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I—FORM FACTOR AND ITS SIGNIFICANCE

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ABSTRACT OF PAPER

Form factor is significant in the study of transformer losses; as is well known, hysteresis loss is small when the form factor is large and *vice versa*. Every wave shape has a definite value of form factor; but the converse is not true, for a particular value of form factor does not indicate a particular wave shape. A wave may contain a third harmonic equal to seventy five per cent. of the fundamental and still have the same form factor as a true sine wave. Form factor, therefore, has no general significance as an indicator of wave form or wave distortion.

A general expression for form factor is derived in terms of the relative amplitudes and phase positions of its harmonic components; curves are drawn showing the variation of form factor with the amplitude and phase of the third harmonic.

Various wave forms are shown, very unlike in appearance, having the same form factor.

FORM factor, f, is the ratio of the r.m.s. value to the average value of an alternating quantity for half a period. The quantity to which form factor refers is usually an alternating electromotive force, in which case $f = E \div E_{av}$. Each particular wave shape has a definite form factor and so to a certain extent form factor indicates the shape of a wave and its departure from a true sine wave. Thus, a sine wave has a form factor 1.1107; a flat wave has a lesser form factor and a peaked wave a greater. If the converse were true and a particular value of form factor indicated one particular wave shape, the form of a wave could be accurately defined in terms of form factor, but, as will be seen later, this is far from being the case.

It is true that, for certain purposes, the value of form factor is significant, as for example in the determination of transformer losses. Hysteresis loss in a transformer depends upon the maximum value of the magnetic flux. But, inasmuch as the flux ϕ is determined by the relation $\phi \propto \int e dt$, the maximum value

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of the flux is proportional to the average value of e and hence to the r.m.s. value divided by form factor; that is

$$\phi_{max} = (E \div f) \times \text{constant}^1.$$

If a transformer is operated at a specified r.m.s. voltage from supply circuits having different voltage wave shapes, the maximum flux and hence the hysteresis loss will, accordingly, have different values for different form factors, becoming greater as the form factor becomes less and *vice versa.* It is well known

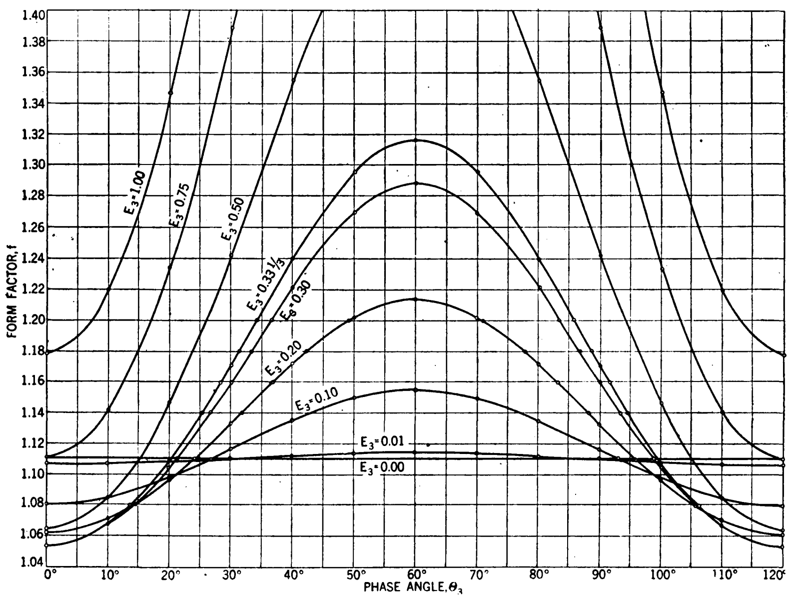

FIG. 1

that a transformer operates less efficiently on a flat wave than on a peaked wave.

If the r.m.s. voltage, E, is increased or decreased in direct proportion to form factor, so that the average voltage, E/f, remains constant, the hysteresis loss in the transformer remains unchanged and this fact is made use of in the determination of transformer losses on a sine-wave basis. For this purpose, the value of form factor can be ascertained by measuring the r.m.s.

