"Ideal" optical delay lines based on tailored-coupling and reflecting, coupled-resonator optical waveguides

Hsi-Chun Liu* and Amnon Yariv

Department of Electrical Engineering, California Institute of Technology, Pasadena, California 91125, USA
**Corresponding author: hliu@caltech.edu*

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We present a design of "ideal" optical delay lines (i.e., constant amplitude and constant group delay over the desired bandwidth). They are based on reflection from coupled-resonator optical waveguides (CROWs). The inter-resonator coupling coefficients are tailored and decrease monotonically with the distance from the input to realize all-pass Bessel filters. The tailored coupling coefficients result in a frequency-dependent propagating distance which compensates for the group velocity dispersion of CROWs. We present a simple formalism for deriving the time-domain coupling coefficients and convert these coefficients to field coupling coefficients of ring resonators. The reflecting CROWs possess a delay-bandwidth product of 0.5 per resonator, larger than that of any kind of transmitting CROW. In the presence of uniform gain, the gain enhanced by slow light propagation and the constant group delay result in efficient and dispersion-free amplifiers.

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Optical delay lines and buffers are key components for optical networks and information processing systems [1]. Delay lines based on conventional optical waveguides are typically very long. The length can be greatly reduced if the group velocity of light is significantly reduced. "Slow light" can be achieved in engineered structures which bounce light back and forth as it propagates. Such structures include grating structures [2,3], photonic crystal waveguides [4], and coupled-resonator optical waveguides (CROWs) [5]. The general characteristic of these slow-light waveguides is a dispersion curve $\omega(\beta)$ whose slope $d\omega/d\beta$, equal to the modal group velocity, is small over a range of frequencies. A major problem in designing delay lines based on these waveguides is the higher-order dispersion, which causes a distortion of signal. Although the second-order dispersion of grating structures and CROWs is zero at the band center, the group velocity approaches zero at frequencies close to the band edges [2,3,5]. Several efforts to cancel or minimize higher-order dispersion have been reported [4,6].

A CROW consists of a chain of weakly-coupled resonators where light propagates by virtue of inter-resonator coupling. The original proposal of CROWs [5] was based on uniform coupling coefficient κ, which leads to a dispersion curve and a group velocity dictated by the coupling coefficient. If the coupling coefficients are allowed to vary along the CROW, the dispersive properties can be further controlled. In the transmission mode, the transfer function T is a function of s, where $s = i(\omega - \omega_0)$ is the frequency detuning from the resonant frequency. Each resonator can be considered as a feedback loop which contributes a pole to the transfer function. An N-resonator CROW is thus an all-pole filter of order N whose transfer function is given by $T(s) = k/p(s)$, where $p(s)$ is a polynomial in s. Filter design approaches can then be applied to derive the coupling coefficients which determine the transfer function of CROWs to achieve desired filter responses [7–9]. For example, Butterworth and Bessel CROWs exhibit maximally flat transmission and group delay, respectively. However, constant amplitude and group delay cannot be achieved by any one of these filters simultaneously, since the amplitude and the phase of all-pole functions are related to each other.

In this Letter, we present an "ideal" optical delay line, which possesses a constant group delay and a constant amplitude transmission over a prescribed bandwidth. It is based on the reflection of a CROW, whose inter-resonator coupling coefficients are tailored to realize an all-pass Bessel filter. The design of all-pass Bessel filters has been explored using microwave equivalent circuit methods [7]. In what follows, we present a simple formalism for deriving the time-domain coupling coefficients and interpret the physics behind the idea.

An all-pass function $[p(s)]^*/p(s)$ preserves the phase of $T(s)$ and has a constant output amplitude of 1. As a result, an all-pass Bessel filter whose $p(s)$ is a Bessel polynomial possesses constant amplitude and maximally flat group delay over a prescribed bandwidth, as shown in Fig. 1(a). The higher-order dispersion is 0 up to the order of N. Such all-pass filters can be realized in the reflection mode of lossless CROWs, as shown in Fig. 1(b). The input energy coupled into the CROW is eventually coupled back as output into the original waveguide since, in the limit of small resonator losses, it is the only exit channel.

Reflecting CROWs can be realized using various kinds of resonators. Figs. 1(c) and 1(d) illustrate reflecting CROWs based on ring resonators and grating-defect resonators, where the output s_r is at the through port of microring CROWs and at the reflection of grating CROWs. The coupling coefficients can be controlled via the gap between ring resonators or the number of holes between adjacent defects. Microring reflecting CROWs have been experimentally demonstrated for the purpose of tunable delay [10]. To realize ideal delay lines, the coupling coefficients need to be derived. The formalism is based on coupled-mode theory.

