Identification of Processes in Closed Loop -- Identifiability and Accuracy Aspects

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IDENTIFICATION OF PROCESS IN CLOSED LOOP IDENTIFIABILITY AND ACCURACY ASPECTS

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IDENTIFICATION OF PROCESSES IN CLOSED LOOP
- IDENTIFIABILITY AND ACCURACY ASPECTS

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It is often necessary in practice to perform identification experiments on systems operating in closed loop. There has been some confusion about the possibilities of successful identification in such cases, evidently due to the fact that certain common methods then fail. A rapidly increasing literature on the problem is briefly surveyed in this paper, and an overview of a particular approach is given. It is shown that prediction error identification methods, applied in a direct fashion will give correct estimates in a number of feedback cases. Furthermore, the accuracy is not necessarily worse due to the presence of feedback; in fact optimal inputs may very well require feedback terms. Some practical applications are also described.

1. INTRODUCTION

The purpose of an identification experiment is to determine the dynamics of a given process. Most processes operate as a part of a control configuration and the inputs to the process are partly determined as feedback from other signals. In many cases, security or production reasons do not permit that the regulators are removed during an identification experiment. In other cases, like for economical and biological systems, the feedback effects may be inherent. Consequently, not seldom identification experiments have to be performed on processes operating in closed loop.

During the last few years there has been a rapidly increasing literature on identification of closed loop systems; and at this symposium a special session is devoted to these problems. The reason obviously is that the task contains some fallacies, and it is not immediately clear when identification can be successfully performed based on closed loop experiments and how it should be done.

The purpose of this paper is to describe the problems which may arise for closed loop identification, to summarize basic results on possibilities and methods and to give an overview over the literature in the field.

Several different approaches have been taken to the problem. Various estimation schemes have been suggested, different feedback configurations have been considered and the systems and models have been parameterized in varying ways. Moreover, sometimes the regulator may be considered as part of what is to be identified, and then the input and output are considered as a joint process. This approach, which has been pursued in a series of interesting papers by Caines and Chan [1], [2], is quite natural, e g for economic systems, where the feedback mechanism is not open for manipulation. For other processes, in particular for industrial ones, it is natural to assume that the regulator is known, and that it can be chosen freely as long as it yields acceptable performance of the output process.

Consequently the picture of approaches, methods and results is quite diverse, and it is of course impossible to give a comprehensive treatment of all aspects in a paper like this. In order to make the paper readable, we have chosen to organize it as follows. In each section we first describe problems, definitions and results which are related to our own activities in the field, [3]-[7]. At the same time we also cover a substantial part of the major results in the literature on closed loop identification. In the second part of each section, we describe and discuss aspects from other papers. In that way the paper can be read as a survey of the field, as well as a fairly complete overview of one of the approaches taken.

In the next section we shall describe the basic problems connected with closed loop identification. Section 3 contains a discussion of identifiability and accuracy. Section 4 contains the main results on identifiability and accuracy aspects, and there also the major survey of other papers is given. Section 5 reviews some applications of closed loop identification.

2. BASIC CONCEPTS

2.1 Preliminaries

The result of an identification experiment depends clearly on several items:

- o the system,
- o the model structure or the model parameterization,
- o the identification method used,
- o the experimental conditions.

System

In this paper a linear, multivariable, discrete-time, stochastic system, S, given on the general form

$$y(t) = G_S(q^{-1})u(t) + H_S(q^{-1})e(t) \tag{2.1}$$

will be considered. The output, $y(t)$, is a vector of dimension n_y and the input, $u(t)$, has dimension n_u. The variables $\{e(t)\}$ are a sequence of independent, random variables with zero mean values and covariances $Ee(t)e^T(t) = \Lambda$. In (2.1) q^{-1} denotes the backward shift operator, $q^{-1}u(t) = u(t-1)$. It is assumed that $G_S(z)$ and $H_S(z)$ are matrices of proper dimensions with rational functions as entries. It is also assumed that $H_S(0) = I$, which implies that $e(t)$ has the same dimension as $y(t)$, but this is no loss of generality. Also let $\det[H_S(Z)]$ have all zeroes strictly outside the unit circle.

Model

To determine a model of the system the function $G(z)$ and $H(z)$ have to be parameterized in a suitable manner by a parameter vector θ. A model corresponding to a certain value of θ is denoted by $M(\theta)$ and is given by

$$y(t) = G_\theta(q^{-1})u(t) + H_\theta(q^{-1})\varepsilon(t) \tag{2.2}$$

where $\{\varepsilon(t)\}$ is a sequence of independent, random vectors with zero mean values and covariances $\hat{\Lambda}$. When θ is varied over the region of feasible values, eq (2.2) represents a family of models denoted by M. This family will sometimes be referred to as the "model structure". The identification problem is to determine the parameter θ so that $M(\theta)$ in some sense suitably describes the system, S, given by eq (2.1).

The parameterization of $G_\theta(q^{-1})$ and $H_\theta(q^{-1})$ can be made in several ways. Frequently used representations are the vector difference equation,

$$A_\theta(q^{-1})y(t) = B_\theta(q^{-1})u(t) + C_\theta(q^{-1})\varepsilon(t) \tag{2.3}$$

and the state space form (here given in the time-invariant innovations representation form),

$$\begin{cases} x(t+1) = A_\theta x(t) + B_\theta u(t) + K_\theta \varepsilon(t) \\ y(t) = C_\theta x(t) + D_\theta\, u(t) + \varepsilon(t) \end{cases} \tag{2.4}$$

Experimental conditions

The input u(t) to the system given by eq (2.1) can be determined in several ways. It can, as in open loop experiments, be chosen freely by the experiment designer. It can also be determined partly from output feedback by a regulator of a given structure etc. On the whole, the manner in which the input is determined will in this paper be referred to as experimental condition, denoted by $\mathcal{X}$. In this concept other experimental conditions such as the sampling rate, the experiment length etc. can be included. These will, however, not be considered here.

In this paper we shall discuss identifiability and accuracy aspects of systems operating in closed loop. A typical such system is shown in fig 2.1.

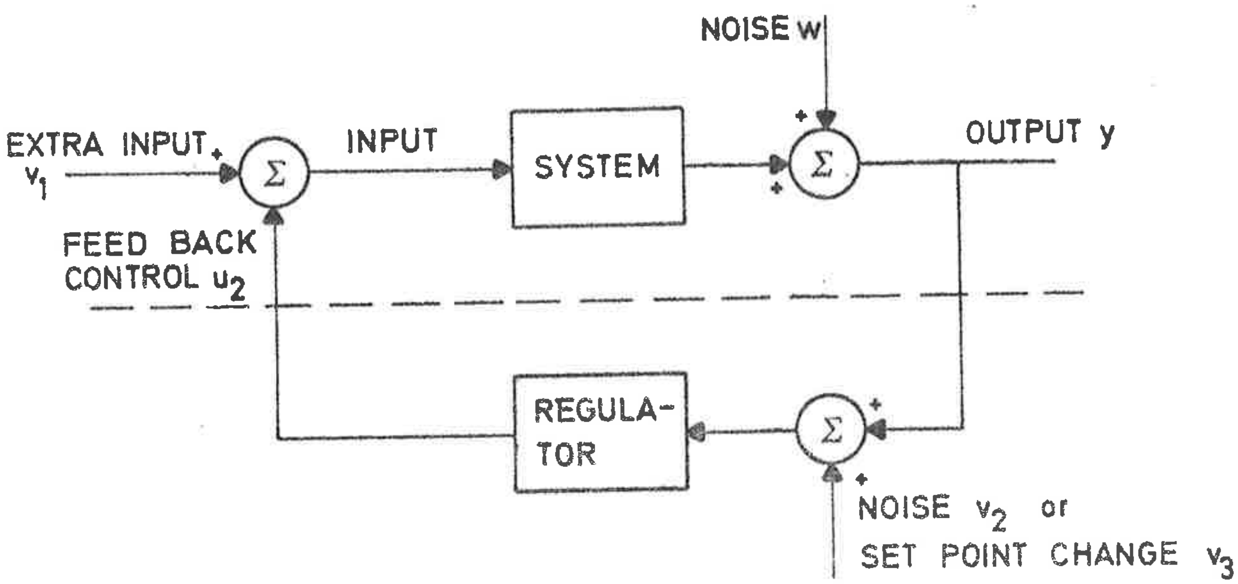

Fig 2.1 Block diagram of a typical feedback system.