1. This constant is 10^8, when E is in volts, divided by $4 \times$ frequency $\times$ cross section of iron in square centimeters $\times$ number of turns embracing it.

voltage by an ordinary voltmeter and the average voltage by means of a commutator[2] and d. c. voltmeter. To make simple the determination of hysteresis loss on a sine wave basis, without the necessity of determining the value of form factor, a special iron-loss voltmeter[3] has been devised by L.W. Chubb. An advantage in the use of this instrument is that it corrects for small variations in frequency as well as in wave form so as to give the loss for a standard frequency and sine wave form; a small error may be introduced, however, by the fact that eddy current loss and hysteresis loss do not follow the same law.

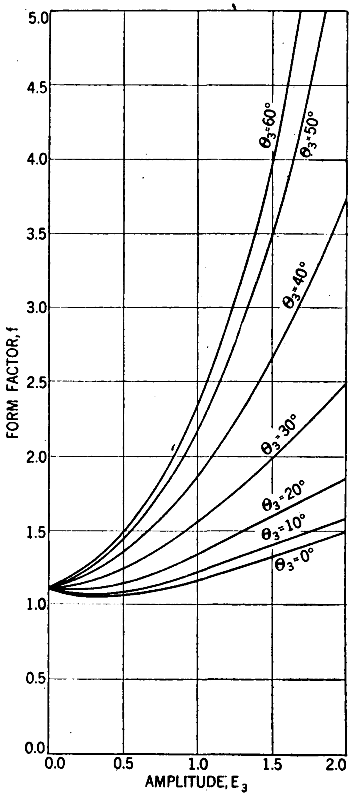

FIG. 2

As has been shown, hysteresis loss depends in a very definite manner upon form factor which, accordingly, *in this connection* has a definite and useful significance. In general, however, form factor has no useful significance as an indication of the shape of a wave or its departure from a sine wave; in fact an irregular wave may have a third harmonic as large as 75 per cent of the fundamental and, if the harmonic is in the proper phase, as will be shown, still have the same form factor as a pure sine wave.

An alternating wave is made up of a fundamental sine wave and harmonics with frequencies that are odd multiples of the fundamental frequency. The form factor of a complex wave depends not only upon the amplitude of these harmonics but upon

2. See, Lloyd and Fisher, "An Apparatus for Determining the Form of a Wave of Magnetic Flux," *Bulletin Bureau of Standards*, Vol. 4, p. 467, 1908; F. Bedell, The Use of the Synchronous Commutator in Alternating Current Measurements, *Journal Franklin Institute*, p. 385, Oct., 1913.

3. Method of Testing Transformer Core Losses, Giving Sine-wave Results on a Commercial Circuit, A.I.E.E. TRANSACTIONS, Vol, 28, p. 417, 1909. See also pp. 432-473.

their relative phase positions. The exact value of form factor is determined as follows:

Let E_1, E_3, E_5, etc., be the r.m.s. values of the several harmonics. The r.m.s. value of the total voltage wave is

$$E = (E_1^2 + E_3^2 + E_5^2 + \quad . \quad . \quad .)^{1/2}$$

This comes from the well-known r.m.s. principle[4] that the r.m.s value of any alternating quantity is the square root of the sum of the squares of its harmonic components, irrespective of their phase positions.

To find the average voltage, let the instantaneous voltage be

$e = E_{1max} \sin x + E_{3\,max} \sin 3(x + \theta_3) + E_{5\,max} \sin 5(x + \theta_5) + \,..$, where x is a variable proportional to time; $x = \omega t$.

