised; this emerges as a new cause of hysteresis. The parameters of the model have natural observable counterparts. This permits numerical calculations, which give us a more vivid picture, and show that the effect of volatility is quantitatively very important.

The analysis is at the industry level. Therefore, the real exchange rate process is specified exogenously, and the response of imports and prices in this industry is derived. It would be desirable to consider many such industries and their feedback on the exchange rate process. But such general equilibrium analysis must await further research, and the partial equilibrium model developed here would remain a useful ingredient of it.

For concreteness, I shall speak of Japanese firms selling in the U. S. market. For simplicity, I shall assume that domestic firms go on functioning in this market; that is, I shall not consider simultaneously the entry and exit decisions of U. S. firms from the home market. It is generally realistic to assume that firms will maintain a presence in their home markets under most conceivable exchange rate fluctuations, but extension of the model to make such decisions endogenous would be a useful item of future research. Finally, I shall assume that at any instant, the firms that are in the market act as price-takers. This is a sensible assumption in many industries even when sunk costs are important for each firm; agriculture, textiles, and many consumer electronics products, and even steel are cases in point. For other industries, such as autos, computers, and aircraft, somewhat less competitive conduct would be more appropriate. For this reason, the case of a Cournot oligopoly should also be studied; this is another important subject for future research.

II. The Model

Let R denote the exchange rate, namely the price of a dollar in yen. This is assumed to follow a Brownian motion (random walk in continuous time):

$$dR/R = \mu\, dt + \sigma\, dz. \tag{1}$$

Here dz is an increment of the standard Wiener process, uncorrelated across time, and at any one instant satisfying

$$E(dz) = 0, \qquad E(dz^2) = dt,$$

where E denotes the expectations operator.

The product in question has stable U. S. demand and supply functions in dollar terms. The net or import demand function in inverse form is $p = P(q)$. The area under it measures the U. S. social surplus. Thus, there is a function $U(q)$ such that $P(q) = U'(q)$ for all q. As usual, $P(q)$ is decreasing, while $U(q)$ is increasing and concave.

There are N Japanese firms that are potential suppliers of imports to the United States. The firms are risk neutral and have rational expectations; they maximize the expected present value of profits in yen. When in the U. S. market, they act as price-takers. Each must incur a sunk capital cost of k dollars to enter this market. This cannot be recouped if the firm should decide to quit at a later date. Moreover, it may have to pay a further l dollars to shut down. If it should decide to reenter at a still later date, it must spend k dollars over again.

Each firm that is active in the U. S. market sells one unit of output per unit time. The variable cost of supplying this unit to the U. S. market is w_n yen for firm n. The firms are labeled so that w_n is increasing.

This setup embodies several special assumptions that deserve comment. Let us begin with two important ones. (1) The exchange rate process is exogenous and of a particular form. While the Brownian motion specification is empirically hard to reject, theoretical general equilibrium reasoning suggests that there should be some reversion to the mean. This would make the model harder to solve, but the effect will be to strengthen the hysteresis. I shall return to this point later. (2) I have neglected depreciation of the entry costs. In my model with discrete firms of fixed size, exponential depreciation can be handled by thinking of it as a maintenance requirement; the firm must spend δk each year to stay in business. This raises the recurrent costs, thus reducing the relative importance of the fixed costs and hence the magnitude of the hysteresis. However, I shall argue later that numerically the effect is modest.

Next a set of relatively minor assumptions. (1) Each firm has a fixed scale and a fixed-coefficient technology. This only matters for numerical results on the exchange rate pass-through, and I shall modify the analysis at that point. (2) Firms are assumed to be risk neutral. To generalize this, the risk-free interest rate should be replaced by an appropriately risk-adjusted rate from a capital asset pricing model; Dixit [1989] shows how the qualitative results are not affected. (3) The entry and exit costs k and l are denominated in dollars while the recurrent costs w_n are in yen. Since the former are costs of setting up or shutting down in the U. S. market, and the latter is the cost of producing in Japan and shipping to the United States, this assumption is probably reasonable. However, the mathematics can readily be adapted to handle alternatives in this regard, and the qualitative features are again unaffected. (4) I assume that k and l are the same for all N firms; at the end of this section I shall discuss the effect of allowing them to differ across firms.

For $n \geq 1$, define

$$p_n = U(n) - U(n-1), \tag{2}$$

the nth firm's marginal contribution to utility. When the number of firms is large and the unit quantity is small in relation to the market, p_n serves as the market price with n active firms. Since $U(n)$ is arbitrary to within an additive constant, we can set $U(0) =$

0, and then

$$U(n) = \sum_{j=1}^{n} p_j. \tag{3}$$

In the same way, it is useful to define a function $W(n)$, being the variable cost of the first n firms. That is,

$$W(n) = \sum_{j=1}^{n} w_j, \tag{4}$$

or

$$w_n = W(n) - W(n-1). \tag{5}$$

Since w_n is increasing, $W(n)$ is increasing and convex.