Several different configurations will be discussed in Section 4. They are essentially covered by the one given in Fig 2.1 and correspond to various assumptions about $v_i, i = 1,2,3$, and about the character of the regulator. We shall throughout assume that the signals u and y are measurable.

Identification method

Finally, the procedure by which θ is determined is called the identification method, I. For systems operating in closed loop different ways to apply the identification methods are possible. One way which always can be applied is to treat the input-output data exactly as if they were obtained from an open loop experiment. This procedure will be called direct identification, I_1. If for example the regulator is linear, noise-free and time-invariant (or alternates between such feedback laws) then an indirect way of estimating the parameters can be applied. The closed loop system can be regarded as a whole and its parameters can be determined by some method. The open loop process parameters can then be determined from the estimated closed loop process parameters using the knowledge of the regulator. This approach will be called indirect identification, I_2. Notice that it is sometimes difficult to refer a specific identification scheme to be either a direct or an indirect identification method.

Further, in some cases it may be advantageous to consider both the input and output processes jointly as the output of a system driven by noise only. This approach may be called joint input-output identification, I_3, and has in particular been pursued by Caines and Chan, [1], [2].

In general it will be assumed in the following that a statistically efficient identification method is used. In particular prediction error identification methods like the maximum likelihood method will be considered. Then the parameter θ is chosen so that a suitable scalar function (e g the determinant or the trace) of the matrix

$$Q_N(S,M(\theta)) = \frac{1}{N}\Sigma[y(t)-\hat{y}(t|t-1;M(\theta))][y(t)-\hat{y}(t|t-1;M(\theta))]^T \qquad (2.5)$$

is minimized, cf [8]. $\hat{y}(t|t-1;M(\theta))$ denotes the linear least squares prediction of y(t) based on data up to time t-1 and using the model $M(\theta)$. The minimizing element is the estimate $\theta(N;S,M,I,X)$. Neglecting initial value effects it is given by

$$\hat{y}(t|t-1;M(\theta)) = [I-H_\theta^{-1}(q^{-1})]y(t) + H_\theta^{-1}(q^{-1})G_\theta(q^{-1})u(t) \qquad (2.6)$$

The "true" prediction $\hat{y}(t|t-1;S)$ is defined analogously. In Ljung [8] it is shown that under mild conditions

$$\theta(N;S,M,I,X) \rightarrow D_I(S,M,I,X) \text{ with prob one as } N \rightarrow \infty$$

where

$$D_I(S,M,I,X) = \{\theta \,|\, \lim_{N\to\infty} \inf \frac{1}{N}\Sigma E|\hat{y}(t|t-1;S)-\hat{y}(t|t-1;M(\theta))|^2 = 0\} \quad (2.7)$$

It is also clear that "desired estimates" are given by the set

$$D_T(S,M) = \{\theta \,|\, G_\theta(z) = G_S(z) \text{ and } H_\theta(z) = H_S(z) \text{ a.e. } z\} \quad (2.8)$$

This set consists of the parameter values that give models $M(\theta)$ with the same transfer function and the same noise characteristics as the system S.

2.2 Basic problems

There are several interesting problems concerning identification of systems operating in closed loop. Some of them will be reviewed here.

Identifiability

The question of identifiability concerns whether or not the open loop characteristics of the system can be obtained as the number of data tends to infinity. The problems when identifying systems operating under closed loop can be illustrated by the following examples.

EXAMPLE 2.1

Consider the system given in Fig 2.2.

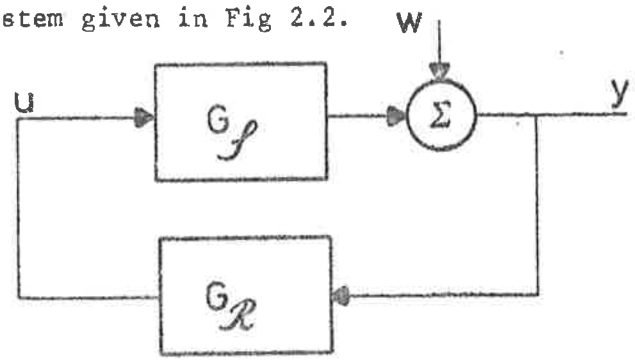

Fig 2.2 A simple closed loop configuration.

Suppose the identification is performed by methods which do not imply a causal structure, e g methods based on correlation analysis and spectral analysis. Then an attempt to identify G_S from measurements of u and y in a direct fashion will give

$$\hat{G}_S = \frac{1}{G_R}$$

i e the inverse of the transfer function of the feedback. This has been pointed out e g by Akaike [9], Åström-Eykhoff [10], and Box-MacGregor [11].

Using other identification methods, e g a prediction error identification method and postulating a causal parametric model, identifiability may be achieved for this type of experimental condition. This is not always true which can be seen from the next example.

□

EXAMPLE 2.2

Consider the system

$$y(t) + ay(t^{-1}) = bu(t^{-1}) + e(t) \tag{2.9}$$

with a proportional regulator

$$u(t) = gy(t)$$

An attempt to estimate the parameters a and b, e g by the least squares method, shows that all parameter estimates

$$\begin{cases} \hat{a} = a + \gamma g \\ \hat{b} = b + \gamma \end{cases}$$

γ arbitrary, give the same value for the identification criterion. For $\gamma \neq 0$ an erroneous description of the open loop system is obtained. Notice in particular that it is of no help to know the regulator parameter g.

If on the other hand identification is considered in a class of models given by

$$y(t) + ay(t-1) = \hat{b}u(t-1) + \varepsilon(t)$$

where a is known, it is obvious that the system (2.9) will be identifiable. □

The examples have shown the following characteristic problems when identifying systems operating in closed loop,

- o the importance to choose an appropriate identification method (see Example 2.1),
- o the importance of using experimental conditions which will guarantee identifiability (in Example 2.2 e g shifting between two proportional regulators with different g will do),
- o the importance of choosing a proper model structure in cases where the experimental conditions do not guarantee identifiability (see Example 2.2).

Testing of identifiability

As will be seen in Section 4 some conditions invoking the true order of the system must be evaluated in order to test identifiability for certain kinds of experimental conditions. Thus a priori information of the order of the system is a crucial point. Moreover, the order cannot be tested a posteriori, i e it is not possible to test after the experiment if the system is actually identifiable. This fact has been pointed out e g by Bohlin [12] and can be illustrated by the following example.

EXAMPLE 2.3

Consider the system

$$A(q^{-1})y(t) = q^{-k}B(q^{-1})u(t) + C(q^{-1})e(t) \qquad (2.10)$$

with the feedback given by

$$F(q^{-1})u(t) = G(q^{-1})y(t) \qquad (2.11)$$

It is obvious that the system

$$[A(q^{-1})+L(q^{-1})G(q^{-1})]y(t) = [q^{-k}B(q^{-1})+L(q^{-1})F(q^{-1})]\,u(t) + C(q^{-1})e(t) \qquad (2.12)$$

with the feedback (2.11) and with an arbitrary polynomial $L(q^{-1})$ will have the same input-output relation in closed loop as the system (2.10). Hence from input-output experiments (under the given experimental conditions: a linear, noise-free, time-invariant regulator) the true system cannot be distinguished. This means that it cannot in general be established a posteriori that the obtained system is identifiable and hence that the estimated model has the same open loop transfer function as the system.

□

Accuracy aspects

So far only identifiability has been considered. In practice also the quality of the estimates, i e the accuracy, for a finite number of data is of interest. It is then an important question how the experimental condition influences on the accuracy. This problem is closely related to optimal experiment design and can be viewed as an extension of the optimal input synthesis problem which has been given much attention. Regarding this problem see e g the survey paper, Mehra [13].

