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Identification of multiple characteristic components with high accuracy and resolution using the zoom interpolated discrete Fourier transform

Qiang Miao^{1,4}, Lin Cong¹ and Michael Pecht^{2,3}

 ¹ School of Mechanical, Electronic and Industrial Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, People's Republic of China
 ² Center for Advanced Life Cycle Engineering (CALCE), University of Maryland, College Park, MD 20742, USA
 ³ Center for Prognostics and System Health Management, City University of Hong Kong, Hong Kong

E-mail: mqiang@uestc.edu.cn

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Abstract

Complex systems can significantly benefit from condition monitoring and diagnosis to optimize operational availability and safety. However, for most complex systems, multi-fault diagnosis is a challenging issue, as fault-related components are often too close in the frequency domain to be easily identified. In this paper, the interpolated discrete Fourier transform (IpDFT) with maximum sidelobe decay windows is investigated for machinery fault feature identification. A novel identification method called the zoom IpDFT is proposed, which combines the idea of local frequency band zooming-in with the IpDFT and demonstrates high accuracy and frequency resolution in signal parameter estimation when different characteristic frequencies are very close. Simulation and a case study on rolling element bearing vibration data indicate that the proposed zoom IpDFT based on multiple modulations has better capability to identify characteristic components than do traditional methods, including fast Fourier transform (FFT) and zoom FFT.

Keywords: prognostics and health management, interpolated DFT, zoom IpDFT, Fourier transform, characteristic component identification

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The rapid development of complex systems such as power plants, high-speed transportation vehicles and high-precision machining centers has been emphasizing the need for condition monitoring and diagnosis so as to maximize operational availability and safety [1]. Therefore, research on prognostics and health management (PHM) has attracted the interest of industry and academia due to its great potential to address these needs [2]. In the process of PHM implementation, data preprocessing and feature extraction are the fundamental modules, since their outputs are used for system health assessment and prediction [2, 3]. However, the complicated structures and working conditions of complex systems may result in multiple faults during their operation that produce multi-frequency signals and lead to a challenging problem called multi-fault diagnosis. In these situations, frequency aliasing arises, and fault-related components may be too close in the frequency domain to be effectively identified for further feature extraction and diagnosis.

The identification of characteristic components is fundamental and important for complex system feature extraction and health assessment [4]. Along with the

⁴ Author to whom any correspondence should be addressed.

development of signal processing, more and more techniques have been introduced to diagnose faults in machinery [5]. The methods used for characteristic component identification or fault character extraction usually can be classified into frequency domain and time domain methods. Time domain methods provide high frequency selectivity and high estimation accuracy, but require computationintensive algorithms to determine the optimal model order [6]. Frequency domain methods use the discrete Fourier transform (DFT) to calculate the spectrum and estimate the frequency parameters of a signal. On account of some inherent drawbacks, the traditional DFT-based approaches have some restrictions in practice. For example, it is hard to obtain accurate frequency, amplitude and phase information about synchronous vibration and its harmonic or subharmonic components because of the leakage and the picket-fence effect of the DFT spectrum [7].

In order to enhance the efficiency and accuracy of fault diagnosis, it is crucial to improve the estimation accuracy of amplitude, frequency and phase of signal for feature extraction in the frequency domain. The method used to deal with this problem is called 'windowing'. One frequency domain method often used for estimating multi-frequency signal parameters under noncoherent sampling is the interpolated DFT (IpDFT) method, which provides very accurate parameter estimates. For example, Ramos and Serra [8] compared the frequency algorithms of IpDFT, Chirp-Z transform, Hilbert transform, STFT, CWT, MUSIC, Sine-fit and Kalman filtering. They determined that the IpDFT algorithm is the most precise, accurate and fastest algorithm. Rife and Vincent [9] proposed a specific approach, called the interpolated fast Fourier transform (IFFT), to dramatically improve the accuracy of the FFT spectrum. Moreover, Jain et al [10] proposed an approximate interpolation algorithm to obtain accurate amplitude, phase and frequency information when a rectangular window is employed. Shi et al [11] proposed the general IFFT for diagnosing faults in large rotating machinery, but the algorithm is very complicated.

The performance of the IpDFT method depends on the window used [12], and it should be noted that the formulas for estimating the parameters of a multi-frequency signal are very complicated for most windows. Among all these windows, the maximum sidelobe decay windows are frequently employed in the IpDFT method. The IpDFT method with maximum sidelobe decay windows leads to very accurate estimates, since the parameters of a multi-frequency signal can be estimated by analytical formulas [13, 14]. Belega and Dallet [15] proposed accurate and simple formulas for estimating the variances of the estimators of the parameters of a multi-frequency signal obtained by the IpDFT method with maximum sidelobe decay windows.

