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Abstract

Despite being critical parameters in many economic fields, the received wisdom, in theoretical and empirical literatures, states that joint identification of the elasticity of capital-labor substitution and technical bias is infeasible. This paper challenges that pessimistic interpretation. Putting the new approach of "normalized" production functions at the heart of a Monte Carlo analysis we identify the conditions under which identification is feasible and robust. The key result is that the jointly modeling the production function and first-order conditions is superior to single-equation approaches in terms of robustly capturing production and technical parameters, especially when merged with "normalization". Our results will have fundamental implications for production-function estimation under non-neutral technical change, for understanding the empirical relevance of normalization and the variability underlying past empirical studies.
JEL Classification: C22, E23, O30, 051.
Keywords: Constant Elasticity of Substitution, Factor-Augmenting Technical Change, Normalization, Factor Income share, Identification, Monte Carlo.

Non Technical Summary

The elasticity of substitution between capital and labor and the direction of technical change are critical parameters in many areas of economics. Despite this, the received wisdom – in both *theoretical* and *empirical* literatures – suggests that identifying the elasticity of substitution with non-neutral technical change is largely infeasible. If so, this would render such debates indeterminate.

Consider theoretical arguments. If production is Cobb-Douglas (i.e., unitary substitution), then technological progress degenerates to the Hicks-Neutral representation. In the case of a non-unitary substitution elasticity, in turn, Diamond et al. (1978) asserted that the elasticity and biased technical change cannot be simultaneously identified. To counter this "impossibility theorem" researchers usually impose specific functional forms for technical progress, e.g., a deterministic (exponential) function and restrictive assumptions about technological progress (e.g. imposing Harrod Neutrality). However, arbitrary ex-ante identification schemes risk spurious ex-post inference.

On the empirical side, despite the huge efforts devoted to their identification, limited consensus has emerged on the value of the substitution elasticity and arguably less on the nature of technical change. This doubtless reflects many practical data problems as well as a priori modeling choices and the performance of various estimators. An added problem, however, is that often the predictions of different elasticity and technical change combinations can have similar implications for variables of interest, such as factor income shares and factor ratios. Notwithstanding, whether factor income movements are driven by high or low substitution elasticities and with different combinations of technical change is profoundly important in terms of their different implications for, e.g., growth accounting, inequality, calibration in business-cycle models, public policy issues etc.

It is legitimate to wonder if standard techniques can separate these effects. It is this key question that we address. To do so, we employ Monte Carlo sampling techniques. Despite their natural appeal in uncovering CES properties, there have been relatively few such studies; reflecting, arguably, the numerical complexity involved and weak results typically reported. Some studies were, for instance, effectively only interested in uncovering single production parameters (e.g., Maddala and Kadane (1966)), leaving researchers unclear as to overall performance. However, more elaborate studies (e.g., Kumar and Gapinski (1974); Thursby (1980)) suggested joint parameter identification was highly problematic (the substitution elasticity seemed especially challenging yielding sometimes highly implausible first and second moments).

Our paper offers a significant improvement over these earlier studies. First, in contrast to the actual US data studies of several other works, we employ a carefully constructed, pre-determined data generation process (DGP). Knowing the exact nature of the data, we can attribute all differences in parameter estimates to the technique used. Thus, we can rank different approaches in terms of their ability to replicate the known DGP and explain that ranking. Second, we consider a more comprehensive range of estimation forms and types than previously (single-equation, system, linear, non-linear, linearized). We also examine a rich source of robustness issues: auto-correlated errors, sample size, the effect of different initial conditions, etc. Finally, we take "normalization" seriously (La Grandville (1989), Klump and de La Grandville (2000)). We find that normalization besides offering several theoretically-consistent advantages, also improves empirical identification.

Our findings are that single equation approaches are largely unsuitable for jointly uncovering technical characteristics. This applies also to our generalized form of the Kmenta approximation (for which we derive some weak technical identification results). Moreover, direct estimation of the non-linear CES does not alleviate identification problems (especially so for high elasticity cases). The key result is the superiority of the system approach (i.e., jointly modeling the production function and first-order conditions) in terms of robustly capturing production and technical parameters. This approach further allows us to highlight the empirical advantages of "normalization".

1 Introduction

The elasticity of substitution between capital and labor and the direction of technical change are critical parameters in many areas of economics; almost all macro or growths model embody some explicit production technology. How useful such models prove to be then on the appropriateness of their technical assumptions.

Why do these parameter matter so much? The value of the substitution elasticity, for example, has been linked to differences in international factor returns and convergence (e.g., Klump and Preissler (2000), Mankiw (1995)); movements in income shares (Blanchard (1997), Caballero and Hammour (1998)), trade and development patterns (e.g., Jones (1965); Duffy and Papageorgiou (2000)); the effectiveness of employment-creation policies (Rowthorn (1999)) etc. Recent work on "normalized"[1] Constant Elasticity of Substitution (CES) functions has also formalized a correspondence between substitution possibilities and growth (La Grandville (1989), Klump and de La Grandville (2000), La Grandville and Solow (2008))[2]. The nature of technical change, on the other hand, matters for characterizing the welfare consequences of new technologies (Marquetti (2003)); labor-market inequality and skills premia (Acemoglu (2002b)); the evolution of factor income shares (Kennedy (1964), Acemoglu (2003)) etc. Moreover, the interdependency between substitution possibilities and technical change has also sparked several interesting debates: e.g., on relating constellations of the substitution elasticity and technical change with the shape of the (local and global) production function, (e.g., Acemoglu (2003), Jones (2005)), and in accounting for medium-run departures from balanced growth (McAdam and Willman (2008)) etc.

Despite the importance of these debates, the received wisdom – in both *theoretical* and *empirical* literatures – suggests that identifying the elasticity of substitution with non-neutral technical change is largely infeasible. If so, this would render such debates indeterminate.

First, consider theoretical arguments. If production is Cobb-Douglas (i.e., unitary substitution), then technological progress degenerates to the Hicks-Neutral representation. In the case of a non-unitary substitution elasticity, in turn, Diamond et al. (1978) asserted that the elasticity and biased technical change cannot be si-

[1]Normalization essentially implies representing the production function in consistent indexed number form.