Now suppose that we start at time $t = 0$ at an initial exchange rate $R_0 = R$ and a number $n_0 = n$ of active Japanese firms. We want to describe the equilibrium of the industry when firms are competitive and have rational expectations. The convenience of these assumptions is that they allow us to find the answer by solving a maximization problem. The equilibrium evolves *as if* to maximize the expected present discounted value of the overall surplus of this industry measured in yen. A proof of this assertion can be constructed along the lines of Lucas and Prescott [1971, 1974] or Jovanovic [1982]. The intuition is evident: competitive firms forecast the evolution of prices correctly, but ignore the pecuniary externalities caused by their entry or exit on the revenues of other firms, just as a mythical expected-surplus-maximizing planner would.[1]

Let ρ be the discount rate used by the Japanese firms. For convergence we need $\rho > \mu$. Also, at instants $t = i$ when the numbers of active firms change, let $[\Delta n_i]_+$ denote an increase, and $[\Delta n_i]_-$ a decrease. Then the maximand is

$$E\left\{\int_0^\infty \{R_t U(n_t) - W(n_t)\} e^{-\rho t} dt - \sum_i R_i \{k[\Delta n_i]_+ + l[\Delta n_i]_-\} e^{-\rho i}\right\}. \tag{6}$$

1. The Japanese firms are assumed to be risk neutral with regard to their yen profits, and this is not the same as risk neutrality in dollars. Therefore, it is important to measure the social surplus in the hypothetical maximization problem in yen. Incidentally, this is what makes the joint problem where U. S. as well as Japanese firms make entry and exit decisions a much harder one.

Let the outcome of this maximization problem—the Bellman value function—be written as $V_n(R)$.

I shall obtain the solution to this dynamic programming problem by thinking of it as a sequence of option pricing problems.[2] For each n we can regard the collection of the first n firms as an asset. While it is held, it produces dividends in the form of the current flow of net surplus, as well as capital gains because the value changes with the stochastic movements of the exchange rate. On the other hand, the asset is an option to buy other assets, namely collections of $(n + 1)$ or $(n - 1)$ firms, by adding or abandoning a firm. The exercise prices of the respective options are k and l dollars, or kR and lR yen. At the exchange rates where it becomes optimal to exercise these options, V_n is linked to V_{n+1} and V_{n-1} by the standard conditions of option pricing. The important new feature, making this problem more difficult than standard option pricing, is that the value of the asset acquired by exercising such an option is again endogenous. It is only by determining all the $V_n(R)$ functions simultaneously that we obtain the solution to the problem.[3]

Over an interval of time (or exchange rate values) where the number of firms remains unchanged at n, the evolution of $V_n(R)$ is given by Itô's Lemma:

$$\begin{aligned} dV_n(R) &= V_n'(R)dR + (\tfrac{1}{2})V_n''(R)(\sigma R dz)^2 \\ &= [\mu R V_n'(R) + (\tfrac{1}{2})\sigma^2 R^2 V_n''(R)]dt + \sigma R V_n'(R)dz. \end{aligned}$$

Therefore, the capital gain is

$$E[dV_n(R)]/dt = \mu R V_n'(R) + (\tfrac{1}{2})\sigma^2 R^2 V_n''(R).$$

The flow dividend is $[RU(n) - W(n)]$. Equating the sum of this and the capital gain to the normal return $\rho V_n(R)$ and rearranging terms, we see that $V_n(R)$ satisfies the following differential equation:

$$(\tfrac{1}{2})\sigma^2 R^2 V_n''(R) + \mu R V_n'(R) - \rho V_n(R) = W(n) - RU(n). \tag{7}$$

2. A fuller discussion of this analogy can be found in Dixit [1989]. For an alternative approach to the rigorous theory, see Bentolila and Bertola [1987].

3. I should also consider transitions from n to $(n + 2)$, $(n + 3)$, . . . or $(n - 2)$, $(n - 3)$, . . . firms. Since $U(n)$ is concave, $W(n)$ is convex, and the costs of changing the numbers of firms are linear, the exchange rates that trigger transitions to $(n + 1)$ or $(n - 1)$ firms are reached before those that trigger multiple transitions. Therefore, the multiple transitions do not take place, except at $t = 0$ when the initial number of firms may be very inappropriate. I have omitted the rigorous treatment of this minor qualification for ease of exposition.

This has the general solution,

$$(8)\quad V_n(R) = A(n)R^{-\alpha} + B(n)R^{\beta} + RU(n)/(\rho - \mu) - W(n)/\rho,$$

where $A(n)$ and $B(n)$ are constants to be determined, and $-\alpha$ and β are roots of the quadratic equation in ξ:

$$f(\xi) \equiv (\tfrac{1}{2})\sigma^2\xi(\xi - 1) + \mu\xi - \rho = 0.$$

Observing that

$$f(0) = -\rho < 0, \qquad f(1) = -(\rho - \mu) < 0, \qquad \text{and} \qquad f''(\xi) > 0,$$

we see that $-\alpha < 0$ and $\beta > 1$.

To interpret the formula (8), note that

$$E(R_t|R_0) = R_0 \exp(\mu t).$$

Therefore,

$$(9)\quad E\int_0^{\infty} \{R_tU(n) - W(n)\} \exp(-\rho t)\, dt = \frac{RU(n)}{\rho - \mu} - \frac{W(n)}{\rho}.$$

In other words, the last two terms on the right-hand side of (8) give the expected present discounted value of maintaining exactly n firms forever, starting with the exchange rate R.[4] The other terms must then be the values of the options to change the numbers of firms. In fact, $A(n)R^{-\alpha}$ is the value of the option to shut down some existing firms, and $B(n)R^{\beta}$ is that of adding some new firms. This interpretation will be developed more fully in the light of the results that follow.