The goal of this paper is to investigate the potential of the IpDFT with maximum sidelobe decay windows in machinery (e.g., gearbox, bearing) feature extraction and condition monitoring. An IpDFT-based method combining the idea of local frequency band zooming-in (i.e. the zoom IpDFT) is proposed in this research to further improve the identification capability of multiple adjacent characteristic components in the frequency domain, which is a challenging issue for complex system condition monitoring and PHM.

The organization of this paper is as follows. Section 2 gives a brief description of the IpDFT. A novel method of zoom IpDFT based on multiple modulations is proposed in section 3 to solve the problem of high frequency resolution in fault characteristic frequency identification. Section 4 further investigates and validates the proposed method with rolling element bearing vibration data. Conclusions are summarized in section 5.

2. The interpolated DFT

2.1. The interpolated DFT with maximum sidelobe decay windows

The maximum sidelobe decay windows are the cosine windows. An *H*-term (H > 1) maximum sidelobe decay window has the most rapidly decaying sidelobes, equal to 6(2H - 1) dB/octave. It can be defined as

$$w(m) = \sum_{h=0}^{H-1} (-1)^h a_h \cos\left(2\pi h \frac{m}{M}\right), \quad m = 0, 1, \dots, M-1$$
(1)

where a_h are the coefficients of the *H*-term maximum sidelobe decay window [15] and *M* is the number of samples. a_h can be expressed as

$$a_0 = \frac{C_{2H-2}^{H-1}}{2^{2H-2}}, \quad a_h = \frac{C_{2H-2}^{H-h-1}}{2^{2H-3}}, \quad h = 1, 2, \dots, H-1$$
 (2)

where $C_m^p = \frac{m!}{(m-p)!p!}$.

The discrete-time Fourier transform (DTFT) of w(m) is given by

$$W(\lambda) = \sin(\pi\lambda) e^{-j\pi\lambda} e^{j\frac{\pi}{M}\lambda} \sum_{h=0}^{H-1} (-1)^h 0.5a_h$$
$$\times \left[\frac{e^{-j\frac{\pi}{M}\lambda}}{\sin\frac{\pi}{M}(\lambda-h)} + \frac{e^{j\frac{\pi}{M}\lambda}}{\sin\frac{\pi}{M}(\lambda+h)} \right], \quad \lambda \in [0, M). \quad (3)$$

For $M \gg 1$, the DTFT of the *H*-term maximum sidelobe decay window can be written as [8]

$$W(\lambda) = \frac{M \sin(\pi \lambda)}{2^{2H-2}\pi \lambda} e^{j\pi\lambda} e^{j\frac{\pi}{M}\lambda} \frac{(2H-2)!}{\prod_{h=1}^{H-1} (h^2 - \lambda^2)}.$$
 (4)

Now, let us consider a multi-frequency signal x(t) with a sampling frequency f_s

$$x(m) = A_0 + \sum_{k=1}^{K} A_k \sin\left(2\pi \frac{f_k}{f_s}m + \varphi_k\right),$$

$$m = 0, 1, \dots, M - 1$$
(5)

where *K* is the number of frequency components; A_k , f_k and φ_k are, respectively, the amplitude, frequency and phase of the *k*th component; A_0 is the offset; and *M* is the number of samples. In order to reduce the leakage error, x(m) is

multiplied by a suitable window sequence w(m). Thus, the DFT of the resulting signal xw = x(m)w(m) is given by

$$X_{W}(\lambda) = A_{0}W(\lambda) + \sum_{k=1}^{K} \frac{A_{k}}{2j} [W(\lambda - \lambda_{k})e^{j\varphi_{k}} - W(\lambda + \lambda_{k})e^{-j\varphi_{k}}], \lambda \in [0, M)$$
(6)

where λ represents the normalized frequency expressed in bins, $W(\lambda)$ is the DTFT of w(m) and $\lambda_k = \frac{f_k}{f_0}$, $f_0 = \frac{f_s}{M}$. If $W(\lambda)$ exhibits sidelobes at a negligible level and if the

If $W(\lambda)$ exhibits sidelobes at a negligible level and if the minimum distance between spectral lines is larger than the main lobe band width (MLBW) expressed in bins, then for $\lambda \cong \lambda_k$, equation (6) becomes

$$X_W(\lambda) \cong \frac{A_k}{2j} W(\lambda - \lambda_k) e^{j\varphi_k}, \qquad k = 1, 2, \dots, K.$$
 (7)

