Identification of Coupled Mathematical Models for Underwater Vehicles

N. Mišković, Z. Vukić and M. Barišić

***Abstract*—The paper presents the procedure for identification of coupled mathematical models for underwater vehicles. The procedure is performed with the use of a simple laboratory apparatus that consists of a webcam placed above the experimental pool. The video recording of the underwater vehicle in motion is then analyzed in order to obtain relative speeds within the camera frame. The experiment uses a simple maneuver which excites the vehicle in all controllable directions (in the horizontal plane). The results have shown that even though the system under observation is nonholonomic, the sway motion occurs due to coupling. This allows for determination of dynamic model in uncontrollable directions. The experimental data also show which terms in a general dynamic model can be omitted when dealing with micro underwater vehicles, in order to preserve the simplicity.**

***Index Terms*—Identification, underwater vehicles, mobile robot dynamics**

I. Introduction

THE interest in underwater vehicles has involved a great number of control engineers mostly due to the challenge of controlling such a complex system. Six degrees of freedom along with coupled and nonlinear behavior makes them difficult to control and model. In order to implement any type of advanced control algorithms, appropriate mathematical model of the system has to be identified. The paper deals with the identification of coupled dynamic models for micro-ROVs.

The paper is organized as follows. Section I. gives a short overview of some interesting methods and procedures used for marine vehicle model identification, and gives a general mathematical model together with its reduction to horizontal plane. Section II. presents a new laboratory apparatus used for identification while Section III. presents the method for determining the parameters that have the greatest influence on the motion that is being identified (i.e. determination of the coupled model structure). Section IV. described the identification procedure and provides parameters for VideoRay Pro II ROV. Section V. concludes the paper.

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A. Some Prior Work

Researchers who are involved with navigation and guidance of underwater vehicles use different methods to identify their system's dynamics. This should be the first step towards designing a complex navigation system. Indeed, control of different degrees of freedom can be accomplished by tuning the controller parameters heuristically, but in order to implement e.g. optimal controllers, the mathematical model is necessary.

Some interesting aspects on identification of underwater vehicles by Ridao *et al.*, [2], [5], and on surface vehicles by M. Caccia *et al.*, [3], [4], can be found in literature. While Caccia uses classical measured data and some estimations to obtain the model of *Charlie* (autonomous surface catamaran), Ridao designed a uniquely patterned bottom of a laboratory test pool in order to localize *Uris* (unmanned underwater vehicle) and thus calculate the speeds which are necessary for model identification. However, both authors identify only uncoupled models of their vehicles. Nevertheless, both provide crucial proof of negligible system parameters and propose improved methods for identifying marine systems' dynamics.

B. Underwater Vehicles' Mathematical Model

Marine vehicles' mathematical models consist of kinematic and dynamic part, [1]. The kinematic model gives the relation between speeds in a body-fixed frame and derivatives of positions and angles in an Earth-fixed frame, see (1). According to terminology in [1], vector of positions and angles of an underwater vehicle ${}^E\boldsymbol{\eta} = [x \quad y \quad z \quad \varphi \quad \theta \quad \psi]^T$ is defined in the Earth-fixed coordinate system (E) and vector of linear and angular velocities ${}^B\mathbf{v} = [u \quad v \quad w \quad p \quad q \quad r]^T$ (surge, sway, heave, roll, pitch and yaw velocity, respectively) is defined in a body-fixed (B) coordinate system.

$$\begin{bmatrix} {}^E\dot{\boldsymbol{\eta}}_1 \\ {}^E\dot{\boldsymbol{\eta}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1({}^E\boldsymbol{\eta}_2) & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{J}_2({}^E\boldsymbol{\eta}_2) \end{bmatrix} \begin{bmatrix} {}^B\mathbf{v}_1 \\ {}^B\mathbf{v}_2 \end{bmatrix} \Leftrightarrow {}^E\dot{\boldsymbol{\eta}} = \mathbf{J}({}^E\boldsymbol{\eta}){}^B\mathbf{v} \qquad (1)$$