We can also formulate some boundary conditions for the endpoints $n = 0$ and N. Since $V_0(R)$ must remain bounded as R goes to zero, we must have $A(0) = 0$. This accords with the above interpretation: when there are no established firms, the shutdown option has zero value. On the other hand, if $n = N$, there are no firms to add, and if R goes to ∞, the eventuality of shutting down any firms becomes remote and unlikely. Therefore, $V_N(R)$ is approximated by the expected present discounted value of exactly N firms, namely the right-hand side of (9). Then, since $\beta > 1$, the solution cannot contain a term in R^{β}. Therefore, we must have $B(N) = 0$. This again accords with the earlier interpretation: when the number of firms is the maximum possible, the option to add more is worthless.

4. If firms have more flexible technology, $RU(n) - W(n)$ will have to be replaced by a more general profit function, and the integral in (9) will not generally have an explicit expression. But the economic interpretation will remain.

The next step is to consider the investment decision. Let I_n be the exchange rate at which it just becomes optimal to introduce the nth firm when previously there were $(n - 1)$. The conditions for this are as follows: the Value-matching condition,

$$V_{n-1}(I_n) = V_n(I_n) - kI_n; \tag{10}$$

and the High-order Contact or Smooth Pasting condition,

$$V'_{n-1}(I_n) = V'_n(I_n) - k. \tag{11}$$

The Value-matching condition is easy to interpret. It equates the value of the option to the value of the asset being acquired *minus* the exercise price. If this failed, arbitrage profits would be possible. The High-order Contact condition is more subtle. Krugman [1988] and Dixit [1989, Appendix A] give heuristic derivations. In financial economics the condition is justified by the argument that if it failed, then moving the critical stock price would raise the value of the option. See the discussion in Merton [1973, footnote 60], Brennan and Schwartz [1985, p. 143], and Majd and Pindyck [1987, footnote 11].

Substitute the functional form of the solution from (8) into (10) and (11). Simplify the resulting equations by defining, for $n = 1, 2, \ldots, N$,

$$a_n = A(n) - A(n - 1), \qquad b_n = B(n - 1) - B(n), \tag{12}$$

or

$$A(n) = \sum_{j=1}^{n} a_j, \qquad B(n) = \sum_{j=n+1}^{N} b_j. \tag{13}$$

Also recall the similar relations between $U(n)$ and p_n, and between $W(n)$ and w_n. Then the Value-matching condition becomes

$$a_n I_n^{-\alpha} - b_n I_n^{\beta} + I_n p_n/(\rho - \mu) - w_n/\rho - kI_n = 0, \tag{14}$$

and the Smooth Pasting condition becomes

$$-\alpha a_n I_n^{-\alpha-1} - \beta b_n I_n^{\beta-1} + p_n/(\rho - \mu) - k = 0. \tag{15}$$

Similar arguments apply to the decision to abandon a firm starting from n. Let D_n be the exchange rate at which this becomes just optimal. Then we have the Value-matching condition

$$V_n(D_n) = V_{n-1}(D_n) - lD_n, \tag{16}$$

and the Smooth Pasting condition

$$V'_n(D_n) = V'_{n-1}(D_n) - l. \tag{17}$$

Using the functional form (8) and simplifying, we have

$$a_n D_n^{-\alpha} - b_n D_n^{\beta} + D_n p_n/(\rho - \mu) - w_n/\rho + l D_n = 0, \tag{18}$$

and

$$-\alpha a_n D_n^{-\alpha-1} - \beta b_n D_n^{\beta-1} + p_n/(\rho - \mu) + l = 0. \tag{19}$$

To complete the solution we must solve, for each $n = 1, 2, \ldots, N$, the four equations (14), (15), (18), and (19) for the four unknowns a_n, b_n, I_n, and D_n. In my earlier paper [Dixit, 1989] only one firm's entry and exit decisions were examined. This required two value functions V_0 and V_1, which were given by functional forms like (8) and led to a set of four equations just like the one above. Therefore, the industry model considered here is a very natural generalization of the firm model of the previous paper.

The analogy can be pushed further if we write the functions $A(n)$, $B(n)$, $U(n)$, and $W(n)$ in terms of a_n, b_n, p_n, and w_n. Then (8) becomes

$$V_n(R) = \sum_{j=1}^{n} \left(a_j R^{-\alpha} + \frac{R p_j}{\rho - \mu} - \frac{w_j}{\rho} \right) + \sum_{j=n+1}^{N} b_j R^{\beta}.$$

Define

$$v_n^I(R) \equiv a_n R^{-\alpha} + R p_n/(\rho - \mu) - w_n/\rho, \tag{20}$$

and

$$v_n^O(R) \equiv b_n R^{\beta}. \tag{21}$$

These are to be interpreted as respective "in" and "out" value functions of the nth firm. Note that we use p_n and w_n, the *marginal* contributions of this firm to utility and costs, respectively. The "in" value function is the capitalized marginal contribution of actually operating the nth firm forever, *plus* the option value of shutting it down. The "out" function is just the option value of activating it. Then we can write $V_n(R)$, the value with n active firms, as the sum of the "in" value functions of all the firms that are in, and the "out" value functions of all the firms that are out:

$$V_n(R) = \sum_{j=1}^{n} v_j^I(R) + \sum_{j=n+1}^{N} v_j^O(R). \tag{22}$$

To complete this interpretation, note that (14) and (15) become just the firm-by-firm Value-matching and Smooth Pasting conditions for entry

$$v_n^O(I_n) = v_n^I(I_n) - k I_n, \tag{23}$$

(24) $$v_n^{O\prime}(I_n) = v_n^{I\prime}(I_n) - k,$$

while (18) and (19) are those for exit

(25) $$v_n^I(D_n) = v_n^O(D_n) - lD_n,$$

(26) $$v_n^{I\prime}(D_n) = v_n^{O\prime}(D_n) - l.$$

Now we can establish a general property of the solution that yields the hysteresis effect. Define