[2]This is termed the "de La Grandville Hypothesis" following La Grandville (1989) and Yuhn (1991). Also, in an earlier contribution, Solow (1956) and Pitchford (1960) showed in the neoclassical growth model that a CES function with an elasticity of substitution greater than one generates sustained growth (even without technical progress).

multaneously identified. To counter this "impossibility theorem" researchers usually impose specific functional forms for technical progress, e.g., a deterministic (exponential) function and restrictive assumptions about technological progress (e.g. imposing Harrod Neutrality). However, arbitrary ex-ante identification schemes risk spurious ex-post inference. Antràs (2004), for instance, suggested that the popular assumption of Hicks-neutral technical progress, coupled with relatively stable factor shares and rising capital deepening biases results towards Cobb-Douglas.

On the empirical side, despite the huge efforts devoted to their identification, limited consensus has emerged on the value of the substitution elasticity and arguably less on the nature of technical change. This doubtless reflects many practical data problems (e.g., outliers, uncertain auto-correlation, structural breaks, quality improvements, measurement errors etc) as well as a priori modeling choices (as just discussed) and the performance of various estimators. An added problem, however, is that often the predictions of different elasticity and technical change combinations can have similar implications for variables of interest, such as factor income shares and factor ratios. Notwithstanding, whether factor income movements are driven by high or low substitution elasticities and with different combinations of technical change is profoundly important in terms of their different implications for, e.g., growth accounting, inequality, calibration in business-cycle models, public policy issues etc.

It is legitimate to wonder if standard techniques can separate these effects. It is this key question that we address. To do so, we employ Monte Carlo sampling techniques. Despite their natural appeal in uncovering CES properties, there have been relatively few such studies; reflecting, arguably, the numerical complexity involved and weak results typically reported. Some studies were, for instance, effectively only interested in uncovering single production parameters (e.g., Maddala and Kadane (1966)), leaving researchers unclear as to overall performance. However, more elaborate studies (e.g., Kumar and Gapinski (1974); Thursby (1980)) suggested joint parameter identification was highly problematic (the substitution elasticity seemed especially challenging yielding sometimes highly implausible first and second moments).

Our paper offers a significant improvement over these earlier studies. First, in contrast to the actual US data studies of Kumar and Gapinski (1974) and Thursby (1980), we employ a carefully constructed, pre-determined data generation process (DGP). Knowing the exact nature of the data, we can attribute all differences in parameter estimates to the technique used. Thus, we can rank different approaches in terms of their ability to replicate the known DGP and explain that ranking. Second, we consider a more comprehensive range of estimation forms and types than previously (single-equation, system, linear, non-linear, linearized). We also examine a rich source of robustness issues: auto-correlated errors, sample size, the effect of different initial conditions, etc. Finally, we take "normalization" seriously (La Grandville (1989), Klump and de La Grandville (2000)). We find that normalization besides offering several theoretically-consistent advantages, also improves empirical identification.

Our findings are that single equation approaches are largely unsuitable for jointly uncovering technical characteristics. This applies also to our generalized form of the Kmenta approximation (for which we derive some weak technical identification results). Moreover, direct estimation of the non-linear CES does not alleviate identification problems (especially so for high elasticity cases). The key result is the superiority of the system approach (i.e., jointly modeling the production function and first-order conditions) in terms of robustly capturing production and technical parameters. This approach further allows us to highlight the empirical advantages of "normalization".

The paper proceeds as follows. Section 2 reviews some relevant technical concepts of the CES function with technical change. The subsequent section briefly appraises existing empirical studies and their apparent lack of robustness. Section 4 discusses the concept of normalization. Section 5 explains the different approaches for estimating the production function and technical change used, whilst the subsequent section elaborates on the Monte Carlo. Sections 7 and 8 present our results and robustness extensions. Section 9 concludes.

2 Background: The CES Production Function and Technical Change

The CES production function – a special type of function rooted in the mathematical theory of elementary mean values (Hardy et al. (1934), p. 13 ff.) – was introduced into economics by Dickinson (1955) and Solow (1956) and further pioneered by Pitchford (1960), Arrow et al. (1961), David and van de Klundert (1965) and others. It takes the form:

$$F\left(\Gamma_t^K K_t, \Gamma_t^N N_t\right) = C\left[\pi\left(\Gamma_t^K K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi)\left(\Gamma_t^N N_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where distribution parameter $\pi \in (0,1)$ reflects capital intensity in production; C is an efficiency parameter and the elasticity of substitution σ between capital K_t and labor N_t is given by the percentage change in factor proportions due to a change in the marginal products (or factor price ratio):

$$\sigma \in (0,\infty) = -\frac{d\log(K/N)}{d\log(F_K/F_N)} \tag{2}$$

Equation (1) nests Cobb-Douglas when $\sigma = 1$; the Leontief function (i.e., fixed factor proportions) when $\sigma = 0$; and a linear production function (i.e., perfect factor substitutes) when $\sigma \longrightarrow \infty$. Finally, when $\sigma < 1$, factors are gross complements in production and gross substitutes when $\sigma > 1$ (Acemoglu (2002a)).[3]

The terms Γ_t^K and Γ_t^N capture capital and labor-augmenting technical progress. To circumvent problems related to Diamond et al. (1978)'s impossibility theorem, researchers usually assume specific functional forms for technical progress, e.g., $\Gamma_t^K = \Gamma_0^K e^{\gamma_K t}$ and $\Gamma_t^N = \Gamma_0^N e^{\gamma_N t}$ where γ_i denotes growth in technical progress associated with factor i, t represents a time trend, and where $\gamma_K = \gamma_N > 0$ denotes Hicks-Neutral technical progress; $\gamma_K > 0, \gamma_N = 0$ yields Solow-Neutrality; $\gamma_K = 0, \gamma_N > 0$ represents Harrod-Neutrality; and $\gamma_K > 0 \neq \gamma_N > 0$ indicates general factor-augmenting technical progress.[4]

As La Grandville (2008) reminds us, the prime motive of introducing the concept of factor substitution was to account for the evolution of income distribution. To illustrate, if factors are paid their marginal products, relative factor income shares

[3] Though there are many plausible data-coherent functional forms, we concentrate on the encompassing CES case. This reflects the power of this functional form in the modern growth literature (e.g., Acemoglu (2008), La Grandville (2008)) and allows us to focus on salient features like the unitary/non-unitary value of the substitution elasticity and the nature of factor-augmenting technical change. Under more flexible functional forms, e.g., the Variable Elasticity of Substitution (VES) (Bairam (1991)) and translog functions, the substitution elasticity becomes time-varying. Substantial numerical problems can arise from the estimation of these forms, and this problem magnifies substantially when incorporating biased technical change. Consequently, the VES appears to have enjoyed limited empirical success, e.g., Genç and Bairam (1998). Therefore, in our exercises, we follow the bulk of the literature in assuming that σ is time-invariant.

