IDENTIFICATION OF RADAR TARGETS IN RESONANCE ZONE: E-PULSE TECHNIQUES

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Abstract—Radar scattering amplitudes contain pole singularities whose importance was recognized in the context of the Singularity Expansion Method: S.E.M. This method uses the fact that the late time domain response $r_t(t)$ of a target, illuminated by an E.M. wave, is mainly defined in a frequency band corresponding to the resonance region of the object. The knowledge of the singularities is useful information for discrimination of radar targets and has been used for different purposes of discrimination and identification. In this paper, we propose a modified scheme of radar target identification. The method presented is based on E-Pulse technique. In practical cases, direct application of classical E-Pulse techniques is not very efficient. Its performances are damaged by the characteristics of the exciting signal (antenna output signal). We propose a modified scheme of E-Pulse technique, which allows more accurate target discrimination and improves radar target identification. This procedure requires the deconvolution of the target response by the antenna signal and the application of an equivalent gaussian impulse excitation. This process has been successfully tested to FDTD simulations and measurements in anechoic chamber.

1. INTRODUCTION

Analysis and identification of objects according to their electromagnetic response remain prone to many researches in radar domain, in particular with regard to detection of buried objects or targets known as stealthies. Development of composite materials, absorbing the electromagnetic waves in the classical radar frequency bands, has returned the problems of detection and identification increasingly complex. These evolutions have directed the axis of research with regard to targets detection, towards a low frequency analysis of the scattered signals. For these frequency bands, the dimension of the object is in the same order than the wavelength associated to the excitation signal. The object is then observed in its resonance zone for which the fluctuation of the energy radiated is significant.

The formalism is based on a representation of the response due to an electromagnetic excitation, whose Baum [1, 2] presented a modeling (in time and frequency domain). This response (1) is composed by an impulsive part or early time, known as forced, followed by a late time $r(t)$, for which the target oscillates freely.

$$r_t(t) = f_e(t) + r(t) \tag{1}$$

The late time response is expressed by a finite sum of damped sinusoids:

$$r(t) = \sum_{n=1}^{N} a_n(\theta) e^{\sigma_n t} \cos\left[\omega_n t + \phi_n(\theta)\right] \quad (2)$$

for $t > T_1$ (the beginning of the late time), (σ_n, ω_n) is the pole of the nth resonance mode of the target described by a damping coefficient and a resonance pulsation, $(a_n$ and $\phi_n)$ are the amplitude and the phase of the nth resonance mode, θ represents the target orientation with the radar observation system and N is the number of modes of the development.

Thus, the resonant behavior of the late time, characteristic of the studied target, can be used in order to define a method of identification. This article presents an effective scheme of radar targets identification, based on E-Pulse technique.

2. CHARACTERIZATION OF EXTRACTIVE SIGNALS OR E-PULSE SIGNALS

The first stage of this study consists in creating a library of resonance poles for a whole targets. Complex Natural Resonance (CNR) can then be characterized as well in time as in frequency domain [3]. In the time domain, CNR's are calculated by Prony's methods [5], by determining the whole couples (σ_n, ω_n) corresponding to the late time response $r(t)$ (2).

In the frequency domain, CNR's correspond to the physical poles associated with the scattered function $F(\omega)$ of the target, defined from its equivalent transfer function $H(\omega)$:

$$H(\omega) = \frac{\vec{E}_d}{\left|\vec{E}_i\right|} = \frac{\mathbf{e}^{-ik_0 r}}{k_0 r} F(\omega) \quad (3)$$

Calculation of the scattered electric field $\vec{E}_d$ uses the resolution of Maxwell integral equations, for example using the method of moments (MoM). Once the whole CNR's of the target are calculated, they can be used to synthesize an extractive or E-Pulse signals, whose formalism is presented in the following paragraph.

2.1. Synthesis of E-Pulse Signals

One tries to construct an extractive signal $e(t)$, characterized by its duration T_e, for which the convolution with the signal $r(t)$ produces a

null function [4]. This convolution can be written as follows:

$$c(t) = e(t)^* r(t) = \int_0^{T_e} e(t')r(t - t')dt' \tag{4}$$

for $t > T_1 + T_e$

After simplification, this relation is written in the form:

$$c(t) = \sum_{n=1}^N a_n(\theta)e^{\sigma_n t}\{A + B\}, \tag{5}$$

With:

$$\begin{cases} A = A_n \cos\left[\omega_n t + \phi_n(\theta)\right] \\ B = B_n \sin\left[\omega_n t + \phi_n(\theta)\right] \end{cases} \tag{5a}$$

and

$$A_n = \mathrm{Re}\left\{E(s_n)\right\} \quad B_n = -\mathrm{Im}\left\{E(s_n)\right\} \tag{5b}$$

$$E(s_n) = TL\left[e(t)\right] = \int_0^{T_e} e(t)e^{-s_n t}dt \tag{5c}$$

Coefficients A_n and B_n are independent of the direction of observation and determine the amplitudes of the convolution product $c(t)$. It is thus possible to synthesize a discriminating signals $e(t)$ in order to give $c(t)$ into the desired form (null function) [4, 6, 7].