The relationship between the frequencies f_k and f_s is given by

$$\frac{f_k}{f_s} = \frac{\lambda_k}{M} = \frac{l_k + \delta_k}{M}, \qquad k = 1, 2, \dots, K$$
(8)

where l_k and δ_k are, respectively, the integer part and the fractional part of λ_k with $\delta_k \in [-0.5, 0.5)$; l_k is the index of the largest discrete spectrum module corresponding to the *k*th component. The IpDFT method is used to estimate δ_k . For this purpose, α_k is defined as

$$\alpha_{k} = \begin{cases} \frac{|X_{W}(l_{k})|}{|X_{W}(l_{k}-1)|}, & \text{if } -0.5 \leqslant \delta_{k} < 0\\ \frac{|X_{W}(l_{k}+1)|}{|X_{W}(l_{k})|}, & \text{if } 0 \leqslant \delta_{k} < 0.5 \end{cases}$$
(9)

From the above expression it follows that δ_k can be estimated by

$$\hat{\delta}_k \cong \begin{cases} \frac{(H-1)\alpha_k - H}{\alpha_k + 1}, & \text{if } -0.5 \leqslant \delta_k < 0\\ \frac{H\alpha_k - H + 1}{\alpha_k + 1}, & \text{if } 0 \leqslant \delta_k < 0.5 \end{cases}$$
(10)

From (8) and (10) the frequency of the kth component can be estimated by

$$\hat{f}_{k} \cong \begin{cases} \left(l_{k} + \frac{(H-1)\alpha_{k} - H}{\alpha_{k} + 1} \right) f_{0}, & \text{if } -0.5 \leqslant \delta_{k} < 0\\ \left(l_{k} + \frac{H\alpha_{k} - H + 1}{\alpha_{k} + 1} \right) f_{0}, & \text{if } 0 \leqslant \delta_{k} < 0.5. \end{cases}$$

$$(11)$$

Using (4) and (7), the amplitude of the kth component can be estimated by

$$\hat{A}_{k} = \frac{2^{2H-1}\pi\delta_{k}|X_{W}(l_{k})|}{M\sin(\pi\delta_{k})(2H-2)!}\prod_{h=1}^{H-1} \left(h^{2}-\delta_{k}^{2}\right).$$
 (12)

From an accurate estimation of the phase associated with the kth component, equation (7) is used in which the window's DTFT is computed by (13)

$$\hat{\varphi}_{k} = \text{phase}\{X_{W}(l_{k})\} - \pi \delta_{k} + \pi \frac{\delta_{k}}{M} - \frac{\pi}{2}\text{sign}(\delta_{k}) - \text{phase}\{W_{0}(-\delta_{k})\}$$
(13)

 Table 1. Estimation of the parameters by the IpDFT.

	Frequency (Hz)	Amplitude	Phase
The first	$\hat{f}_1 = 20.2869$	$\hat{A}_1 = 1.5004$	$\hat{\varphi}_1 = 0.2898$
The second component	$\hat{f}_2 = 90.7004$	$\hat{A}_2 = 2.0000$	$\hat{\varphi}_2 = 0.4976$
The third component	$\hat{f}_3 = 171.7000$	$\hat{A}_3 = 1.7000$	$\hat{\varphi}_3 = 0.1002$

where

$$W_0(\lambda) = \sum_{h=0}^{H-1} (-1)^h 0.5a_h \left[\frac{\mathrm{e}^{-\mathrm{j}\frac{\pi}{M}\lambda}}{\sin\frac{\pi}{M}(\lambda-h)} + \frac{\mathrm{e}^{\mathrm{j}\frac{\pi}{M}\lambda}}{\sin\frac{\pi}{M}(\lambda+h)} \right],$$

 $\lambda \in [0, M)$

 $sign(\cdot)$ is the sign function, and

$$\operatorname{sign}(\delta_k) = \begin{cases} -1, & \text{if } -0.5 \leqslant \delta_k < 0\\ 1, & \text{if } 0 \leqslant \delta_k < 0.5 \end{cases}$$

2.2. Investigation of the IpDFT using a simulation example

In order to explore the accurate parameter estimation of a multi-frequency signal by the IpDFT method with maximum sidelobe decay windows, a simulated time domain signal containing three components is analyzed as follows:

$$x(n) = \sum_{k=1}^{3} A_k \sin\left(2\pi n \frac{f_k}{f_s} + \varphi_k\right) + \operatorname{rand}(s),$$

$$n = 0, 1, \dots, N - 1$$
(14)

where $A_1 = 1.5$, $f_1 = 20.3$ Hz, $\varphi_1 = 0.3$, $A_2 = 2$, $f_2 = 90.7$ Hz, $\varphi_2 = 0.5$, $A_3 = 1.7$, $f_3 = 171.7$ Hz, $\varphi_3 = 0.1$ and rand(s) denotes normally distributed white noise. The original sampling frequency is $f_s = 4096$ Hz and the total number of samples is N = 512. Table 1 shows the estimation of the parameters of x(t) by the IpDFT method with maximum sidelobe decay windows; \hat{f}_k , \hat{A}_k and $\hat{\varphi}_k$ are the frequency, amplitude and phase estimations of the signal, respectively. The results in table 1 indicate that the IpDFT can estimate the parameters of the signal with a high degree of accuracy (as shown in table 1) through comparison with the actual parameters (i.e. f_k , A_k and φ_k) of the signal.