[4] Neutrality concepts associate innovations to related movements in marginal products and factor ratios. An innovation is Harrod-Neutral if relative input shares remain unchanged for a given capital-output ratio. This is also called labor-augmenting since technical progress raises production equivalent to an increase in the labor supply. More generally, for $F(X_i, X_j, ..., A)$, technical progress is X_i-augmenting if $F_A A = F_{X_i} X_i$.

($sh^{K/N}$) and relative marginal products are (dropping time subscripts):

$$\frac{rK}{wN} = sh^{K/N} = \frac{\pi}{1-\pi}\left(\frac{\Gamma^K K}{\Gamma^N N}\right)^{\frac{\sigma-1}{\sigma}} \tag{3}$$

$$\frac{F_K}{F_N} = \frac{\pi}{1-\pi}\left[\left(\frac{K}{N}\right)^{-\frac{1}{\sigma}}\left(\frac{\Gamma^K}{\Gamma^N}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{4}$$

where r and w represent the real interest rate and real wage, respectively.

Thus, capital deepening, ceteris paribus, assuming gross complements (gross substitutes) reduces (increases) capital's income share:

$$\frac{\partial\left(sh^{K/N}\right)}{\partial\left(K/N\right)} \begin{array}{l} < 0 \text{ for } \sigma < 1 \\ = 0 \text{ for } \sigma = 1 \\ > 0 \text{ for } \sigma > 1 \end{array} \tag{5}$$

and reduces its relative marginal product:

$$\frac{\partial\left(F_K/F_N\right)}{\partial\left(K/N\right)} < 0 \ \forall\sigma \tag{6}$$

Likewise, a relative increase in, say, capital-augmentation assuming gross complements (gross substitutes) decreases (increases) its relative marginal product and factor share:

$$\frac{\partial\left(F_K/F_N\right), \partial\left(sh^{K/N}\right)}{\partial\left(\Gamma^K/\Gamma^N\right)} \begin{array}{l} < 0 \text{ for } \sigma < 1 \\ = 0 \text{ for } \sigma = 1 \\ > 0 \text{ for } \sigma > 1 \end{array} \tag{7}$$

Accordingly, it is only in the gross-substitutes case that, for instance, capital augmenting technical progress implies capital-biased technical progress (i.e., in terms of (7), raising its relative marginal product for given factor proportions). Naturally, as can verified from (5) and (7), the relations between the substitution elasticity, technical bias and factor shares evaporates under Cobb-Douglas.[5]

These conditions illustrate the very real potential for identification problems. For

[5]As an aside: if the growth of capital deepening matches that of technical bias, then stable factor shares can arise for any non-unitary substitution elasticity.

example a rise in the labor share could be equally well explained by a rise [or fall] in capital deepening in efficiency units depending on whether production exhibits gross-complements [or gross substitutes]. Failure to properly identify the nature of the substitution elasticity in the first instance will thus seriously deteriorate inference on biased technical change on a given dataset.

3 Empirical Studies on the Substitution Elasticity and Technical Bias

Despite the centrality of the substitution elasticity and technical biases in many areas of economics, and the huge efforts devoted to their identification, there seems little empirical consensus on their value and nature. **Table 1** summarizes some well-known empirical studies for the US: we observe a variety of augmentation forms and elasticity values.[6] Despite its pervasive use, we observe limited support for Cobb-Douglas and for above-unitary substitution elasticities in general.

We briefly review reasons for such heterogeneity in results. This will also help to clarify our contribution.

(a) *Data quality and data consistency.*

Several papers (e.g., Berndt (1976), Antràs (2004), Klump et al. (2007)) put a strong emphasis on the selection of high-quality, consistent data. Problems nevertheless remain endemic to production function estimation: e.g., the correct measurement of the user cost and capital income, the possible use of quality-adjusted measures for factor inputs, neglect of capital depreciation and the aggregate mark-up, the treatment of indirect taxes, assumptions about self-employed labor income, measurement of capacity utilization rates, and so on.

[6] The substitution elasticity tends to be greater when estimated from aggregate time series than from micro (firm, industry) cross-section/panel studies.

(b) *Choice of estimating equation*[7]

On the conceptual side there is the problem of how exactly the production parameters are to be estimated. Single equation, two- and three-equation system approaches are competing. Single equation estimates usually concentrate either on the production function or on the one of the first-order conditions of profit maximization, whilst system approaches combine them exploiting cross-equation restrictions.

The estimation of the production function alone is generally only accomplished with quite restrictive assumptions about the nature of technological progress. Antràs (2004), for instance, argued that the popular assumption of Hicks-neutral technical progress, coupled with a relatively stable factor share and rising long-run capital deepening biases results towards Cobb-Douglas (famously advocated by Berndt (1976) for US manufacturing). Furthermore, the elasticity of substitution estimated from the first-order condition with respect to labor seems to be systematically higher than that with respect to capital.[8] Single equation estimates (based on factor demand functions) may be systematically biased, since factor inputs depend on relative factor prices that again depend on relative factor inputs (see David and van de Klundert (1965), p. 369; Willman (2002)).

Two-equation systems that estimate demand functions for both input factors as in Berthold et al. (2002) should alleviate such a systematic simultaneous equation bias. However, since two-equation systems usually do not explicitly estimate a production function (with the nature of technological progress restricted by a priori assumptions), identification remains problematic. The benefit of a three-equation System is that it treats the first-order conditions of profit maximizing jointly, containing cross-equation parameter constraints, which may facilitate the joint identification of the technical parameters.

[7] Although an important issue in itself, we do not consider the effect of adjustment costs in identifying production and technology parameters (e.g., Caballero (1994)). To pursue this would require agreement on the functional form of such adjustment costs and distributed lag structure for factor demands and technology. As Chirinko (2008) notes, most studies of production parameters are, as here, performed using long-run or frictionless concepts and are generally to be preferred for capturing deep production characteristics.

[8] This is also our finding. We rationalize this as being due to the differential shock process on capital and labor returns (see section 7). Discussing this in a dynamic setting, Berndt (1991) suggests it also relates to the less rapid adjustment of capital stock relative to labor.

(c) *Estimation Method*

There are a variety of econometric techniques applicable to estimate production parameters. Some of these follow from the specification of the problem – such as the application of OLS to the first-order conditions or linearized variants of the production function (e.g., Kmenta (1967)); non-linear methods to the CES function itself; IV or full-information approaches to the System approach.

Although all these issues are relevant, in our case we construct the data ourselves allowing us to abstract from (a) above. However, we address the other points by considering (Monte Carlo) estimation using different sample sizes, single equation and system approaches, linear and non-linear methods, and normalized and non-normalized specifications. Thus, our exercise considers many issues related to past estimation and identification practices.