$$\mathbf{J}_1({}^E\boldsymbol{\eta}_2) = \begin{bmatrix} c\psi\, c\theta & -s\psi\, c\phi + c\psi\, s\theta\, s\phi & s\psi\, s\phi + c\psi\, s\theta\, c\phi \\ s\psi\, c\theta & c\psi\, c\phi + s\psi\, s\theta\, s\phi & -c\psi\, s\phi + s\psi\, s\theta\, c\phi \\ -s\theta & c\theta\, s\phi & c\theta\, c\phi \end{bmatrix}$$

$$\mathbf{J}_2({}^E\boldsymbol{\eta}_2) = \begin{bmatrix} 1 & s\phi\, t\theta & c\phi\, t\theta \\ 0 & c\theta & s\phi \\ 0 & s\phi\, c^{-1}\theta & c\phi\, c^{-1}\theta \end{bmatrix}$$

As stated before, the dynamic model of underwater vehicles is highly coupled and nonlinear. The main reasons for this are not only the rigid-body dynamics but hydrodynamic influences also. A general dynamic equation for underwater vehicles is given with (2).

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} + \boldsymbol{\tau}_d \qquad (2)$$

Matrix $\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A$ represents the sum of the rigid-body mass and added mass matrix. Matrix $\mathbf{D}(\mathbf{v})$ is a damping matrix, which is diagonal and usually represented with a linear and quadratic term. Matrix $\mathbf{C}(\mathbf{v}) = \mathbf{C}_{RB}(\mathbf{v}) + \mathbf{C}_A(\mathbf{v})$ represents the sum of the rigid-body and added mass Coriolis matrix, vector $\mathbf{g}$ represents gravitational and buoyancy forces, vector $\boldsymbol{\tau}$ consists of external forces and moments that act upon the underwater vehicle and $\boldsymbol{\tau}_d$ is the disturbance vector.

C. Mathematical Model in the Horizontal Plane

This paper addresses the issue of identification of coupled models in the horizontal plane, i.e. heave, roll and pitch motions are not observable. Therefore, the general mathematical model should be reduced to observable states only. Under the assumption that $w = \phi = \theta = 0$, the following kinematic (3) and dynamic (4) equations of motion can be written.

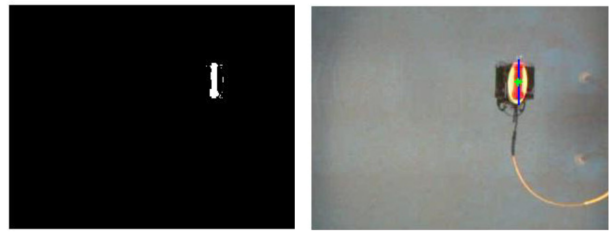
$$\begin{bmatrix} {}^E\dot{x} \\ {}^E\dot{y} \\ {}^E\dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^Bu \\ {}^Bv \\ {}^Br \end{bmatrix} \qquad (3)$$

$$\underbrace{\begin{bmatrix} m - X_{\dot{u}} & -X_{\dot{v}} & -my_G \\ -X_{\dot{v}} & m - Y_{\dot{v}} & mx_G \\ -my_G & mx_G & I_z - N_{\dot{r}} \end{bmatrix}}_{\mathbf{M}_{RB}+\mathbf{M}_A} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} +$$
$$\underbrace{\begin{bmatrix} 0 & 0 & -m(x_G r + v) + Y_{\dot{v}} v \\ 0 & 0 & -m(y_G r - u) - X_{\dot{u}} u \\ m(x_G r + v) - Y_{\dot{v}} v & m(y_G r - u) + X_{\dot{u}} u & 0 \end{bmatrix}}_{\mathbf{C}_{RB}(\mathbf{v})+\mathbf{C}_A(\mathbf{v})} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \qquad (4)$$
$$-\underbrace{\begin{bmatrix} X_u + X_{u|u|}|u| & 0 & 0 \\ 0 & Y_v + Y_{v|v|}|v| & 0 \\ 0 & 0 & N_r + N_{r|r|}|r| \end{bmatrix}}_{\mathbf{D}(\mathbf{v})} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} \tau_X \\ \tau_Y \\ \tau_N \end{bmatrix}$$

Fig. 1. *VideoRay* micro-ROV, VideoRay LLC

a) b)

Fig. 2. a) Experimental setup and b) view from the camera above the pool



a) b)

Fig. 3. a) Isolated red marker and b) original picture fused with orientation and position of the red marker

The dynamic model that is given with (1) is general in a way that it assumes the centre of gravity is not coincident with centre of mass, added mass matrix is not completely diagonal and the damping matrix contains quadratic terms.

