Identification of the De-synchronization, Synchronization and Forced Oscillation Phenomenon of a Nonlinear System

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Abstract—The phenomena of de-synchronization, synchronization, and forced oscillation has been investigation using describing function theory for a two input and two output nonlinear system containing saturation-type nonlinearities and subjected to high-frequency deterministic signal for the purpose of limit cycle quenching. The analytical results have been compared with the results of digital simulation Matlab-Simulink for a typical example varying the nonlinear element.

Keywords—Signal stabilization, limit cycle, forced oscillation, dither, saturation.

I. INTRODUCTION

R ecognition of nonlinear self-oscillations or limit cycles in multidimensional nonlinear systems as they are indeed has had a long history, and is closely related to the system stability [1-23]. Engineers are continually involved in the design of system simply to ensure that it meets the performance criterion, which strictly excludes the existence of limit cycles [1], [3], [4], [6] and [8]. One of the important and interesting methods of extinguishing such limit cycle is by the employment of high-frequency signal to the nonlinear system input. The high-frequency signal is usually called dither. The use of dither to turn limit cycles off is referred to as signal stabilization. In many cases the introduction of an extra signal is less expensive than actually replacing the nonlinear element [9]. It has been extensively investigated by Olden-burger and his students [9], among the first to discover this phenomenon experimentally and subse-quently, to provide analytical justification. However, these are for single-input and single-output (SISO) systems with both deterministic and random inputs. Other notable works on signal stabilization of SISO systems can be seen from the wealth of literatures [4], [6], [9], [13-16].

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There are, however, a large number of practical industrial problems with two- or higher-dimensional nonlinear control configurations [1], [4], [18], [19] and the analysis of signal stabilization there has a huge significance in its own right. Unfortunately, relatively small amount of work has been published on forced oscillation/signal stabilization of multidimensional systems and hence addressed here for a two-dimensional system subjected to a deterministic dither. The describing function (DF) method provides a convenient tool and by virtue of its inherent approximations leads to a significant reduction in the complexity of analysis [1-10], [13-17], [22] and [23]. The dual input describing function (DIDF) is analogous to the conventional describing function as far as the manner of using is concerned. Investigation of signal stabilization via describing function theory can be executed in two stage process [20]. First, by use of DIDF theory, the dither and the original nonlinear element are replaced with an equivalent nonlinear element, whose form implicitly accounts for the presence of dither, but which no longer explicitly displays the dither signal. Second, the resulting system is made the object of a DF analysis to reveal the presence or absence of limit cycle [5], [9], [13-17] and [20]. The variation of amplitude and frequency of limit cycle with variation of forcing signal amplitude (phenomena of forced oscillations, synchronization and de-synchronization) has been analyzed. The exact magnitude of dither for which limit cycle is extinguished (i.e. synchronization) or induced (i.e. de-synchronization) has been found. The technique is derived from the basic concept of DIDF, incremental input describing function (IDF) and relationship between system variables. Apart from directness of application, the method outlined has the notable advantage that it brings out the influence of individual system (effect of interaction/coupling) on the forced oscillation parameter, and can be applied to a higher-dimensional system [10] and [13].

This technique also forms the basis of computer algorithms for predicting limit cycle/forced oscillation [10]. This rather simple investigation scheme has been illustrated through examples and comparison of results with digital simulation without loss of generality [13] and [21].

The system has also been simulated using MATLAB 7.0 for forced oscillation investigation.

II. IMPORTANT NOTATIONS

B amplitude of high frequency external signal (dither)

C_1, C_2 amplitude of outputs of the two subsystem

G_1, G_2 transfer functions of linear elements

N_1, N_2 describing functions (DF) of nonlinear elements

N_{1i}, N_{2i} incremental input DFs of the nonlinear elements

N_{1eq} two sinusoidal input DF (DIDF) of nonlinear element N_1

$\mathcal{N}$ the multitude of natural numbers

$\mathcal{Z}$ the multitude of whole numbers

$\mathcal{R}$ the multitude or real numbers

X_1, X_2 amplitude of input to nonlinear elements

X_1', X_2' amplitude of input to nonlinear elements when the system exhibits forced oscillations

Y_1, Y_2 amplitude of output (fundamental) of nonlinear elements

U_1, U_2 inputs of the two subsystems

ω_f frequency of high frequency external signal

ω_B frequency of self oscillations (limit cycle)

Subscripts 1 and 2, correspond to quantities of two subsystems S_1 and S_2, respectively

III. DISCONNECTION OF NONLINIAR PROCESSES

In a multivariable process, the multitude of input numbers is likely to affect the evolution of output numbers. The purpose of disconnection is to limit as much as possible the effect of one input on one output, thus allowing turning the process into a set of mono-variable systems that evolve separately, because the commands are not interactive.

The functioning of this system requires a similar number of input and outputs, preferably equal.

A. Disconnection using a regulator

A simplified approach to this problem is if $H(s)=C(sI-A)^{-1}B$ is the transfer matrix of a process that corresponds to (*A, B, C*) it is sufficient (Fig.1) to place a regulator $R(s)$ before the process so that the transfer matrix of the new process $D(s)=H(s)R(s)$ is diagonal.
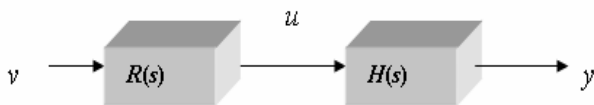

Fig.1: Disconnection through insertion of a regulator.