(27) $$\phi_n(R) = v_n^I(R) - v_n^O(R),$$

and note that it satisfies

(28) $$(\tfrac{1}{2})\sigma^2R^2\phi_n''(R) + \mu R\phi_n'(R) - \rho\phi_n(R) + Rp_n - w_n = 0.$$

From (20) and (21) we can construct the general shape of $\phi_n(R)$, as is shown in Figure I. Then (23)–(24) say that I_n is given by a tangency between $\phi_n(R)$ and the ray kR through the origin, while (25)–(26) say that D_n is given by a tangency between $\phi_n(R)$ and $-lR$. For such a configuration, $\phi_n(R)$ must be convex at D_n and concave at I_n. Using this in (28) yields, for each $n = 1, 2, \ldots, N$,

(29) $$w_n + (\rho - \mu)kI_n < p_nI_n,$$

and

(30) $$p_nD_n < w_n - (\rho - \mu)lD_n.$$

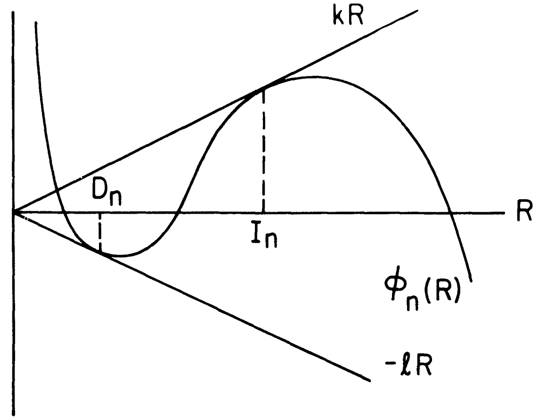

FIGURE I
Determination of Entry and Exit Triggering Exchange Rates

The first inequality says that the market price at the entry point of the nth firm exceeds the full flow cost (the variable cost *plus* the interest on the entry capital).[5] Similarly, the second inequality says that at the exit point of the nth firm, the market price is so low that the operating loss exceeds the interest on the abandonment cost. In the language of financial economics, an option is not exercised if it is only just "in the money."

The idea is very similar to that for a single firm discussed in Dixit [1989]. However, analysis of industry equilibrium makes a difference of degree. When the nth firm on the verge of entry contemplates future developments, it rationally calculates that if the exchange rate moves upward, firms $(n + 1)$ etc. will enter, and the dollar price will fall. A similar difference occurs for a firm contemplating exit.

The two inequalities (29) and (30) embody the idea of "hysteresis." If we draw I_n and D_n as functions of n, they form a band of inaction. For any n, if the exchange rate fluctuates between I_n and D_n, neither new entry nor exit of foreign firms occurs. If the rate rises to I_{n+1} and another firm enters, it will stay unless the rate falls to D_{n+1}.

For the firm's model in Dixit [1989] I computed numerical solutions for a wide range of parameter values, and found that the entry and exit trigger points were wide apart even when the costs k and l and the uncertainty parameter σ were quite small. We shall soon see that the same is true of the band of inaction in the industry model. Therefore, if the exchange rate starts out in the middle of the band, entry and exit are relatively rare events. However, once they occur, they are even harder to reverse.

To conclude this section, consider the effects of altering two of the assumptions of the model. First, I assumed above that foreign firms differed in their variable costs, but all had the same sunk costs. Consider the opposite extreme, where the firms differ only in their sunk costs. What this changes is the relative order of entry and exit. With common sunk costs, the firms enter in the order of increasing variable cost, and exit in the reverse order. Thus, if we know that there are n firms in the industry, we can be sure that they are the ones with the smallest values of w. When firms differ in their sunk costs, those with the smallest sunk costs will be the first to enter *and* to leave. Suppose that the normal state had prevailed, and now let there be a period of overvaluation. Foreign firms will

5. Since the costs of entry and exit are in dollars, the appropriate rate of interest is $(\rho - \mu)$.

enter in the order of increasing sunk costs. Next let the exchange rate go down again. The firms with the smallest sunk costs will exit, and those with large sunk costs will hang in. Finally, let the rate start to rise again. The new entry will be of firms with low sunk costs again. Then the foreign firms in the market will be of two types, the new entrants who have low sunk costs, and the holdovers from the previous overvaluation, with much higher sunk costs. When firms differed only in their variable costs, the state of the industry could be described simply by the number of firms that were in. With differing sunk costs, the state must be described by listing the whole set of firms that are in. Now there are 2^N states, that many value functions, and transitions to consider. The model gets much more complicated.

Finally, consider what happens if the real exchange rate process shows mean-reversion to a normal value. Suppose that the rate has reached a higher level, and the next foreign firm in line is contemplating entry. It balances the profits to be obtained over the next short interval of time against the value of the option to wait. If there is mean-reversion, the future path of the exchange rate is more likely to evolve in an unfavorable direction. Therefore, the firm will require greater current profit to trigger its entry. The reverse argument applies if the rate has reached a low level. In other words, the band formed by I_n and D_n will be wider, so hysteresis will be more pronounced, if the exchange rate process has mean-reversion.