2.2. A Construction of the Discriminating Signal

In order to interpret physically the nature of the discriminating signals (particularly those of the E-Pulse type), it is possible to represent the exciting wave by a sum of two components, one known as "forcing" $e_f(t)$ which excites the target and the other $e_e(t)$ known as extinctive which inhibits the response due to $e_f(t)$ [4]:

$$e(t) = e_f(t) + e_e(t), \quad \text{with}: \begin{cases} e_f(t) \ \ 0 \leq t \leq T_f \\ e_e(t) \ \ T_f \leq t \leq T_e \end{cases} \tag{6}$$

The choice of $e_f(t)$, can be more or less arbitrary but conditions the form of $e_e(t)$ which can be expressed using the basis functions $f_m(t)$:

$$e_e(t) = \sum_{m=1}^M \alpha_m f_m(t), \quad \text{with}: M = 2N \tag{7}$$

where α_m are the weighting coefficients associated to the basis functions $f_m(t)$.

The weighting coefficients are calculated by solving an equation system deduced from (6) and (7).

2.3. Synthesis of an E-Pulse Signal Harmonic Domain

The constraint which yields the null result to convolution (4), imposes (5c):

$$E(s_n) = E(s_n^*) \equiv 0 \quad \text{for} \ \ 1 \le n \le N \tag{8}$$

Condition (8) leads to:

$$E_e(s_n) = -E_f(s_n) \tag{9}$$

and

$$\sum_{m=1}^{2N} \alpha_m F_m(s_n) = -E_f(s_n) \tag{10}$$

which can be put into a matrix form:

$$\begin{bmatrix} F_1(s_1) & F_2(s_1) & \cdots & F_{2N}(s_1) \\ \cdot & \cdot & \cdots & \cdots \\ F_1(s_N) & F_1(s_N) & \cdots & F_{2N}(s_1) \\ F_1(s_1^*) & F_2(s_1^*) & \cdots & F_{2N}(s_1^*) \\ \cdot & \cdot & \cdots & \cdots \\ F_1(s_N^*) & F_2(s_N^*) & \cdots & F_{2N}(s_1^*) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{2N} \end{bmatrix} = \begin{bmatrix} -E_f(s_1) \\ \cdot \\ \cdot \\ -E_f(s_N) \\ -E_f(s_1^*) \\ \cdot \\ \cdot \\ -E_f(s_N^*) \end{bmatrix}$$

$$\text{with}: \ \ F_m(s) = TL\{f_m(t)\} \ \ \text{and} \ \ E_f(s) = TL\{e_f(t)\} \tag{11}$$

One chooses to fix the "forcing" component to be zero. The matrix equation becomes homogeneous and a non-trivial solution exists only if the matrix determinant is null. The difficulties associated with roots extraction of (8) can be overcome by a judicious choice of basis sub-functions:

$$f_m(t) = \begin{cases} g[t - (m-1)\Delta] & \text{if} \ (m-1)\Delta \le t \le m\Delta \\ 0 & \text{elseif} \end{cases} \tag{12}$$

The choice of $g(t)$ is not very significant, provided that the function has a Laplace transformation. In this case:

$$\begin{aligned} F_m(s) &= e^{-(m-1)s\Delta}G(s) \\ G(s) &= TL\{g(t)\} \end{aligned} \tag{13}$$

Noticing that: $F_m(s) = F_1(s)e^{-(m-1)s\Delta}$ the E-Pulse spectrum is then equal to:

$$E(s) = F_1(s) \sum_{m=1}^{2N} \alpha_m z^{m-1}, \quad Z = e^{-s\Delta} \tag{14}$$

The roots of $E(s)$ result from the polynomial equation:

$$\sum_{m=1}^{2N} \alpha_m z^{m-1} \equiv 0 \qquad (15)$$

which can be also written in a matrix form:

$$\begin{bmatrix} 1 & z_1 & z_1^2 & .. & z_1^{2N-1} \\ . & . & . & .. & . \\ 1 & z_N & z_N^2 & .. & z_N^{2N-1} \\ 1 & z_1^* & z_1^* & .. & z_1^{*2N-1} \\ . & . & . & .. & . \\ 1 & z_N^* & z_N^{*2} & .. & z_N^{*2N-1} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ . \\ . \\ . \\ . \\ . \\ . \\ \alpha_{2N} \end{bmatrix} = \begin{bmatrix} 0 \\ . \\ . \\ 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix} \qquad (16)$$

This equation with a second null member has a non-trivial solution (different to 0) only when the matrix determinant is equal to zero. The matrix being of the Vandermonde type, its determinant is equal to:

$$\text{Det} = \prod_{\substack{i=1..2N \\ j=1..N-1 \\ i>j}} (z_i - z_j) \qquad (17)$$

The only null possible terms are:

$$z_i - z_i^* = 0 \iff z_i = z_i^* \iff 2\sin(\omega_i \Delta) = 0 \text{ so } \Delta = \frac{p\pi}{\omega_i}$$
$$p = 1, 2, 3, \ldots \text{et } 1 \leq i \leq N \qquad (18)$$

Thus, the E-Pulse signal duration depends only on the imaginary part of one of the natural frequencies. The minimal T_e value is calculated by choosing the greatest value of ω_i.

$$(T_e)_{\min} = \frac{2N\pi}{\sup[\omega_i]_{i=1\ldots N}} \qquad (19)$$

Once Δ determined, the amplitudes of the basic functions are calculated using the theory of determinants. The system solution gives:

$$\alpha_m = (-1)^m P_{(2N-1)-(m-1)} \qquad (20)$$

where $P_{(2N-1)-(m-1)}$ is a product sum with degree equals $(2N-1)-(m-1)$.