2.3 Other formulations and problems

The preliminaries given in Section 2.1 cover most of the approaches in the literature. The system and models are almost always taken to be linear. Usually more specific linear models than (2.2) are considered, e g (2.3), (2.4) ([14],[15],[16]), impulse response models ([17],[18], [19]) or transfer function models ([9],[19]-[22]), cf Table 4.1 below. Most experiment configurations studied are covered by Fig 2.1. There may be some more specific ones discussed, and it should be realized that identifiability properties may be critically dependent upon what is known about the special structure of the system versus the regulator. Some schemes, like indirect identification use that v_1, rather than u is measureable. The most diverse situation is for the choice of estimation scheme, I. Apart from prediction error identification methods, (2.5), of which least squares is a special case, correlation and spectral analysis methods are often discussed. They cannot be used in a direct identification fashion, cf Example 2.1, but several special schemes have been deviced and discussed, e g Akaike [9], [17] and Box and McGregor [11].

Problems, in addition to those listed in Section 2.2, which have been discussed are basically the problem to detect whether feedback is present and to analyse causal relationships between the recorded signals. These questions are treated, e g in [2].

3. IDENTIFIABILITY AND MEASURES OF ACCURACY

3.1 Identifiability

The concept of identifiability has been given several different definitions in the literature. The most common approach is to relate the identifiability property to consistency of the parameter estimate $\hat{\theta}(N)$. The "true" parameter θ_0 is then said to be identifiable, if the sequence of estimates $\hat{\theta}(N)$ converges to θ_0 in some stochastic sense, see, e g, Åström- Bohlin [23], Staley-Yue [24] and Tse-Anton [25]. In this paper the definitions introduced in Ljung et al [4] will be used: (D_T is the set defined by (2.8)).

DEFINITION 1: The system S is said to be <u>System Identifiable under M, I and X</u>, SI(M,I,X), if $\theta(N; S,M,I,X) \to D_T(S,M)$ with prob one as $N \to \infty$ [i.e. $\inf_{\theta \in D_T} |\theta(N,S,M,I,X) - \theta| \to 0$ w.p. 1 as $N \to \infty$].

DEFINITION 2: The system S is said to be <u>Strongly System Identifiable under I and X</u>, SSI(I,X), if it is SI(M,I,X) for all M such that $D_T(S,M)$

is non-empty.

DEFINITION 3: The system S is said to be Parameter Identifiable under M, I and X, $PI(M,I,X)$, if it is $SI(M,I,X)$ and $D_T(S,M)$ consists of only one element.

These definitions make no reference to any "true" parameter value, θ_0, but should be regarded as consistency-oriented since the requirement that $D_T(S,M)$ is non-empty implies that there is a "very good" model available among the set of models M. If the set of permitted values θ contains a "true" parameter θ_0 then the definition of parameter identifiability is equivalent to the definitions given by e g Tse-Anton [25], Glover-Willems [26] and Bellman-Åström [27].

Notice that $PI(M,I,X)$ is always implied by $SI(M,I,X)$ if $D_T(S,M)$ consists of only one point. This condition on $D_T(S,M)$ involves neither I nor X, and is the problem of canonical representation of transfer functions as indicated earlier. It turns out to be convenient to treat this problem separately and study the identifiability properties for different experimental conditions, identification methods and model structures by considering $SI(M,I,X)$.

Clearly, a necessary condition on M to achieve $SI(M,I,X)$ is that $D_T(S,M)$ is non-empty. If the system is $SSI(I,X)$, this condition is also a sufficient condition on M for $SI(M,I,X)$. In that case the fact that the system may operate in closed loop does not add any extra difficulties when choosing appropriate model structures M. Experimental conditions that give $SSI(I,X)$ therefore are equivalent to open loop from the view point of identifiability.

3.2 Criteria of accuracy

The reasons for performing parameter identification may for example be: determination of certain parameter values which have some specific physical meaning, control system design, or prediction.

The result of an identification is very often used for all the above purposes. This immediately leads to the conclusion that it is not possible to give a single criterion of accuracy covering all situations. Discussions about different criteria and references to other works on this subject can be found in Mehra [13], and, e g, Söderström et al [6].

One common measure of accuracy is to use the covariance matrix P of the parameter estimates (it will be assumed that the estimates are unbiased). An alternative is to use Fisher's information matrix J, see e g Kendall-Stuart [28] for a definition. In eq (2.3) $\varepsilon(t)$ can be regarded as a function of $\hat{\theta}$. Let $\varepsilon'(t)$ denote the derivative of $\varepsilon(t)$ with respect to $\hat{\theta}$ and evaluated in θ_0. Then the information matrix can be written as

$$J = N\ E\{\varepsilon'(t)^T\ \Lambda^{-1}\ \varepsilon'(t)\} \tag{4.1}$$

where N is the number of data. For statistically efficient methods like the maximum likelihood method the covariance matrix P of $\hat{\theta}$ asymptotically satisfies

$$P = J^{-1} \tag{4.2}$$

For an arbitrary identification method the Cramér-Rao inequality, see Cramér [29] and for this particular case also Åström [30], can be stated as

$$P \geq J^{-1} \tag{4.3}$$

It will be generally assumed in the following that only efficient methods are considered so that P is minimal, i e (4.2) is satisfied. It is then equivalent to use P or J as a measure of the accuracy.

It is, however, advantageous to have a scalar measure. In this paper we shall take $\det J = (\det P)^{-1}$, which is commonly used, e g Mehra [13] and Nahi and Napjus [31]. The larger the value of det J is, the better the identification result.

4. INFLUENCE OF FEEDBACK ON IDENTFIABILITY AND ACCURACY

From the discussion of the previous sections, it follows that all the three items, the model structure M, the identification method I and the experimental conditions X will have influence on the identifiability of a system as well as on the accuracy when it is identifiable. Some general results concerning the influence of M and I are given in Söderström et al [6]. In general it can be said that the maximum likelihood method (= minimizing det Q in (2.5)) is always efficient. Further, if this procedure fails to give identifiability, then no other method, based on measurements of y and u, will be successful.

In this section the influence of X will be discussed. In one sense the experimental conditions will have a greater influence on the results. If it is found that an X does not yield identifiability or acceptable accuracy, then the whole identification experiment has to be repeated. In contrast, if M (or I) was not chosen in a suitable manner it is possible to try other M (or I) on the same data.

4.1 Influence on identifiability

It follows from (2.6) and (2.7) that the set D_I consists of those θ for which

$$\liminf_{N\to\infty} \frac{1}{N} \sum_{1}^{N} |L_\theta(q^{-1})y(t) - M_\theta(q^{-1})u(t)|^2 = 0 \tag{4.1}$$

where

$$L_\theta(z) = H_\theta^{-1}(z) - H_S^{-1}(z) \tag{4.2)a}$$

$$M_\theta(z) = H_\theta^{-1}(z)\, G_\theta(z) - H_S^{-1}(z)\, G_S(z) \tag{4.2)b}$$

Clearly, $D_T \subset D_I$ (D_T defined by (2.8)), since for $\theta \in D_T$ we have $L_\theta(z) = M_\theta(z) \equiv 0$. However, if the system is not SI under the current experimental conditions, then there exists a $\theta^* \in D_I$ such that $\theta^* \notin D_T$. This means that the non-zero filters $L_{\theta^*}(z)$ and $M_{\theta^*}(z)$ are such that (4.1) holds. This defines, essentially, a linear, time invariant, noisefree relationship between y and u,

$$M_{\theta^*}(q^{-1})\, u(t) \sim L_{\theta^*}(q^{-1})\, y(t) \tag{4.3}$$

Therefore only if there is a feedback like (4.3) we may loose SI. This condition is the most general statement about System Identifiability, but as it stands it is fairly implicit. We shall therefore proceed to give an exact explicit condition for SI, which should cover most practical cases of interest.