III. Numerical Results

In this section I obtain some numerical solutions for the industry equilibrium. A configuration of parameters to constitute a central case is established, and some variations around it are considered. The choice of parameters is governed by concerns of simplicity and rounding as much as by reality. However, the picture is a fair representation of some U. S. import-competing industries in the 1980s.

Let us begin by establishing hypothetical stationary conditions around which uncertainty will be introduced. Suppose that R would equal 1 in such a state; this makes it easier to think of percentage of over- and undervaluations. Suppose that 150 is the stationary value for the Yen/Dollar rate; then this amounts to measuring Japanese values in a new unit of 150 Yen. I shall usually give both sets of exchange rate numbers.

Suppose that the U. S. dollar price p under stationary conditions would be 1. For the median Japanese firm, $w = 0.95$, or 142.5 Yen. Take $k = 2$, and $\rho = 0.025$, so $\rho k = 0.05$, and the annualized sunk cost is 5 percent of the full cost for the median firm. This seems a relatively modest recognition of the existence of sunk costs.[6] I shall consider the effect of changing the mix of sunk and variable costs later. I set the exit cost l at zero; required costs of quitting a market are probably more important in the European context than the United States.

Suppose that the maximum number of Japanese firms is $N = 100$. Since $w + \rho k = 1 = p$ for the median firm, under stationary conditions half of the Japanese firms will sell in the U. S. market. Suppose that the total demand in the U. S. market at this price is 200 units, and domestic firms supply 150 units. Thus, the import share in the stationary state is 25 percent.

Now consider what would happen at other prices. Suppose that the U. S. demand function is

$$Q^d = 360 - 160p,$$

which has an elasticity of 0.8 at $p = 1$, and the domestic firms' supply function is

$$Q^s = 110 + 40p,$$

implying a supply elasticity of 0.3 at $p = 1$. Then the net or import demand function is

$$q = 250 - 200p,$$

which finally gives

$$p_n = 1.25 - n/200. \tag{31}$$

We can substitute this into the U. S. demand function, and obtain an expression for the import share s_n when there are n Japanese firms:

$$s_n = n/(160 + 0.8n). \tag{32}$$

Next we need the profile of variable costs of all Japanese firms. In the central case I shall let them vary by about 10 percent around

6. I suggested that exponential depreciation can be regarded as maintenance expenditure and added to the recurrent costs. If $\delta = 10$ percent per year, w changes to 1.15, and the proportion of sunk costs in total costs falls to 4 percent. This reduces the width of the hysteresis bands by a small amount.

the median value. Therefore,

$$w_n = 0.85 + n/500. \tag{33}$$

The linearity of the p_n function comes from the assumed linearity of the U. S. import demand function; that of w_n embodies an assumption of a uniform distribution of Japanese firms over the range of variable costs. The choice seems harmless at this level of analysis, but the numerical computing can be altered to handle alternative assumptions without any difficulty.

Finally, let us introduce uncertainty. In the central case I take $\mu = 0$ and $\sigma = 0.1$. This means that the variance of the real exchange rate is 1 percent per year. Since the standard deviation of Brownian motion increases as the square root of time, the standard deviation of the exchange rate is 10 percent for one year and 20 percent over four years. These magnitudes are broadly consistent with the historical observations in Frankel and Meese [1987, pp. 123–25].

With these values Figure II shows the trigger exchange rates for entry and exit, I_n and D_n, as functions of n. For greater visual

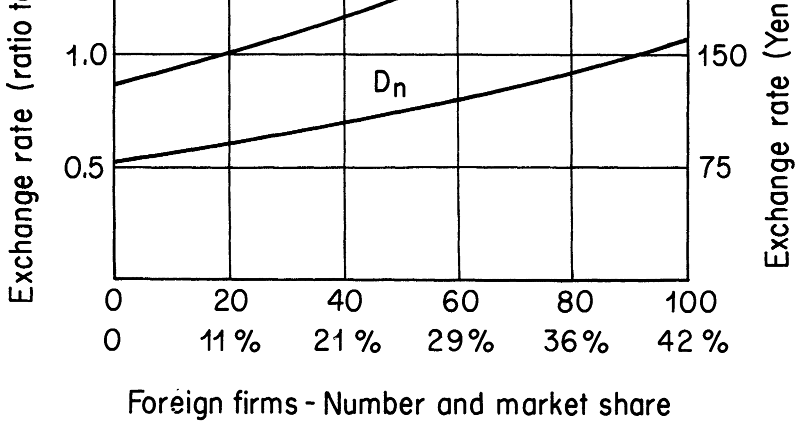

FIGURE II
Trigger Exchange Rates for Entry and Exit

appeal I show these as continuous curves and not as step functions. The exchange rates are shown as proportions of the normal or stationary levels on the left-hand scale, and in Yen/$ on the right-hand scale. The import share s_n is shown along with n on the horizontal axis. Note that as n increases, p_n falls and Q^d, the total quantity bought in the U. S. market, increases. Therefore, s_n does not increase linearly with n.

The first point to note is the width of the gap between I_n and D_n. As n increases from 1 to 100, the ratio I_n/D_n increases from 1.61 to 1.78. At the normal level of $n = 50$, the exchange rate that induces one more firm to enter is 68 percent higher than the rate that induces one firm to exit. An equilibrium with 50 firms is compatible with any exchange rate between 0.75 (112 yen/dollar) and 1.25 (187 yen/dollar). Looked at the other way around, the normal exchange rate of 1 (150 yen/dollar) is compatible with any number of firms from 20 to 90. If history brought about a particular state of affairs, it can prevail as an equilibrium until the exchange rate moves out of the band formed by I_n and D_n.