$$E_{av} = \frac{1}{\pi}\int_0^\pi e dx = -\frac{1}{\pi}\Big[E_{1max} \cos x\Big]_0^\pi$$

$$-\frac{1}{\pi}\Big[\frac{1}{3}E_{3max} \cos 3\ (x + \theta_3)\Big]_0^\pi - \ .\ .\ .$$

$$= \frac{2}{\pi}(E_{1max} + \frac{1}{3}E_{3max} \cos 3\theta_3 + \frac{1}{5}E_{5\,max} \cos 5\ \theta_5 + \ldots.),$$

$$= \frac{2\sqrt{2}}{\pi}(E_1 + \frac{1}{3}E_3 \cos 3\theta_3 + \ldots.)$$

The form factor is, accordingly

$$f = \frac{E}{E_{av}} = 1.1107\ \frac{(E_1^2 + E_3^2 + \ldots.)^{1/2}}{(E_1 + \frac{1}{3}E_3 \cos 3\theta_3 + ..)}$$

In this equation E_1, E_3, etc., may represent either r.m.s. or maximum values.

In the following discussion the effect of the third harmonic only will be considered. Corresponding results for any other harmonic may be obtained in a like manner and will vary with the order of the harmonic; but in view of the results here shown and

4. A proof of this principle is given on p. 391, " The Principles of the Transformer," 1896, by F. Bedell.

the fact that the results for any other harmonics would obviously be of the same character, such a laborious study does not seem worth while.

It is seen that form factor varies with the phase as well as with the amplitude of the third harmonic; that is, in the preceding equation there are three variables, E_3, θ_3 and f. Figs. 1, 2 and 3 are plotted by assigning constant values to each one of these

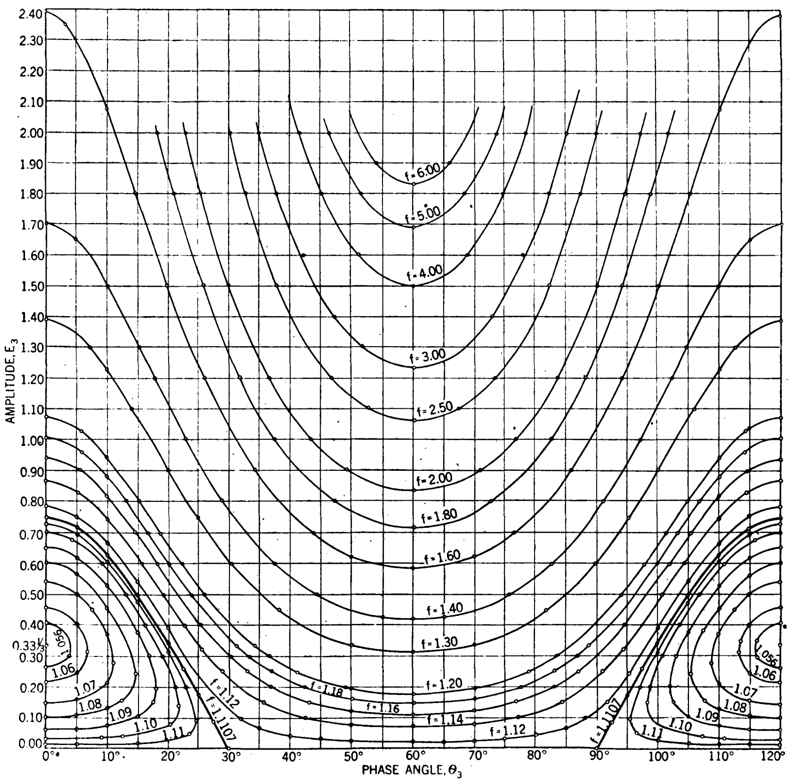

FIG. 3

three variables in turn and evaluating the equation so as to obtain the law of variation between the other two variables. Each figure will be discussed separately; it will be seen, however, that the remarks made in connection with any one figure apply to all.

In Fig. 1, each curve shows the variation of form factor with the phase angle θ_3, of the third harmonic when the harmonic has a definite value, namely, 0, 0.01, 0.10, 0.20, 0.30, 0.33, 0.50, 0.75 and 1.00 times the fundamental. The variation of θ_3 from 0° to 120° represents all possible values, for beyond these limits the curves repeat themselves.