Let the input signal, u(t), to the process be given by

$$u(t) = F_i(q^{-1})\, y(t) + K_i(q^{-1})\, v(t) \qquad i=1,\ldots,r \tag{4.4}$$

where F_i and K_i are linear rational functions of the delay operator q^{-1}. These filters are changed during the identification experiment between r (different) ones, so that each is used a non-zero proportion of the total time. The variable v is a vector of dimension n_v and it is an external disturbance to the input signal. Comparing with Fig 2.1 we see that v can be considered as an additional extra input signal v_1 (measurable), as noise v_2 in the regulator, as set point changes in the regulator, v_3, or as a combination of these effects. To include the case where v is correlated with the system noise e, assume that it can be written

$$v(t) = \tilde{v}(t) + R(q^{-1})\, e(t) \tag{4.5}$$

where R is a causal filter. Before we give the result, we may remind the readers of the concept of persistenly exciting signals, see, e g Åström--Bohlin [23] and Mayne [32]. We say that a signal $\tilde{v}(\cdot)$ is persistently exciting (p e) of order m if

$$\delta I < \frac{1}{N} \Sigma V_m(t) V_m(t)^T < \frac{1}{\delta} I \qquad N \geq N_0 \tag{4.6}$$

where

$$V_m(t) = \text{col } (\tilde{v}(t),\ldots,\tilde{v}(t-m+1)) \tag{4.7}$$

It is clear that if v is a stochastic process with full rank innovations, then it is p e of any finite order.

Theorem 1

Consider the system S, (2.1) with input signal (4.4), X, where $\tilde{v}$ is p e of any finite order. The model set may consist of arbitrary time invariant linear models as in (2.2). We assume there is a time delay either in the system (and model) or in the regulator, i e $G(0)F_i(0)=0$. The identification method I is a prediction error identification method (2.5) applied in a direct fashion. Then S is Strictly System Identifiable, SSI(I,X) if and only if

$$\text{rank} \begin{bmatrix} K_1(z) & \ldots\ldots & K_r(z) & F_1(z) & \ldots. & F_r(z) \\ 0 & \ldots\ldots & 0 & I & \ldots. & I \end{bmatrix} = n_y + n_u \quad \text{a.e. } z \tag{4.8}$$

[The dimension of K_i is $n_u|n_v$, of F_i $n_u|n_y$, of 0 $n_y|n_v$ and of I $n_y|n_y$, so the dimension of the matrix in (4.8) is $(n_u+n_y)|r(n_v+n_y)$.] □

This theorem is an extension of the results in Ljung et al [4]. The proof of the extended version is in Söderström et al [33].

Obviously, a necessary condition for (4.8) to hold is that

$$r \geq (n_y + n_u)/(n_y + n_v) \tag{4.9}$$

On the other hand, if (4.9) holds then K_i and F_i can always be chosen so that (4.8) is satisfied.

Let us discuss some special cases of the result. First we see that if $n_v = n_u$ and $K_1(z)$ is non-singular, then (4.8) is satisfied for any r (including r=1) regardless of the feedback filters F_i. Therefore, if the number of independent "extra inputs" is equal to the number of inputs to the system, then we always have SSI, no matter what the feedback might be. This fact is by now also widely recognized, see below. If there are no extra inputs, $n_v=0$, then (4.8) can be satisfied if and only if

$$r \geq 1 + n_u/n_y \tag{4.10}$$

by a proper choice of feedback laws. This means that by shifting between sufficiently many regulators we can also obtain SSI even if no extra input signal is allowed. This fact was pointed out in [4]. For example if $n_u=n_y$, then it is sufficient to use two regulators. These shall be chosen so that det $(F_1(z)-F_2(z)) \not\equiv 0$, which is quite a mild condition. In certain applications additional input would require a very good regulator in order to keep the output variance sufficiently low. In such cases it may be easier to find several different regulators that yield acceptable output variance, than to find one close to the optimum. Then the way of obtaining SSI by shifting regulators is of importance.

We may repeat that when (4.8) is satisfied, i e when the system is SSI, there is no additional difficulties in finding an appropriate model structure due to the feedback terms. The whole identification procedure can be performed as though the data was collected during open loop operation. In fact, the open loop situation is just a special case in the theorem ($F_i=0$). It can also be remarked that when the disturbance signal is measurable, indirect identification may be applied, as described in Section 2. Then the transfer function(s) from v to y are determined (or, if v=0, y is modelled as a time series) and knowing the regulator(s) the open loop dynamics is solved for. However, it can be shown, cf Ljung et al [4] and [33], that the identifiability properties for this procedure are exactly the same as those for direct identification using (2.5), i e Theorem 1 is valid also for indirect identification.

When (4.8) is not satisfied, then the system is not SSI. It may, however, be SI for certain model structures, but these can be characterized only in terms of fairly complex expressions involving the matrix in (4.8) and the structure of the true system S. Neither can it be tested a posteriori from data only, whether the chosen M actually yielded SI, cf Example 2.3.

An important case when SSI is impossible is when the regulator is time invariant, linear and noisefree and no extra input is added, i e $r=1$, $n_v=0$. We shall further investigate this case for $n_u=n_y=1$ and M being difference equation models.

Let the system be given by

$$S:\ y(t)+a_1y(t-1)+\ldots+a_{n_a}\ y(t-n_a)=b_1u(t-k-1)+\ldots+b_{n_b}u(t-k-n_b)+ \\ +\ e(t)+c_1e(t-1)+\ldots+c_{n_c}\ e(t-n_c) \qquad (4.11)$$

$$k\geq 0,\ a_{n_a}\neq 0,\ b_{n_b}\neq 0,\ c_{n_c}\neq 0$$

and the regulator is

$$u(t)=-f_1u(t-1)-\ldots-f_{n_f}u(t-n_f)+g_0y(t)+\ldots+g_{n_g}y(t-n_g) \qquad (4.12)$$

$$f_{n_f}\neq 0,\ g_{n_g}\neq 0.$$

Define the polynomials A(z), B(z), C(z), F(z) and G(z) by

$A(z) = 1 + a_1 z + \ldots + a_{n_a} z^{n_a}$ C(z), F(z) analogously

$B(z) = b_1 z + \ldots + b_{n_b} z^{n_b}$

$G(z) = g_0 + g_1 z + \ldots + g_{n_g} z^{n_g}$

Assume that F(z) and G(z) are relatively prime and that there is no common factor to all the polynomials A(z), B(z) and C(z). Further assume that the polynomials C(z) and $A(z)F(z) - z^k B(z)G(z)$ have exactly n_p common factors. Then we have the following result.

Theorem 2

(Söderström et al [5]) Consider the system given by (4.11) with the feedback $\mathcal{X}$ (4.12). Let the class of models $\mathcal{M}$ also be given by (4.11), where the coefficients a_i, b_i and c_i are estimated using the identification method (2.5); $\mathcal{I}$. Then $\mathcal{S}$ is SI($\mathcal{M},\mathcal{I},\mathcal{X}$) if and only if

$$\max(n_f - n_b,\ n_g + k - n_a) - n_p \geq 0 \qquad (4.13)$$

□

A similar result is shown in Vorchik [34]; cf also Box and McGregor [35]. In [5] further results are given for the case when the time delay and the orders of the model do not coincide with those of the system.

Remark 1

In order to test identifiability of a system some condition invoking the true order of the system must be evaluated. Thus a priori information of the order of the system is a crucial point. Moreover, the order cannot be tested a posteriori, i e it is not possible to test after the experiments if the system really was identifiable as was illustrated by the Example 2.3.

Remark 2

The results can for example be applied to the special case when a minimum variance regulator, [36], is used. It can then easily be shown that (4.13) reduces to

$k \geq n_c + 1.$

4.2 Influence on the accuracy

From Section 4.1 it follows that good identifiability properties are obtained for a number of feedback situations. It is, however, sometimes claimed that even if identification of systems operating in closed loop is theoretically possible, the obtained estimates are in practice very poor, due to the effect of the feedback terms in the input signal. Normally, the reason for feedback control is to decrease the variance of the output. Then, naturally, also the information contents of the measured data decreases and the estimates obtained have worse accuracy. For this reason closed loop experiments may be inferior to open loop ones. However, this comparison is clearly unfair. Often the limiting variable for an identification experiment is the output variance. Due to security or production reasons this variance must be kept below a certain value. Therefore different experimental conditions should be compared for the same output variance.