Generally, an important index of the spectrum analysis is frequency resolution, Δf , i.e. the minimum identifiable space between two frequencies in the spectrum. The relationship between frequency resolution, Δf , sampling frequency, f_s , sampling period, T, and the number of samples, N, can be expressed as follows:

$$\Delta f = \frac{f_s}{N} = \frac{1}{TN}.$$
(15)

The FFT algorithm is a very commonly used spectrum analysis method. In the FFT method, frequency resolution, Δf , has a strong impact on the accuracy of spectrum analysis. Take the second component of the simulated signal described by (14) as an example. That is, $x_2(n) = A_2 \sin(2\pi n \frac{f_2}{f_s} + \varphi_2)$, n =0, 1, ..., N-1. Let the number of samples N be 42, 128, 256,



Figure 1. Comparison of bias of the frequency estimation.

512, 1024, 2048 and 4096. Define the bias of the frequency estimation as

bias =
$$\left| \frac{f_2 - \hat{f}_2}{f_2} \right| \times 100\%.$$
 (16)

Figure 1 shows the bias of the frequency estimation using the FFT method and the IpDFT method under different numbers of samples.

The experimental results indicate that the method of IpDFT can estimate the parameters of the signal with higher accuracy (i.e. very small bias) compared with the FFT, especially when the number of samples is limited. The same conclusion can be obtained in the estimation of bias of amplitude and phase.

3. The zoom IpDFT

3.1. The problem of frequency resolution

In the process of multi-frequency signal analysis, it is necessary to achieve very high resolution in the case where the frequencies of certain components are very close. Assume a simulated signal containing two frequency components (e.g., the frequencies are 110 and 112 Hz, and the amplitudes are 1 and 0.5, respectively). The simulated signal is given as

$$x(n) = \sum_{k=1}^{2} A_k \sin\left(2\pi n \frac{f_k}{f_s} + \varphi_k\right) + \operatorname{rand}(s),$$

$$n = 0, 1, \dots, N - 1$$
(17)

where $A_1 = 1$, $f_1 = 110$ Hz, $\varphi_1 = 0$, $A_2 = 0.5$, $f_2 = 112$ Hz, $\varphi_2 = 0$ and rand(s) denotes normally distributed white noise. The original sampling frequency is $f_s = 1024$ Hz and the total number of samples is N = 1024.

Figures 2(a) and (b) show the analysis results obtained using the methods of FFT and IpDFT, respectively. As is well known, the FFT can be seen as a rectangular window DFT, and in this paper the IpDFT method is based on a cosine window DFT called the maximum sidelobe decay window. In the previous section, the IpDFT demonstrates high accuracy in the estimation of signal parameters. However, it failed to distinguish between two frequency components in this





Figure 2. Comparison of the frequency estimation of small difference components using the FFT and the IpDFT methods.

example compared with the FFT method. That is because the main-lobe width of the maximum sidelobe decay window is wider than the main-lobe width of the rectangular window in the condition of the same window width, which leads to poor frequency resolution with the IpDFT. In order to distinguish the two frequency components with the IpDFT method, we need to improve the frequency resolution.

According to equation (15), there are two approaches to improve the frequency resolution. The first is to reduce the sampling frequency, f_s , and the second is to increase the number of samples, N (i.e. the intercepted length of sequence). The former reduces the analysis range of the frequency and cannot improve the frequency resolution of the high frequency band (of course, if the signal information in the high frequency band is not important, a filter may be used to filter the high frequency component, and then the proper low sampling frequency can be taken). The latter must increase the length of the data window, which not only affects operation speed, but also demands larger internal memory storage capacity.

The increasing demand for computation time and memory space with the IpDFT method is a challenging issue for implementation of real-time condition monitoring and diagnosis in embedded systems. Sometimes, such demands cannot be met. Meanwhile, when the number of samples of IpDFT does not change, the location of the frequency point of the base-band analysis does not change either, and the flexibility of the spectrum analysis is poor. Since the base band of IpDFT has difficulty increasing the frequency resolution, the zoom IpDFT is proposed in this paper. The fundamental idea is to locally magnify certain bands of the signal spectrum, i.e. to increase the spectral density locally around a certain frequency of interest.