When $E_3 = 0, f = 1.1107$ for all values of θ_3. When $E_3 > 0.75$, $f > 1.1107$ for all values of θ_3. When $E_3 < 0.75$ (the usual case), f may be greater or less than 1.1107, according to the value of θ_3.

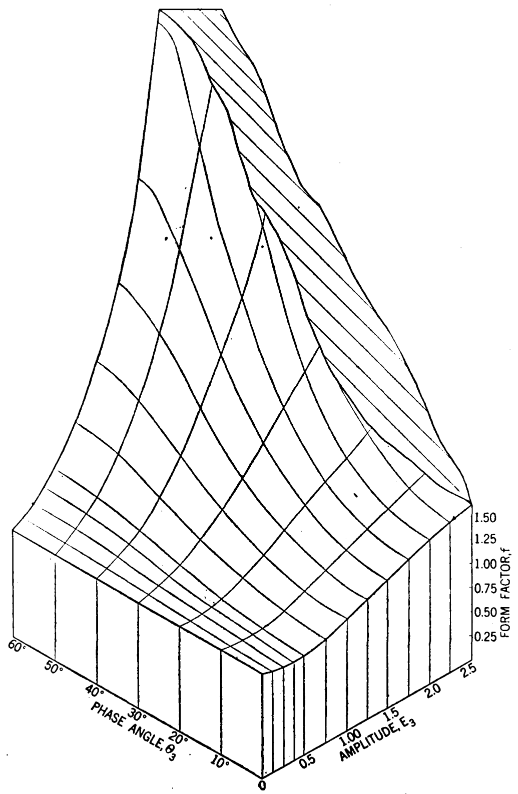

FIG. 4—VARIATION OF FORM FACTOR WITH AMPLITUDE AND PHASE OF THIRD HARMONIC

The maximum value of f occurs when $\theta_3 = 60$ deg. and this maximum is infinite[5] when $E_3 = 3.00$.

The minimum value of form factor occurs when $\theta_3 = 0$ deg. or 120 deg., and the lowest value of this minimum is 1.0537 when $E_3 = 0.33\frac{1}{3}$. The minimum value is 1.1107 when $E_3 = 0.75$, and is greater than 1.1107 when $E_3 > 0.75$.

5. When $E_3 > 3$, the value of f passes through infinity for two values of θ_3, one greater and one less than 60 deg.

Fig. 2 shows the variation of form factor with the amplitude of the third harmonic, when θ_3 is 0 deg., 10 deg., 20 deg., 30 deg., 40 deg., 50 deg. and 60 deg. As in Fig. 1, it is seen that, when $E_3 = 0, f = 1.1107$. When $E_3 < 0.75, f < 1.1107$ for small phase angles. The minimum value of f is 1.0537.

In Fig. 3, each curve is drawn for a constant form factor and shows the corresponding relation between E_3 and θ_3. The heavy curve is drawn for $f = 1.1107$, corresponding to a sine wave of