Such a simple approach involves the following:

- It is very likely that modal, unforeseeable or un controllable numbers may appear. This is very dangerous, especially if the numbers are unstable. We will avoid simplifying poles or unstable zeros between the functions of the $H(s)$ and $R(s)$ processes.
- The degree of the numerator must be smaller or equal to the degree of the denominator.
- When we have an inverted matrix $H^*(s)$, the right of the matrix defined by:

$$\exists l \in N, H(s)H^*(s) = Is^{-1}, \qquad (1)$$

with the unit matrix I defined by $\Re^n x \Re^n$, an obvious solution would be to consider:

$$R(s)=H^*(s)D^*(s), \qquad (2)$$

where $D^*(s)$ is the chosen diagonal matrix.

This case, which is very rare in practice, requires finding the minimum degree inverted matrix of the right side of $H(s)$. The solution we have reached from our first situation is not acceptable if by using it we will be introducing poles or unstable zeros.

These situations bring out the fact that before we use such a disconnection method, we should find out the status reaction so we can assure the stability of our system. Therefore, the method has lost its advantage that allowed us to simplify the calculus of the regulator or command.

B. Disconnection in linear processes

We will consider the model described by these status equations:

$$\dot{x} = Ax + Bu,$$
$$y = Cx, \qquad (3)$$

where: $x \in \Re^n$, $y \in \Re^m$, $u \in \Re^m$, the B and C matrixes have maximum degree and the y output can be controlled, state that can be expressed through the condition:

$$class[CB, CAB, ..., CA^{n-1}B]=m. \qquad (4)$$

The objective is to establish a reaction after

$$u=Kx+Lv, \qquad (5)$$

where, v is the new input vector, which disconnects the system, such that the input v_i does not depend on the output v_i.

As a result, it is enough to determine the L and K matrixes, so that the transfer matrix of the $H_B(s)$ system is a diagonal:

$$H_B(s)=C[sI-(A+BK)]^{-1}BL. \qquad (6)$$

In practice, solving a matrix equation in L and K is not simple at all.

If we note with C_i the i line of the controlled C matrix, the scalar output y_i can be stated by the condition:

$$Class(C_iB, C_iAB, ..., C_iA^{n-1}B)=1. \qquad (7)$$

Because the y output of the system is controlled, it is the same as y_i. In other words, if condition (4) is verified, then for every i belonging to $\{1,..., m\}$, there is a d_i that belongs to $\{0,1,..., n-1\}$, such that:

$$C_i A^{d_i} B \neq 0, \forall \alpha \in N, \alpha < d_i, C_i A^{\alpha} B = 0. \qquad (8)$$

In this case, a solution to our problem can be found by successively deriving relationships (3) and (5), thus obtaining for our i output the following equation system:

$$\dot{y}_i = C_i(A+BK)x + C_iBLv \qquad = C_iA_x,$$

$$\ddot{y}_i = C_iA(A+BK)x + C_iABLv \qquad = C_iA^2x,$$

$$\ldots \tag{9}$$

$$y_i^{(di)} = C_iA^{d_i-1}(A+BK)x + C_iA^{d_i-1}BLv = C_iA^{d_i}x,$$

$$y_i^{(d_i+1)} = C_iA^{d_i}(A+BK)x + C_iA^{d_i}BLv$$

or writing the relationship for each output:

$$
\begin{bmatrix} y_1^{(d_1+1)} \\ \ldots \\ y_i^{(d_i+1)} \\ \ldots \\ y_m^{(d_m+1)} \end{bmatrix} = \left[\begin{pmatrix} C_1A^{d_1+1} \\ \ldots \\ C_iA^{d_i+1} \\ \ldots \\ C_mA^{d_m+1} \end{pmatrix} + \begin{pmatrix} C_1A^{d_1} \\ \ldots \\ C_iA^{d_i} \\ \ldots \\ C_mA^{d_m} \end{pmatrix} BK \right] x + \begin{bmatrix} C_1A^{d_1} \\ \ldots \\ C_iA^{d_i} \\ \ldots \\ C_mA^{d_m} \end{bmatrix} BL, \tag{10}
$$

or as a matrix:

$$y^* = (A^* + B^*K)x + B^*Lv. \tag{11}$$

If the B^* has an invert matrix, then we will choose:

$$L = (B^*)^{-1}, \; K = -(B^*)^{-1}A^*, \tag{12}$$

That leads to the equation $y^* = v$, or to the input-output equation:

$$\forall i \in \{1, \ldots, m\}, Y_i(s) = \frac{1}{s^{d_i+1}} V_i(s). \tag{13}$$

If the matrix B^* has an invert, then this is a necessary and sufficient decompression condition. However, we must see the structure of the curled system we obtained and verify if instable poles have been introduced.

The previous study requires few comments:
- The status reaction that leads to decompression is calculated having a certain model in mind. Thus the decompression is not that rigorous as it would be if we were to calculate having a précis model in mind.
- The appearance of modal, stabile and unnoticeable values in our curled system can be disturbing, in which case there could be important differences between the transitory regime of the process and that of the model.
- If we have modal, stabile and unnoticeable values, the simplified model that we obtain will not be accepted. One useful rule, that helps avoid big errors, requires that we determine the transfer matrix of the system and check to see if our calculus was made through simplification of the poles and instable zeros.
- One general rule states that we must check the whether our curled system can be observed or not, and also check the stability of the unnoticeable part.
- When we cannot measure all parts, we must use a status estimator, in which case we must make a stable reaction.

C. Decompression for nonlinear processes

The previous method can be generalized for the decompression of nonlinear processes, by making the process linear. The evolution is described using the following equation system:

$$\dot{x} = f(x) + G(x)u, \tag{14}$$
$$y = h(x)$$

where $f : \Re^n \to \Re^n, G : \Re^n \to \Re^{nxm}, h : \Re^n \to \Re^m$, output values vector y is completed with fictive outputs, in order to have the same vector dimension just like the command vector.

Assuming that the functions f, G and h are continuous and derivable, our method is, just like for the linear model, to derive every output y, thus determining the command vector.