This procedure will be thus used to characterize an identification method of radar targets.

3. VALIDATION OF E-PULSE METHOD: FDTD SIMULATIONS

We propose to apply the E-Pulse method to perfectly conducting targets which have been subject of electromagnetic modeling and measurements in an anechoic chamber. The targets under test are:

- a dipole with length $L = 0.15\,\mathrm{m}$ and radius $a : L/a = 300$
- a conducting sphere with radius $r = 0.177\,\mathrm{m}$
- a metallic plate with side $a = 1\,\mathrm{m}$

The first results are presented in the case of signals calculated by FDTD simulations. In that case, the transfer function of the transmitting and receiving antennas corresponds to those measured on a time domain UWB instrumentation.

For each target, Figures 1a, 2a and 3a represent the E-Pulse functions deduced from the theoretical CNR's calculated by SEM. The target responses $r(t)$ and the convolution products $c(t)$ are superposed on Figures 1b, 2b and 3b.
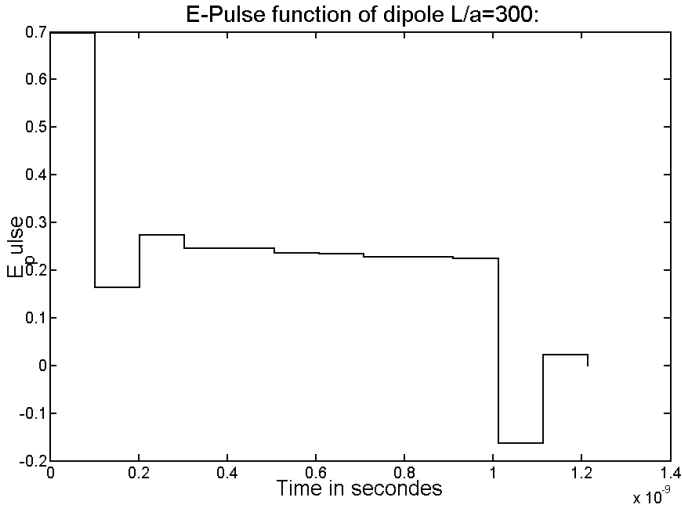
3.1. Case of a Conducting Dipole

For the conducting dipole, $N = 6$ CNR's are used to construct the E-Pulse function, which is plotted on Figure 1a.

Figure 1b shows that the convolution product tends toward zero for $t > T_L = T_1 + T_e$. The E-Pulse function thus synthesized seems to confirm theoretical results of paragraph 2 (cancellation of $c(t)$ for an extractive E-Pulse signal). In order to validate more accurately the method, this analysis is extended to other targets defined previously.

3.2. Case of a Conducting Sphere and a Metallic Plate

In case of the conducting sphere and the metallic plate, $N = 5$ and $N = 6$ CNR's are respectively used to construct the E-Pulse functions. If the convolution analysis between the target response and its associated E-Pulse function seems to give good results in the case of the conducting dipole (cancellation of convolution for $t > T_L = T_1 + T_e$), this result is not verified for the other targets (conducting sphere and metallic plate). Therefore, no identification or discrimination seems to be possible from these observations.

