# Lawrence Berkeley National Laboratory Recent Work 

## Title

I = 3/2 nN RESONANCES IN THE 1900 MeV REGION
Permalink
https://escholarship.org/uc/item/0m728026
Author
Cutkosky, R.E.

## Publication Date

1976-06-01

RECEIVED<br>CAWRENCE<br>BERKGAEY LABORATORY

## JUL 291976

LIERARY AND $I=3 / 2 \pi N$ RESONANCES IN THE 1900 MeV REGION. $*$, DOCUMENTS SECTION $3 / 2$ N
R. E. Cutkosky and R. E. Hendrick Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
and
R. L. Kelly

Lawrence Berkeley Laboratory, Berkeley, California 94720

June 1976
*Supported by U. S. Energy Research \& Development Administration.


This is a Library Círculating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

## DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

## $\angle B \angle-41858$,

June 1976

## I $=3 / 2 \pi \mathrm{n}$ RESONANCES IN THE 1900 MeV REGION*

R. E. Cutkosky and R. E. Hendrick

Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
and

Lawrence Berkeley Laboratory, Berkeley, California 94720

## Abstract

Using the available world data for $\pi N$ scattering, we have performed a partial wave analysis for center of mass energies between 1550 and 2150 MeV . We report here results for $I=3 / 2$ resonances observed in the 1900 MeV region: $\mathrm{F}_{35}(1885), \mathrm{F}_{37}(1945), \mathrm{F}_{31}(1940)$, and $\mathrm{D}_{35}$ (1925). The presence of the $\mathrm{D}_{35}(1925)$ is difficult to accommodate in conventional baryon models. We describe tests performed to insure that the data require a $D_{35}$ (1925), and briefly discuss alternative models accommodating such a state.

[^0]The structure of baryons has been studied with increasing interest in recent years. The $S U(6) \times O(3)$ harmonic oscillator model proposed by Greenberg, and its relativistic ${ }^{2}$ and diquark ${ }^{3}$ variations, have had notable success in reproducing the observed baryon mass spectrum. More recent developments such as the "dual string" ${ }^{4}$ and 'bag" ${ }^{5}$ models of baryons have again focused attention on the spectrum of baryon resonances.

The primary source of information to test such models comes from partial wave analyses. We report here the first results of a $\pi \mathrm{N}$ partial wave analysis between 1550 and 2150 MeV . In particular, we concentrate on resonances observed in the $I=3 / 2$ channel around 1900 MeV . Clear resonant states have been observed in the $F_{35}, F_{37}, P_{31}$ and $D_{35}$ partial waves, and the resonance parameters have been determined. The even parity states at this mass are easily accommodated by the harmonic oscillator quark models in a $\left[56,2^{+}\right] \operatorname{SU}(6) \times O(3)$ supermultipiet. However, the odd parity $D_{35}(1925)$ is not readily accommodated by conventional $S U(6) \times O(3)$ models. We have performed several additional tests which support the $\mathrm{D}_{35}$ (1925), and present a baryon model which provides a $\mathrm{D}_{35}$ resonance at this energy.