4 "Normalization"

The importance of explicitly normalizing CES functions was discovered by La Grandville (1989), further explored by Klump and de La Grandville (2000), Klump and Preissler (2000), La Grandville and Solow (2006), and first implemented empirically by Klump et al. (2007). Normalization starts from the observation that a family of CES functions whose members are distinguished only by different elasticities of substitution need a common benchmark point. Since the elasticity of substitution is originally defined as point elasticity, one needs to fix benchmark values for the level of production, factor inputs and for the marginal rate of substitution, or equivalently for per-capita production, capital deepening and factor income shares.

Following Klump and Preissler (2000) we start with the definition of the elasticity of substitution in the case of linear homogenous production function $Y_t = F\left(\Gamma_t^K K_t, \Gamma_t^N N_t\right) = \Gamma_t^N N_t f\left(k_t\right)$ where $k_t = \left(\Gamma_t^K K_t\right) / \left(\Gamma_t^N N_t\right)$ is the capital-labor ratio in efficiency units. Likewise $y_t = Y_t / \left(\Gamma_t^N N_t\right)$ represents per-capita production in efficiency units. The substitution elasticity can be expressed as,

$$\sigma = -\frac{f'(k)\left[f\left(k\right) - kf'(k)\right]}{kf''(k)\, f\left(k\right)} \tag{8}$$

This definition can then be transformed into a second-order partial differential equation in k having the following general CES production function as its solution:

$$y_t = a\left[k_t^{\frac{\sigma-1}{\sigma}} + b\right]^{\frac{\sigma}{\sigma-1}} \Rightarrow Y_t = a\left[\left(\Gamma_t^K K_t\right)^{\frac{\sigma-1}{\sigma}} + b\left(\Gamma_t^N N_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{9}$$

where parameters a and b are two arbitrary constants of integration with the following correspondence with the parameters in equation (1): $C = a\left(1+b\right)^{\frac{\sigma}{\sigma-1}}$ and $\pi = 1/\left(1+b\right)$.

A meaningful identification of these two constants is given by the fact that the substitution elasticity is a point elasticity relying on three baseline values: a given capital intensity $k_0 = \Gamma_0^K K_0/\left(\Gamma_0^N N_0\right)$, a given marginal rate of substitution $\left[F_K/F_N\right]_0 = w_0/r_0$ and a given level of per-capita production $y_0 = Y_0/\left(\Gamma_0^N N_0\right)$. For simplicity and without loss of generality, we scale the components of technical progress such that $\Gamma_0^K = \Gamma_0^N = 1$. Accordingly, (1) becomes,

$$\begin{aligned} y_t &= C\left[\pi\left(\Gamma_t^K K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi)\left(\Gamma_t^N N_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \Rightarrow \\ &= Y_0\left[\pi_0\left(\frac{\Gamma_t^K K_t}{K_0}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{\Gamma_t^N N_t}{N_0}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{10}$$

where $\pi_0 = r_0 K_0/\left(r_0 K_0 + w_0 N_0\right)$ is the capital income share evaluated at the point of normalization.

As mentioned earlier, normalization is implicitly or explicitly used in all production functions. Special cases of (10) are those used by Rowthorn (1999), Bentolila and Saint-Paul (2003) or Acemoglu (2002, 2003), where $N_0 = K_0 = Y_0 = 1$ is implicitly assumed[9], or $N_0 = K_0 = 1$ by Antràs (2004). Caballero and Hammour (1998), Blanchard (1997) and Berthold et al. (2002) work with a version of (10) where in addition to $N_0 = K_0 = 1$, $\frac{\partial \log\left(\Gamma_t^N\right)}{\partial t} = \gamma_N > 0$, $\frac{\partial \log\left(\Gamma_t^K\right)}{\partial t} = \gamma_K = 0$ is also assumed (i.e., Harrod-Neutral). We also note that for constant efficiency levels $\Gamma_t^K = \Gamma_t^N = 1$ our normalized function is formally identical with the CES function that Jones (2005) proposed for the characterization of the "short term".[10]

Moreover, we now see that the parameters of (10) have a clear, unambiguous interpretation in terms of the point of normalization.[11] The normalized function

[9]As we demonstrate in section 7.1 one consequence of the $N_0 = K_0 = Y_0 = 1$ normalization case is the counterfactual outcome that the real interest rate at the normalization point is equal to the capital income share.

[10]This long-run production function is then considered Cobb-Douglas with constant factor shares of π_0 and $1-\pi_0$ with a constant exogenous growth rate. Actual behavior of output and factor input is modeled as fluctuations around "appropriate" long-term values.

[11]The advantages of rescaling input data to ease the computational burden of highly non-linear

defines all production functions that belong to the same family, i.e., all CES production function that share common baseline point and are distinguished by different elasticities of substitution. Only across production functions belonging to the same family does the following growth theoretic properties of the CES production hold (Klump and de La Grandville (2000)); (1) when two countries start from a common initial point, the one with the higher elasticity of substitution will experience, ceteris paribus, a higher per-capita income; (2) any equilibrium values of capital-labor and income per head are an increasing function of σ.

Non-normalized functions, by contrast, lack these properties since each non-normalized CES function with a different elasticity of substitution belongs to a different family and are therefore unsuitable for comparative static analysis. This arises because the parameters of the non-normalized function are not "deep": besides on the point of normalization they also depend on σ (i.e., comparing (10) with (1)):

$$C(\sigma, \bullet) = Y_0 \left[\frac{r_0 K_0^{1/\sigma} + w_0 N_0^{1/\sigma}}{r_0 K_0 + w_0 N_0} \right]^{\frac{\sigma}{\sigma-1}} \tag{11}$$

$$\pi(\sigma, \bullet) = \frac{r_0 K_0^{1/\sigma}}{r_0 K_0^{1/\sigma} + w_0 N_0^{1/\sigma}} \tag{12}$$

Hence, maintaining C and π as constants, each non-normalized function (1), corresponding to different values of σ, goes through a different point of normalization belonging to different families.

Although there is a clear correspondence between the parameters of the non-normalized and normalized production function, the estimation of the latter offers some advantages. An appropriate choice of the normalization point links the distribution parameter π_0 directly to the factor income shares at that point. Hence, a suitable choice for the point of normalization may markedly facilitate the identification of deep technical parameters as it allows pre-fixing them for estimation.