II. Experimental Setup

In this paper, we present a new laboratory apparatus for marine vehicle model identification. Using the data obtained from this apparatus, a coupled mathematical model of *VideoRay Pro® II* ROV, VideoRay LLC will be identified.

A. VideoRay Micro-ROV

The real time system used for identification of coupled dynamics is the VideoRay Pro micro submersible shown in Fig. 1. Its dimensions are 355mm x 228mm x 215mm and it weighs about 4 kg. Heading sensor is a magnetic compass with 2° quantization. Also, it is equipped with a depth pressure sensor and two video cameras: the color one in front with the tilt and focus option, and the black and white one which is stationary. The vehicle is connected to the control board with a tether that causes substantial disturbance that should be compensated.

The ROV has three thrusters (port, starboard and vertical), hence it presents a nonholonomic system controllable in heave, surge and yaw direction. This means that the input $\boldsymbol{\tau}_Y = 0$. The only force acting in sway direction could be the one from disturbance, but we exclude this possibility due to laboratory conditions.

B. Laboratory Pool

The experimental setup that was used to obtain the position

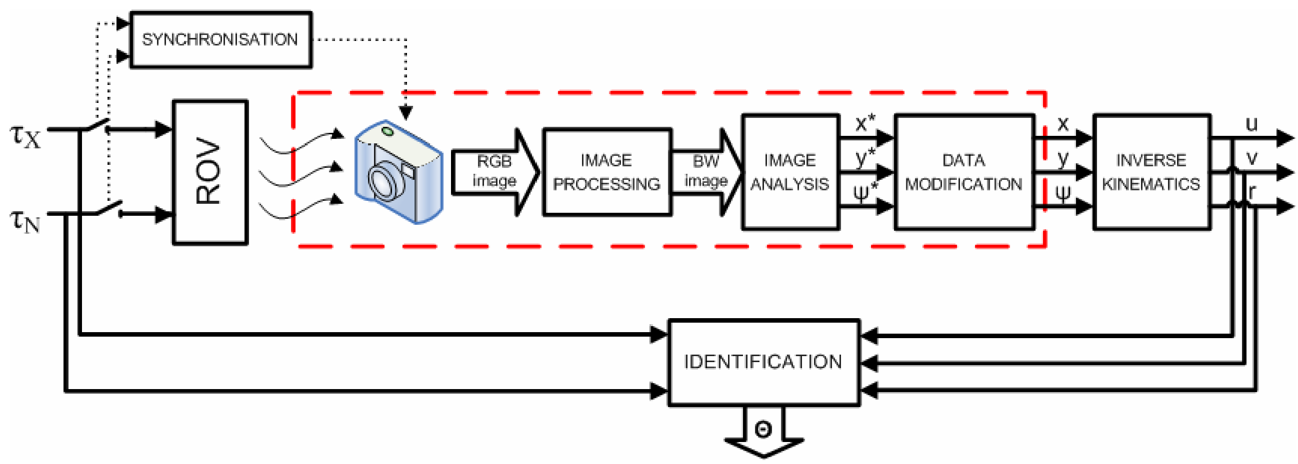


Fig. 4. Data acquisition scheme

and orientation of the vehicle is a laboratory pool (3.5 m in diameter, 1.2 m depth) and a webcam that was placed directly above the swimming pool, see Fig. 2a). A red marker was placed on top of the ROV so that its position and orientation within the camera frame could easily be extracted from the recorded video (Fig. 2b)). Since the depth of the pool is small, the identification procedure was performed only in two dimensions - surge, yaw and sway are the motions taken into consideration. This is important since only surge and yaw are controllable, and in the paper it will be demonstrated that due to coupling of motions, sway can be identified also.

C. Data Acquisition

As it was mentioned earlier, a web camera was placed above the pool in order to localize the ROV. Prior to that, the ROV was marked with a red line so it could be easily detected from the video.