We note with $h^T(x) = [h_1(x), \ldots, h_m(x)]$ and with $h^0_{i,x} = h_{i,x}$ and for $r \geq 1$:

$$h_i^r(x) = h_{i,x}^{r-1^T}(x)f(x),$$
$$G_i^r(x) = h_{i,x}^{r-1^T}(x)G(x), \tag{15}$$

with d_i such that $G_i^{d_i+1}(x) \neq 0$ and $\forall \alpha \in N^+, \alpha \leq d_i, G_i^\alpha(x) = 0$ the outputs are:

$$y_i = h_i^0(x),$$
$$y_i^{(1)} = h_i^1(x),$$
$$\ldots \tag{16}$$
$$y_i^{(d_i)} = h_i^{d_i}(x),$$
$$y_i^{(d_i+1)} = h_i^{d_i+1}(x) + G_i^{d_i+1}(x)u$$

or by writing the relationship for every output as a matrix:

$$
\begin{bmatrix} y_1^{(d_1+1)} \\ \ldots \\ y_i^{(d_i+1)} \\ \ldots \\ y_m^{(d_m+1)} \end{bmatrix} = \left[\begin{pmatrix} h_1^{d_1+1}(x) \\ \ldots \\ h_i^{d_i+1}(x) \\ \ldots \\ h_m^{d_m+1}(x) \end{pmatrix} + \begin{bmatrix} G_1^{d_1+1}(x) \\ \ldots \\ G_i^{d_i+1}(x) \\ \ldots \\ G_m^{d_m+1}(x) \end{bmatrix} \right] u. \tag{17}
$$

Relationship (17) can also be written as:

$$y^* = h^* + G^*u, \tag{18}$$

and if the matrix G^* is irreversible, d_i indexes are invariant and we choose a reaction where:

$$u = (G^*(x))^{-1}[v - h^*(x)], \tag{19}$$

that will lead, just like in the linear case, to the relationship $y^* = v$, where v is a new input of the system or:

$$\forall i \in \{1, \ldots, m\}, y_i^{(d_i+1)} = v_i, \tag{20}$$

The system we obtain will not only be decompressed but also linear. Moreover, besides the validity conditions we have

already talked about (indexes d_i invariants and $G^*(x)$ irreversible), if the system we have made linear through decompression has a smaller dimension than in the beginning, then we must see if the unnoticeable part of the system is stabile.

If it is not, the command will not be accepted, but it is possible for us to try another decompression using a different representation of the process (for example using different outputs or introducing question elements on the inputs).

The main disadvantage of the method is that it is very hard to actualize the status estimators when almost all parts of the system cannot be measured.

IV. SIGNAL STABILIZATION

In this section, we consider a two-dimensional nonlinear system configuration as represented in Fig.2 with two inputs U_1 and U_2 and the two outputs C_1 and C_2 and saturation as the only nonlinearities present in both the subsystems S_1 and S_2.

This nonlinearity is not uncommon. For example, frequently the valves used as actuators in process control applications give rise to nonlinearity as a result of actuator saturation, where the limit corresponds to a fully open or closed valve. Actuator saturator may lead to a large "overshoot" inducing a limit cycle [22].

This particular system has been used earlier by the authors for prediction of limit cycle parameters [9-13]. The characteristics of nonlinear elements used in the examples considered are shown in Fig.3. It is a general class of two-dimensional system developed by the author [11], [12] considering the coupling effect between subsystems and relationships between individual parameters of significance within the subsystem. The system claims to be more suitable for the analysis of limit cycle/signal stabilization.

The system shown in Fig.2 exhibits a limit cycle in the autonomous state [8-13]. We now examine the possibility of quenching the limit cycle by injecting a high-frequency dither. The dither can be injected either at u_1 or u_2 or at both the inputs simultaneously.

However, for the present investigation we confine attention to the case when the dither, $B\sin\omega_f t$, (ω_f is at least 10 times greater than limit cycle frequency [19]) is injected at u_1 only while u_2 is kept unexcited from external sources. When the dither amplitude at u_1 is gradually enhanced, the system would exhibit forced oscillations.

The signals at various points in the system would then be composed of signals of frequency (ω_f), signals of frequency of limit cycle (ω_S) and the combination frequencies, $k_1\omega_f \pm k_2\omega_S$ where k_1, along with k_2 assume various integer values. However, with increase of the dither amplitude B, the frequency of limit cycle (ω_S) would also gradually change [4], [9] and [14]. For a certain amplitude of dither, *synchronization* would occur i.e., the limit cycle would vanish and the system would exhibit forced oscillations at the dither frequency of only [4], [9] and [20].

If subsequently the amplitude B is gradually reduced, a point may be reached at which the limit cycle would reappear and the system would exhibit forced oscillations once again. This phenomenon has been termed as *de-synchronization* [10].

The analysis of such oscillation even in a relatively simple two-dimensional nonlinear system is exceedingly complex.

This section presents analysis of these phenomena based on the following assumptions:
(a) The external signal is impressed on system only at u_1 (cf. Fig.2).
(b) The linear elements composing various loops of the system possess low-pass characteristics (filter hypothesis) [1], [4], [9], [13-16] and [20].