Overall, we can say that normalization: (a) is necessary for identifying in an economically meaningful way the constants of integration which appear in the solution to the differential equation from which the CES function is derived; (b) helps to distinguish among the various functional forms, which have been developed in the CES literature; (c) is necessary for securing the basic property of CES production in the context of growth theory, namely the strictly positive relationship between the substitution elasticity and the output level given the CES function's representation as a

regressions has been the subject of some study (e.g., ten Cate (1992)) albeit in an atheoretical context.

“General Mean” of order $(\sigma - 1)/\sigma$ for two production factors (see La Grandville and Solow (2006)); (d) is convenient when biases in the direction of technical progress are to be empirically determine[12]; finally, and especially relevant in our context; (e) normalization may alleviate the estimation of the deep parameters (making the estimated function also suitable for comparative static analysis).

5 Estimation forms to identify the substitution elasticity and technical change

We consider the following estimation types: the linear first-order conditions of profit maximization; a Kmenta linear approximation of the CES function exploiting normalization; the non-linear CES production function; non-linear system estimation incorporating the CES function and the first-order conditions (FOCs) conditions jointly (the system). Within these estimation types, we consider OLS, IV, non-linear least squares, and system estimation methods. We implement different values of substitution and technical biases, normalized and non-normalized forms, as well as different sample sizes.[13]

[12]Normalization also fixes a benchmark value for factor income shares. This is important when it comes to an empirical evaluation of changes in income distribution arising from technical progress. If technical progress is biased in the sense that factor income shares change over time the nature of this bias can only be classified with regard to a given baseline value (Kamien and Schwartz (1968)). As pointed out by Acemoglu (2002, 2003), the neoclassical theory of induced technical change regards such biases as necessary market reactions to changes in factor income distribution; the interaction of factor substitution and biased technical change is then responsible for the relative stability of long term factor income shares.

[13]We confine ourselves to constant-returns production functions. This is largely done to be consistent with much of the aggregate evidence (e.g., Basu and Fernald (1997)). However, the incorporation of non-constant returns would also require a consistent explanation of the source, nature and disbursement of those non-constant returns and thus an appropriate structure for the aggregate and intermediate goods supply side system and corresponding factor demands. We leave this open for future work.

5.1 Linear Single Equation Forms

5.1.1 Estimation using the First Order Conditions of Profit Maximization

Given CES function (1), the standard FOCs of profit maximization yield:

$$\text{K\_FOC}: \log\left(\frac{Y_t}{K_t}\right) = \alpha_1 + \sigma\log(r_t) + \gamma_K(1-\sigma)t \tag{13}$$

$$\text{N\_FOC}: \log\left(\frac{Y_t}{N_t}\right) = \alpha_2 + \sigma\log(w_t) + \gamma_N(1-\sigma)t \tag{14}$$

$$\text{Factor Prices:} \log\left(\frac{K_t}{N_t}\right) = \alpha_3 + \sigma\log\left(\frac{w_t}{r_t}\right) + (\gamma_N-\gamma_K)(1-\sigma)t \tag{15}$$

$$\text{Factor Shares:} \log\left(\frac{K_t}{N_t}\right) = \alpha_4 + \frac{\sigma}{1-\sigma}\log\left(\frac{S_t^N}{S_t^K}\right) + (\gamma_N-\gamma_K)t \tag{16}$$

Where $\alpha_i(\sigma,\pi,C)' s$ are constants, γ_N and γ_K are the growth rates of labor and capital augmenting technical progress, $S^{N,K}$ are the shares of labor and capital in total income.

These equations represent the FOC with respect to capital and labor respectively, the remaining two are combinations thereof. All can be used to estimate σ. However, the first two only admit estimates of technical progress terms contained by their presumed FOC choice (in that sense technical progress terms, are by definition, not separately identifiable). The last two, in turn, capture only overall technical bias. Despite their obvious drawbacks, these forms are common: e.g., equation (13) has been widely used in the investment literature (e.g., Caballero (1994)) and (14) was the form used by Arrow et al. (1961) amongst others.

5.1.2 The Kmenta Approximation

The Kmenta (1967) approximation is a Taylor-series expansion of the CES production function around a unitary substitution elasticity.[14] Its main merit is therefore the computational simplicity associated with the approximation. Its main drawback (so far) is that tractability requires a purely Hicks Neutral representation.

Applying the Kmenta approximation to the normalized CES production function (10) yields,

[14] It is worth noting this can be taken also as an initial step towards the development of the translog model (although it seems Kmenta never received credit for it).

$$\begin{aligned}
\log\left(\frac{Y_t}{Y_0}\right) &= \pi_0 \log\left(\frac{K_t}{K_0}\right) + (1-\pi_0)\log\left(\frac{N_t}{N_0}\right) \\
&+ \frac{(\sigma-1)\,\pi_0\,(1-\pi_0)}{2\sigma}\left[\log\left(\frac{K_t/K_0}{N_t/N_0}\right)\right]^2 \\
&+ \pi_0\left[1+\frac{(\sigma-1)\,(1-\pi_0)}{\sigma}\log\left(\frac{K_t/K_0}{N_t/N_0}\right)\right]\gamma_K\,(t-t_0) \\
&+ (1-\pi_0)\left[1-\frac{(\sigma-1)\,\pi_0}{\sigma}\log\left(\frac{K_t/K_0}{N_t/N_0}\right)\right]\gamma_N\,(t-t_0) \\
&+ \frac{(\sigma-1)\,\pi_0\,(1-\pi_0)}{2\sigma}\left[\gamma_K-\gamma_N\right]^2(t-t_0)^2
\end{aligned} \tag{17}$$

As before, we assume $\Gamma_t^K = e^{\gamma_K(t-t_0)}$ and $\Gamma_t^N = e^{\gamma_N(t-t_0)}$, which ensures $\Gamma_{t_0}^K = \Gamma_{t_0}^N = 1$. In the Hicks neutral representation ($\gamma_K = \gamma_N = \gamma$) the three bottom rows of (17) - i.e., total factor productivity - simplify to $\gamma\,(t-t_0)$.