The scheme of data acquisition system is shown in Fig. 4. The 'Synchronization' block is used to ensure that a frame is recorded, and that control signals are sent once every sample time (100 ms). Once the synchronization is achieved, the procedure can be described as follows:

- Acquire an RGB image from the camera (Fig. 2b)) and separate it to a red, green and blue component;
- Transfer the image to a binary equivalent (Fig. 3.a)) where detection of the red color results in a logical 1 (white) and everything else results in a logical 0 (black).
- Find the centroid of the group of white pixels (this is the position of the ROV)
- Find the orientation of the group of white pixels (this is the orientation of the ROV). Now we obtain Fig 3b) where the camera image is augmented with ROV's position (green circle) and orientation (blue line).
- Perform inverse kinematics on the data using (3), to obtain linear and angular speeds that are required for model identification.

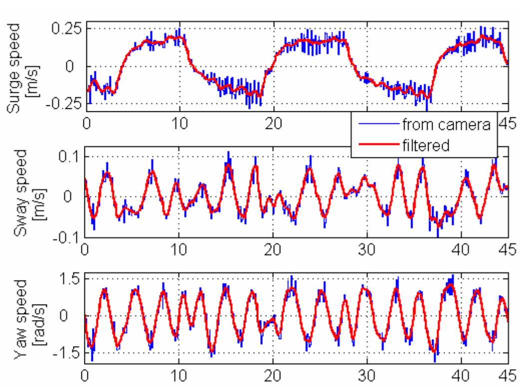


Fig. 5 Data obtained from camera (the "S-maneuver")

An example of obtained velocities using camera data is shown in Fig. 5. Raw data from camera are naturally noisy, therefore they should be filtered. We used a Golay filter as it was proposed in [2].

D. The "S-maneuver"

In order to obtain a coupled mathematical model, the underwater vehicle was driven in the "S-maneuver" illustrated in Fig. 6. This maneuver consists of driving the ROV with a constant surge force while changing the yaw moment from positive to negative constant value. The exact procedure is performed for the same surge force only in the opposite direction.

When this type of motion is applied, surge and yaw motions are performed simultaneously, and the data collected from the camera clearly show that the vehicle is also performing sway motion (see Fig. 5). Since the vehicle is not equipped with thrusters for sway motion, and there is no external disturbance, the conclusion is that the model is highly coupled.

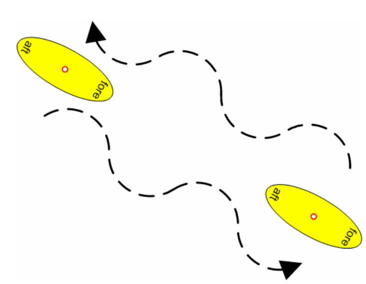


Fig. 6. The "S-maneuver"

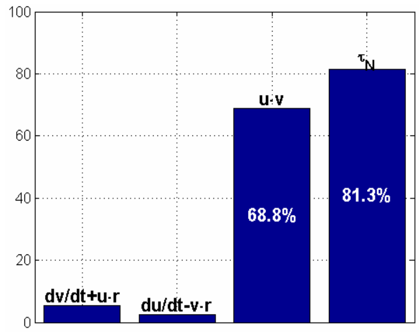


Fig. 7. Correlation coefficients for yaw motion

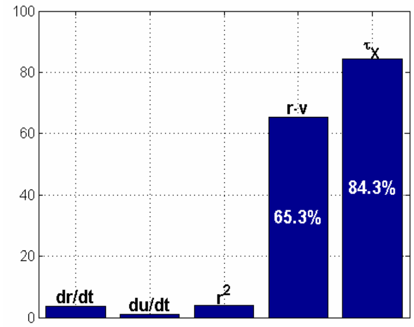


Fig. 8. Correlation coeff. for surge motion

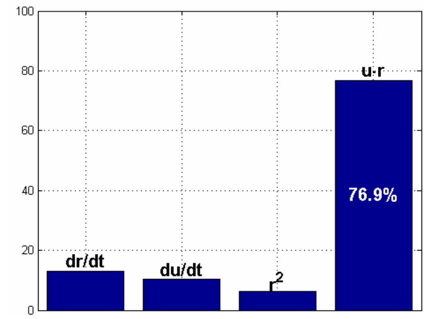


Fig. 9. Correlation coeff. for sway motion

III. Deciding on the Model

The following part will provide methods for determining model parameters that are negligible in comparison to dominant ones. The simplest way is to find the correlation coefficient at zero lag between the observed speed and all other factors that appear in the corresponding equation, [8].