3.2. Basic principle of zoom IpDFT

The zoom FFT (ZFFT) technique is a very important method in the field of spectrum analysis [16]. It is a kind of FFT technique with frequency band expansion, and it is also known as local spectrum enlargement. It can provide a local frequency of interest with higher frequency resolution, just like a zoom lens zooming in on a portion of a whole picture. The basic



Figure 3. The implementation of zoom IpDFT based on multiple modulations.

principle of the zoom IpDFT method is similar to ZFFT, with the FFT being replaced by the IpDFT. The experimental results in figure 1 indicate that the IpDFT is better than the FFT under satisfactory frequency resolution conditions. Therefore, the zoom IpDFT has the advantages of both higher parameter estimation accuracy (like the IpDFT) and higher frequency resolution (like ZFFT). This paper mainly discusses the zoom IpDFT method based on multiple modulations.

The implementation of the zoom IpDFT can be summarized by the steps shown in figure 3. The steps include digital frequency shift, digital low-pass filtering, re-sampling, IpDFT transforming and weighted correction.

Given the original sampling frequency, f_s , the frequency components over $f_s/2$ of the continuous signal x(t) are first eliminated via an anti-superposing low-pass filter, then resampled with the frequency f_s to get the discrete data x(n)including N points. Suppose the central frequency of the local band intended to be enlarged is f_0 and the frequency bandwidth is B. Then the second step is to carry multiple modulations on the signal x(n), which is realized by multiplying the digital signal x(n) by $e^{-j2\pi n f_0/f_s}$. Therefore, the signal after the frequency shift $x_0(n)$ can be expressed as follows:

$$x_0(n) = x(n)e^{-j2\pi nf_0/f_s}$$

= $x(n)\cos(2\pi nf_0/f) - jx(n)\sin(2\pi nf_0/f).$ (18)

According to the frequency shift nature of the discrete Fourier transform, the discrete spectrum of $x_0(n)$ is

$$X_0(k) = X(k + L_0)$$
(19)

where $L_0 = Nf_0/f_s$ represents the order number of the spectral line when the frequency is f_0 ; $X_0(k)$ is the spectrum after the frequency shift; and X(k) is the spectrum before the frequency shift.



Figure 4. Comparison of the frequency and amplitude estimation using different methods.

Then, all of the frequency components of the signal are filtered after the frequency shift except for a narrow band B around f_0 via a low-pass filter:

$$Y_0(k) = X_0(k)H(k) = X(k+L_0)H(k),$$

$$k = 0, 1, 2, \dots, N-1$$
(20)

where H(k) is the response of the low-pass filter. The output time domain signal of the low-pass filter is

$$y_0(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y_0(k) W_N^{-nk} = \frac{1}{N} \sum_{k=0}^{N-1} Y_0(k) e^{-j2\pi nk/N}.$$
 (21)

Through re-sampling at a new sampling frequency, $f'_s = f_s/D$, the frequency resolution is enhanced *D* times while reducing the sampling frequency and keeping the original length of record in seconds. That is, the multiple of refining is *D*, and a new discrete signal $x_r(n) = y_0(Dn)$ can be obtained by this process. By applying the complex IpDFT computation to the signal $x_r(n)$, the spectral line after refining can be received with its central frequency as f_0 and bandwidth as $B = f_s/2D$ [16].

3.3. Comparison of the zoom FFT and zoom IpDFT using a simulation example

In order to explore the parameter estimation accuracy of a multi-frequency signal by the zoom IpDFT method with maximum sidelobe decay windows, a simulated signal containing four components is analyzed as follows:

$$x(t) = \sum_{k=1}^{4} A_k \sin(2\pi f_k t) + \operatorname{rand}(s).$$
(22)

Here, $A_1 = 1$, $f_1 = 180.5$ Hz, $A_2 = 2$, $f_2 = 181$ Hz, $A_3 = 3$, $f_3 = 181.4$ Hz, $A_4 = 7$, $f_4 = 181.9$ Hz and rand(s) denotes a normally distributed white noise. The original sampling frequency is $f_s = 1024$ Hz, the number of samples is N =256 and D = 30. The central frequency of the local band, f_0 , is 178 Hz. According to the equation $B = f_s/2D$, the bandwidth B is about 17 Hz. Figure 4 shows a comparison of



Figure 5. Test rig for bearings.

frequency and amplitude estimation using the zoom FFT and zoom IpDFT, respectively.

As shown in figure 4, both the zoom FFT method and the zoom IpDFT method can increase the resolution of the spectrum and obtain good results in frequency estimation. However, for the zoom FFT, the frequency estimation result is not clear compared with the zoom IpDFT, especially at the first signal component frequency of 180.5 Hz. Figure 4 also demonstrates that the amplitude estimation with the method of zoom FFT is not accurate due to the energy leakage of the base frequency. Thus, the amplitude error is larger compared with the zoom IpDFT, and it cannot be used for quantitative analysis. Overall, the zoom IpDFT can achieve better performance in characteristic frequency identification.