Then the interesting questions to ask are:

- o Are open loop experiments inherently better than closed loop ones giving the same output variance?
- o Is there any systematic difference between various experimental conditions that all yield identifiability?

In Söderström et al [6],[7], these two problems are studied by means of simulations and analysis of low order systems. The answers are quite clear for these examples: Open loop experiments are not necessarily better than closed loop ones that give the same output variance, and there is no systematic difference between different experimental conditions that yield identifiability. In fact, it can be shown that the same information matrices can be obtained within each class of feedback configurations, like when an extra input signal is injected, when the regulator alternates between different feedback laws or when a linear regulator of sufficiently high order is used.

Encouraged by these results, we proceed to optimize the experimental conditions in order to obtain as good accuracy of the parameter estimates as possible. We shall consider the following classes of experimental conditions.

$X(\underline{O})$: Open loop experiments. The covariance function (or the spectrum) of the input signal is at the experiment designer's disposal.

$X(\underline{S}(r,n))$: Shifts between r linear regulators of order n. The parameters of the regulators, and the proportion of the total time each one is used are at disposal.

$X(\underline{E}(n))$: A linear regulator of order n and an extra input signal. The parameters of the regulator as well as the covariance function of the extra input is at disposal.

To choose the optimal experimental conditions within the class $X(0)$ is of course the well-known problem of optimal input design, which has been studied in a great number of papers, e g Levin [37], Levadi [38], Gagliardi [39], Aoki-Staley [40], Nahi-Napjus [31], Goodwin et al [41], Keviczky-Bányasz [42], Goodwin-Payne [43], Ljubojević [44] and Mehra [13]. In these papers the possibility to change the experimental conditions by for example introducing a feedback has not been considered.

Unfortunately, it is technically very involved to consider optimization of X for a general system. We shall therefore analyse the problem only for a first order system,

$$S_1:\quad y(t) + ay(t-1) = b\, u(t-1) + e(t) + ce(t-1) \qquad (4.14)$$

Assume that the model set M is chosen as

$$M_1:\quad y(t) + \hat{a}\, y(t-1) = \hat{b}\, u(t-1) + \varepsilon(t) + \hat{c}\, \varepsilon(t-1) \qquad (4.15)$$

As measures of the accuracy the determinant of the information matrix, det J, given in Section 3 will be used. When no restriction is put on the variances of the input and the output it would be favourable to have very large signals. This is of course for different reasons not possible in practice. Therefore the cases of constrained output variance,

$$C_1:\quad E\, y^2(t) \le \lambda^2(1+\delta_1), \qquad \delta_1 \ge 0$$

and constrained input variance,

$$C_2:\quad E\, u^2(t) \le \lambda^2\delta_2/b^2, \qquad \delta_2 \ge 0$$

will be considered. Of these constraints it seems more realistic to put bounds on the variance of the output.

EXAMPLE 4.1

Consider the system S_1 (4.14) where $Ee^2(t) = 1$. Assume that det J is maximized subject to the constraint C_1. Then it can be shown, [7],[60], that the optimal experimental condition in the class $X(E(n))$ is obtained as

$$u(t) = \frac{a-c}{b}\, y(t) + v(t) \qquad (4.16)$$

where $v(t)$ is white noise with variance $\delta_1(1-c^2)$. It is consequently sufficient to take n=0.

In the class $X(S(r,n))$ the optimal experimental condition will be

$$u(t) = \frac{a}{b}\left(1 - \frac{c}{1+\delta_1} \pm \frac{\sqrt{\delta_1(1-c^2+\delta_1)}}{1+\delta_1}\right) y(t) \qquad (4.17)$$

(the + sign and − sign are used 50% of the time each). Hence r can be taken as 2 and n as 0.

Both (4.16) and (4.17) give the optimal value

$$\det J = N^3 \frac{\delta_1^{\;2}}{b^2(1-c^2)^3}$$

It can also be shown that no open loop input can yield this value, unless in the degenerated case a=c. □

Although we consider it to be more natural to require that the variance of the output is bounded than that the variance of the input is bounded, it is instructive to see how this change of constraint will influence the result.

EXAMPLE 4.2

Consider again the system S_1, (4.14) with $c = \hat{c} = 0$. Let the criterion still be det J but change the constraint to C_2. Then optimization in the classes $X(0)$, $X(S(r,n))$, $X(S(1,1))$, $X(E(n))$ will give the same optimum, namely

$$\det J = N^2 \frac{\delta_2(\delta_2+1-a^2)}{b^2(1-a^2)^2}$$

The optimum can be obtained in class $X(0)$ if

$$u(t) = \frac{1}{1+aq^{-1}} w(t)$$

where w(t) is white noise with variance $\delta_2(1-a^2)/b^2$.

In the class $X(S(r,n))$ the optimum is obtained with shifts between the two proportional regulators (each used 50% of the total time)

$$u(t) = \frac{-a\,\delta_2 \pm \sqrt{\delta_2(1+\delta_2-a^2)}}{b(1+\delta_2)}$$

□

The analysis has several important implications. In the first place it shows that feedback terms may have to be included in the input signal in order to obtain optimal accuracy. The optimal input problem, as usually discussed, therefore can be said not to be well posed. Inclusion of feedback terms is by no means any problem in practice, and therefore the optimal input should be sought in the classes $X(S(\cdot,\cdot))$ or $X(E(\cdot))$

rather than in X(0).

One might discuss the generality of the results in Example 4.1. It is difficult to carry out the analysis in the general case, but it seems plausible that it is always favourable to control the process with a minimum variance controller and add extra input so that the constraint on the output variance is met. Now, to know the minimum variance controller is essentially equivalent to know the system itself and then there is no need for identification. This is the usual dilemma of optimal input synthesis: The system must be known in order to design the optimal input signal. However, the following rule-of--thumb can be given: Decrease the output variance as much as possible using a feedback regulator. This makes use of the available a priori information about the system in a sensible manner. Then add an extra perturbation signal (e g white noise) so that the output variance reaches its allowed value. Alternatively, change the regulator parameters in two different ways so that the bound on the variance of the output is met. Then shift between these two regulator settings. One way to obtain a suitable regulator in the first place if the process dynamics are unknown could be to use a self-tuning regulator, Åström and Wittenmark [45].

4.3 Survey of other contributions

A number of papers dealing with the problem of identifying a process operating in closed loop are now available. So far most of them discuss the problem from the identifiability point of view. In the following the references will be briefly reviewed. In a short review like this it is not possible to give full details. Notions like identifiability etc will therefore be used without giving the definitions that were used in each particular paper. Nevertheless it is most often intuitively clear what is meant. In Table 4.1 an attempt is made to systemize the contributions according to the cases discussed and the methods and model structures used. The papers are reviewed in chronological order.

Fisher [14] discusses identifiability of continuous, deterministic, linear state space systems with no extra perturbation and with a time invariant, noise free feedback. A least squares approach to the parameter estimation is taken. Necessary and sufficient conditions for identifiability are in this case that the control law is non-linear in the states, and that the system is completely controllable.

The problem of identification of closed loop systems has been treated by Akaike [9], [17]. There are also several applications using his ideas, e g Otomo et al [46] and Itoh et al [47]. Akaike points out that cross spectral analysis requires, that the input is measured without noise and that the noise and the input are independent. This last condition is violated for closed loop systems. If there is an extra perturbation, i e $v_1 \neq 0$, cross spectral analysis can, however, be used in a special way. Akaike treats particularly the case when there is an unmeasurable disturbance in the feedback loop. Then ordinary cross spectral analysis fails. Instead the problem is solved by introducing a causal time domain model. Direct identification is used with an impulse response model with a finite number of parameters.