Because of the low-pass characteristics of the linear elements, the components of high-frequency signal at C_1 and C_2 would be very small. Hence, the component of the high-frequency signal at the input to the nonlinear element, N_1, would be equal to the magnitude of the dither at u_1. It may be noted that just prior to de-synchronization, the system would be exhibiting a forced harmonic oscillation, and consequently, this phenomenon which is relatively easier to analyze and is, therefore, considered first.
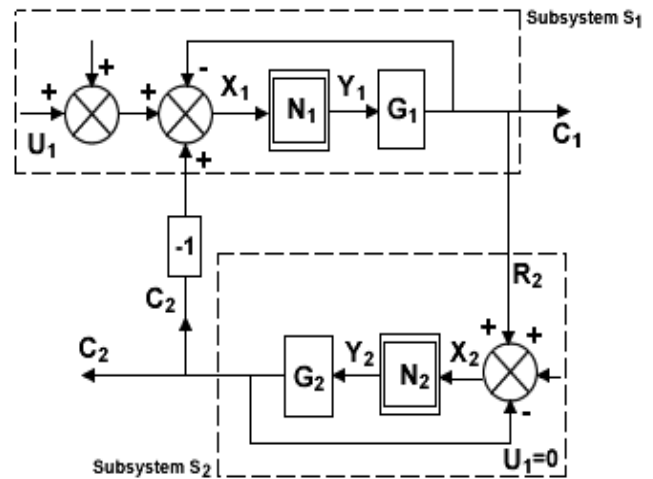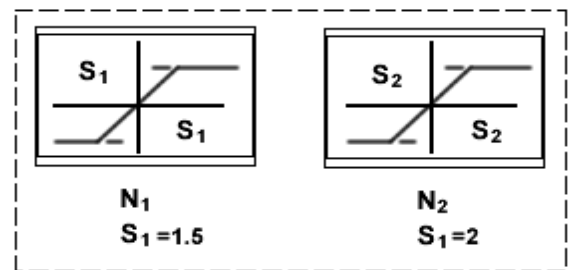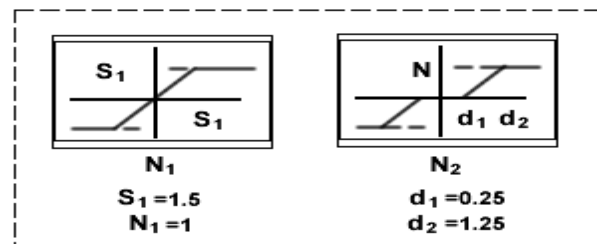

Fig.2: A general 2x2 nonlinear systems.

Fig.3: Characteristics of nonlinear elements used in Examples 1 and 2.

A.De-synchronization

Let the system in Fig.2 be subjected to a dither, *Bsinω_f t*, and consider the situation for a reasonably large *B,* when the limit cycle has been quenched and, consequently, the system is exhibiting a harmonic oscillation at dither frequency (*ω_f*). Since the frequency of the dither is high, the magnitude of C_1 and C_2 can be assumed to be negligibly small. Hence, it follows that under these conditions, the inputs to the nonlinear elements N_1 and N_2 can be approximated, respectively, as (i) *Bsinω_f t,* and (ii) a vanishingly small signal. It has been shown in earlier works for SISO system [16-18], [20] that when *B* is gradually reduced the self-oscillations reappear at a point at which the forced oscillations become unstable and that this in-stability can be predicted by employing IDFs [4] and [20]. Hence, in two-dimensional case also the limiting values of *B* at which the self-oscillations reappear can be obtained by replacing the nonlinear elements N_1 and N_2 by their IDFs, N_{1i} and N_{2i}, for vanishingly small signals superposed on the finite amplitude signals of frequency *ω_f* at their respective inputs. The linearised system is shown in Fig.4 and conditions for the stability limit can be obtained in a straightforward manner.
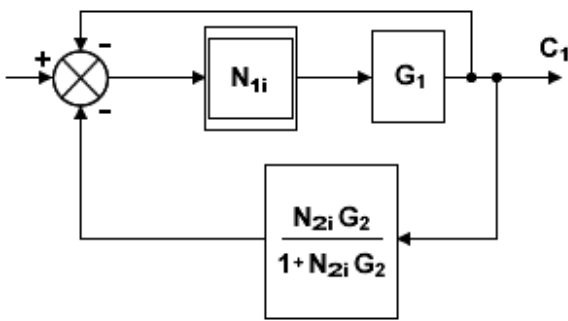

Fig.4: Equivalent linearization for incremental signals for the system of Fig.2.

The condition for self-oscillations to just reappear is thus obtained as

$$1 + N_{1i}G_1(j\omega)\left[1 + \frac{\left(N_{2i}G_2(j\omega)\right)}{1+\left(N_{2i}G_2(j\omega)\right)}\right] = 0 \,. \qquad (21)$$

As shown above, the magnitudes of the high-frequency signals at the inputs to the nonlinear elements N_1 and N_2 are approximated by *Bsinω_f t* and zero, respectively. Hence, it follows that in Eq.(21), N_{1i} is the slope at the origin of the modified characteristic of N for an input *Bsinω_f t*, while N_{2i} is the slope at the origin of the characteristic of N_2 [1], [4], [23]. The following examples illustrate the procedure for determining the value of *B* for which desynchronisation would take place and self-oscillations would reappear.

Example 1. Considering the system of Fig. 2, where $G_1(s)=2/s(s+1)^2$, $G_2(s)=1/s(s+4)$ and the two nonlinear elements have ideal saturation characteristics as shown in Fig.2a. Since the value of N_{2i} for small signals is equal to unity, Eq.(21) leads to

$$1 + G_1(j\omega)N_{1i}\left[1 + \frac{G_2(j\omega)}{1+G_2(j\omega)}\right] = 0 \,. \qquad (22)$$

Substituting $G_1(j\omega)$ and $G_2(j\omega)$ and separating in real and imaginary parts finally yields:

$$\omega^4 - 10\omega^2 + \left(8N_{1i}+1\right) = 0$$

and

$$4N_{1i} + 6\omega^4 - \left(6+2N_{1i}\right)\omega^2 = 0 \,.$$

Simultaneous solution of the above equations yields: N_{1i} = 0.51 (critical).

The IDF for the saturation characteristic with given s_1 (= 1.5) is given by [1], [4], [17] and [20]

$$N_{1i} = \frac{2}{\Pi}\left(sin^{-1}\frac{1.5}{B}\right).$$

Hence, the amplitude *B* of the dither that would make the IDF equal to the critical value of 0.51 is found to be *B*=2.09.