Amalgamated np elastic and charge-exchange scattering data at 26 momenta in the range $0.8 \leq \mathrm{p}_{1 \mathrm{ab}} \leq 2.0 \mathrm{GeV} / \mathrm{c}$ were partial wave analyzed by a combination of the accelerated convergence expansion (ACE) technique for energy-independent fitting and a hyperbolic dispersion relation technique for the resolution of ambiguities and the imposition of s-channel analyticity constraints. Amalgamated data at each energy were made by fitting a surface to the world data in a $-150 \mathrm{MeV} / \mathrm{c}$ wide momentum range, interpolating the data in the central part of the range
( $-50 \mathrm{MeV} / \mathrm{c}$ wide) into fixed angular positions (with $3^{\circ}$ spacing) at the central momentum, and then combining the data statistically while allowing for normalization errors, momentum calibration errors, discrepancies between experiments, correlated interpolation errors, and various other systematic effects. ${ }^{7}$ We made single energy fits to these data using a parametrization in which each invariant amplitude is represented as a sum of a fixed "Born term" containing explicit contributions from nearby singularities and a fitted polynomial term constructed according to the ACE prescription. ${ }^{8}$ The Born term contains contributions from Pomeron and peripheral di-pion exchange, ${ }^{9}$ and also from Reggeized $\rho, f, N$ and $\Delta$ ex-
change. The parametrization (similar to that described in Ref. 10) is analytic in the full physical sheet of the $\cos \theta$-plane, with asymptotic power behavior $(\cos \theta)^{\alpha(s)}$, for large $|\cos \theta|$, and has no sharp cut-off in angular momentum. A smooth truncation of the fitted polynomial was accomplished by minimizing the quantity $x^{2}=x^{2}+\Phi$, where $\Phi$ is a convergence test function of the type used in Ref. 10. The actual search parameters were the values of the lower partial waves, with the higher waves being obtained by the ACE method of analytic extrapolation. The effective number of parameters used varied from about 30 at the lowest energies to about 50 at the highest. At each energy $50-100$ searches for local minima were made, with randomly generated starting points spread over large areas of the Argand diagrams, using a modified version of the CERN minimizer MIGRAD. ${ }^{11}$ The best-fit values of $x^{2}$ per degree of freedom at each energy were typically between 1.0 and 1.3 , and in the worst case ( $1590 \mathrm{MeV} / \mathrm{c}$ ) were as large as 1.6. Groups of local minima which were statistically indistinguishable were combined into clusters by a technique of weighted
averaging. A covariance matrix for each cluster was formed by adding the intra-cluster dispersion to a weighted average of the MIGRAD co-
variance estimates. To allow for possible systematic biases these were formed assuming that a $x^{2}$-deviation of $\Delta x^{2}=5$ corresponded to a single standard deviation, rather than the usual $\Delta x^{2}=1$. The number of clusters at each momentum varied from about 3 to 15 depending on the completeness of the available data.

In order to resolve the ambiguities we reconstructed the invariant amplitudes along 4 hyperbolic curves in the physical region of the s-t plane and fit the reconstructed amplitudes with a parametrization based on hyperbolic dispersion relations (HDR). ${ }^{12}$ We supplemented our partial wave results with the results of Carter et al.$^{13}$ and Ayed et al.$^{14}$ at higher and lower energies, the higher energy points being given reduced weights. Starting from several candidate "paths" we minimized $x^{2}$ with respect to all possible combinations of clusters in groups of up to five neighboring energies. This procedure selected a unique (though sometimes poorly determined) cluster at cach encrgy. To check and further stabilize these results, the input amplitudes at each energy in turn were deleted and HDR predictions were generated. These predictions, with enlarged errors, were then combined with the scattering data for a second round of energy independent fitting in which all the original local minima were used as starting points. The clusters resulting from this second fitting are shown in Fig. 1 for the four $I=3 / 2$ waves which resonate near 1900 MeV . The real and imaginary parts of each data point shown in fig. 1 are correlated, and there is also an estimated correlation coefficient of about one-third for each pair of adjacent points.

Resonances have been parametrized using a smoothly varying background term and a modified Breit-Wigner resonance term for the partial wave amplitudes:

$$
T_{\ell}=T_{B}+S_{B} \cdot T_{R}
$$

$T_{R}$ is the resonance amplitude

$$
T_{R}=\frac{a_{1} \phi_{e l}(s)}{s_{0}-s-i_{1} \phi_{e}(s)-i \sum_{k} a_{k+1} \phi_{k}(s)}
$$