Consider the CROW in Fig. 1(b), which consists of N identical resonators and is coupled to an input waveguide. For an input s_{in} at a frequency ω, the steady-state coupled-mode equation can be written as $\boldsymbol{A}\boldsymbol{a} = \boldsymbol{v}_{in}$, where

$$\mathbf{A} = \begin{bmatrix} s+\frac{1}{\tau_{e1}} & i\kappa_1 & 0 & 0 & \cdots & 0 \\ i\kappa_1 & s & i\kappa_2 & 0 & \cdots & 0 \\ 0 & i\kappa_2 & s & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & s & i\kappa_{N-1} \\ \cdot & \cdot & \cdot & \cdot & i\kappa_{N-1} & s \end{bmatrix} \quad (1)$$

is an $N \times N$ tridiagonal coupling matrix, $\boldsymbol{a} = [a_1 \quad a_2 \quad \cdots \quad a_N]^T$ denotes the complex mode amplitudes of the resonators, and $v_{\text{in}} = [-i\mu_i s_{\text{in}} \quad 0 \quad \cdots \quad 0]^T$ is the input. The coupling between input waveguide and the first resonator is modeled by an external loss $1/\tau_{e1}$ and the coupling $\mu_1 = \sqrt{2/\tau_{e1}}$. The equation is the same as the equation of transmitting CROWs in [9], except that $1/\tau_{e2} = 0$.

The transfer function of reflection can be written as

$$R(s) = \frac{s_r}{s_{\text{in}}} = 1 - \mu_1^2[A^{-1}]_{1,1} = \frac{p_N - \mu_1^2 p_{N-1}}{p_N}, \quad (2)$$

where p_k for $k = 1, 2, \ldots, N$ is defined as the determinant of the bottom-right $k \times k$ submatrix of A and is a polynomial in s with a leading term s^k. The polynomials p_1 through p_N satisfy the recursive formulas [9]

$$\begin{aligned} p_N &= \left(s + \frac{1}{\tau_{e1}}\right) p_{N-1} + \kappa_1^2 p_{N-2}, \\ p_{N-1} &= s p_{N-2} + \kappa_2^2 p_{N-3} \\ &\vdots, \\ p_2 &= s p_1 + \kappa_{N-1}^2, \\ p_1 &= s. \end{aligned} \quad (3)$$

As an example we consider an all-pass Bessel filter of order $N = 6$, whose transfer function is given by $R(s) = [p(s)]^*/p(s)$, where is a Bessel polynomial, $p(s) = s^6 + 4.495s^5 + 9.622s^4 + 12.358s^3 + 9.92s^2 + 4.672s + 1$. The group delay of $R(s)$ is maximally flat between $\Delta\omega = -1$ and 1. Since the coefficients of are real, $[p(s)]^*/p(-s)$. Comparing $R(s)$ with Eq. (2), we obtain $p_6 = p(s)$ and $\mu_1^2 p_5 = p(s) - p(-s)$. Since the leading coefficient of every polynomial p_k is 1, $\mu_1^2 = 8.990$ and $p_5 = s^5 + 2.749s^3 + 1.039s$. With p_6 and p_5, all the coupling coefficients can be extracted step by step, using Eq. (3). The extracted coefficients are $(1/\tau_{e1}, \kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5) = (4.495, 2.622, 1.207, 0.824, 0.632, 0.463)$, which decrease monotonically from the input. Finally, we multiply the coefficients by a bandwidth parameter B to choose the bandwidth, which leads to maximally flat delay between $\Delta\omega = -B$ and B. The group delay is inversely proportional to B. The spectra of reflection and group delay are shown in Fig. 1(a).

To understand the physics behind reflecting Bessel CROWs, we plot the CROW propagation band as a function of distance. CROWs with uniform coupling coefficient κ form a constant height band between $\omega_0 - 2\kappa$ and $\omega_0 + 2\kappa$. Frequencies within the CROW band propagate freely while those outside evanesce exponentially with distance. CROWs with tailored coupling coefficients correspond to a distance-dependent CROW band whose thickness is $4\kappa(z)$, where $\kappa(z)$ is the local coupling coefficient. Fig. 2(a) shows the modulated CROW band of an $N = 20$ reflecting Bessel CROW, whose bandwidth decreases monotonically from the input. An input signal at a given frequency propagates into the CROW until it reaches the band edge where it is reflected back. The red lines in Fig. 2(a) indicate the propagating distances at $\Delta\omega/B = 0.14$, and 2. Fig. 2(b) plots the field distribution at these frequencies. At $\Delta\omega = 0$, light propagates to the last resonator. As the frequency moves away from the resonant frequency, the propagating distance decreases. The dependence of the propagating distance on frequency compensates for the group velocity dispersion of CROWs, whose group delay increases monotonically from the band center to the band edge, and results in a constant group delay.