Example 2. Consider the system of Example 1 but the characteristics of nonlinear elements are as shown in Fig. 3b. The value of dither amplitude for desynchronisation is found to be *B*=1.5. It is important to note that the above analysis is based on the assumption that the amplitude of signal X_S (self-oscillation) is zero at all points in the system. However, once the amplitude of self-oscillation is different from zero, the signals at the various points would represent forced oscillations.

Consequently, the frequency of self-oscillation that the system would eventually sustain after desynchronisation would be different from the one predicted above.

So, the above analysis predicts only the critical amplitude *B* at which the process of desynchronisation sets in.

B. Forced oscillations

We now present an analysis of the forced oscillations in the system of Fig.2 when it exhibits self-oscillations while being subjected to a high-frequency input *Bsinω_f t* at u_1. As a consequence of assumed low-pass characteristics of linear elements, the input to the nonlinear element N_1 is composed of dither signal of frequency *ω_f* and self-oscillating signal of fre-quency *ω_S*, while the input to the nonlinear element N_2 is composed only of self-oscillating signal of frequency *ω_S*. Consequently, an analysis of the components of the frequency of self-oscillation, *ω_S*, can be visualized as the analysis of the system of Fig.5a.

The system of Fig.5a is obtained by replacing the nonlinear element N_1 in Fig.1 by its modified characteristics [3], [4], [9], [13-16], [20] determined by the component of the frequency *ω_f* at its input. In view of the low-pass characteristics of the linear elements and the high-frequency of dither, this component can be approximated as *Bsinω_f t*.

The dither frequency should be much greater compared to self-oscillation frequency and the frequency ratio is considered irrational so that the DIDF will depend only on amplitude of two signals [1], [4], [18] and [20]. The component of frequency *ω_f* at the input to the non-linear element N_2 is negligibly small and therefore, the characteristics of the element N_2 in Fig.5a would remain unaltered.

The system of Fig.5a can subsequently be analyzed for possible self-oscillations by employing the techniques developed in [4], [10]. If such an analysis shows the presence of self-oscillation for the system of Fig.5a, then the system of Fig.2 would exhibit forced oscillation of the frequencies ω_S and ω_f. A rigorous analysis of such a system is extremely complex. However, if the whole system is assumed to exhibit an oscillation predominantly at a single frequency and if the loops possess low-pass characteristics, then a simpler analysis, based on harmonic balance approach can be developed along the following lines.

The characteristic equation in frequency domain is obtained as

$$G_1(j\omega)N_{1eq}+G_2(j\omega)N_2+2G_1(j\omega)G_2(j\omega)N_{1eq}N_2=-1.0 \quad (23)$$
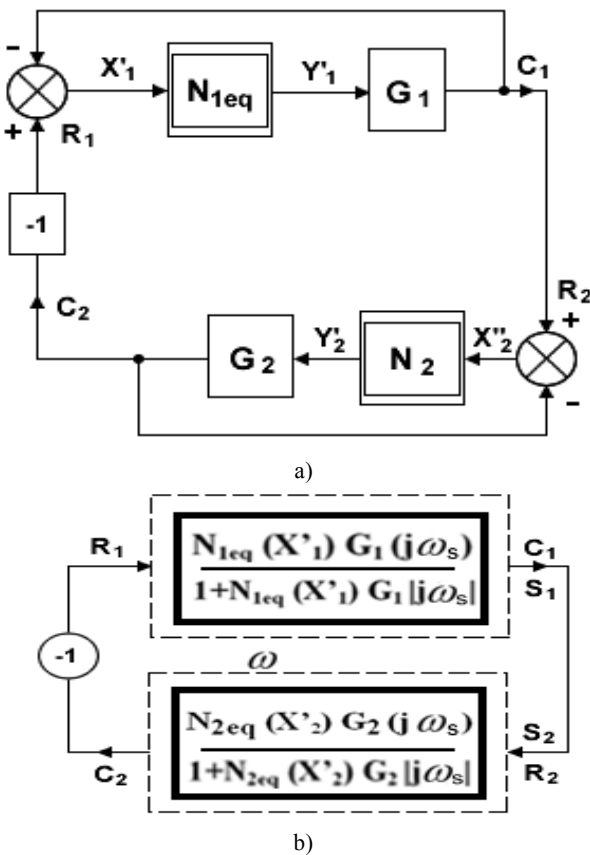

a)

b)

Fig.5: (a) Equivalent system for analyzing forced oscillations of Fig. 2 with external input at U_1.
(b) Linearised equivalent for the system of Fig. 2 for analyzing forced oscillation.

The three unknowns, X_1, X_2 and w require three independent equations for their evaluation. Separating the real and imaginary parts only two independent equations can be developed.

The characteristic equation alone is not sufficient for analysis of self-oscillation in multidimensional systems. However, representing the system of Fig.5a alternatively as in Fig.5b, the following conditions must be fulfilled for ensuring harmonic balance.

(i) The phase condition

$$\theta_{c1}+\theta_{c2}=180^0 \; . \quad (24a)$$

where θ_c = loop angle of subsystems.

(ii) The gain condition:

$$(C_1/R_1)(C_2/R_2)=1 . \quad (24b)$$

(iii) The amplitude ratio condition

$$\frac{X_1'}{X_2'}=\frac{\left|1+N_2G_2(j\omega)\right|}{\left|N_{1eq}G_1(j\omega)\right|} \; . \quad (24c)$$

It may be noted that N_{1eq} and X_1' are related through the DF expression for the modified characteristic of the element N_1 (DIDF), while N_2 and X_2' are related through the DF expression of the element N_2. Eq.(24) constitutes three equations for the solution of the three unknowns ω, X_1', X_2'.