4. Case study

In this paper, real motor bearing data picked up with a sampling frequency of 12k Hz by an accelerometer placed at the drive end of the motor housing were used to validate the proposed method. Three kinds of bearing conditions were considered in this case study, namely the normal condition, the inner race fault condition and the outer race fault condition. The accelerometers under the normal conditions and inner race fault conditions were placed at the 12 o'clock position, while under the outer race fault conditions, they were placed at the 6 o'clock position. The test rig is shown in figure 5. Single point faults were introduced to normal bearings using electrodischarge machining with a fault diameter of 0.007 inches and a fault depth of 0.011 inches. The specifications of a bearing are shown in table 2. The shaft rotation speed, f_r , varied from 1730 to 1797 rpm. The characteristic frequencies of the bearing were calculated by the following formulas [17]:

$$f_I = 5.4152 \times f_r \tag{23}$$

$$f_0 = 3.5848 \times f_r.$$
(24)

Here, f_I and f_O are the inner race fault characteristic frequency and outer race fault characteristic frequency, respectively. There is a total of 12 data sets, including 4 normal bearings, 4 inner race fault bearings, and 4 outer race fault bearings at different rotation speeds and work loads. Table 3 shows their corresponding characteristic frequencies.

Table 2. Motor bearing specifications (inches).

Inside diameter	0.9843
Outside diameter	2.0472
Thickness	0.5906
Ball diameter	0.3126
Pitch diameter	1.537

Table 3. Fault characteristic frequencies of bearing at different rotation speeds and loads.

Motor load (HP)	Motor speed (rpm)	Inner race $f_I(\text{Hz})$	Outer race $f_O(\text{Hz})$
0	1797	162.2	107.4
1	1772	160.0	105.9
2	1750	157.9	104.6
3	1730	156.1	103.4

4.1. Identification of fault characteristic frequencies at a motor speed of 1797 rpm and load HP 0

In this section, vibration signals, including normal bearing data, inner race fault data and outer race fault data at a motor speed of 1797 rpm and load of HP 0, are used to validate the proposed method. Figure 6 shows the original vibration signals of normal data, inner race fault data and outer race fault data. In order to enhance the computing efficiency, each piece of data with 0.2 s is selected for envelope spectrum analysis. Figure 7 shows the identification of fault-related characteristic components at 1797 rpm and HP 0.

Figure 7(a) shows no fault characteristic frequency when the bearing was under normal conditions. However, bearings with an inner race fault or an outer race fault can be identified via their corresponding fault characteristic frequencies (including harmonics), as shown in figures 7(b)and (c). The estimations of these frequencies are almost consistent with the corresponding theoretical calculations (see table 3).

4.2. Identification of fault characteristic frequencies under different working conditions

In this section, we consider the influence of different working conditions, including different rotation speeds and different loads. Figure 8 shows the analysis results using vibration data from a bearing with an outer race fault under different working conditions. It can be observed that the proposed zoom IpDFT can identify the outer race fault characteristic frequency, f_O , and its harmonics. In addition, figure 9 gives the analysis results using a bearing with an inner race fault under different working conditions, which also shows good potential for the identification of a characteristic component, f_I .

4.3. Comparison with the traditional FFT in fault characteristic identification using the IpDFT

Usually, the number of samples should be chosen according to expert experience, working condition or calculation speed when utilizing the FFT or the IpDFT. A larger number



Figure 6. The original signals at a motor speed of 1797 rpm and load HP 0.



Figure 7. Identification of fault-related characteristic components in envelope spectra at 1797 rpm and HP 0.

of samples requires more computation time and memory space, while a smaller number of samples may result in a loss of accuracy. Thus, it is very important to identify the characteristic component with the appropriate number of samples. For comparison, vibration data at a motor speed of 1797 rpm and load HP 0 were analyzed by FFT and IpDFT, with the number of samples from 500 to 4000. In the comparison study, only the primary characteristic frequency and its amplitude were estimated for the sake of convenience. In table 3, the outer race fault characteristic frequency is



Figure 8. Identification of characteristic components of bearing with outer race fault in envelope spectra.

107.4 Hz, and the inner race fault characteristic frequency is 162.2 Hz. The comparison results are shown in figures 10 and 11 for the bearings with an outer race fault and an inner race fault, respectively.