This result can be mainly explained by the characteristics of the excitation signal (signal measured at the antenna output). Indeed the formalism used by Baum to model the late time response of the target requires an impulse excitation of Dirac type. In our case, the response
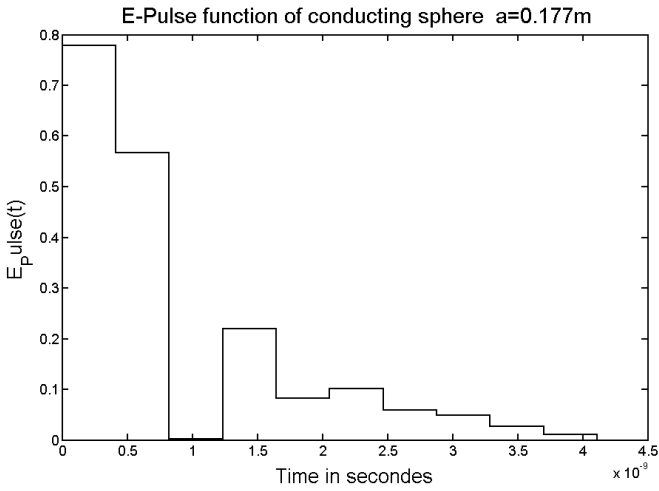
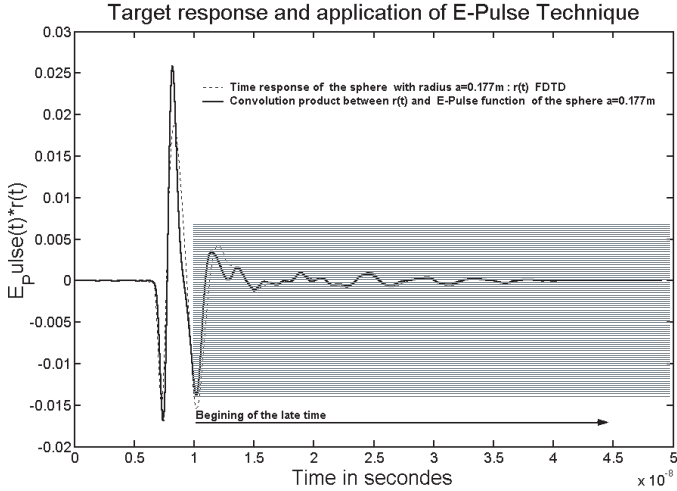**Figure 1a.** A *E*-Pulse associated with a conducting dipole.
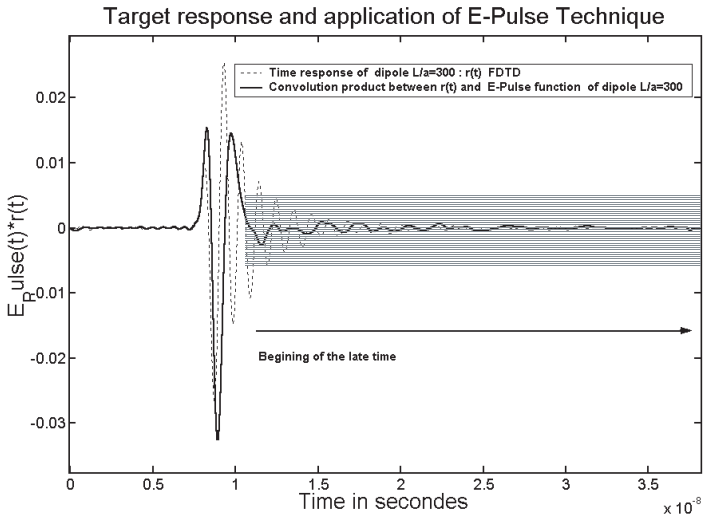
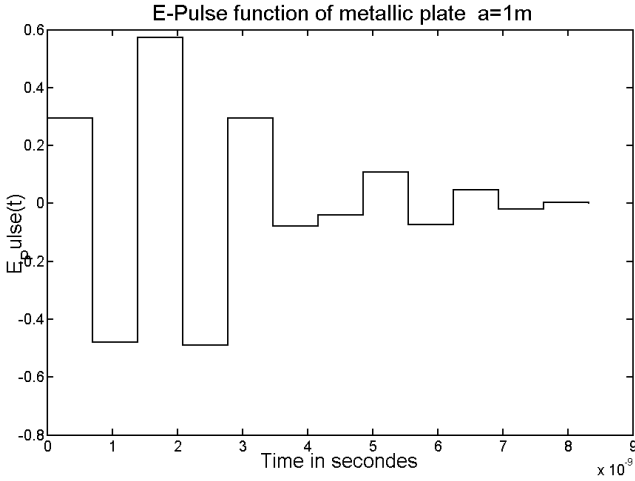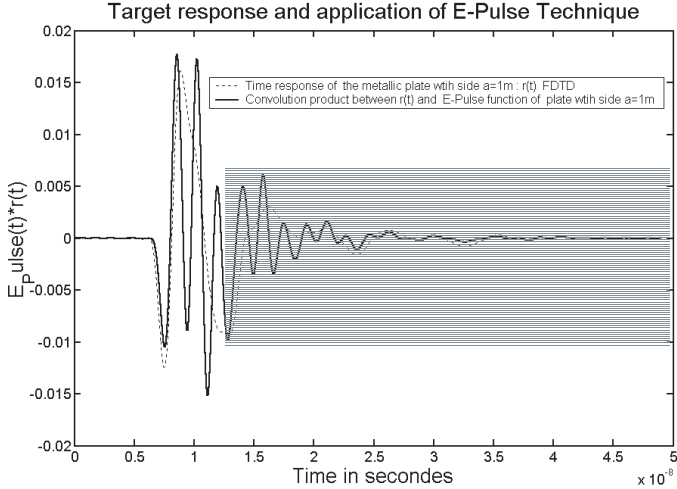**Figure 1b.** Time response of the conducting dipole and convolution product with the *E*-Pulse function.