Example 3. Consider again the system of Example 1. Substituting $G_1(j\omega)$, $G_2(j\omega)$ in Eqs.(24a) and (24b) finally yields

$$N_2=\frac{\omega^2\left(11-3\omega^2\right)+\sqrt{\omega^2\left(11-3\omega^2\right)^2-8\left(1-\omega^2\right)^2\left(\omega^2+16\right)}}{4\left(1-\omega^2\right)} \quad (25)$$

and

$$N_{1eq}=\left(\frac{\omega^2-1}{8}\right)N_2+\frac{9\omega^2-\omega^4}{8} \; . \quad (26)$$

Again, substituting $G_1(j\omega)$, $G_2(j\omega)$ in Eq.(24) finally yields

$$\frac{X_1'}{X_2'}=\frac{\sqrt{\omega^2\left(\omega^2+16-4N_2\right)+2N_2^2}}{\omega\sqrt{\left(\omega^2+16\right)}} \; . \quad (27)$$

Furthermore, the relations between X_1' and N_{1eq} and, X_2' and N_2 are obtained from the given nonlinear characteristics as [2], [4]

$$N_{1eq}\left(X_1',B\right)=\frac{2}{\Pi X_1'}\int_{-\infty}^{\infty}\frac{sin(S_1u)}{u^2}J_0(Bu)J_1\left(X_1'u\right)du, \quad (28)$$
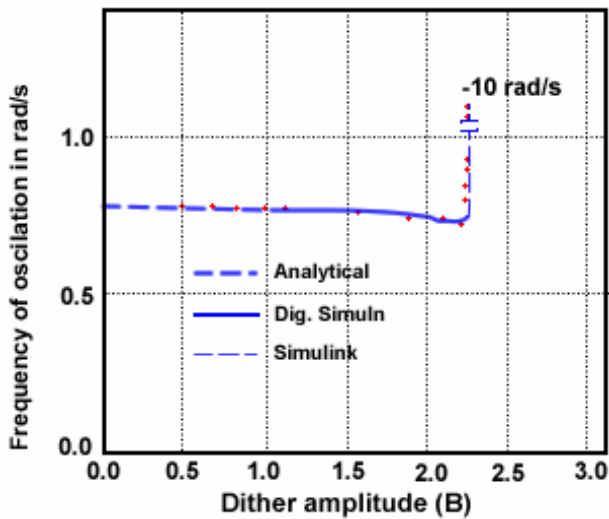
where J_0, and J_1 are Bessel's function of first kind of order 0 and 1, respectively.

We also note that the input to nonlinear element N_2 can be approximated by a signal of frequency of self-oscillation alone. Therefore, the gain for the nonlinear element N_2 would be defined by its DF:

$$N_2=\frac{2}{\Pi}\left[sin^{-1}\frac{2}{X_2'}+\frac{2}{X_2'}\sqrt{\left(1-\left(\frac{2}{X_2'}\right)\right)}\right] \; . \quad (29)$$
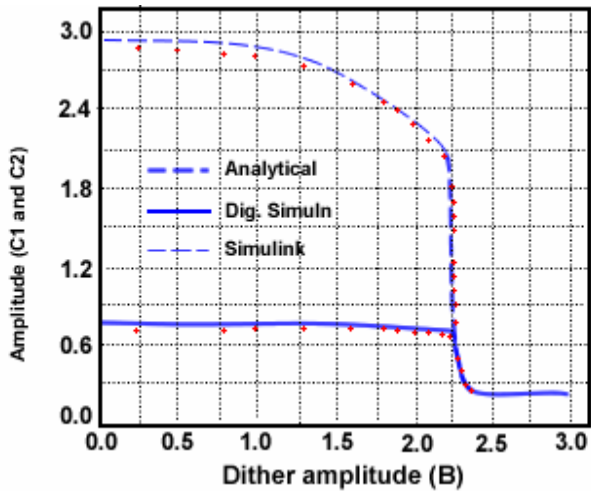
a)

b)

Fig.6: (a) Variation of C_1 and C_2 with dither amplitude, illustration of limit cycle quenching (signal stabilization), results of Example 3. (b) Variation of frequency with dither amplitude results of Example 3.

The procedure for evaluation of the frequency of oscillations and other parameters is executed in the following sequential steps:

(a) certain value of ω is assumed;
(b) Eq.(25) yields a value of N_2;
(c) consequently Eq.(26) yields N_{1eq};
(d) subsequently Eq.(27) yields a value of X_1'/X_2';
(e) for the N_2 and N_{1eq} obtained in steps (b) and (c) above and for a particular value of B an alternative ratio X_1'/X_2' can be obtained from Eqs.(28) and (29);
(f) steps (a)-(e) are repeated for several assumed values of ω, while keeping the value of B a fixed number.

The frequency for which the ratio X_1'/X_2' can be obtained by two alternative means are equal is the frequency of self-oscillation of the system.

The other variables associated with self-oscillations can, subsequently be calculated. For example, if the frequency of oscillation is found out, N_{1eq} and N_2 can be determined from Eqs.(25) and (26). From these values X_1' and X_2' and hence C_1

and C_2 can be calculated. For various values of B, this procedure is repeated and the variations of C_1, C_2 and co for various B are depicted in Fig.6 along with the results of digital simulation. The digital simulation technique used is similar to the Subramanian's work on SISO system [21] and also used by the authors' earlier work on limit cycle prediction for two-dimensional autonomous system [13].

The dither frequency chosen in the work is 10 rad/s. Fig.7 depicts the buildup of subsystem output C_1 at B =1.0, which shows the periodic nature of oscillation. The forced oscillation has less settling time. The low-frequency demand signal is the excitation signal used to initiate the oscillation.