Figure 10(a) shows a comparison of the characteristic frequency identification of a bearing with an outer race fault using the FFT and the IpDFT methods, which indicates that the IpDFT can accurately identify the outer race fault characteristic frequency and provide stable results with small bias when the number of samples changes from 500 to 4000. However, the FFT method has poor performance compared with the IpDFT, and the bias is large for some numbers of samples. Figure 10(b) shows a comparison of amplitude estimation of characteristic frequency, which can represent the severity of an outer race fault. As shown in figure 10(b), the amplitude of the characteristic frequency with the IpDFT is steady, but the amplitude with FFT fluctuates as the number of samples changes. Figure 10(b) also shows that the amplitude of the characteristic component with the IpDFT is larger, which means that the energy leakage of the fault characteristic frequency is less. Overall, the IpDFT can identify the outer race fault characteristic frequency and reflect the severity of a fault more accurately and steadily than the FFT.

Figure 11 shows a comparison of the characteristic component identification of a bearing with an inner race fault using the FFT and the IpDFT methods. In figure 11(a), the characteristic frequency identification error with the IpDFT at 500 samples is larger than the one with the FFT method,

which is mainly due to the fact that frequency aliasing happens. However, the IpDFT can identify the inner race fault characteristic frequency accurately and steadily when the number of samples changes from 1000 to 4000. Thus, we can get the same conclusion as from figure 10.

4.4. Fault characteristic frequency identification with zoom IpDFT

Most mechanical systems are composed of many elements. The fault characteristic frequency identification we discussed above is a special case, with the fault in the bearing being seeded artificially and uniquely (i.e. just one fault at any time). However, multiple faults exist in many mechanical systems, which means that there is a need for multi-fault diagnosis. Furthermore, frequency aliasing arises, and the fault-related frequencies are too close to be effectively distinguished under certain circumstances. For example, the rotor winding fault is a common failure mode in an induction motor. It is known that the rotor winding fault results in an extra current characteristic component with a frequency $(1 \pm 2 \text{ slip ratio})$ of fundamental frequency. Under normal load conditions, the slip ratio is very small (a few per cent, even less than one per cent). Therefore, the frequency of the stator fundamental current is very close to the frequency of the rotor fault current component [18].

If we want to identify two adjacent frequency components, a large number of samples are required to get enough frequency resolution with the IpDFT or FFT. However, a large number of points may not always be available due to the restrictions



Figure 9. Identification of characteristic components of bearing with inner race fault in envelope spectra.



Figure 10. Comparison of characteristic component identification of bearing with outer race fault using the FFT and the IpDFT methods.



Figure 11. Comparison of characteristic component identification of bearing with inner race fault using the FFT and the IpDFT methods.



Figure 12. The constructed signal and its components.



Figure 13. Comparison of characteristic component identification using the FFT, the zoom FFT and the zoom IpDFT.

of calculation speed and calculation ability for hardware. The proposed zoom IpDFT in this research is an alternative when dealing with such problems.

To explore the potential of the zoom IpDFT in bearing fault characteristic identification, a signal x(t) is constructed to simulate the existence of two components that are adjacent in the frequency domain. The mixed signal x(t) is composed of two parts: one is the raw inner race fault data, $x_i(t)$, at a motor speed of 1797 rpm and load HP 0, and the other is a masking signal, $x_m(t)$, that may come from the gear meshing. The simulated signal, $x_m(t)$, and the mixed signal, x(t), can be written as

$$x_m(t) = A_1(1 + A_2 \cos(2\pi f_2 t)) \sin(2\pi f_1 t)$$
(25)

$$x(t) = x_i(t) + x_m(t)$$
 (26)

where $A_1 = 0.435$, $f_1 = 29.95$ Hz, $A_2 = 0.75$ and $f_2 = 158.4$ Hz. Therefore, the frequency of the simulated masking signal is 158.4 Hz, which is very close to the theoretical inner race fault characteristic frequency of 162.2 Hz. The sampling frequency of the simulated masking signal is 12 kHz.

Figure 12 shows the mixed signal and its components. Figure 13 shows the analysis results of the mixed signal x(t) with the zoom IpDFT method, together with a comparison study using the FFT and the zoom FFT.

Figure 13(a) shows the results of characteristic frequency identification using the traditional FFT method. The FFT failed to identify two frequency components at 162.2 and 158.4 Hz. According to figures 13(b) and (c), both the zoom FFT and the zoom IpDFT can identify the characteristic frequencies. The performance of the zoom IpDFT is better than the zoom FFT because the energy or the amplitude of the characteristic frequencies has less leakage, and the

zoom IpDFT can provide more accurate information to reflect the severity of a fault. It should be noted that the actual characteristic frequency of the bearing inner race fault should be around 161.8 Hz (according to figure 7(c)), and the zoom IpDFT provides more accurate estimation of frequencies (i.e. 158.4 and 161.6 Hz) compared with the zoom FFT (i.e. 158 and 162 Hz).