Contrast these results with the effects of sunk costs alone. Suppose that the initial conditions are normal, and 50 foreign firms are in this market. The fifty-first firm knows $p_{51} = 0.995$ and $w_{51} = 0.952$. Remember that $k = 2$ and $\rho = 0.025$. If the exchange rate makes a once-and-for-all unexpected jump to R, entry for this firm is profitable as long as

$$0.995R - 0.952 > 0.025 \times 2, \qquad \text{or } R > 1.007.$$

We saw above that when the exchange rate follows Brownian motion with $\sigma = 0.1$, the trigger rate for this firm's entry is much higher, namely about 1.25. In other words, the ongoing uncertainty is a much more potent generator of hysteresis than sunk costs alone.

The two distinct aspects of hysteresis worth re-emphasis are as follows: (i) very large deviations of the exchange rate from the normal are needed to alter the numbers of foreign firms and the extent of import penetration. (ii) Once such a change in import share has occurred, even larger deviations of the exchange rate in the other direction are necessary to restore the former condition.

Start from a normal initial state, with $R = 1$ and $n = 50$. No entry or exit will occur unless the exchange rate changes by 25 percent during the year. This is 2.5 times the standard deviation over one year, and therefore an event of probability 1 percent over this horizon.[7] Over four years the standard deviation is twice as large, and the probability rises to over 21 percent. But suppose that an even more extreme fluctuation occurs: let R rise by 40 percent to 210 yen/dollar, which is as much as has been claimed for the real overvaluation of the dollar in the early 1980s. Then n rises to 64; that is, the import share goes up from the normal level of 25 percent to 30.3 percent. That is quite a small effect of a very large overvaluation. However, once this occurs, the entering foreign firms will hold on even if the exchange rate then returns to the normal level. It will need a large drop to 0.82 (123 yen/dollar) to drive out the sixty-fourth firm, and not until the rate drops to 0.75 (112 yen/dollar) will all the newly entered firms exit.

Of course these results, and therefore the significance of the hysteretic effect, depend on the parameter values chosen for the central case. Therefore, I examined the response of the system to changes in these parameters around the central values. A sample of the results is reported in Figures III–V, and Table I.

If either k or σ is increased, the I_n curve shifts up and the D_n curve shifts down, so the band they form is widened. This is shown schematically in Figure III, as is the difference this makes to the effects of an overvaluation. The I_n and D_n curves for the central case are shown dashed, and the ones after the parameter change are shown undashed. Suppose that the normal exchange rate is at R_0 with N_0 firms, and let the rate rise to the point R_1. In the central case, this led to an increase in the number of foreign firms, to the point N_2. When either k or σ increases and the bands are wider, the increase is only to N_3. However, a greater correction to the exchange rate is now needed to get these firms to leave. The rate must fall to R_3 rather than R_2 to start any exit at all, and to R_5 rather than R_4 to restore the normal import share.

Table I gives the corresponding quantitative information. All the experiments start with a 40 percent overvaluation, and the first line of the table recapitulates the results for the central case stated above. In the first variation, k is doubled from 2 to 4, while the profile of w_n is offsettingly lowered to keep the normal full cost at the median equal to 1. The initial import penetration is reduced—we get 57 firms instead of 64—but a much greater subsequent undervaluation is necessary to start exit. In fact, the rate has to fall to 101 yen/dollar to restore normality. The second variation

7. This is not quite true, because there exist sample paths that cross the bands during the year and return to within those limits by the end of the year, but the rough calculation will suffice here.

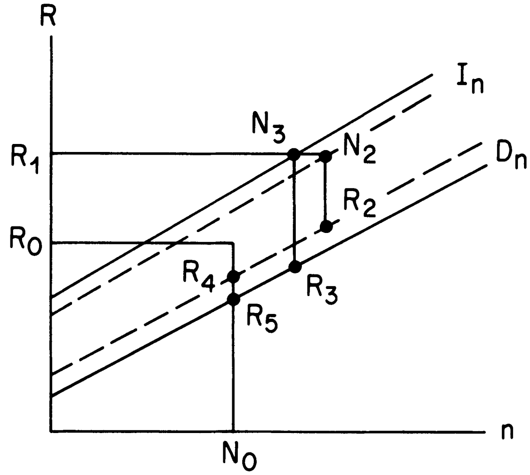

FIGURE III
Effects of Higher k or σ

involves halving σ to 0.05. This has the opposite effect, and the import penetration is much more "easy-come-easy-go."

The parameter values, whether in the central case or in the variations, are all at best semirealistic. Therefore, I would not attach great significance to the precise numbers of import shares or the exchange rates that trigger changes in the shares. However, the exercises do suggest that the phenomenon is likely to have considerable quantitative significance.

