

## IDENTIFICATION AND CONTROL OF MECHANICAL SYSTEMS

Vibration is a significant issue in the design of many structures including aircraft, spacecraft, bridges, and high-rise buildings. This book discusses the *control of vibrating systems*, integrating structural dynamics, vibration analysis, modern control, and system identification. Integrating these subjects is an important feature in that engineers will need only one book, rather than several texts or courses, to solve vibration/control problems.

The book begins with a review of the fundamentals in mathematics, dynamics, and control that are needed for understanding subsequent materials. Chapters then cover recent developments in aerospace control and identification theory, including virtual passive control, observer and state–space system identification, and data-based controller synthesis. Many practical issues and applications are addressed, with examples showing how various methods are applied to real systems. Some methods show the close integration of system identification and control theory from the state–space perspective, rather than from the traditional input–output model perspective of adaptive control.

This text will be useful for advanced undergraduate and beginning graduate students in aerospace, mechanical, and civil engineering, as well as for practicing engineers.

Dr. Jer-Nan Juang is the Principal Scientist at the Structural Dynamics Branch, NASA Langley Research Center. He is also an Adjunct Professor at George Washington University, Duke University, and Virginia Polytechnic Institute and State University.

Dr. Minh Q. Phan is Associate Professor of Engineering at Thayer School of Engineering, Dartmouth College.

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**JER-NAN JUANG**

NASA Langley Research Center

**MINH Q. PHAN**

Dartmouth College



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*To Lily, Philo, Derek, and my parents*  
JNJ

*To Suzu, Daniel, my mother, and to the memory of my father*  
MQP

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## Preface

This book is based on a series of lecture notes developed by the authors. The first author has used part of the notes for two graduate-level classes in System Identification and Control of Large Aerospace Systems at the Joint Institute for Advancement of Flight Sciences, George Washington University at NASA Langley Research Center for the past 10 years. The second author has used part of the notes for senior and first year graduate-level courses in Dynamics and Control of Mechanical Systems and System Identification at Princeton University and Dartmouth College since 1995. There are many reasons that motivated the writing of this book; some of them are outlined below.

First, the lecture notes received overwhelming response from the students taking these courses, with many urging us to turn these materials into a textbook. When developing the notes, we tried to place emphasis on the fundamentals and clarity of presentation. Second, the subject matter is important in practice, but it is challenging both for students to learn and for us to teach because it is an integration of several disciplines: structural dynamics, vibration analysis, modern control, and system identification. The primary goal is for students to learn what these tools are without having to take a separate course for each subject and how they are brought together to solve a vibration control problem. Third, although there are many excellent textbooks dedicated to each of the individual disciplines, there are none that bring all of the disciplines together in this specific area of system identification and control of vibrating systems. Recently, there have been several textbooks that integrate dynamics and control, and others that deal with vibrations and control. In all cases, little attention, if any, is paid to system identification, which is still viewed as an advanced and separate topic. If one takes a course in system identification, it is likely that the course's emphasis will be on adaptive estimation and control for single-input single-output models for which there are many textbooks available. In this book, the emphasis is on the multi-input multi-output state-space models that are common in aerospace vibration control practice. We believe that the topic of state-space system identification has reached the critical point of maturity at which it has become useful as a practical tool and should no longer be treated as an advanced topic. Fourth, the book contains a number of recent and useful developments in aerospace control and identification theory. These include virtual passive control, observer and state-space identification, and data-based controller synthesis. Some of these new developments show the close integration of system identification and control theory as done from the state-space perspective, not from the traditional input-output model perspective of adaptive control.