From (1) it can be seen that in general case, yaw motion depends on $\dot{v}+ur$, $\dot{u}-vr$, uv and τ_N; surge motion depends on $\dot{r}$, $\dot{u}$, r^2, rv and τ_X; and sway motion depends on $\dot{r}$, $\dot{u}$, r^2 and ur. Figures 7, 8 and 9 show correlation coefficients for motions in the three degrees of freedom and obvious candidates for their models at zero lag for 6 different data sets. It is clear that yaw motion correlates to τ_N and uv; surge motion correlates to τ_X and rv; while sway motion correlates only to the coupled term ur.

From figures 7, 8 and 9 we can make the following conclusions:

- Parameter $X_{\dot{v}}$ from added mass matrix is negligible – the added mass matrix is diagonal. This is common for underwater vehicles, [1].
- The centre of buoyancy is practically equivalent to the centre of gravity ($x_G = y_G = 0$) in the horizontal plane. This is true for micro-ROVs.

Using this information and dynamic equation (1), we conclude that the following models for surge, yaw and sway motion are appropriate.

$$\dot{r} = \underbrace{\frac{N_r}{I_z - N_{\dot{r}}}}_{\alpha_1} r + \underbrace{\frac{1}{I_z - N_{\dot{r}}}}_{\alpha_2} \tau_N - \underbrace{\frac{X_{\dot{u}} - Y_{\dot{v}}}{I_z - N_{\dot{r}}}}_{\alpha_3} uv \quad (5)$$

$$\dot{u} = \underbrace{\frac{X_u}{m - X_{\dot{u}}}}_{\beta_1} u + \underbrace{\frac{1}{m - X_{\dot{u}}}}_{\beta_2} \tau_X + \underbrace{\frac{m - Y_{\dot{v}}}{m - X_{\dot{u}}}}_{\beta_3} rv \quad (6)$$

$$\dot{v} = \underbrace{\frac{Y_v}{m - Y_{\dot{v}}}}_{\gamma_1} v - \underbrace{\frac{m - X_{\dot{u}}}{m - Y_{\dot{v}}}}_{\gamma_2} ur \quad (7)$$

IV. Identification

The identification method that is used in the paper is the least-squares (LR) method, [9]. In concordance to the derived models, the regression vectors used to fit input-output data are augmented with a constant (δ_x) in order to exclude all possible model uncertainties and external disturbances from influencing real model parameters, as in [2]. The general matrix form for LR identification is given with (8), where $\mathbf{\Theta}$ is a column vector of parameters that are to be identified.

$$\frac{1}{T}\begin{bmatrix} x(1)-x(0) \\ x(2)-x(1) \\ \vdots \\ x(N)-x(N-1) \end{bmatrix} = \begin{bmatrix} x(0) & F(0) & x_2(0)x_3(0) & 1 \\ x(1) & F(1) & x_2(1)x_3(1) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x(N) & F(N) & x_2(N)x_3(N) & 1 \end{bmatrix} \begin{bmatrix} \mathbf{\Theta} \\ \\ \\ \delta_x \end{bmatrix} \quad (8)$$

If yaw motion is identified then $x = r$, $x_2 = u$, $x_3 = v$ and $F = \tau_N$; for surge motion $x = u$, $x_2 = r$, $x_3 = v$ and $F = \tau_X$; while for sway motion $x = v$, $x_2 = u$, $x_3 = r$ and $F = 0$. The identified parameters are shown in Table I.

A. Modified Identification

The identified parameters β_3 and γ_3 should be inverse and reciprocal (see (6) and (7)). As it can be seen from Table I. they are inverse, but the reciprocity is not satisfied. Since both of the identified parameters are close to 1, we will make the following assumptions:

1. $\beta_3 = -\gamma_3 = 1$, i.e. added mass terms in surge and sway direction are equal
2. $\alpha_3 = 0$, i.e. yaw motion is not coupled to other two motions (this comes as a direct consequence of assumption number 1)

The results of identification with fixed values of α_3, β_3, γ_3 are shown in Table II.