where $\phi_{e l}(s)$ is an elastic phase space factor and $\phi_{k}(s)$ are inelastic phase space factors for the $\pi n N$ channel and for quasitwo body channels such as $N A, \rho N, \eta N, u N$, and $\rho \Delta$. The coefficients $a_{i}$, and $s_{o}$, are free parameters determined in the fitting procedure. The background amplitude is
$T_{B}=\left(S_{B}-1\right) / 2 i$ where $S_{B}=0,10$;
$D_{t}=\left\{1+b_{2} y \mp i \phi_{e l}\left(b_{1}+y b_{3}\right)\right\}\left(1 \pm \phi_{e l} \sum_{k} \phi_{k}\left[(1-y) b_{2 k+2}^{2}+y b_{2 k+3}^{2}\right]\right\}$
with $y=\left(\sqrt{s-\alpha}-\sqrt{s_{1}-a}\right) /\left(\sqrt{s-\alpha}+\sqrt{s_{1}-\alpha}\right)$. The coefficients $b_{i}$ are free parameters, $s$ is the center of mass energy squared, $s_{1}$ is the threc body threshold energy, $\left(M+2 \mu_{n}\right)^{2}$, and a is a fixed parameter determining the onset of the cut in the left-hand $s$-plane. The parameters $s_{o}, a_{i}$ and $h_{i}$ were determined for each partial wave resonance by a standard least squares fit to the partial wave data. Resonance parameters such as the mass, width, elasticity, pole position and residue were determined from a series of best fits. Errors on these parameters were estimated by a Monte Carlo procedure of varying the partial wave data with correlated Gaussian distributions around their central values, and by separately varying the background parametrizations. The resonance mass is defined here to be the real center of
mass energy at which the real part of the denominator of $T_{R}$ vanishes. The width definition we adopt is,

$$
\Gamma_{M}=\left.\frac{2 I m n_{R}(E)}{\left\lvert\, \frac{d}{d E} R e n_{R}(E)\right.}\right|_{E=\text { Nass }}
$$

where $D_{R}$ is the denominator of $T_{R}$, and $E=\sqrt{s}$. The elasticity is the ratio of the elastic width to the total width, defined in terms of $T_{R}$ to be $x=\left[-\operatorname{lm}\left(T_{R}{ }^{-1}\right)\right]^{-1}$. The pole position is the complex energy at which the denominator of $T_{R}$ vanishes; and the residue of the pole includes the factor $S_{B}$.

Figure 1 shows the fits and parameters for the $I=3 / 2$ resonances observed in the 1900 MeV energy region. Two systematic features of these resonances are the sizeable difference between the real part of the pole position and the resonance mass, and the negative phasés of the residues. These effects can be largely accounted for by the energy dependence of the width, although the backgrounds also contribute negative phases, especially for $P_{31}$. Another regular feature of the data is the imaginary pole position being somewhat less than $r_{M} / 2$, probably a result of the local definition of $r_{M}$. We also observed that the pole position is generally more stable than the mass or width parameters, ${ }^{15}$ and that its imaginary part is usually closer to the "eyeball" half-width than is $\Gamma_{M} / 2$.

Three well established even parity resonances were observed: $F_{35}{ }^{(1885)}$, $\mathrm{F}_{37}$ (1945), and $\mathrm{P}_{31}$ (1940). Previously quoted widths ${ }^{16}$ are somewhat smaller than those we report, but this discrepancy is probably due to the use of different definitions for the width. Conventional $\operatorname{SU}(6) \times 0(3)$ models place these 3 resonances in a $\left[56,2^{+}\right]$supermultiplet.

A significant result of our analysis is the confirmation of a $D_{35}$ resonance at 1925 MeV . Other analyses ${ }^{16}$ have also reported a $\mathrm{n}_{35}$ resonance near 1900 MeV . Because the $\mathrm{D}_{35}$ (1925) is not consistent with conventional $\mathrm{SU}(6) \times O(3)$ models (see discussion below) we have performed tests to insure that it is really required by the $\mathfrak{N}$ scattering data. New single energy fits were made to the amal gamated scattering data at three energies in the resonance region, with the $\mathbf{D}_{35}$ constrained to a smooth background value and the HDR constraints omitted. At two of the energies, 1804 MeV and 1900 MeV , the amplitudes are particularly well determined because charge exchange cross section and polarization data exist. Twenty randomly-started searches were made at each energy, in the neighborhood of modified partial wave values adjusted through use of the partial wave covariance matrix to accommodate a non-resonant $\mathrm{D}_{35^{\circ}}$. At 1900 MeV the best $\mathrm{X}^{2}$ for a fit to the 226 data was 210 , to be compared with 176 obtained for the best fit using HDR constraints, and the $x^{2}$ for $103 \pi^{+} \mathrm{p}$ data increased from 75 to 102 . At 1804 MeV and 1856 MeV somewhat larger increases in $x^{2}$ were found, with the increase at 1804 distributed more evenly among different types of data. In fits with the $D_{35}$ resonance removed, there was a tendency for the $G_{39}$ partial wave to develop anomalous ancestor-like behavior. These direct tests confirmed earlier studies based on the partial wave covariance matrix, which showed that the resonant behavior of the ${ }^{0} 35$ was associated with observable quantities rather than with unobservable phases.