With the predetermined normalization point, the advantage of (17) over the Kmenta approximation of the non-normalized CES is that, since all variables appear in indexed form, the estimates are invariant to a change in units of measurement. Another advantage is that in the neighborhood of the normalization point (i.e., $K_t = K_0, N_t = N_0$) and without σ deviating "too much" from unity, as the approximation also assumes, the terms including the normalized capital intensity and multiplying linear trend have only second order importance and, without any significant loss of precision, can be dropped, yielding,

$$\begin{aligned}
\log\left(\frac{Y_t/Y_0}{N_t/N_0}\right) &= \pi_0 \log\left(\frac{K_t/K_0}{N_t/N_0}\right) + \underbrace{\frac{(\sigma-1)\,\pi_0\,(1-\pi_0)}{2\sigma}}_{a}\left[\log\left(\frac{K_t/K_0}{N_t/N_0}\right)\right]^2 \\
&+ \underbrace{\left[\pi_0\gamma_K + (1-\pi_0)\,\gamma_N\right]}_{b}(t-t_0) \\
&+ \underbrace{\frac{(\sigma-1)\,\pi_0\,(1-\pi_0)}{2\sigma}\left[\gamma_K-\gamma_N\right]^2}_{c}(t-t_0)^2
\end{aligned} \tag{18}$$

Equation (18) yields 4 parameters, $\pi_0, \widehat{a}, \widehat{b}, \widehat{c}$, for 4 primitives, $\pi_0, \sigma, \gamma_K, \gamma_N$. Using π_0 allows us to exactly identify σ from composite parameter a. However, without a priori information on which one of two technical progress components dominates and, in addition, that the signs of estimates a and c are (or are constrained to be) the same, one cannot identify γ_K and γ_N. This leads to the following weak identification result:

for $\gamma_N > \gamma_K$ we obtain $\qquad \gamma_N = \widehat{b} + \pi_0 \sqrt{\frac{\widehat{c}}{\widehat{a}}}$ and $\gamma_N = \widehat{b} - (1-\pi_0)\sqrt{\frac{\widehat{c}}{\widehat{a}}}$

for $\gamma_N < \gamma_K$ we obtain $\qquad \gamma_N = \widehat{b} - \pi_0 \sqrt{\frac{\widehat{c}}{\widehat{a}}}$ and $\gamma_N = \widehat{b} + (1-\pi_0)\sqrt{\frac{\widehat{c}}{\widehat{a}}}$

Given this, although the Kmenta approximation can be used to estimate σ, it cannot effectively identify the direction of the biased technical change.

Finally, note, if $\sigma = 1$ then Taylor expanded forms (17) and (18) naturally reduce to Cobb-Douglas. Furthermore, when $\sigma \neq 1$ and technical progress deviates from Hicks neutrality, factor augmentation introduces additional curvature into the estimated production function via the quadratic trend both in (17) and (18) and, in addition, in (17) via the term where capital intensity multiplies the linear trend.

5.2 The System Approach

A still relatively rarely used framework for the estimation of aggregate CES production functions is the supply-side system approach (i.e., production function plus FOC's). Its origin goes back to Marschak and Andrews (1947) in the context of cross-section analysis, and in the time-series context by Bodkin and Klein (1967).

Since, normalization is implicitly or explicitly employed in all CES production function, we define the production system as explicitly normalized. To be empirically applicable, however, *the point of normalization* must be defined in terms of the underlying data. If the DGP were deterministic, this would be unproblematic: every sample point would be equally suitable for the point of normalization.[15] However, if the DGP is stochastic this is not so, because the production function does not hold exactly in any sample point. Therefore, to diminish the size of stochastic component in the point of normalization we prefer to define the normalization point in terms of sample averages (geometric averages for growing variables and arithmetic ones for factor shares).

[15] It is straightforward to show that the point of normalization can be shifted from point t_0 to any point $t_1 \geq t_0$ so that

$$\begin{aligned} Y_t &= Y_0 \left[\pi_0 \left(\frac{e^{\gamma_K (t-t_0)} K_t}{K_0} \right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0) \left(\frac{e^{\gamma_N (t-t_0)} N_t}{N_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= Y_1 \left[\pi_1 \left(\frac{e^{\gamma_K (t-t_1)} K_t}{K_1} \right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_1) \left(\frac{e^{\gamma_N (t-t_1)} N_t}{N_1} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where $\pi_1 = \pi_0 \left[\frac{K_1/K_0}{Y_1/Y_0} e^{\gamma_K (t_1 - t_0)} \right]^{\frac{\sigma-1}{\sigma}}$ equalling capital income share at point t_1.

However, due to the nonlinearity of the CES function, the sample average of production need not exactly coincide with the level of production implied by the production function with sample averages of the right hand variables *even with a deterministic DGP*. Therefore, following Klump et al. (2007), we introduce an additional parameter ξ whose expected value is around unity (we call this the normalization constant[16]). Hence, we can define $Y_0 = \xi\overline{Y}$, $K_0 = \overline{K}$, $N_0 = \overline{N}$, $\pi_0 = \overline{\pi}$ and $t_0 = \overline{t}$ where the bar refers to the respective sample average (geometric or, as in the last two, arithmetic).

The normalized system can be written as follows:

$$\log(r) = \log\left(\overline{\pi}\frac{\overline{Y}}{\overline{K}}\right) + \frac{1}{\sigma}\log\left(\frac{Y/\overline{Y}}{K/\overline{K}}\right) + \frac{\sigma-1}{\sigma}\left(\log(\xi) + \gamma_K\left(t-\overline{t}\right)\right) \tag{19}$$

$$\log(w) = \log\left((1-\overline{\pi})\frac{\overline{Y}}{\overline{N}}\right) + \frac{1}{\sigma}\log\left(\frac{Y/\overline{Y}}{N/\overline{N}}\right) + \frac{\sigma-1}{\sigma}\left(\log(\xi) + \gamma_N\left(t-\overline{t}\right)\right) \tag{20}$$

$$\log\left(\frac{Y}{\overline{Y}}\right) = \log(\xi) + \frac{\sigma}{\sigma-1}\log\left[\overline{\pi}\left(e^{\gamma_K\left(t-\overline{t}\right)}\frac{K}{\overline{K}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\overline{\pi})\left(e^{\gamma_N\left(t-\overline{t}\right)}\frac{N}{\overline{N}}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{21}$$

Compared to single-equation approaches, the system offers some advantages. From an economic standpoint the system embodies the assumption that the data reflect both optimizing behavior and technology, while single equation approaches capture only one of these aspects. From the econometric standpoint the system, containing cross-equation parameter constraints, increases the degrees of freedom and may enhance efficient estimation and parameter identification. An advantage of the normalized system over the non-normalized system, in turn, is that the distribution parameter $\overline{\pi}$ has a clear data-based interpretation. Therefore, it can either be pre-fixed before estimation or, at least, the sample average can be used as a very precise initial value of the distribution parameter. Likewise a natural choice for the initial value of normalization constant, ξ, is one. Estimated values of these two parameters should not deviate much from their initial values without casting serious doubts on the reasonableness of estimation results. In the non-normalized case, by contrast, no clear guidelines exist in choosing the initial values of distribution parameter π and efficiency parameter C. In the context of non-liner estimation

[16] Only in the log-linear case of Cobb-Douglas would one expect ξ to exactly equal unity. Hence, in choosing the sample average as the point of normalization we lose precision because of the CES's non-linearity. If, alternatively, we choose the sample mid-point as the normalization point, we should also lose because of stochastic (and in actual data, cyclical) components that would also imply non-unitary ξ.

this may imply a significant advantage of the normalized over the non-normalized system. We examine this in section 7.1.