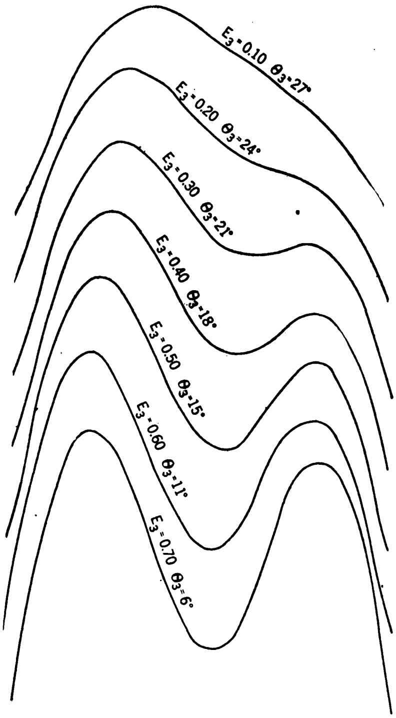

FIG. 5—WAVES WITH FORM FACTOR, $f = 1.1107$

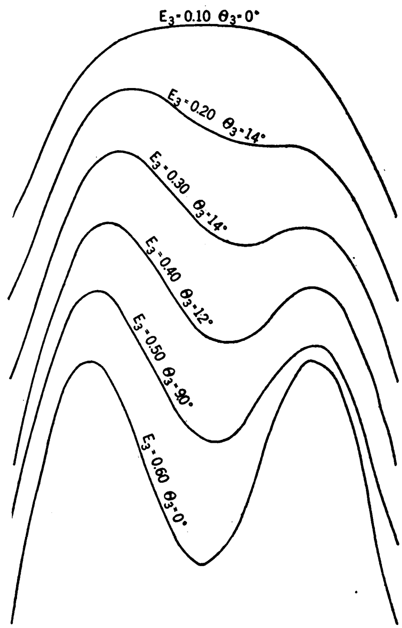

FIG. 6—WAVES WITH FORM FACTOR, $f = 1.08$

electromotive force; E_3 may have any value between 0.75 (when $\theta_3 = 0$ deg. or 120 deg.) and zero. If either $E_3 > 0.75$ or $\theta_3 > 30$ deg., $f > 1.1107$. A form factor less than 1.1107 is obtained only when $E_3 < 0.75$ and $\theta_3 < 30$ deg. or $\theta_3 > 90$ deg.; for these cases there are two values of E_3 for each value of θ_3. The curves shrink to a single point when f has its minimum value, 1.0537, corresponding to $E_3 = .33\frac{1}{3}$ and $\theta_3 = 0$ deg. or 120 deg.

Fig. 4, drawn in isometric projection, shows the variation of form factor with both E_3 and θ_3. Cross sections of Fig. 4, taken parallel to each pair of axes in turn, would give curves as shown in Figs. 1, 2 and 3.

To see whether or not there is any similarity of appearance in wave shapes that have the same form factor, Figs. 5, 6 and 7 were drawn by H. Papazian. The wave shapes in each figure have the same form factor, the corresponding values of E_3 and θ_3 being indicated in each case. Fig. 5 shows wave shapes having $f = 1.1107$, the same as a sine wave. Figs. 6 and 7 show wave shapes having $f = 1.08$ and 1.14, a little more and a little less than a sine wave, respectively. The curves show no distinguishing characteristics by which it is possible to tell whether a certain curve has the same form factor as a sine wave or one that is greater or less.

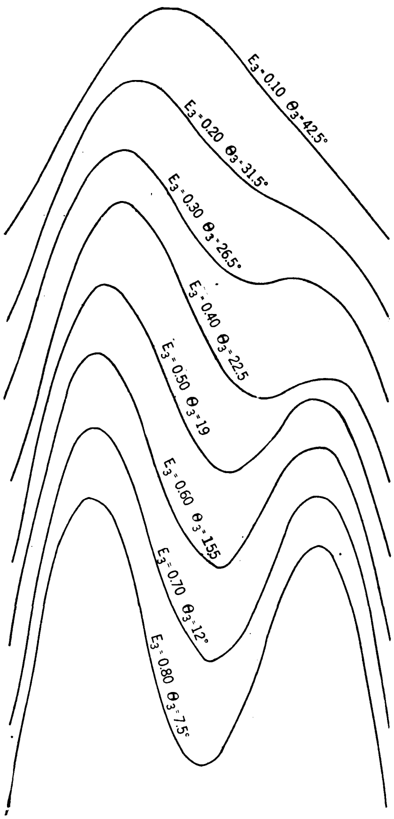

FIG. 7—WAVES WITH FORM FACTOR, $f = 1.14$

The use of five places may seem useless in designating the form factor of 1.1107 for a sine wave, but in plotting the curves here given, particularly those shown in Fig. 3, it was necessary to carry many of the calculations thus far. The results were, in places, inconsistent and unintelligible when computations were less accurate; inspection of Fig. 3 will show that the form factor for a sine wave is a critical value and a slight change in this value makes a great difference in the character of the curve.

In the following paper will be discussed other factors than form factor for indicating the amount of distortion of a wave from a standard sine wave.