Priestley [19] treats the problem of estimating the process transfer function from data consisting of records of the input and the output from a closed loop system with additive disturbances. No known perturbation is available and the feedback is linear. Direct identification is used. The least squares method is compared with the weighted least squares method.

In the survey paper on identification by Åström and Eykhoff [10] identification of closed loop systems is only discussed very briefly. An example is given for the case of noise free, linear, time invariant regulator without extra perturbation. In this case a noncausal method will give the inverse of the regulator as the model. However, a method using a causal model may or may not give the correct model, cf Example 2.1 and 2.2.

The most general discussion is given by Bohlin [12]. This paper is concerned with the basic limitations of identification and the practical implications of the mathematical assumptions involved. Thus also identification of processes operating in closed loop during the experiments is treated. Bohlin remarks that there are very simple but realistic cases when identification is impossible under closed loop conditions. However, a closed loop during the experiments does not necessarily prevent identification of the open loop characteristics. In the case of noise free, linear feedback without any extra perturbation the process is not identifiable without a priori knowledge of the structure. A chosen structure cannot be validated by the data, cf Example 2.3.

In Bohlin [48] the ambiguity of the maximum likelihood method is treated. It is demonstrated that if the regulator is noise free, linear, time invariant and if no extra perturbation is injected only the noise transfer function for the closed loop system is identifiable. When the structure is known, identifiability of the open loop transfer function is secured if there is a one to one correspondance between the parameters of the open loop transfer function and the closed loop noise transfer function. No explicit conditions for the identifiability of the open loop dynamics are given.

Schultze [49] treats the case with an extra perturbation v_1 or with an unknown disturbance v_2. The choice of model structure and identification criterion is also discussed. It is demonstrated that this choice is of crucial importance. For the closed loop case the difference equation model for the process must include possibilities to handle correlated disturbances. Direct identification is proposed. An application to a ball mill is presented.

Box and MacGregor [11],[18] first show that correlation methods fail for closed loop systems. If an extra input signal is available however identification is possible. They also discuss what can be done in the case of no extra input by modelling the process as a pure time series using the output (i e indirect identification). They propose a test for feedback by testing the cross covariance between the input and output. Applications to polymer viscosity and paper machine data are given.

Caines and Wall [50] discuss parameter estimation of closed loop systems and particularly the case with an unknown disturbance in the feedback loop. They state that direct identification using the loss function for the open loop maximum likelihood estimation will not give the maximum likelihood estimates for the closed loop case. Instead it is proposed to estimate the parameters in the model consisting of the inputs and outputs expressed as time series of the noise sources. This idea has been pursued by Caines and Chan, see below.

Leonhard [51] dicusses the identification of systems operating in closed loop. Indirect identification with the least squares method is used. It is stated that the open loop transfer function can be obtained if an extra perturbation v_1 is injected or when there is a disturbance v_2 acting in the feedback loop. An example using direct identification is also given.

Eykhoff [52] discusses briefly identification of closed loop systems in connection with least squares estimation. The case with noise free feedback and with a perturbation signal is treated. Indirect and direct identification are proposed as the two existing possibilities. It is mentioned that difficulties can arise in indirect identification because it is not always trivial to solve for the open loop dynamics from the closed loop transfer function.

Glover [15] is mainly concerned with parameter identifiability of continuous and discrete time state space systems. A recommended way to model a system with feedback is to write down the state space equations for the open loop system and then modify these equations with the feedback law. The closed loop system is then expressed in terms of the open loop parameters and the parameters of the feedback law. The identifiability questions for the unknown parameters of this closed loop system can then be answered as for the open loop case.

Goodwin, Payne and Murdoch [53] are mainly concerned with the synthesis of optimal test signals for closed loop identification. A variant of the case with an extra perturbation is treated. According to our notion indirect identification is used.

Lindberger [54], [55] has discussed problems related to the identification of closed loop systems. In principle the case with unmeasurable disturbances in the feedback loop is treated. Lindberger uses indirect identification according to our notion. He also proposes a strategy in order to try to achieve identifiability by successively increasing the complexity of the regulator and performing one experiment with each regulator until a reasonable model is obtained.

Panich and Trachevskii [56] derive identifiability conditions in the case of a linear noisefree feedback without any extra input assuming special structures of the system and of the regulator making least squares identification applicable. Necessary conditions turn out to be that the lag of the system is large enough or that the controller is complex enough. Simulations illustrate what happens for different number of delays in the system and for different controllers.

Vorchik et al [57] consider the identification of a closed loop system with disturbances in the feedback loop which may be correlated with the system disturbances. The disturbances are assumed to be uncorrelated in time. Conditions on the signals and the regulator are derived under which the parameters of the system and/or the regulator converge to the true values and under what conditions the estimates are efficient. These results are extended in Vorchik [34] to cases where the structure of the disturbances are more complex. Identifiability conditions are also given for the case when there is no disturbance source in the feedback loop.

Rödder [20],[21] gives conditions on which signals that must exist and which signals that have to be measured to be able to determine the open loop characteristics from closed loop experiments by correlation (spectral) analysis. In general cases the regulator has to be known. Also estimates of the obtained accuracies are given.

Phadke and Wu [58] present a procedure for the identification of a multivariable system described by a vector difference equation model which is similar to the one proposed by Caines and Chan [1]. The procedure consists of two steps. First a multivariate time series model is fitted to the input and output series. The open loop characteristics can then be determined from this model. The conditions are that there is a disturbance source in the feedback loop and that the plant (or feedback) has at least one lag. The procedure is applied to the identification of a multivariable model of a blast furnace.

Thöm and Krebs [22] compare correlation analysis and parameter estimation when applied to the identification of closed loop systems. They discuss the possibilities to use correlation analysis and conclude that e g generalised least squares and maximum likelihood methods are preferable.

Kurz and Isermann [59] consider several different closed loop configurations and discuss under what conditions the open loop characteristics are identifiable. A two-stage (on-line) identification procedure is used. In the first step correlation analysis is performed. The obtained estimates of the correlation functions are then used in a least squares parameter estimation. The different cases are illustrated by simulations trying to show the achievable accuracy in the different cases.

Box and MacGregor [35] study the effects on the estimation of characteristics of systems operating in closed loop of optimality and suboptimality of the regulator, of the influence or an additional input signal and of lags in the systems. In particular, they consider two problems, the estimation of parameters in the dynamic and stochastic parts of the system and the estimation of only those functions of these parameters which occur in the control equation. Identifiability conditions are given showing that the conditions for the latter case are much weaker.

Caines and Chan [1] and especially [2] give a very complete treatment of the problem with feedback in the input signal. They present a definition of feedback between an ordered pair of multivariable processes. It provides statistical criteria for the detection of feedback. The proposed identification procedure starts with a multivariable time series, modelling the input and output. It is assumed that there is a disturbance in the feedback loop and that there is at least one lag either in the system or in the regulator. From the obtained model the system and regulator characteristics can be derived. The papers includes an application to economic model building. An application to power systems identification is described in [2] and further developed in Caines and Sinha [61].

Graupe [62] considers the identification of closed loop feedback systems of different configurations and discussed consistency of the obtained parameter estimates.

Wellstead and Edmunds [63] consider the identifiability problem in the case when the system has uncorrelated disturbances. They also discuss the case when there is no lag either in the system or in the regulator. It is proposed that the identification in this case can be performed using an instrumental variable method.

So far the papers reviewed have mainly been concerned with the identification of the open loop dynamics from closed loop experiments with constant regulators. Identifiability problems for closed loop systems appear also naturally in many adaptive control situations. For such problems the feedback is generally timevarying in a very subtle way. Some papers have discussed such problems, e g Turtle and Phillipson [64], Saridis and Lobbia [16], Lobbia and Saridis [65], Balakrishnan [66], Ljung and Wittenmark [67].

Captions to Tables 4.1 and 5.3

Parantheses indicate that only a short discussion is given.

Feedback cases

EI: Extra input, the signal v_1 in Fig 2.1 is non-zero.

NR: Noise in the regulator, the signal v_2 in Fig 2.1 is non-zero.