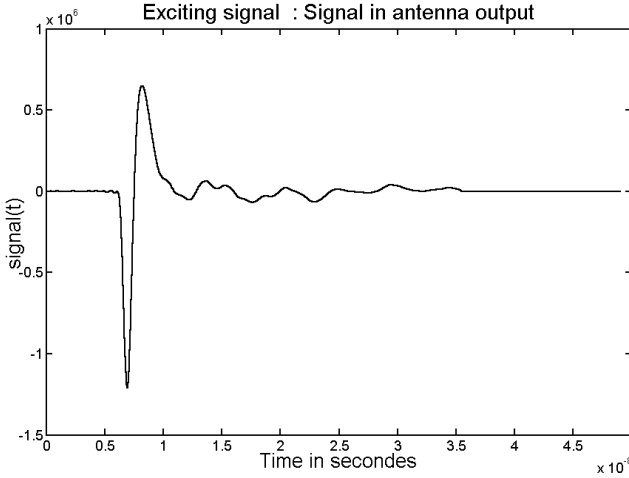**Figure 2a.** A *E*-Pulse associated with a conducting sphere.

Figure 2b. Response of a conducting sphere and application of *E*-Pulse method.

Figure 3a. A *E*-Pulse associated with a metallic plate.

Figure 3b. Response of a metallic plate and and application of *E*-Pulse method.

Figure 4. Experimental exciting signal measured in antenna output. So the signal available at the antenna output has a shape nearer of a derived Gaussian than a Dirac pulse.

of a target given by (2) is the result of a convolution product between the impulse response of the target and an exciting signal of Figure 4:

$$\begin{cases} r(t) = s_{\text{exc}}(t)^* h_{\text{cible}}(t) \\ s_{\text{exc}}(t) : \text{ exciting signal} \\ h_{\text{cible}}(t) : \text{ impulse response of the target} \end{cases} \tag{21}$$

However the signal available at the antenna output rather has shape of a derived gaussian signal.

### 3.3. Improvement of the E-Pulse Method: Gaussian Excitation

In order to improve the E-Pulse technique, we propose to apply a gaussian signal as excitation. This procedure will first require deconvolution of the target response by the antenna signal. Then, we apply the convolution of the resulting signal by a gaussian impulse. This process can be summed up by the synoptic of Figure 5.

In this configuration the exciting signal is represented on Figure 6a.

The application of this technique to the targets under test clearly improves the previous results. (Figures 6b and 6c) and allows to define a discrimination factor which criteria is specified in the following
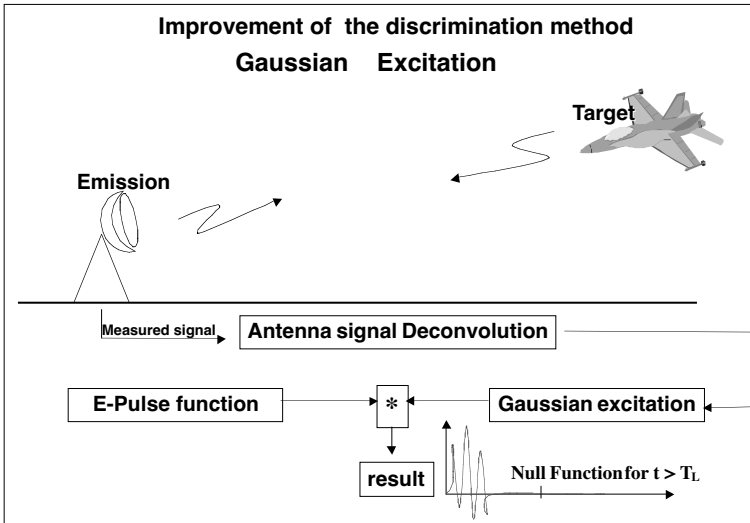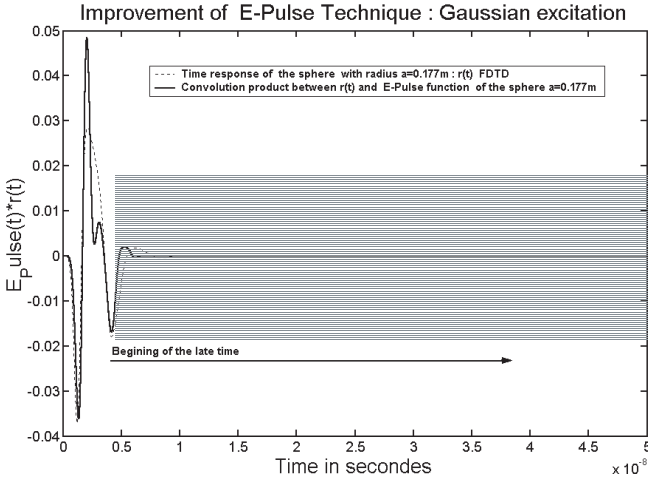
Figure 5. Synoptic of a modified scheme of radar target identification.
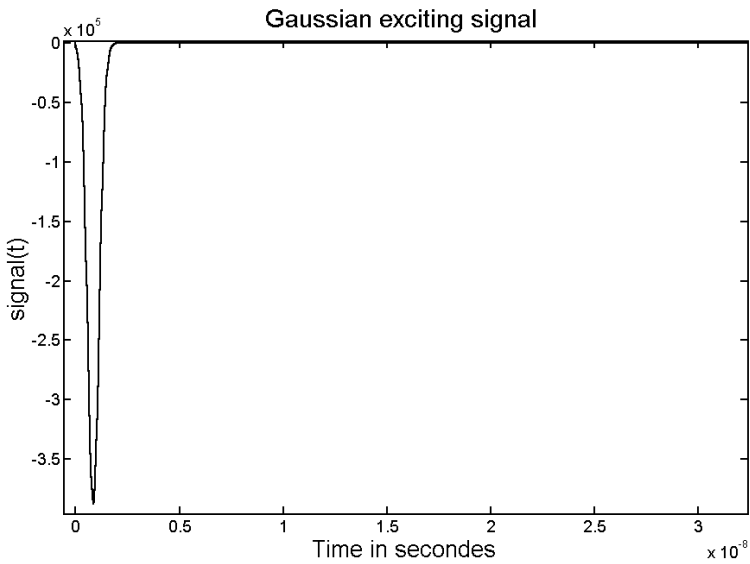
Figure 6a. Representation of the exciting gaussian signal.