The system was also simulated through MATLAB 7.0 for predicting the above phenomenon at various dither values. The simulation results are also shown in Fig.6. The analytical results have excellent agreement with simulation results.

Example 4. Consider the same system of Example 2. The results from simulation and analytical technique are compared in Fig.8. It can be seen that the simulation provides a good match with the frequency and amplitude of oscillation. Synchronization occurs at B=1.865.
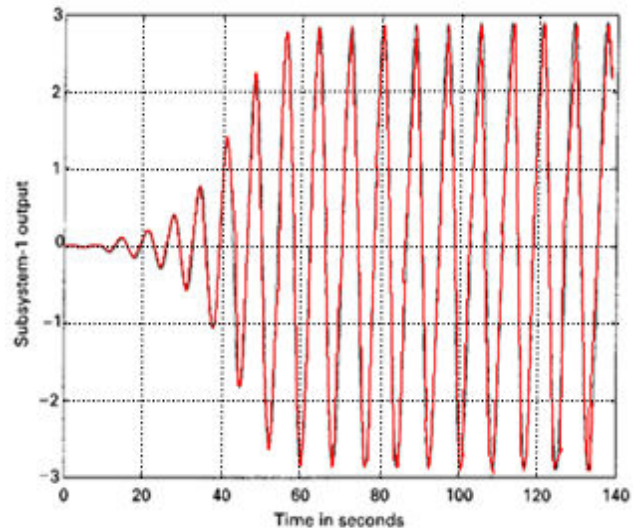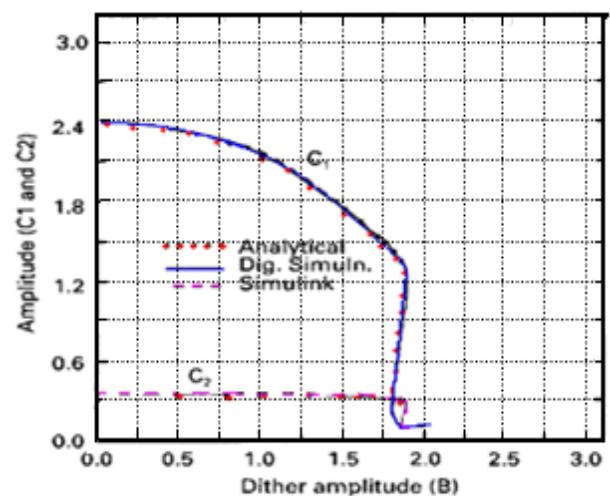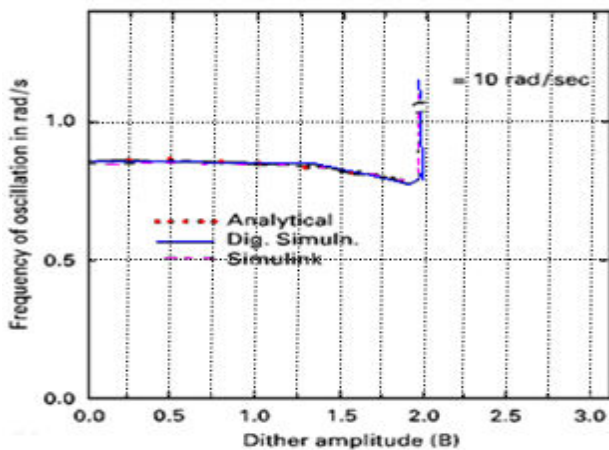

Fig.7: Build up of oscillation is subsystem-1.

a)

b)
Fig.8: (a) Variation of C_1 and C_2 with dither amplitude, illustration of limit cycle quenching (signal stabilization), results of Example 4.
(b) Variation of frequency with dither amplitude. Results of Example 4.

V. AQUERING INPUT – OUTPUT DATA

A solid identification requires the use of a command signal of the process with the most frequencies. This signal will be overlapped on the static value of the command which corresponds to the functioning point around which we want to identify the process. Usually, we use as a excitation signal a SPAB of low amplitude.

A. Acquisition protocol

To be able to correctly identify the static amplification of the dynamic process, at least one of the impulses of the SPAB signal must be higher than the develop time of the system to be identified.

This leads to the following condition:

$$p\,N\,T_e \geq t_c , \qquad (30)$$

where: T_e is the sampling period, p is the frequency divider; pT_e is the period of the SPAB generator (the frequency of the clock is equal to f_e/p); N is the length of the generator's register and t_c is the development time of the system.

The amplitude of the SPAB generator must not surpass the static command signal (a few percentages).

The acquisition is usually done with the help of a computer equipped with an acquisition board of input- output data, which contains analog- numeric and numeric- analog convertors. The board is simulated on the computer which gives away both the SPAB signal and the continuous component that corresponds to the functioning point.

The identification program must generate, with the help of a PC, SPAB signals, must generate a continuous command signal to define the functioning point and it also must perform the acquisition of the input and output process.

There are two connection configurations of the process:

1. *Open circuit process*, the easiest case. In this situation, the excitation signal overlapped on the command signal of the functioning point is directly applied to the process. The launching of the acquisition protocol must be done as soon as the process is found in a stationary regime.

2. *Closed circuit process*, where there are two situations. In the first case, if the excitation of the process is overlapped on the output of the regulator, case where the SPAB signal is combined with the output of the regulator, the acquisition of real input and output process will be started. The transfer between y_0 and y1 will be identified. The SPAB signal will be combined with the reference, thus defining the functioning point and acquiring the input and output data of the process.

It is better that a "weak" regulator is used (without a derived action and a week proportional action), but keeping an integral action that will help maintain the functioning point.

