

## I. THE LAW OF CAUSALITY.

§ 1. We start with the assumption that *everything that exists, and everything that happens, exists or happens as a necessary consequence of a previous state of things*. If a state of things is repeated in every detail, it must lead to exactly the same consequences. Any difference between the results of causes that are in part the same, must be explainable by some difference in the other part of the causes.

This assumption, which may be called the law of causality, cannot be proved, but must be believed; in the same way as we believe the fundamental assumptions of religion, with which it is closely and intimately connected. The law of causality forces itself upon our belief. It may be denied in theory, but not in practice. Any person who denies it, will, if he is watchful enough, catch himself constantly asking himself, if no one else, why *this* has happened, and not *that*. But in that very question he bears witness to the law of causality. If we are consistently to deny the law of causality, we must repudiate all observation, and particularly all prediction based on past experience, as useless and misleading.

If we could imagine for an instant that the same complete combination of causes could have a definite number of different consequences, however small that number might be, and that among these the occurrence of the actual consequence was, in the old sense of the word, accidental, no observation would ever be of any particular value. Scientific observations cannot be reconciled with polytheism. So long as the idea prevailed that the result of a journey depended on whether the power of Njord or that of Skade was the stronger, or that victory or defeat in battle depended on whether Jove had, or had not, listened to Juno's complaints, so long were even scientists obliged to consider it below their dignity to consult observations.

But if the law of causality is acknowledged to be an assumption which always holds good, then every observation gives us a revelation which, when correctly appraised and compared with others, teaches us the laws by which God rules the world.

We can judge of the far-reaching consequences it would have, if there were conditions in which the law of causality was not valid at all, by considering the cases in which the effects of the law are more or less veiled.

In inanimate nature the relation of cause and effect is so clear that the effects are determined by observable causes belonging to the condition immediately preceding, so that the problem, within this domain, may be solved by a tabular arrangement of the several observed results according to the causing circumstances, and the transformation of the tables into laws by means of interpolation. When, however, living beings are the object of our observations, the case immediately becomes more complicated.

It is the prerogative of living beings to hide and covertly to transmit the influences received, and we must therefore within this domain look for the influencing causes throughout the whole of the past history. A difference in the construction of a single cell may be the only indication present at the moment of the observation that the cell is a transmitter of the still operative cause, which may date from thousands of years back. In consequence of this the naturalist, the physiologist, the physician, can only quite exceptionally attain the same simple, definite, and complete accordance between the observed causes and their effects, as can be attained by the physicist and the astronomer within their domains.

Within the living world, communities, particularly human ones, form a domain where the conditions of the observations are even more complex and difficult. Living beings hide, but the community deceives. For though it is not in the power of the community either to change one tittle of any really divine law, or to break the bond between cause and effect, yet every community lays down its own laws also. Every community tries to give its law fixity, and to make it operate as a cause; for instance, by passing it off as divine or by threats of punishment; but nevertheless the laws of the community are constantly broken and changed.

Statistical Science which, in the case of communities, represents observations, has therefore a very difficult task; although the observations are so numerous, we are able from them alone to answer only a very few questions in cases where the intellectual weapons of historical and speculative criticism cannot assist in the work, by independently bringing to light the truths which the communities want to conceal, and on the other hand by removing the wrong opinions which these believe in and propagate.

§ 2. An isolated sensation teaches us nothing, for it does not amount to an observation. Observation is a putting together of several results of sensation which are or are supposed to be connected with each other according to the law of causality, so that some represent causes and others their effects.

By virtue of the law of causality we must believe that, in all observations, we get essentially correct and true revelations; the difficulty is, to ask searchingly enough and to understand the answer correctly. In order that an observation may be free from every other assumption or hypothesis than the law of causality, it must include a perfect

description of all the circumstances in the world, at least at the instant preceding that at which the phenomenon is observed. But it is clear that this far surpasses what can be done, even in the most important cases. Real observations have a much simpler form. By giving a short statement of the time and place of observation, we refer to what is known of the state of things at the instant; and, of the infinite multiplicity of circumstances connected with the observation we, generally, not only disregard everything which may be supposed to have little or no influence, but we pay attention only to a small selection of circumstances, which we call *essential*, because we expect, in virtue of a special hypothesis concerning the relation of cause and effect, that the observed phenomenon will be effect of these circumstances only.

Nay, we are often compelled to disregard certain circumstances as *unessential*, though there is no doubt as to their influencing the phenomenon; and we do this either because we cannot get a sufficient amount of trustworthy information regarding them, or because it would be impracticable to trace out their connection with the effect. For instance in statistical observations on mortality, where the age at the time of death can be regarded as the observed phenomenon, we generally mention the sex as an essential circumstance, and often give a general statement as to residence in town or country, or as to occupation. But there are other things as to which we do not get sufficient information: whether the dead person has lived in straitened or in comfortable circumstances, whether he has been more or less exposed to infectious disease, etc.; and we must put up with this, even if it is certain that one or other of these things was the principal cause of death. And analogous cases are frequently met with both in scientific observations and in everyday occurrences.