case of a conducting sphere:

Improvement of E-Pulse Technique : Gaussian excitation

Figure 6b. Conducting sphere response associated with a gaussian impulse and application of *E*-Pulse method.

case of a metallic plate:

Improvement of E-Pulse Technique : Gaussian excitation
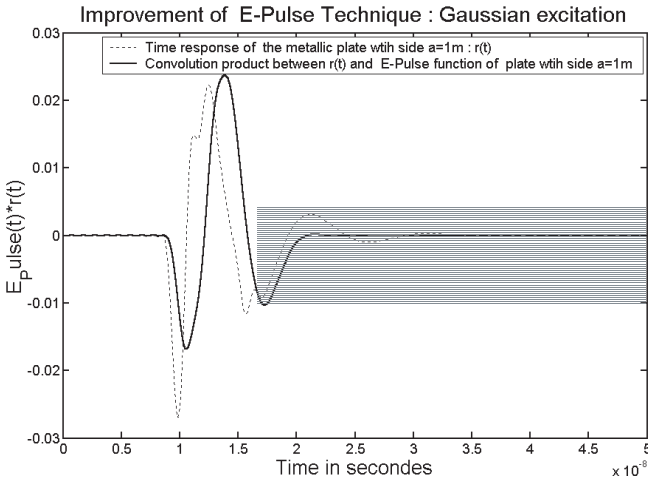
Figure 6c. Metallic plate response associated with a gaussian impulse and application of *E*-Pulse method.

Figure 7. Convolution of a conducting dipole response and E-Pulse associated with a conducting sphere.

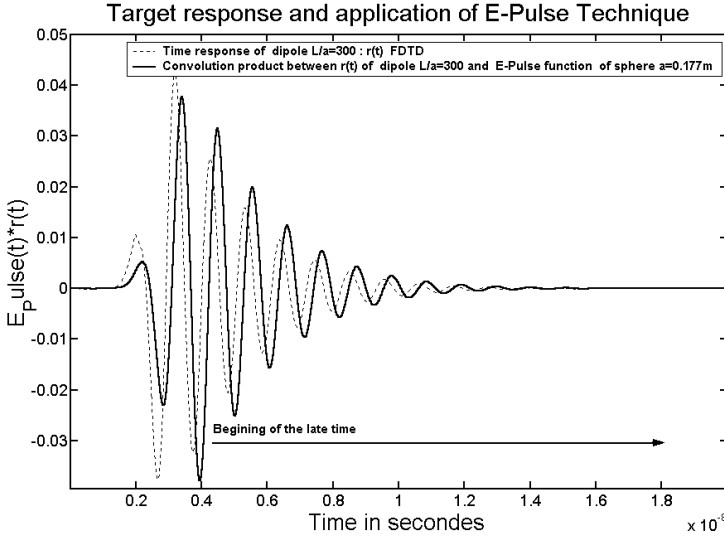paragraph. This time, $c(t)$ tends more quickly toward the null function, than the previous cases.

4. DEFINITION OF A DISCRIMINATION FACTOR (EXTRACTION DISCRIMINATION NUMBER)

The results presented in the preceding paragraph are examples when the E-Pulse functions considered are synthesized from CNR's of the studied target. In order to check that this function is characteristic of the object, one considers the case of Figure 7 when the dipole response is convoluted to the E-Pulse function constructed with the CNR of the conducting sphere of radius $a = 0.177$ m.

   Then, one can observe that there is no cancellation of the convolution product for $t > T_L = T_1 + T_e$. Thus, it is possible to define a discrimination number EDN [8–12] (E-Pulse Discrimination Number) which measures the relative power (compared to power of the E-Pulse function) corresponding to the end of the convolution product.

This number is defined by following expression:

$$\text{EDN} = \frac{\displaystyle\int_{T_L}^{T_L+W} |c(t)|^2\, dt}{\displaystyle\int_0^{T_e} |E\text{-pulse}(t)|^2\, dt} \qquad (22)$$

where W represents the width of the time window, which corresponds to 95% of the power of the convolution product from $t > T_L$.