5. Conclusions

In machinery condition monitoring, the identification of faultrelated characteristic components is a crucial step to realize feature extraction for further diagnosis. In this paper, the IpDFT with maximum sidelobe decay windows is investigated for its identification of characteristic components in the frequency domain. A novel method based on the IpDFT is then proposed to combine the idea of local frequency band zooming-in, which is called the zoom IpDFT. In this process, the IpDFT with maximum sidelobe decay windows can estimate the components of a signal accurately and stably. The proposed zoom IpDFT based on multiple modulations provides better performance in signal parameter estimation with high accuracy and frequency resolution, especially in a situation of multi-fault diagnosis in which frequency aliasing exists and fault-related components may be too close in the frequency domain to be effectively identified.

To validate the proposed zoom IpDFT method, a series of experiments was conducted in this paper. It was tested using vibration data collected from a motor bearing at a rotation speed of 1797 rpm and a load of HP 0. The method can identify fault-related characteristic components in the frequency spectrum. It was then used to identify the characteristic components at different loads and rotation speeds and the results showed good potential in characteristic frequency identification. Comparison studies of the Fourier transform and the IpDFT method using bearing vibration data with inner race fault and outer race fault showed that the IpDFT method can provide distinct results in characteristic frequency identification with low frequency bias and accurate frequency amplitude estimation. Later, a mixed signal containing two adjacent fault characteristic frequencies was constructed to compare the proposed zoom IpDFT with other methods including the FFT and the zoom FFT. The comparison results of the mixed signal using the FFT, the zoom FFT and the zoom IpDFT can identify the two fault characteristics with good accuracy and frequency resolution.

It should be noted that the vibration signals from rolling element bearings usually exhibit pseudo-cyclostationarity, which means that there exists stochastic variation in the spacing of the bursts generated by the local faults on rolling elements and races [4]. This effect causes random slip during bearing operation, and leads to a certain degree of variation in the frequency domain. Therefore, the proposed method in this research is most applicable to the separation of discrete frequency components, which is more popular in gear systems.

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References

- Wang X, Zi Y and He Z 2009 Multiwavelet construction via an adaptive symmetric lifting scheme and its applications for rotating machinery fault diagnosis *Meas. Sci. Technol.* 20 045103
- [2] Pecht M 2008 Prognostics and Health Management of Electronics (Hoboken, NJ: Wiley)

- [3] Sheppard J W, Kaufman M A and Wilmer T J 2009 IEEE Standards for prognostics and health management *IEEE Aerosp. Electron. Syst. Mag.* 24 34–41
- [4] Antoni J and Randall R B 2002 Differential diagnosis of gear and bearing faults ASME J. Vib. Acoust. 124 165–71
- [5] Liu J, Wang W, Golnaraghi F and Liu K 2008 Wavelet spectrum analysis for bearing fault diagnostics *Meas. Sci. Technol.* 19 015105
- [6] Marple S L 1987 Digital Spectral Analysis with Applications (Englewood Cliffs, NJ: Prentice-Hall)
- [7] Sundararajan D 2001 The Discrete Fourier Transform: Theory, Algorithms and Applications (Singapore: World Scientific Publishing)
- [8] Ramos P M and Serra A C 2009 Comparison of frequency estimation algorithms for power quality assessment *Measurement* 42 1312–7
- [9] Rife D C and Vincent G A 1970 Use of the discrete Fourier transform in the measurement of frequencies and levels of tones *Bell Syst. Tech. J.* 49 197–228
- [10] Jain V K, Collins W L and Davis D C 1979 High-accuracy analog measurements via interpolated FFT *IEEE Trans. Instrum. Meas.* 28 113–22
- [11] Shi D F, Qu L S and Gindy N N 2005 General interpolated fast Fourier transform: a new tool for diagnosing large rotating machinery J. Vib. Acoust. 127 351–61
- [12] Offelli C and Petri D 1992 The influence of windowing on the accuracy of multifrequency signal parameters estimation *IEEE Trans. Instrum. Meas.* 41 256–61
- [13] Nutall A H 1981 Some windows with very good sidelobe behavior *IEEE Trans. Acoust. Speech Signal Process.* 29 84–91
- [14] Belega D and Dallet D 2009 Choice of the acquisition parameters for frequency estimation of a sine wave by interpolated DFT method *Comput. Stand. Interfaces* 31 962–8
- [15] Belega D and Dallet D 2009 Multifrequency signal analysis by interpolated DFT method with maximum sidelobe decay windows *Measurement* 42 420–6
- [16] Hoyer E and Stork R 1977 The zoom FFT using complex modulation Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processingvol 2 78–81
- [17] Loparo K A, Case Western Reserve University, Bearing Data Center, Seeded Fault Test Data http://www.eecs.case.edu/laboratory/bearing/
- Bellini A, Filippetti F and Franceschini G et al 2002 On-field experience with online diagnosis of large induction motors cage failures using MCSA *IEEE Trans. Ind. Appl.* 38 1045–53