To realize reflecting Bessel CROWs in ring resonators, we convert the time-domain coupling coefficients to the field coupling coefficients in the coupling regions. We consider silicon ring resonators as an example. The mode index and group index of the Si waveguides are respectively 2.4 and 4. The radii of the rings are selected as 20 μm so that one resonant wavelength is 1570.8 nm and the free spectral range f_{FSR} is 597 GHz. The relation between the field coefficient η and the coupling coefficient κ is given by $\eta = \sin(\kappa/f_{\text{FSR}})$ for inter-resonator coupling and $\eta_i = \sqrt{2\sin(1/\tau_{e1}f_{\text{FSR}})/[1 + \sin(1/\tau_{e1}f_{\text{FSR}}]}$ at the input [9]. We choose $B = \omega_{\text{FSR}} \cdot 0.003$ and $B = \omega_{\text{FSR}} \cdot 0.03$, which lead to field coupling coefficients of (0.395, 0.0494, 0.0228, 0.0155, 0.0119, 0.0087) and (0.926, 0.474, 0.226, 0.155, 0.119, 0.087), respectively. The spectra of reflection and group delay are shown in Figs. 3(a) and 3(b). The spectra are ideal for weaker coupling coefficients

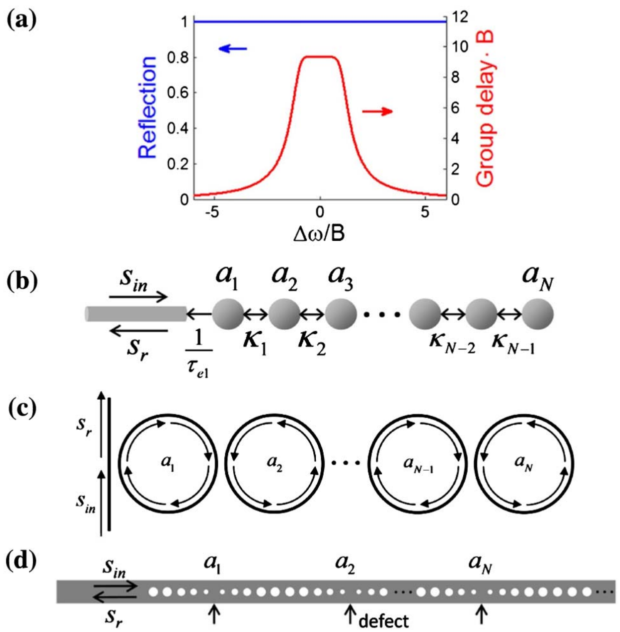

Fig. 1. (Color online) (a) Spectra of reflection and group delay of an $N = 6$ reflecting Bessel CROW. (b) Schematic drawing of a reflecting CROW. (c), (d) Reflecting CROWs based on ring resonators and grating-defect resonators, respectively.

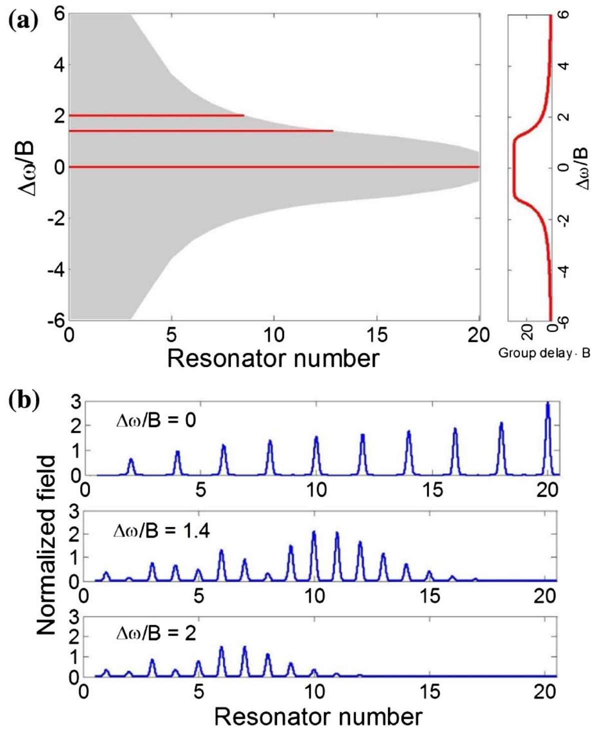

Fig. 2. (Color online) (a) (left) CROW propagation band as a function of distance of an $N = 20$ reflecting Bessel CROW. Red lines: Propagation distance for $\Delta\omega/B = 0$, 1.4, and 2. (right) Group delay spectrum. (b) Field distribution along the CROW for $\Delta\omega/B = 0$, 1.4, and 2.