Theoretically, the latter must be null for $t > T_L = T_1 + T_e$. In practice, one calculates EDN factor for an unspecified target and its corresponding E-Pulse function. This value is taken as reference. It corresponds to the least EDN factor and will be used for definition of the normalized factor (EDR), compared to the minimal value of EDN coefficient:

$$\text{EDR} = 10 \log_{10} \left\{ \frac{\text{EDN}}{\min(\text{EDN})} \right\} \qquad (23)$$

As examples, one applies this method to series of conducting sphere whose radius vary from $0.15\,\text{m}$ to $0.25\,\text{m}$ (Figure 8). The sphere considered as reference has a radius equal to $r = 0.2\,\text{m}$. Application of E-Pulse technique allows to calculate several EDN factors from which the smallest indeed corresponds to conducting sphere with radius $r = 0.2\,\text{m}$.

## 5. VALIDATION OF THE E-PULSE METHOD: MEASUREMENT RESULTS

This paragraph presents an application of the modified scheme of E-Pulse technique, to measurements. The results concern three measured conducting targets.

The measurements are made directly in time domain using an UWB laboratory measurement system developed by CELAR (French defense organization) in an anechoic chamber.

- a sphere with radius $r = 0.1\,\text{m}$
- a square plate with side $a = 0.6\,\text{m}$
- a cylinder with dimensions $L = 0.618\,\text{m}$ $r = 0.125\,\text{m}$

The EDN's are calculated for each reference targets and are compared to those obtained with others targets. All E-Pulses used for the EDN's calculation are determined thanks to theoretical CNR's.

For the conducting sphere, the target response with a real and a gaussian excitations are presented graphically (Figures 9a and 9b). Also the convolution product $c(t)$ between the E-Pulse function of the
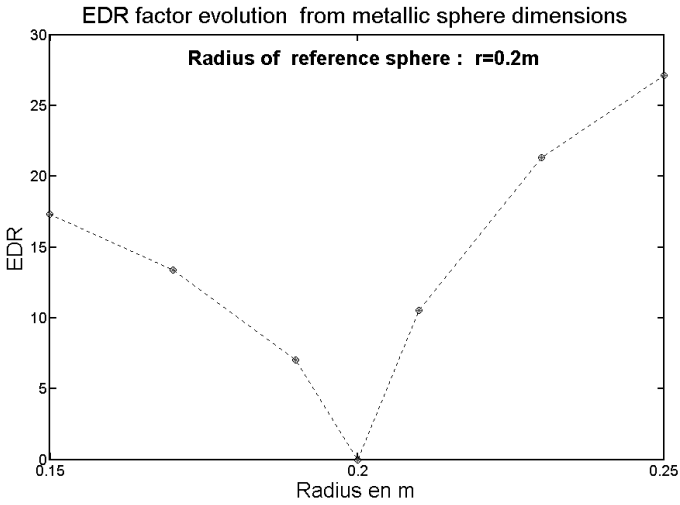
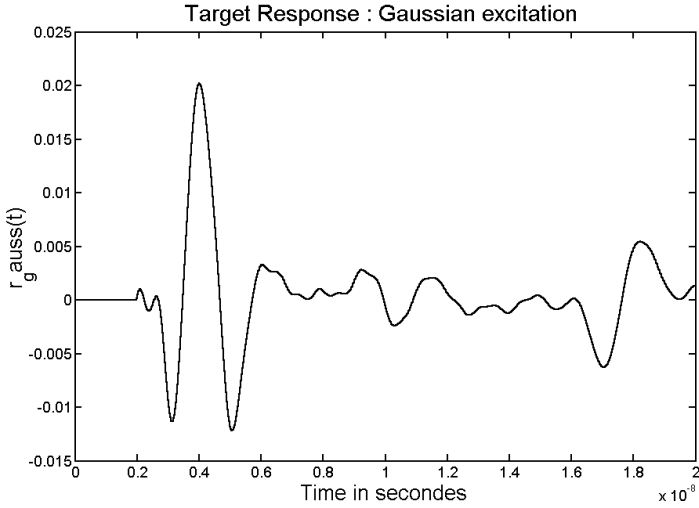**Figure 8.** Evolution of E.D.R factor from radius of a conducting sphere.
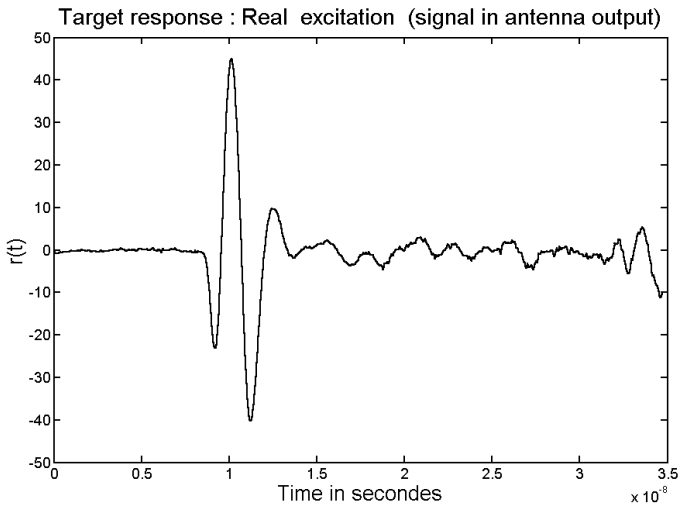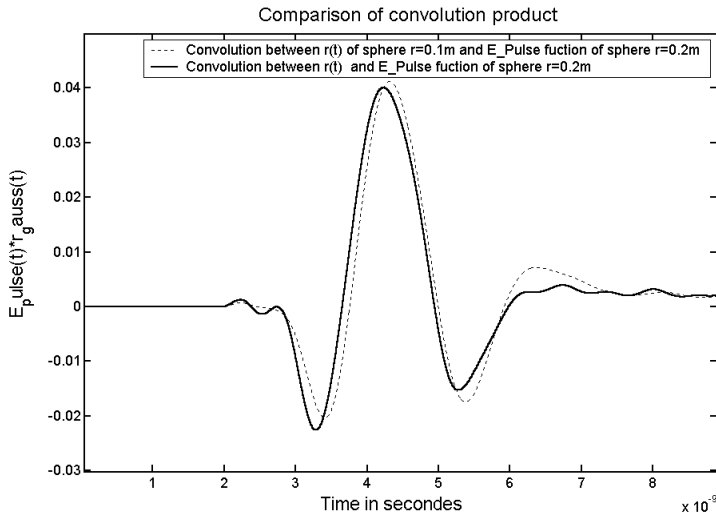