The second case is when the excitation is applied on the reference. If n is large enough ($n \geq 4$), an anti-distortion filter is usually enough:

$$y_f(t) = \frac{y(t) + y(t-1) + ... + y(t-n+1)}{n} , \quad \forall\, t \in \mathcal{N}. \qquad (31)$$

In case we take too many samples, we must turn our attention towards choosing the clock/ tact (p) frequency divisors of the SPAB signal. The frequency (p) divisor used for the SPAB signal must be a multiple of n, so that the final sampling frequency f_e will remain a multiple of the clock frequency of the SPAB signal.

The clock frequency of the SPAB signal is given by the relationship:

$$f_{SPAB} = \frac{1}{n} f_a , \qquad (32)$$

and the final sampling frequency by:

$$f_e = \frac{1}{n} f_a = \left(\frac{p}{n}\right) f_{SPAB} , \qquad (33)$$

where (p/n) must be a whole value.

B. Anti-distortion filters

A conversion of continuous signals higher than $0,5f_e$ will introduce a distortion of the sample signal from 0 to $0,5f_e$. To avoid this situation, we must introduce an *analogical anti-distortion filtering* between the measured output of the process and the acquisition board.

This is possible by using a pass-down filter that allows diminishing the signal components which have the frequency higher than $0,5f_e$. If we not respect the rule, we might encounter serious errors during the identification process. Even if we obtain discrete models that are validated, they will not be representative for the actual behavior of the process (of the continuous model of the process).

C. Over- sampling

Choosing the sampling frequency for the regulation-command systems is done depending on which passing belt we desire for our closed system. Whatever the way of telling

what our desired performances are, they can always be connected to the passing belt of the closed-curled system.

The many situations we have encountered in practice lead to the use of a acquisition frequency which is a multiple of the sampling frequency in the regulation loop.

For example:
- in may numeric control and command systems, the cadence of the acquisitions is bigger than the sampling frequency in the regulation loop;
- in the systems which use a sampling period higher than 1 in their regulation loops, analogical anti-distortion filtering is very difficult and we must use a numeric filtering technique;

An acquisition must be made with a frequency multiple of the sampling frequency (or vice-versa);

$$f_a = n f_e, \qquad (34)$$

where: f_a is the acquisition frequency, n is the frequency report and f_e is the sampling frequency of the regulation loop. The discrete signal obtained after sampling for the f_a frequency is passed through an anti- distortion numeric filter that attenuates all frequencies higher than $0,5f_e = 0,5f_a/n$. All n samples obtained are gathered at once.

D. Conditioning the signals

The elimination of the continuous component: The structures of the models used for identification are dynamic models (variations of the input data depending on the variations of the output data around the functioning point). For a correct identification, we must remove from the input-output data the continuous components (functioning point) or the slow derives of the functioning point.

There could be two cases:
1. Eliminating the stationary continuous components.
This can be done in two stages:
(a) Calculating the medium values (MV) of the input- output files I/O (input-output).
(b) Deducting the medium value from the I/O files and creating a new file:

$$y'(t) = y(t) - VM(y) , \ u'(t) = u(t) - VM(u) , \quad \forall t \in \mathcal{N}. \ (35)$$

2. Eliminating the un-stationary continuous components: we replace the files of the input/ output measures with the variations of the measured, filtered I/O measures:

$$y'(t) = \frac{y(t) - y(t-1)}{1 + f_1 q^{-1}} , \ u'(t) = \frac{u(t) - u(t-1)}{1 + f_1 q^{-1}} , \forall t \in \mathcal{N}. \quad (36)$$

with $-0,5 \le f1 \le 0$; but other types of filters can be used.
These functions are usually assured by the identification program we use (MATLAB).

If the identification process contains a pure integrator, then if we know about the existence of an integrator, we can use this preliminary information to reduce the complexity of the model we have to identify.

Two methods can be used:

1. The input is replace by its integral, the output remains the same:

$$y'(t) = y(t), \ u'(t) = \frac{u(t)}{1 - q^{-1}} , \quad \forall t \in \mathcal{N}. \qquad (37)$$

2. We replace the output by its variations and the input remains the same:

$$y'(t) = y(t) - y(t-1), u'(t) = u(t) , \forall t \in \mathcal{N}. \qquad (38)$$

We can filter $y'(t)$ and $u'(t)$. These functions can be obtained with the help of the identification program we use (MATLAB).

We then scale the inputs and outputs. The vector $\phi(t)$ contains the following components $y(t), y(t-1), \ldots$ (or variables correlated to the outputs) and $u(t), u(t-1), \ldots$:

$$\phi^T(t) = [-y(t), -y(t-1), \ldots, u(t), u(t-1), \ldots], \quad \forall t \in \mathcal{N}. \quad (39)$$

One the other hand, the adaptation amplification $K(t)$ has the following expression:

$$K(t) = \sum_{i=1}^{t} \phi(i-1)\phi^T(i-1) + \frac{1}{\delta} I , \ \forall t \in \mathcal{N}, \ (\delta << 1) \ (40)$$

If the level of $u(t), u(t-1), \ldots$ is different from the level of $y(t), y(t-1), \ldots$, the amplification matrix will not be balanced, which will lead to different convergence speeds for $\hat{a}_i(t)$ şi $\hat{b}_i(t)$ parameters. As a consequence, scaling the input/ output files is necessary. A modification of the identified static amplification will be the result. Thus the values of the parameters will have to be divided or multiplied so that we can obtain a model that has the same static amplification as the model. This scaling function is done through the identification program we use.

VI. CONCLUSION

Comparison of analytical results with the results of digital simulation of the example considered, shows that the simplifying assumptions made in the analysis lead to results of acceptable accuracy. In addition, the method of analysis also aids the conceptual visualization of the mechanism leading to these interesting phenomena.

However, the signal stabilization for the system comprising several interconnected subsystems exhibiting limit cycle at different frequencies are yet to be explored and this method of analysis may be appended by Neural Network model [15].

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