B. Validation of the Results

For validation of results, an error function $J = \frac{1}{N}\sum_{k=1}^{N} e^2(k)$ is used, where $e(k)$ is the difference between the simulated and real value in step *k*, and *N* is the number of samples. As it can be seen from Tables I and II, the quality of identification was not worsened by constraining α_3, β_3 and γ_3 to a fixed value. For yaw and sway models, the error function J became smaller, while for surge model it is insignificantly larger. In addition to that, more precise identification of the parameters is gained – standard deviations have decreased. These results let us conclude that the assumptions made are valid.

Another test which should prove the correctness of the identified model is checking some natural properties of the system's matrices, [1]: the overall mass matrix should be symmetric and positive definite (9), the overall Coriolis matrix should be inversely symmetric (10) and the damping matrix should be positive definite (11). All of these are fulfilled.

$$\mathbf{M}+\mathbf{M}_A=\left(\mathbf{M}+\mathbf{M}_A\right)^T>0 \tag{9}$$

$$\mathbf{C}(\mathbf{v})+\mathbf{C}_A(\mathbf{v})=-\left[\mathbf{C}(\mathbf{v})+\mathbf{C}_A(\mathbf{v})\right]^T \tag{10}$$

$$\mathbf{D}>0 \tag{11}$$

The final identification results for the VideoRay ROV in the horizontal plane are shown with the following matrix form:

$$\underbrace{\begin{bmatrix} 8.2372 & 0 & 0 \\ 0 & 8.2372 & 0 \\ 0 & 0 & 0.0372 \end{bmatrix}}_{\mathbf{M}+\mathbf{M}_A}\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{r}\end{bmatrix}+$$
$$\underbrace{\begin{bmatrix} 0 & 0 & -8.2372v \\ 0 & 0 & 8.2372u \\ 8.2372v & -8.2372v & 0 \end{bmatrix}}_{\mathbf{C}(\mathbf{v})+\mathbf{C}_A(\mathbf{v})}\begin{bmatrix}u\\ v\\ r\end{bmatrix}-\underbrace{\begin{bmatrix} -4.4283 & 0 & 0 \\ 0 & -14.9571 & 0 \\ 0 & 0 & -0.0997 \end{bmatrix}}_{\mathbf{D}}\begin{bmatrix}u\\ v\\ r\end{bmatrix}=\begin{bmatrix}\tau_X\\ 0\\ \tau_N\end{bmatrix}$$

Figures 10, 11 and 12 show results obtained by identification – red lines present the real data obtained from camera, while blue lines present simulated data (using the identified models). Fig. 10 shows the results for the yaw motion. It is easily seen that the yaw motion is much faster than motions in other directions.

Fig. 11 shows the results for surge motion. The green dotted line presents simulated data when an uncoupled model is assumed ($\beta_3=0$). From this figure it is obvious that a coupled model describes the vehicle's dynamics much better.

Fig. 12 shows the results for sway motion It can easily be seen that even though the vehicle was not excited in sway direction, the model can be identified.

V. Conclusion

In the paper we have shown that coupled motion should not be neglected when dealing with underwater vehicles. The data obtained from the described apparatus explicitly showed that a motion which cannot be controlled (sway) appears when the vehicle is forced to surge and yaw motion.

The paper describes the use of correlation coefficients for determining the coupling between motions. This method allowed us to conclude the following: the centre of buoyancy is in the same place in the horizontal plane as the centre of gravity (this is true for small underwater vehicles), and that the added mass terms $X_{\dot{u}}$ and $Y_{\dot{v}}$ can be considered equal while dealing with micro-ROVs (this can also be considered as a consequence of small ROV dimensions). Finally, comparison between coupled and uncoupled models is given. This comparison clearly proves that coupled model describes input-output data much better then the uncoupled one, and therefore the procedure should be used in all cases when a more precise underwater vehicle model is required.