LTI: Linear, time-invariant, noisefree regulator without extra input.

NL: Non-linear regulator.

LTV: Linear time-varying regulator.

Identification method (cf Section 2.1)

D: Direct identification

Table 4.1 Classification of some papers dealing with theoretical aspects of identification of closed loop systems.

Author(s)	Feedback cases					Methods					Systems		Model structures			
	EI	NR	LTI	NL	LTV	D	ID	JIO	PE	COR	SI SO	MI MO	SSP	DE	IR	FR
Akaike [9],[17]	x	x				x	(x)	(x)	x	x	x	x			x	x
Bohlin [12],[48]	x		x			x	(x)		x	(x)	x			x		
Box-McGregor [11],[18]	x	x	x				x			x	x			x	x	
Box-McGregor [35]	x		x				x		x		x			x		
Caines-Chan [1],[2]		x						x	x		x	x		x		
Caines-Wall [50]		x						x	x		x	x		x		
Defalque et al [68]	x								x	x	x		x		x	
Eykhoff [52]	x		(x)			x	x		x		x			x	(x)	
Fisher [14]				x		x			x		x		x			
Froisy et al [69]			x			x			x		x			x		
Glover [15]	(x)		x				x		x		x	x	x			
Goodwin et al [53]	x						x		x		x			x		
Graupe [62]	x	x				x			x		x			x		
Gustavsson et al [3]	x	x	x	(x)	x	x	x		x	(x)	x	x		x		
Kurz-Isermann [59]	x	x	x			x	x			x	x			x		
Leonhard [51]	x	x	x			(x)	x		x		x			x		

Table 4.1, continued.

Author(s)	Feedback cases					Methods					Systems		Model structures			
	EI	NR	LTI	NL	LTV	D	ID	JIO	PE	COR	SI SO	MI MO	SSP	DE	IR	FR
Lindberger [54],[55]		x	(x)				x		x		x			x		
Ljung et al [4]					x	x	x		x		x	x	(x)	(x)	(x)	(x)
Panich-Trachevskii [56]			x			x			x		x			x		
Phadke-Wu [58]		x						x	x		x	x		x		
Priestly [19]		x	x			x			x	x	x				x	x
Rödder [20],[21]	x	x	x				x			x	x					x
Saridis-Lobbia [16]	x					x			x		x		x			
Schulze [49]	x	x				x			x	(x)	x			x		
Schwalb [70]	x						x			x	x					x
Söderström et al [5]			x			x			x		x			x		
Söderström et al [6]	x	x	x		x	x	x		x		x	x	(x)	x	(x)	(x)
Thöm-Krebs [22]		x				x	x		x	x	x			x		x
Wellstead-Edmunds [63]		x	x			x			x		x			x		
Vorchik [34]	x	x	x			x			x		x			x		
Vorchik et al [57]		x				x			x		x			x		

ID: Indirect identification

JIO: Joint input-output identification

PE: Prediction error identification method (e g ML or LS)

COR: Correlation or spectral analysis

System

SISO: Single input single output

MIMO: Multiple input multiple output

Model structures

SSP: State space model

DE: Difference equation model

IR: Impulse response model

FR: Frequency response model

5. APPLICATIONS

In this section two applications of identification of processes operating in closed loop during the experiments will be described. Other applications found in the literature will be surveyed in the second part of this section.

5.1 Applications to ship dynamics and to a laboratory process

EXAMPLE 5.1

This application is concerned with the identification of ship dynamics. A model of the ship dynamics is needed for example when designing autopilots or for simulation. In this case it is valuable to be able to use data from closed loop experiments for the modelling, since the ship then can operate under fairly normal conditions even while the experiments are carried out.

Two experiments with a fully loaded 255 000 tdw supertanker, T/T Sea Swift, are compared. The ship is 329 m long and has a maximum speed of 16 knots. The measurements were made in the Indian Ocean. The weather conditions, the trim of the ship and the water depth were about the same in the two experiments. The dynamics from requested rudder angle to the ship's heading angle was determined. The first experiment was performed in open loop, the second one in closed loop with a proportional regulator and with an additive rudder disturbance in order to secure the identifiability. The open loop experiment lasted 78 minutes, the closed loop one 59 minutes. The sampling interval was chosen to 10 seconds. The variance of the input signal was approximatively 4 times larger in the closed loop experiment. On the other hand the variance of the heading angle for the closed loop experiment was ca one fourth of the corresponding variance for the open loop experiment. In Fig 5.1 the inputs and outputs are shown for the two experiments. The experiments are described in more detail in Källström [71].

The dynamics from the requested rudder angle to the differenced heading angle was determined in a model form like (4.11) using the maximum likelihood method. (The reason for using the differenced heading angle as the output instead of the angle itself, is that due to the physical knowledge of the process it is known that there is a pure integrator in the process, when no wind is present, viz. the heading angle is the integrated angular velocity of the ship [72].)

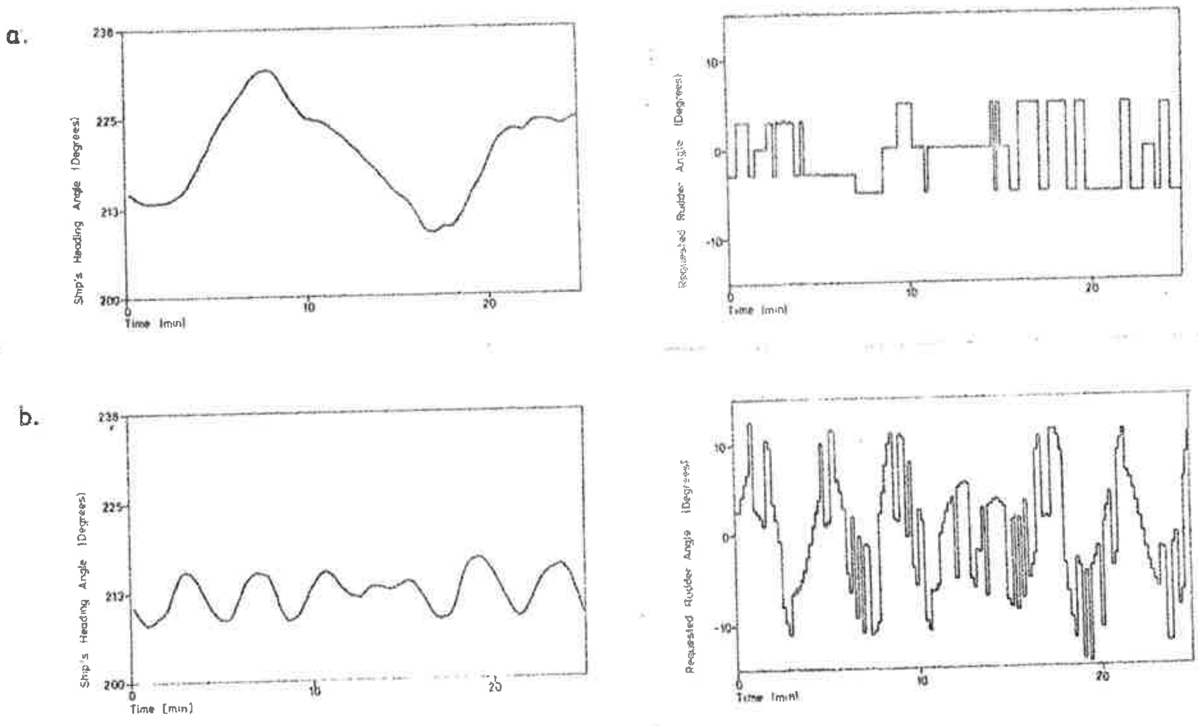

Fig 5.1 a) Requested rudder angle and heading angle for the open loop experiment. b) Same for the closed loop experiment.

For both experiments statistical tests indicated that a third order model was appropriate. Also cross correlation functions between the residuals and the input indicated a third order model. The parameters of the models are given in Table 5.1 together with the estimated accuracies of the parameters.