Finally, the normalized non-linear CES production in isolation is given by equation (21).

6 Methodology: The Monte Carlo experiment

The Monte Carlo (MC) consists of M draws of simulated stochastic processes for labor (N_t), capital (K_t), labor- (Γ_t^N) and capital- (Γ_t^K) augmenting technology from which we derive equilibrium output (Y_t^*), observed output (Y_t) and real factor payments (w_t and r_t), for a given set of parameter values and shock variances.

We assume that the log of capital and labor follow an I(1) process:

$$\log(N_t) = n + \log(N_{t-1}) + \varepsilon_t^N \tag{22}$$

$$\log(K_t) = \kappa + \log(K_{t-1}) + \varepsilon_t^K \tag{23}$$

where n and κ represent their mean growth rate respectively, implying that both variables are random walks with drift. Initial values were set as $N_0 = K_0 = 1$ (although we re-examine the sensitivity of results to the initial values of the variables in section 7.1). Both ε_t^K and ε_t^N (i.e., shocks to labor supply and capital accumulation) are assumed to be normally distributed i.i.d error terms with zero mean and standard errors $se\left(\varepsilon_t^K\right)$ and $se\left(\varepsilon_t^N\right)$.

As in most applications, technical progress functions are assumed to be exponential functions with a deterministic and stochastic component (around a suitable point of normalization):

$$\Gamma_t^K = \Gamma_0^K e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}, \quad \Gamma_t^N = \Gamma_0^N e^{\left(\gamma_N(t-t_0)+\varepsilon_t^{\Gamma^N}\right)}$$

where Γ_0^K and Γ_0^N are arbitrary initial values for technology which we also set to unity for simplicity. Shocks to technical progress are assumed to follow $\varepsilon_t^{\Gamma^K} \sim N\left(0, se\left(\varepsilon_t^{\Gamma^K}\right)\right)$ and $\varepsilon_t^{\Gamma^N} \sim N\left(0, se\left(\varepsilon_t^{\Gamma^N}\right)\right)$.

Once the DGP for production factors and technology are defined, we derive equilibrium output from the normalized CES function:

$$Y_t^* = Y_0^* \left[\pi_0 \left(\frac{K_t}{K_0} e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{N_t}{N_0} e^{\left(\gamma_N(t-t_0)+\varepsilon_t^{\Gamma^N}\right)}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{24}$$

We call this "equilibrium" output to distinguish it from the observed output obtained from the national accounts identity. The reason for this, as we shall see, is that we need to define this equilibrium output value in order to obtain values for factor payments from which we then obtain "observed" output series (that we then use to estimate the different models).

Real factor payments are then obtained from (24) using the respective FOC's:

$$\begin{aligned}\frac{\partial Y_t^*}{\partial K_t} &= \pi_0\left(\frac{\Gamma_t^K Y_0^*}{\Gamma_0^K K_0}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t^*}{K_t}\right)^{\frac{1}{\sigma}} e^{\varepsilon_t^r} \\ &= \pi_0\left(\frac{Y_0^*}{K_0} e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t^*}{K_t}\right)^{\frac{1}{\sigma}} e^{\varepsilon_t^r} = r_t \qquad (25)\\ \frac{\partial Y_t^*}{\partial N_t} &= (1-\pi_0)\left(\frac{\Gamma_t^N Y_0^*}{\Gamma_0^N N_0}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t^*}{N_t}\right)^{\frac{1}{\sigma}} e^{\varepsilon_t^w} \\ &= (1-\pi_0)\left(\frac{Y_0^*}{N_0} e^{\left(\gamma_N(t-t_0)+\varepsilon_t^{\Gamma^N}\right)}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t^*}{N_t}\right)^{\frac{1}{\sigma}} e^{\varepsilon_t^w} = w_t \qquad (26)\end{aligned}$$

that is, factor returns equal their marginal product times a multiplicative i.i.d error term that represent shocks that temporarily deviate factor payments from equilibrium, $\varepsilon_t^r \sim N(0, se(\varepsilon_t^r))$, $\varepsilon_t^w \sim N(0, se(\varepsilon_t^w))$.

Note that these FOCs are derived directly from the CES function and hence factor payments reflect the parameter values of the production function including the substitution elasticity and technical progress.

We then obtain "observed" output using the accounting identity:[17]

$$Y_t \equiv r_t K_t + w_t N_t \qquad (27)$$

Combining (27) with (24) yields:

$$\frac{Y_t}{Y_t^*} = \eta_t e^{\varepsilon_t^r} + (1-\eta_t) e^{\varepsilon_t^w} \qquad (28)$$

where $\eta_t = \dfrac{\pi_0\left(\frac{K_t}{K_0} e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}\right)^{\frac{\sigma-1}{\sigma}}}{\pi_0\left(\frac{K_t}{K_0} e^{\left(\gamma_K(t-t_0)+\varepsilon_t^{\Gamma^K}\right)}\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)\left(\frac{N_t}{N_0} e^{\left(\gamma_N(t-t_0)+\varepsilon_t^{\Gamma^N}\right)}\right)^{\frac{\sigma-1}{\sigma}}}$.

This makes clear that series $\frac{Y_t}{Y_t^*}$ is both stationary and driven directly by the two stochastic errors with time-varying (homogenous of degree one) weights (with

[17]We abstract from any aggregate mark-up or pure profit component.

the weights themselves dependent on stochastic technical progress and the factor indices).

This observed value from (27) is then used to estimate the production function using the different estimation methods previously described. The reason why we proceed this way instead of simply adding a stochastic shock to (24) and then obtaining the FOCs with a shock as in (25) and (26) is that our simulated data has to be fully consistent: the shares of capital and labor must sum to unity. Had we proceeded using this alternative way, nothing would have ensured that this condition is met because our generated data is stochastic (these stochastic shocks may make factor shares deviate from values consistent with national-accounting identities). Hence, in our DGP we have shocks to labor supply, capital accumulation, technology, and factor markets and consistency with national-accounting practice is achieved.