TABLE I
EFFECTS OF 40 PERCENT OVERVALUATION (210 Y/$)

	Entry		Rate needed to start exit		Rate needed to restore $n = 50$	
Experiment	n	%	Ratio	Y/$	Ratio	Y/$
Central case	64	30.3	0.816	122	0.750	112
k doubled	57	27.7	0.700	105	0.670	101
σ halved	74	33.7	0.975	146	0.822	123
w_n range halved	66	31.0	0.820	123	0.750	112
w_n horizontal	68	31.7	0.815	122	0.750	112
p_n range halved	72	33.1	0.820	123	0.750	112
$\mu = -2\%$	49	24.6	0.801	120	0.801	120
$\mu = +2\%$	78	35.1	0.822	123	0.680	102

Figure IV shows the effect of flattening the profile of w_n or p_n around the median point. The two have similar effects, but different causes. The steepness of the w_n profile captures the extent of heterogeneity in the foreign firms' costs. So making the profile flatter means assuming that these firms are more alike. The steepness of p_n is just the slope of the U. S. demand curve for imports. Thus, making it flatter means making our domestic demand or supply or both more elastic. In either case, the I_n and D_n curves become flatter around the median point. In the experiment that commences with a given overvaluation, this means greater import penetration. In the figure this takes the economy to the point N_3 rather than N_2. Correspondingly, the subsequent fall needed to start the exit process is to R_3 rather than R_2. Here we have two offsetting effects, and the net result is ambiguous. In Table I, for the particular experiments of halving the slopes of each of the profiles in turn, we see this general picture but some numerical differences. In the w case, the extra initial entry beyond the central case level is negligible. For the p case it is somewhat larger. In either case the exchange rate drop needed to start exit is little different than that in the central case. The case where all Japanese firms have the same variable costs, so the w_n schedule is horizontal, is also shown. Again the quantitative effect is negligible. In this case, the

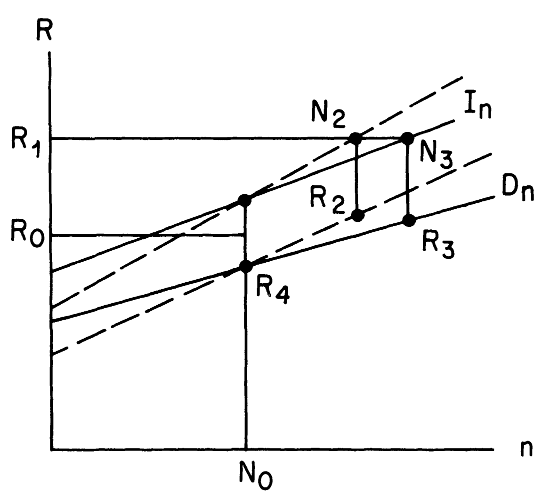

FIGURE IV
Effects of Flatter w or p Profiles

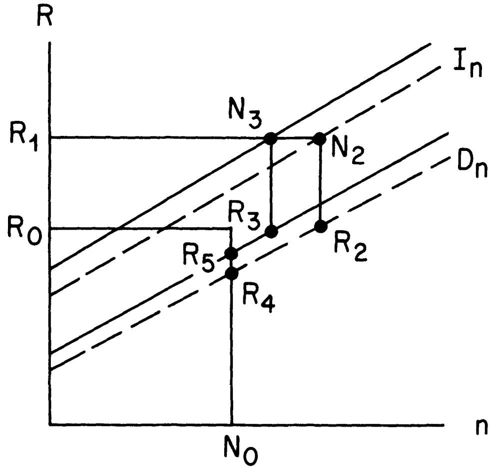

FIGURE V
Effects of Negative μ

equilibrium numbers of Japanese firms that enter or exit are determinate, but their actual identities are arbitrary.[8]

Next I consider the effect of changing the trend parameter. If μ becomes negative, the I_n and D_n functions are uniformly higher, as shown in Figure V. In response to any given dollar appreciation, new Japanese firms are more reluctant to enter the U. S. market, and in response to any subsequent fall of the dollar, they are more ready to leave. This is because they look ahead to the trend that reduces their competitiveness. In the specific case of $\mu = -0.02$, we see from Table I that a 40 percent dollar appreciation does not lead to any new entry at all.

In many industries the Japanese share of the U. S. market has risen at the same time that the dollar has fallen relative to the yen, so this result needs a little explanation. In the aggregate the yen undergoes real appreciation when Japan has a higher rate of technical progress in the tradables sector. What happens to the competitiveness of a specific Japanese firm or industry in this process depends on how its rate of technical progress compares with the average. Thus, even when μ is -2 percent per year, a firm whose

8. The case where w_n is horizontal can be interpreted as a model of a firm with hiring and firing costs of labor. For an explicit treatment of this application, see Bentolila and Bertola [1987] and Caplin and Krishna [1986].

w is decreasing at 4 percent per year will expect an improving trend of competitiveness, and be more eager to enter the U. S. market and more difficult to dislodge.

With this in mind, I have also shown the case of $\mu = +0.02$, representing the more progressive Japanese firms or industries. The quantitative effect is quite substantial; an initial 40 percent dollar overvaluation raises the import penetration to 35 percent.

Finally, consider what would happen if each firm was not restricted to a unit output and fixed input coefficients. Technically, this would be handled by replacing the flow profit $(Rp - w)$ by a more general profit function $\Pi(Rp,w)$. The crucial property of hysteresis as captured in equations (27) and (28) would remain. But the added flexibility would make foreign firms more willing to enter in response to a temporarily favorable exchange rate, and more ready to remain in the market despite temporary adversity. This would lower the I_n and D_n curves. However, much more complicated numerical solutions would be needed to determine the size of the effect.

IV. Exchange Rate Pass-Through

How do changes in the exchange rate affect the U.S. market price? With hysteresis it is necessary to consider this question in three separate phases: one where the exchange rate is appreciating and new foreign firms are entering; another where the rate is depreciating and established foreign firms are leaving; and the third where the exchange rate moves within the band and the number of foreign firms is constant.