	Open loop experiment	Closed loop experiment
$\hat{a}_1$	-2.031 ± 0.051	-2.012 ± 0.073
$\hat{a}_2$	1.322 ± 0.074	1.337 ± 0.088
$\hat{a}_3$	-0.297 ± 0.037	-0.330 ± 0.034
$\hat{b}_1$	-0.649 ± 0.110	-0.526 ± 0.132
$\hat{b}_2$	-1.381 ± 0.230	-2.146 ± 0.248
$\hat{b}_3$	1.355 ± 0.196	1.702 ± 0.273
$\hat{c}_1$	-1.426 ± 0.058	-1.284 ± 0.088
$\hat{c}_2$	0.603 ± 0.041	0.484 ± 0.057
$\hat{\lambda}$	0.078	0.095

Table 5.1 The estimated parameters of the third order difference equation models of the ship dynamics. The b-parameters are scaled with a scale factor 100.

The models do not differ very much. The zeros and the poles of the corresponding continuous models are given in Table 5.2.

	Open loop experiment	Closed loop experiment
Zeros	-0.031 0.31	-0.038 0.25
Poles	0.0020 -0.062 ± 0.036 i	0.0015 -0.056 ± 0.051 i

Table 5.2 Poles and zeros of the continuous models of the ship dynamics.

There is one unstable mode in the model. This is not unexpected since it was known in advance that the ship might be unstable under certain loading conditions. Furthermore a pair of complex poles occur. These modes may be due to nonlinear effects in the ship dynamics. Another explanation may possibly be the dynamics of the rudder servo.

In conclusion we may say that the two experiments are quite comparable both regarding results and accuracies. However during the closed loop experiments the variance of the output was considerably less. The disturbances in the course of the ship would be quite acceptable for longer experiments, when the control loop is closed.

EXAMPLE 5.2

In this case a laboratory process, a bar with a rolling ball is identified. The angle of the bar and the position of the ball can be measured. The control variable is the voltage of the motor driving the bar around.

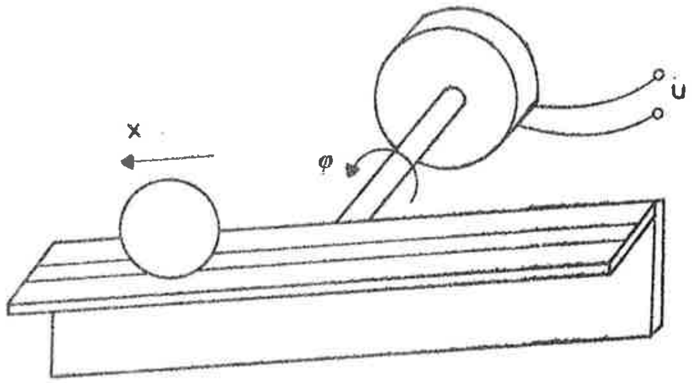

Fig 5.2 The process: A bar, which can be rotated with a motor around a horisontal axis, applied in its midpoint. A metal ball rolls freely along the bar.

In this case we were interested in the dynamics from the angle φ to the position x. It is clear from basic physical laws that this dynamics is a double integrator: $G(s) = 1/s^2$. Obviously, the identification experiment cannot be performed in open loop. With no control, the ball will very quickly roll off the bar. A simple PD-controller feeding back x to the voltage of the motor was used. A disturbance was added to the setpoint of this controller, so that the ball was made to roll from one end of the bar to the other. Inputs (φ) and outputs (x) are shown in Fig 5.3.

The sampling interval was 0.04 s and the experiment lasted 20 sec. The parameters of a second order difference model (4.11) were determined using the maximum likelihood method. The model was converted to continuous time, and an amplitude Bode plot was drawn, Fig 5.4. It is seen that in a wide frequency range the model gives a good description of the system $1/s^2$. The "bad" behaviour for very low frequencies is

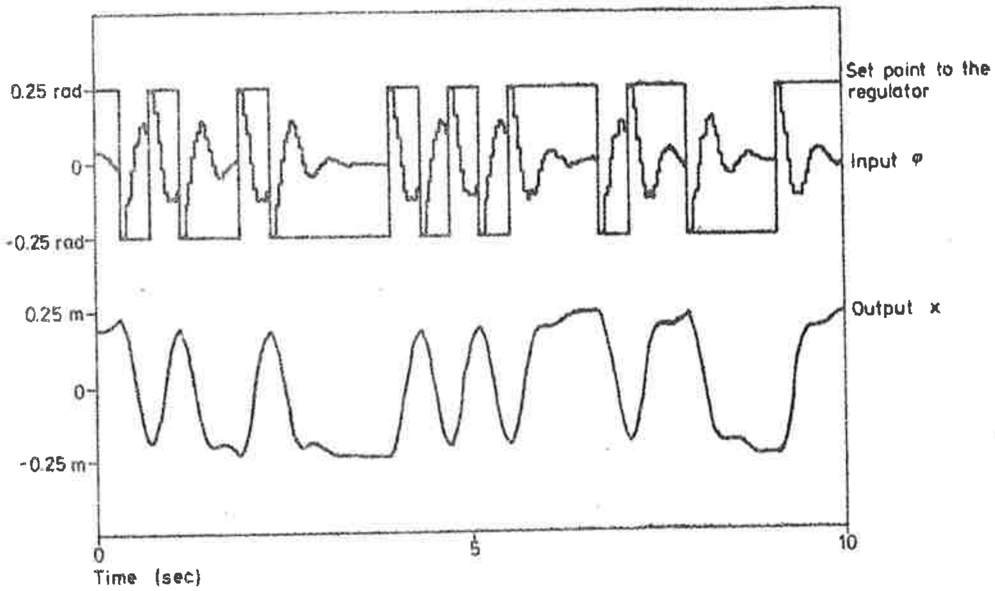

Fig 5.3 Inputs (φ) and outputs (x) for the identification experiment.

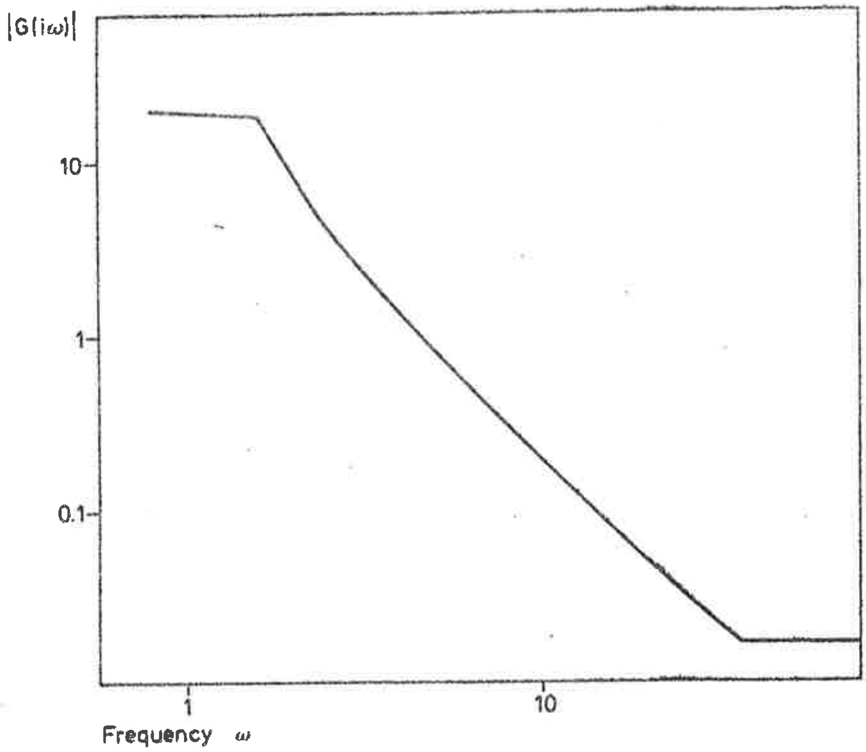

Fig 5.4 Amplitude Bode-plot of the frequency response of the obtained model, converted to continuous time.