Our fits to other partial waves will be described in a more detailed publication later, but we mention here results which are relevant to the

SU(6) $\times O(3)$ classification scheme. We see all the $\pi N$ resonant states which are conventionally assigned to the leading trajectory ( $70,1^{-}$) and $\left(56,2^{+}\right)$supermultiplets and to the $\left(56,0^{+}\right)$radial excitation supermultiplet. ${ }^{6}$ By supplementing our fits with some higher energy points from Ayed, et al., we further identify most of the ( $70,3^{-}$) states. Some additional states are also seen. We have fits with $S_{31}$ resonances at 1635 MeV and 2060 MeV , and $\mathrm{P}_{33}$ resonances at 1690 and 2100 McV . Neither partial wave is easy to fit above 1850 MeV and could have additional unresolved structure. We reproduce a known $\mathrm{D}_{33}$ resonance at 1690 McV , but have no evidence for a second resonance. We would not be able to see a $\mathrm{D}_{33}$ resonance in the 1900 MeV region if its elasticity were less than 0.1 .

The $I=1 / 2$ partial waves are more erratic, because the $\pi^{-} p$ elastic data are less precise than the $\pi^{+} p$ data and because there is not much charge exchange data. In the $0_{13}$ partial wave we find a second resonance at about 1655 MeV , a higher resonance at about 2070 MeV , and structure at 1900 MeV which is consistent with (but does not require) a resonance of elasticity $=0.06$ at 1900 McV . We find an $S_{11}(1630)$ and a $\mathrm{r}_{11}(1810)$, but above about 1800 MeV these two partial waves are somewhat erratic (and statistically highly correlated) so there could be additional resonances with elasticities up to about 0.2 . In general, odd parity resonances with $\mathbf{j} \leq 3 / 2$, which could be assigned to a $\left(56,1^{-}\right)$or to a second $\left(70,1^{-}\right)$ supermultiplet, are not required by our analysis, but are also not excluded.

Since we believe the existence of the $D_{35}$ (1925) to be quite certain, we have further investigated its phenomenological implications. $A D_{35}$
resonance can be accomodated by either a ( $70,3^{-}$) or a ( $56,1^{-}$) supermultiplet. We consider a ( $70,3^{-}$) assignment for the $D_{35}$ (1925) unlikely, because the mass of the observed state seems to be too low. Assuming a unit Regge slope, a recurrence of the $S_{31}(1635)$ nember of the (70, $1^{-}$) would be expected near 2200 MeV ; such a mass would be consistent with the masses of other states assigned to the ( $70,3^{-}$) provided the symmetry-breaking pattern is similar to that in the ( $70,1^{-}$). In fact, our $D_{35}$ partial wave amplitudes, when combined with higher energy values from Ayed, et al., are consistent with the existence of a second $D_{35}$ resonance in the 2200 to 2400 MeV region. Therefore we consider the $\mathrm{D}_{35}(1925)$ to be a member of a new supermultiplet, a $\left(56,1^{-}\right)$, which would also contain $S_{11}, D_{13}, S_{31}$, and $D_{33}$ resonances. Our present analysis is consistent with the existence of these four resonances, but does not require them.

The familiar 3 -quark harmonic oscillator model ${ }^{1,2}$ does not predict a $\left(56,1^{-}\right)$supermultiplet in the 1900 MeV region. Dual string models, ${ }^{4}$ however, suggest the existence of additional daughter states degenerate with those on leading trajectories. To pursue this idea in a phenomenological model, we propose a model in which baryons are made up of four massless partons, three of which are the usual valence quarks, and the fourth is a neutral, colorless, scalar "monad" which represents the energy and momentum carried by gluon fields or non-valence quarks. The leading term in the potential energy is proportional to the minimum possible length of "colored strings" required to join the partons; at least one string is attached to each quark, at least 2 strings are attached to the monad, and a vertex with three intersecting strings is allowed. This potential energy favors states in which the partons lie along a line. If there is
a quark at each end of the line, a single string can join all the partons, otherwise a doubled string is required.