A. *Future Work*

One of the assumptions made in this paper was that the damping matrix $\mathbf{D}(\mathbf{v})$ does not depend on the speed of the underwater vehicle. This has proven to be true. The experiments in this paper were made under constant surge and yaw speeds (in different directions), therefore the identified model is linear with regards to damping at this particular speed. Future work will concentrate on augmenting the coupled model with quadratic damping terms in matrix **D**.

TABLE I
YAW MODEL PARAMETERS

PARAMETER	α_1	α_2	α_3	J
MEAN	-2.4395	25.968	31.969	0.045
ST. DEV. [%]	-3.39	2.88	5.05	

SURGE MODEL PARAMETERS

PARAMETER	β_1	β_2	β_3	J
MEAN	-0.5531	0.121	0.8994	0.485
ST. DEV. [%]	-12.84	7.13	12.18	

SWAY MODEL PARAMETERS

PARAMETER	γ_1		γ_3	J
MEAN	-1.6282		-0.9092	0.434
ST. DEV. [%]	-12.57		-10.41	

TABLE II
YAW MODEL PARAMETERS

PARAMETER	α_1	α_2	α_3	J
MEAN	-2.6798	26.8758	0	0.041
ST. DEV. [%]	-3.68	3.07	0	

SURGE MODEL PARAMETERS

PARAMETER	β_1	β_2	β_3	J
MEAN	-0.5376	0.121	1	0.489
ST. DEV. [%]	-11.56	6.88	0	

SWAY MODEL PARAMETERS

PARAMETER	γ_1		γ_3	J
MEAN	-1.8158		-1	0.399
ST. DEV. [%]	-5.23		0	

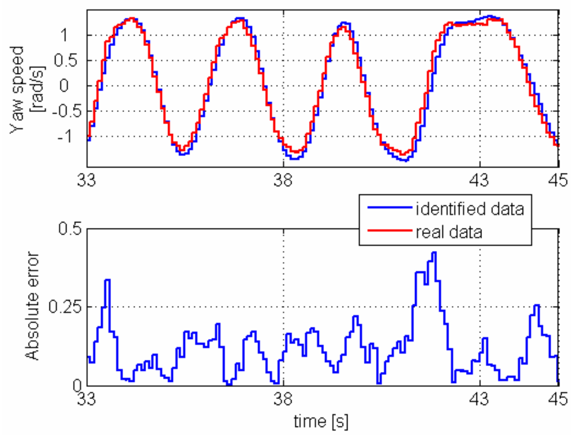


Fig. 10. Results for yaw motion

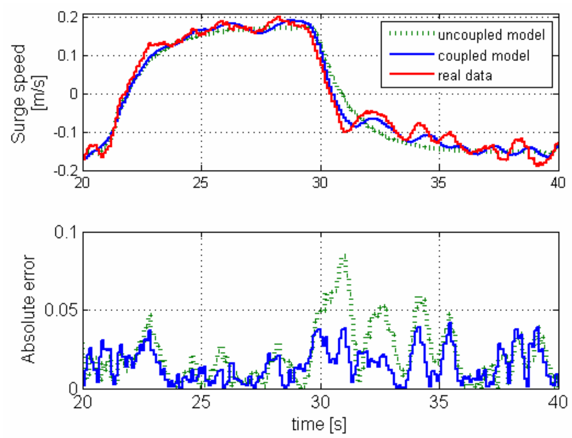


Fig. 11. Results for surge motion

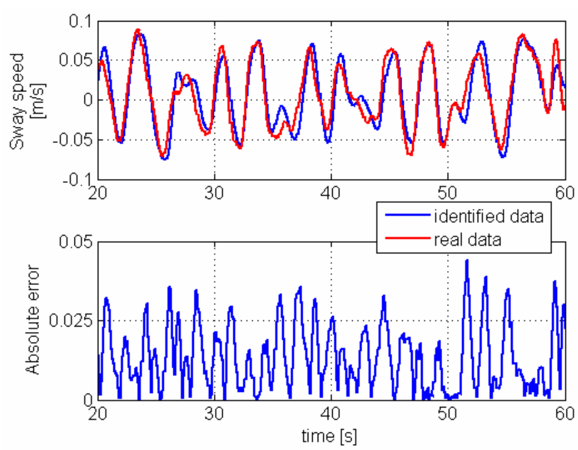


Fig. 12. Results for sway motion

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