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The book begins with a review of basic mathematics (Chaps. 1 and 2) that is needed for understanding the subsequent materials. Often, the source of difficulty for some students is the lack of knowledge of certain specific areas of mathematics. We wish to eliminate this kind of difficulty up front by making the mathematical review part of the course, rather than including it as an appendix as in most other textbooks. Next, we describe basic modeling techniques in Chap. 3, including Newton's laws, D'Alembert's principle, the principle of virtual work, Hamilton's principle, Lagrange's equations, Gibbs–Appell equations, and Kane's equations. Of course, in a single chapter it is not possible to go into depth on each of the above topics, but we do not think that this is necessary for our present purpose. By having many modeling techniques in one place, we hope to bring out the essence of each of the techniques and, more importantly, show how one modeling technique is related to another. This kind of global perspective is important, and a beginner should not be biased toward one or another. Dedicated texts addressing each of the modeling techniques are available and should be used for further studies. The finite-element method presented in Chap. 4 is a very popular technique for the numerical solution of complex problems in engineering. In continuous systems, the formulations describing the system response are governed by partial differential equations as described in Chap. 3. The exact solutions of the partial differential equations satisfying all boundary conditions are possible for only relatively simple systems such as a uniform beam. Numerical techniques must be introduced to discretize the partial differential equations to turn them into linear ordinary differential equations to predict the system response approximately, i.e., the finite-element method. Next, in Chap. 5 we present the core materials of any standard linear vibrations text. We do not create separate chapters for single- and multiple-degree-of-freedom systems, and we do not place any undue emphasis on single-variable systems. Instead, we move quickly from the single-degree-of-freedom to the multiple-degree-of-freedom case and show how both cases can be treated within a common framework. In Chap. 6 the subject of control is introduced, starting with designs that have clear physical interpretation by taking advantage of the second-order differential equations of system dynamics. Modern control is often criticized for its abstract tendency. This chapter shows what can be done if one wishes to stay as close as possible to the physics and what the advantages and disadvantages are with this approach. The next three chapters contain some of the most basic results of modern control theory. Chapter 7 introduces the notion of the state–space model that provides a common platform for the treatment of a wide variety of systems. Chapter 8 addresses the topic of state-feedback control. Some of the key results associated with classical optimal control are presented here. Chapter 9 focuses on dynamic feedback control including the observer-based state-feedback approach. For the most part we do not make a great deal of distinction between continuous-time and discrete-time representations. There are cases in which the mathematics are simpler in continuous time than in discrete time, but the most basic results in continuous-time control have their equivalent discrete-time counterparts. One should see this similarity right away, not after taking a course in linear system theory, which is usually taught in continuous time, and then see it again in a course in digital control theory some time later. By showing how the two approaches are actually similar, we hope that the readers will be able to see when they are different and why. Chapter 10 gives basic concepts and properties of state–space system identification. Computation of system Markov parameters (pulse-response time history) is described. Most time-domain methods in

modal testing are based on the pulse-response time history to identify modal parameters such as system frequencies, damping ratio, and mode shapes at the sensor points. Here identification techniques are derived that provide the state–space models needed for the design of a modern controller. In Chap. 11, the problem of predictive control is addressed. Originating in the chemical process community, predictive control has found its way into aerospace control problems. Some of the latest results in the integration of system identification and predictive control to produce the so-called data-based controller synthesis are presented here.

This book can be used in a junior-, senior-, or first-year-graduate-level course. Very minimal background is required other than elementary differential equations and linear algebra. When used at the graduate level, certain specific directions can be dealt with in more detail at the discretion of the instructor. The materials here provide a firm formulation so that the instructor's own research can easily be incorporated into such a course. Thus, the students will get exposed to the instructor's research with proper understanding and insight to the solution procedure for a general vibration/control problem. In our opinion, the ability to see complicated things in the simplest way across disciplinary boundaries is a prerequisite in creating new and fundamental knowledge. We believe that, when properly presented, many complicated techniques will look simple, and, being far from perfect, we hope that our attempt at this is a useful one.

We probably would not have been able to write this book or develop a number of techniques presented here without considerable support from our organizations including NASA Langley Research Center, George Washington University, Princeton University, and Dartmouth College. We sincerely acknowledge the influence of our colleagues John L. Junkins, Richard W. Longman, Lucas G. Horta, Leonard Meirovitch, Daniel J. Inman, Earl H. Dowell, Raymond G. Kvaternik, and our undergraduate and graduate students over the years. Their friendship and encouragement provided the motivation for writing this text.

We want to thank our family members, Lily Juang, Philo Juang, Derek K. Juang, Suzu Phan, and Daniel Phan for their support and understanding during those numerous dislocations in both time and space. Last, but not least, we would like to thank our parents for their love and support.

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