For configurations in which the partons form two clusters, anasymptotic calculation gives

$$
M^{2}=8 K T[L+1 / 2+\sqrt{2}(N+1 / 2)]+O(1 / L)
$$

where $L$ is the orbital angular momentum and $N$ is the number of radial nodes. Also, $T$ is the tension of the strings, and $K$ is their number: $K=1$ gives the single-string configuration, and $K=2$ the doubled string configuration. For $K=1$ we have the usual quark-diquark states, ${ }^{3}\left(56, \mathrm{~L}_{\text {even }}^{+}\right) 1$ or ( $70, \mathrm{~L}_{\text {odd }}^{-}$) while $\mathrm{K}=2$ leads to $\left(56, \mathrm{~L}^{ \pm}\right)_{2}$ supermultiplets. The new supermultiplet $\left(56,1^{-}\right)_{2}$ is nearly degenerate with the nornal $\left(56,2^{+}\right)_{1}$. Note also that the radially excited $\left(56,0^{+}\right)_{1}^{*}$ has a lower energy than the $\left(56,2^{+}\right)_{1}$, which agrees with the observed masses of the $P_{11}$ and $P_{33}$ resonances.

We acknowledge valuable discussions with J. W. Alcock, Y. A. Chao, D. P. Hodgkinson, R. G. Lipes and J. C. Sandusky, and also their aid in developing the computer programs which have been used in this analysis.

## references

1. O. W. Greenberg, Phys. Rev. Letters 13, 598 (1964); 0. W. Greenberg and M. Resnikoff, Phys. Rev. 163, 1844 (1967).
2. R. P. Feynman, M. Kislinger and F. Ravndal, Phys. Rev. D3, 2706 (1971).
3. A. N. Mitra and D. L. Katyal, Nucl. Phys. B5, 308 (1968); D. B. Lichtenberg, Phys. Rev. 178, 2197 (1969).
4. See, for example, S. Mandelstam, Physics Reports 13C, 260 (1974); I. Bars and A. J. Hanson, Phys. Rev. D13, 1744 (1976).
5. A. Chodos, et al., Phys. Rev. D9, 3471 (1974), Phys. Rev. D10, 2599 (1974); W. A. Bardeen, et al., Phys. Rev. D11, 1094 (1975).
6. R. Horgan and R. II. Dalitz, Nucl. Phys. B66, 135 (1973); R. H. Dalitz in New Directions in Hadron Spectroscopy (ANL-HEP-CP-75-58), eds. S. L. Kramer and E. L. Berger, p. 383, 1975.
7. D. P. Hodgkinson et al., LBL-3048, 1974.
8. R. E. Cutkosky and B. B. Deo, Phys. Rev. 174, 1859 (1968).
9. J. W. Alcock and W. N. Cottingham, Nuc1. Phys. B41, 141 (1972) and private communication.
10. R. E. Cutkosky et al., Nucl. Phys. B102, 139 (1976).
11. F. James and M. Roos, MINUIT, CERN D 506/D 519, 1971.
12. Y. A. Chao et al., Phys. Letters 57B, 150 (1975).
i'3. J. R. Carter et al., Nucl. Phys. B58, 378 (1973).
13. R. Ayed and P. Bareyre, private communication to the Particle Data Group, 1974.
14. Particle Data Group, Rev. Mod. Phys. 48, S 147 (1976).
15. Particle Data Group, ibid., s18.3-S186


Fig. 1. $I=3 / 2 \mathrm{nN}$ amplitudes which resonate in the 1900 MeV region. Results of energy independent fitting are shown as data points; energy dependent partial wave fits are shown as curves. The energy dependence of each wave is shown by plotting the real and imaginary parts vs. energy in alignment with the Argand plot, and by arrows on the Argand plot at multiples of 50 MeV . Values of the resonance parameters defined in the text are shown, as well as the pole.position, $W$, and the pole residue, $R$.

## LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATÖRY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720


[^0]:    *Supported by U. S. Energy Research \& Development Administration.