In order to obtain a perfect observation it is necessary, moreover, that our sensations should give us accurate information regarding both the phenomenon and the attendant circumstances; but all our senses may be said to give us merely approximate descriptions of any phenomenon rather than to measure it accurately. Even the finest of our senses recognizes no difference which falls short of a certain finite magnitude. This lack of accuracy is, moreover, often greatly increased by the use of arbitrary round numbers for the sake of convenience. The man who has to measure a race-course, may take into account the odd metres, but certainly not the millimetres, not to mention the microns.

§ 3. *Owing to all this, every actual observation is affected with errors.* Even our best observations are based upon hypothesis, and often even on an hypothesis that is certainly wrong, namely, that only the circumstances which are regarded as essential, influence the phenomenon; and a regard for practicability, expense, and convenience makes us give approximate estimates instead of the sharpest possible determinations.

Now and then the observations are affected also by *gross errors* which, although

not introduced into them on purpose, are yet caused by such carelessness or neglect that they could have been, and ought to have been, avoided. In contradistinction to these we often call the more or less unavoidable errors *accidental*. For accident (or chance) is not, what the word originally meant, and what still often lingers in our ordinary conception of it, a capricious power which suffers events to happen without any cause, but only a name for the unknown element, involved in some relation of cause and effect, which prevents us from fully comprehending the connection between them. When we say that it is accidental, whether a die turns up "six" or "three", we only mean that the circumstances connected with the throwing, the fall, and the rolling of the die are so manifold that no man, not even the cleverest juggler and arithmetician united in the same person, can succeed in controlling or calculating them.

In many observations we reject as unessential many circumstances about which we really know more or less. We may be justified in this; but if such a circumstance is of sufficient importance as a cause, and we arrange the observations with special regard to it, we may sometimes observe that the errors of the observations show a regularity which is not found in "accidental" errors. The same may be the case if, in computations dealing with the results of observations, we make a wrong supposition as to the operation of some circumstance. Such errors are generally called *systematic*.

§ 4. It will be found that every applied science, which is well developed, may be divided into two parts, a theoretical (speculative or mathematical) part and an empirical (observational) one. Both are absolutely necessary, and the growth of a science depends very much on their influencing one another and advancing simultaneously. No lasting divergence or subordination of one to the other can be allowed.

The theoretical part of the science deals with what we suppose to be accurate determinations, and the object of its reasonings is the development of the form, connection, and consequences of the hypotheses. But it must change its hypotheses as soon as it is clear that they are at variance with experience and observation.

The empirical side of the science procures and arranges the observations, compares them with the theoretical propositions, and is entitled by means of them to reject, if necessary, the hypotheses of the theory. By induction it can deduce laws from the observations. But it must not forget — though it may have a natural inclination to do so — that, as shown above, it is itself founded on hypotheses. The very form of the observation, and especially the selection of the circumstances which are to be considered as essential and taken into account in making the several observations, must not be determined by rule of thumb, or arbitrarily, but must always be guided by theory.

Subject to this it must as a rule be considered best, that the two sides of the science should work somewhat independently of one another, each in its own particular

way. In what follows the empirical side will be treated exclusively, and it will be treated on a general plan, investigating not the particular way in which statistical, chemical, physical, and astronomical observations are made, but the common rules according to which they are all submitted to computation.

## II. LAWS OF ERRORS.

§ 5. Every observation is supposed to contain information, partly as to the phenomenon in which we are particularly interested, partly as to all the circumstances, connected with it, which are regarded as essential. In comparing several observations, it makes a very great difference, whether such essential circumstances have remained unchanged, or whether one or several of them have changed between one observation and another. The treatment of the former case, that of *repetitions*, is far simpler than that of the latter, and is therefore more particularly the subject of our investigations; nevertheless, we must try to master also the more difficult general case in its simplest forms, which force themselves upon us in most of the empirical sciences.

By *repetitions* then we understand those observations, in which all the essential circumstances remain unchanged, in which therefore the results or phenomena should agree, if all the operative causes had been included among our essential circumstances. Furthermore, we can without hesitation treat as repetitions those observations, in which we assume that no essential circumstance has changed, but do not know for certain that there has been no such change. Strictly speaking, this would furnish an example of observations with systematic errors; but provided there has been no change in the care with which the essential circumstances have been determined or checked, it is permissible to employ the simpler treatment applicable to the case of repetitions. This would not however be permissible, if, for instance, the observer during the repetitions has perceived any uncertainty in the records of a circumstance, and therefore paid greater attention to the following repetitions.

§ 6. The special features of the observations, and in particular their degree of accuracy, depend on causes which have been left out as unessential circumstances, or on some overlooked uncertainty in the statement of the essential circumstances. Consequently no speculation can indicate to us the accuracy and particularities of observations. These must be estimated by comparison of the observations with each other, but only in the case of repetitions can this estimate be undertaken directly and without some preliminary work. The phrase *law of errors* is used as a general name for any mathematical expression representing the distribution of the varying results of repetitions.