Figure 9a. Response of a conducting sphere associated with a real excitation.

Figure 9b. Response of a conducting sphere associated with a gaussian excitation.

Figure 9c. Convolution of E-Pulse signal and response of conducting sphere considered as reference.

reference sphere ($r = 0.1\,\mathrm{m}$) and another with radius $r = 0.2\,\mathrm{m}$ is represented. The whole coefficients EDN are consigned in Tables 1 to 3.

Case of a conducting sphere with radius $r = 0.1\,\mathrm{m}$

The reference target is the conducting sphere of radius $a = 0.1\,\mathrm{m}$. The EDN coefficient calculated equals to: $\mathrm{EDN} = -17.04\,(\mathrm{dB})$.

Table 1. EDN coefficients of testing targets with the conducting sphere $a = 0.1\,\mathrm{m}$ as reference signal.

Testing targets	E.D.N in dB
sphere with radius $a = 0.1\,\mathrm{m}$	**−17.04**
sphere with radius $a = 0.17\,\mathrm{m}$	−14.83
sphere with radius $a = 0.2\,\mathrm{m}$	−13.22
sphere with radius $a = 0.4\,\mathrm{m}$	−13.08
disc with radius $a = 0.2\,\mathrm{m}$	−12.05

Case of a conducting cylinder

The reference target is the conducting cylinder with dimensions: $L = 0.618\,\mathrm{m}$, $r = 0.125\,\mathrm{m}$. The EDN coefficient calculated equals to: $\mathrm{EDN} = -34.41\,(\mathrm{dB})$.

Table 2. EDN coefficients of testing targets with the conducting cylinder as reference signal.

Testing targets	E.D.N in dB
Conducting cylinder	**−34.41**
sphere with radius $r = 0.17\,\mathrm{m}$	−24.50
sphere with radius $r = 0.2\,\mathrm{m}$	−21.91
sphere with radius $r = 0.4\,\mathrm{m}$	−22.16
sphere with radius $r = 0.1\,\mathrm{m}$	−10.19
disc with radius $r = 0.2\,\mathrm{mm}$	−17.38

Case of a metallic plate

The reference target is the metallic plate conducting with side $a = 0.6\,\mathrm{m}$. The EDN coefficient calculated equals to: EDN$=-25.55\,(\mathrm{dB})$.

Table 3. EDN coefficients of testing targets with the metallic plate as reference signal.

Testing targets	E.D.N in dB
metallic plate with side $a = 0.6\,\mathrm{m}$	**-25.55**
sphere with radius $r = 0.17\,\mathrm{m}$	-21.11
sphere with radius $r = 0.2\,\mathrm{m}$	-20.49
sphere with radius $r = 0.4\,\mathrm{m}$	-15.58
sphere with radius $r = 0.1\,\mathrm{m}$	-9.41
disc with radius $r = 0.2\,\mathrm{m\,m}$	-11.06

Analysis of the results shows that for each target studied, the EDN coefficient calculated is minimum for E-Pulse function corresponding to the measured target. The simulation and the efficiency of our modified E-Pulse method results are checked. These results can be improved by decreasing the duration of the exciting Gaussian impulse, in order to approach the theoretical impulse response of the target.

6. CONCLUSION

The results obtained from this study have shown potential use of resonance poles libraries of radar targets. This information can thus be integrated in discrimination and identification devices. The exposed method consists, from the CNR of an object, in the synthesis of extractive signals (characteristic of the studied object), which, after convolution with the late time response, provide a null function. In practical cases, direct application of E-Pulse techniques is not very efficient. Their performances are damaged by the characteristics of the exciting signal (antenna signal). We have proposed a modified scheme of E-Pulse technique in order to improve radar target identification. It consists firstly in the deconvolution of the target response by the antenna signal, and then, the convolution of the resulting signal by a gaussian impulse. This process has been successfully tested to FDTD simulations and measures in anechoic room and can be applied to an operational device of identification.

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