In the model of this paper, where each foreign firm sells exactly one unit of the good in the U. S. market, the answer in the third phase is trivial and that in the first two phases is distorted away from reality. When the number of foreign firms is constant at n, the U. S. dollar price is constant at p_n irrespective of the exchange rate, that is, the pass-through is zero. In the first phase, as R increases beyond the entry-triggering level, not only will new foreign firms enter, but also the previously established firms will expand. The dollar price will fall on both counts. The model ignores the second effect, and therefore underestimates the pass-through. During the phase where the dollar is depreciating and the foreign firms are leaving, in reality the existing firms will contract and the dollar price will go up on both counts. Again my model underestimates the pass-through.

TABLE II
PASS-THROUGH WHEN FOREIGN FIRMS ENTER

Calculation no.	n	I_n	p_n	$-\hat{p}/\hat{I}$
1	50	1.2526	1.0000	0.63
	55	1.3021	0.9750	
2	70	1.4691	0.9000	0.65
	75	1.5318	0.8750	

I shall first consider the simple unit-scale model in the entry and exit phases, to obtain an idea of the lower bound for the pass-through coefficient. Then I shall separately examine the supply response of a fixed number of firms, to find the plausible magnitude of the coefficient when the exchange rate moves within the hysteretic band, and the correction necessary during the entry and exit phases.[9]

Table II shows the effect when entry is occurring. The calculations are for the central values of the parameters, and are made at the normal point and a point corresponding to a much higher import penetration. The respective values of the pass-through coefficients are far from zero, and indeed closer to one. Table III shows the corresponding calculations as the exchange rate depreciates and the number of foreign firms decreases to the normal level. Here the coefficients are higher still. Similar calculations for variations of the parameters around the central case do not change this conclusion significantly. Recall that the supply response of the existing firms will move these coefficients even closer to one.

TABLE III
PASS-THROUGH WHEN FOREIGN FIRMS EXIT

Calculation no.	n	D_n	p_n	$-\hat{p}/\hat{D}$
1	75	0.8891	0.8750	0.80
	70	0.8576	0.9000	
2	55	0.7722	0.9750	0.77
	50	0.7465	1.0000	

9. A full treatment should consider the interaction between the supply responses of the existing firms and the entry and exit of new ones. A given dollar depreciation may cause less exit, because staying in at a lower scale is now possible.

Next consider the effect of the supply response of the existing firms. The nth firm has the variable cost w_n yen, and faces a yen market price of pR. Let its profit function be $\Pi(pR,w_n)$, so its supply function is $\Pi_1(pR,w_n)$. Let $q(p)$ be the U. S. import demand function. With a fixed number n of foreign firms, the equilibrium condition is

$$q(p) = \sum_{j=1}^{n} \Pi_1(pR,w_j). \tag{34}$$

As R changes, we have

$$q'(p)dp = \sum_{j=1}^{n} \Pi_{11}(pR,w_j)(pdR + Rdp). \tag{35}$$

Suppose that the U. S. import demand elasticity is constant and equal to ϵ, and that each foreign firm's supply elasticity is constant and equal to η. Thus,

$$\epsilon = -pq'(p)/q(p);$$

and for all j,

$$\eta = pR\Pi_{11}(pR,w_j)/\Pi_1(pR,w_j).$$

Substituting into (35) and simplifying, we have

$$-\epsilon q(p)\hat{p} = \eta \sum_{j=1}^{n} \Pi_1(pR,w_j)(\hat{R} + \hat{p}),$$

where $\hat{p} = dp/p$ etc. Using (34), we have the pass-through coefficient

$$-\hat{p}/\hat{R} = \eta/(\eta + \epsilon). \tag{36}$$

If we take $\eta = 0.3$, the same as the value chosen for the domestic firms's supply elasticity in constructing the central case of previous numerical calculations, and the value of ϵ that emerges from the import demand function stipulated there, namely $\epsilon = 4$, we have the pass-through coefficient equal to 0.07. If we raise all firms' supply elasticities to 1, we have $\eta = 1$ and $\epsilon = 7.2$, so the pass-through coefficient rises to 0.14. Even if we raise only the Japanese firms' supply elasticity to 1, the coefficient is only 0.2. In other words, while the exchange rate fluctuates in the band where the number of foreign firms stays constant, we should expect those firms to absorb most of the effect of any dollar depreciation. It should be emphasized that there is no claim of any strategic or predatory motive on the part of the foreign firms; the calculation is the standard comparative statics of a competitive industry.

These sample calculations suggest that the pass-through coefficient in the phase where the number of foreign firms is constant is quite small, perhaps close to zero, while its values in the phases with entry or exit are much larger, perhaps close to one. In other words, in the early phases of a rise or a fall in the exchange rate, we get very little pass-through, while in the later stages as the entry or exit bands are crossed, the effect is much larger. Such breaks in the relation between the exchange rate and the dollar prices of importables could provide a test of the hysteresis hypothesis for various industries.

V. Concluding Remarks

I hope this paper makes a useful start in the important task of examining the effects of exchange rate fluctuations on an industry where sunk costs are important for each firm. At various points I indicated how the model can be extended in future research. Building a general equilibrium model that will explain real exchange rate fluctuations and the behavior of aggregate import prices is perhaps the most difficult. In the meantime, caution must be exercised when interpreting the results of the industry analysis in a wider context.

Princeton University

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