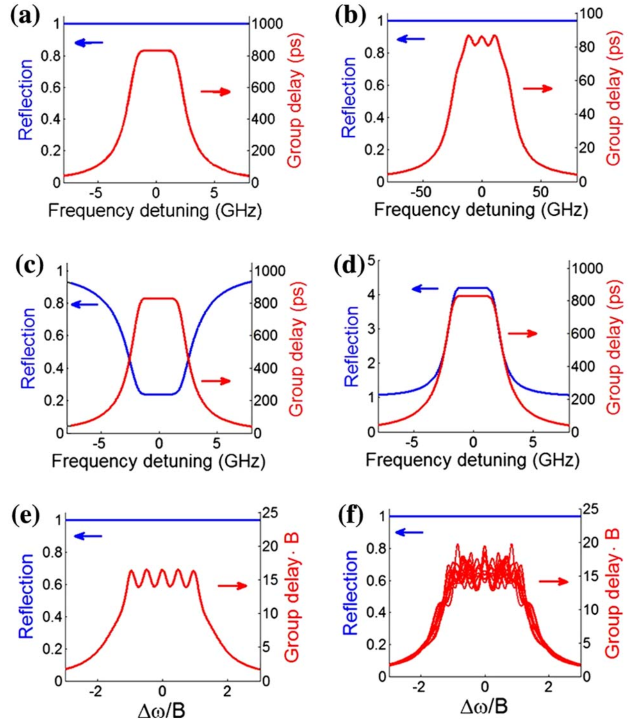

Fig. 3. (Color online) (a–d) Spectra of reflection and group delay of $N = 6$ reflecting Bessel CROWs based on ring resonators. (a) $B = \omega_{\mathrm{FSR}} \cdot 0.003$. (b) $B = \omega_{\mathrm{FSR}} \cdot 0.003$. (c), (d) $B = \omega_{\mathrm{FSR}} \cdot 0.003$ with a uniform loss and gain of 1 dB/cm respectively. (e),(f) $N = 10$ reflecting Bessel CROWs with disorder of coupling coefficients. (e) $\kappa_4' = 1.05\kappa_4$. (f) $\kappa_i' = \gamma_i \kappa_i$ for all i.

when $B = \omega_{\mathrm{FSR}} \cdot 0.003$. For the case of $B = \omega_{\mathrm{FSR}} \cdot 0.003$, there are oscillations in the group delay spectrum since the coupling coefficient at the input is close to the maximal coupling coefficient of the ring resonators, $\omega_{\mathrm{FSR}}/4$.

Up to this point we have considered only lossless resonators. For lossy resonators with constant loss rates, the total loss is proportional to the group delay. Since the group delay is flat, the loss is also flat within the bandwidth, and the definition of ideal delay lines is still satisfied. Fig. 3(c) shows the spectra of the same microring CROW in Fig. 3(a) with a propagation loss of 1 dB/cm. On the other hand, if the resonators are pumped with uniform gain, the amplification is proportional to the group delay and is also flat, as shown in Fig. 3(d). The enhanced reflection gain by slow light results in efficient and dispersion-less amplifiers.

One important parameter of optical delay lines is the delay–bandwidth product (DBP), $\Delta f \cdot \tau$, which represents the number of bits that can be stored. The DBP of reflecting Bessel CROWs is approximately 0.5 per resonator, as can be evaluated in Figs. 1(a), 2(a), and 3(a). The upper bound of DBP per resonator of CROWs in the transmission mode is 0.5, resulting from a total phase shift of π provided by a pole. Because each resonator in a reflecting CROW contributes both a pole and a zero to the transfer function, the upper bound is 1. Therefore, the DBP per resonator of reflecting Bessel CROWs is larger than that of any kind of transmitting CROW.

Although the delay capability of reflecting CROWs is larger, reflecting CROWs are more sensitive to fabrication disorder of coupling coefficients and resonant frequencies. Any imperfection in a reflecting CROW scatters light twice as it takes a round trip. The cavity between the imperfection and the end of the CROW causes Fabry—Perot-type oscillations, as shown in Fig. 3(e). Fig. 3(f) shows the spectra of 10 different $N = 10$ reflecting Bessel CROWs under disorder of coupling coefficients. The modified coupling coefficients are given by $\kappa_i' = r_i \kappa_i$, where γ_i is a Gaussian-distributed random variable with a standard deviation of 0.03. The effect of disorder in resonant frequencies is similar. Under disorder, the average of the delay spectra is still optimally flat among all kinds of reflecting CROWs.

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