The MC therefore proceeds in the following steps, each of which is repeated M times:

1. *Obtain capital, labor, and technology series using (22)-(23) for sample period T.*

2. *Using these series, generate values for equilibrium output and factor payments using (24)-(26).*

3. *Obtain an observed output series from (27).*

4. *Estimate the parameters of the model using the different estimation approaches explained in section 5 making use of the observed value for output and the series for capital, labor and factor payments (i.e., the series available to the econometrician).*

Table 2 lists the MC parameters. We set the distribution parameter to 0.4.[18] The substitution elasticity ranges from a low 0.2 and 0.5, to a near Cobb-Douglas (0.9) value and a value exceeding unity, 1.3.

The technical progress parameters are set so as to sum to a reasonable value of 2% growth per year across the different augmentation forms.[19] As in the bulk of theoretical and empirical studies, we assume broadly constant technical progress growth rates. To assume time-varying growth rates, mimicking models of "directed"

[18] We also experimented with values of 0.3 and 0.6, but this made no qualitative difference to the results; accordingly, we kept its value fixed across all experiments to reduce the volume of results.

[19] We performed experiments where the values did not sum up to 2% per year, with values as large as 4%. This did not make any qualitative difference to the results of the experiment.

technical change (see Kennedy (1964), Samuelson (1965), Zebra (1998), Acemoglu (2002a), 2003) would require, for instance, agreement on the nature of the economy's "innovation possibilities frontier" alongside an explicit framework of imperfect competition. Although we address related issues in Section 8.3 below, we leave a detailed analysis open for future research.

We assume labor supply grows at an average rate of 1.5% per year (roughly the value for US population growth). We then set the capital stock so that (in equation 23) the drift parameter κ equals the drift of labor supply growth n plus the trend growth of labor-augmenting technical progress, γ_N. This ensures that technical progress increases per-capita output independently from the nature of factor augmentation. This formulation allows us to analyze cases in which the evolution of factor shares is notionally consistent with a balanced growth path (i.e. for $\gamma_N = 0.02$, $\gamma_K = 0.00$), and cases for which capital and labor shares are (stochastically) increasing or decreasing, hence covering a wide set of formulations for factor shares.[20]

To avoid counter-factual volatility of the simulated data, we paid due attention to the standard errors of the shocks. We chose a value of 0.1 for the capital and labor stochastic shocks.[21] For the technical-progress parameters we used a value of 0.01 when the technical progress parameter is set to zero, so that the stochastic component of technical progress does not dominate. When technical progress exceeds zero we used a value of 0.05 to capture the likelihood that when technical progress is present it may also be subject to larger shocks.[22]

For the case of wage and rental prices we resorted to real data and used the value of the standard deviation of, respectively, their de-trended and demeaned values in the US economy over 1950-2000.[23] The value for real wages data is 0.05 and 0.3 for capital income, reflecting the larger volatility of user costs. This differential will have important implications for the relative success of the first order conditions using OLS, as will be discussed later. Accordingly, we also repeated the experiments where we equate these variances and where we use an instrumental variables (IV)

[20]We also set κ exogenously to 3% but this, again, did not affect the interpretation of results in any significant way.

[21]This is approximately the standard error of labor and capital equipment around a stochastic trend with drift for US data from 1950 to 2005. If we consider all capital stock, i.e. including infrastructures, the standard error is around 0.05. Hence, we reproduced the results using this smaller variance specification for K_t but this did not affect our conclusions.

[22]Nevertheless, we also replicated the results assuming a zero shock when technical progress is zero and also equal shocks for both components. This, again, did not have any significant effect on the results of the experiment.

[23]We use Bureau of Economic Analysis national accounts data.

estimator.

Finally, we consider sample sizes of 25-100 data points (years) with the number of MC draws set to 5,000.[24]

7 Results

Of the cases in Table 2, to keep results manageable, we mostly report those relating to the empirically more relevant T=50 horizon and the combinations $\gamma_N = 0.015$ and $\gamma_K = 0.005$ and $\gamma_N = 0.005$ and $\gamma_K = 0.015$ (**Tables 3a** and **3b**). All other cases are available on request, although there is no qualitative difference in the interpretation of results (from those shown). We report the median values of the estimated coefficients across the 5,000 draws and the 10% and 90% percentiles.[25]

In terms of the OLS FOC's (the first four columns of Tables 3a and 3b) we generally see poor tracking properties except perhaps at the near-Leontief $\sigma = 0.2$ case. The estimated substitution elasticity tends to get trapped around 0.5 as the true value is increased.[26] Estimates of technical progress also appear badly captured. The exception is the FOC with respect to labor: here the substitution elasticity is estimated quite precisely (with a slight deterioration of performance for the $\sigma = 1.3$ case) as is the growth rate of technical progress.

The reason why one OLS approach dominates can be traced to equations (25) and (26): the presence of a stochastic component in factor returns that represents measurement error (or simultaneity bias). In such cases, we know the probability limit of the estimator tends to its true value depending on the noise-to-signal ratio (i.e., the variance of the error process $V(\varepsilon_t)$ over that of the independent variable $V(X_t)$, which in our case is either r_t or w_t):[27]

[24] The non-linear estimations (i.e., direct CES estimation and that of the system) require initial (parameter) conditions. Following Thursby (1980) we set the initial parameter values to those obtained from OLS estimates of first order conditions. For the technical progress parameters we used the labor and capital FOCs 14 and 13. For σ we used the OLS estimation of the ratio between the capital and labor FOC 15. The nature of the non-linear results remains very robust to whichever rule we used.

[25] We report the median rather than the mean because in some of the nonlinear estimation methods one cannot rule out abnormal estimation outcomes in some of the draws, which can skew the results substantially. For maximum transparency, moreover, we ran these MC experiments without any distorting, non-replicable user interference: we never imposed any sign or bounds restriction on any of the parameters. Not with standing, the tables produced relatively few non-standard outcomes.

[26] Researchers disposed towards high or above-unitary substitution elasticities (e.g., Caballero and Hammour (1998)) may draw comfort from these results given that many of the OLS systematically under-estimate the elasticity of substitution, with that bias increasing in the true elasticity.

[27] This argument only applies to the case of two factors of production. The direction of the

$$\text{p}\lim \widehat{\beta} = \frac{\beta}{1 + V(\varepsilon_t)/V(X_t)}$$

Consider the joint interest-rate/capital marginal-product condition:

$$\frac{\partial Y_t^*}{\partial K_t} = \pi_0 \left(\frac{Y_0^*}{K_0} \Gamma_t^K \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t^*}{K_t} \right)^{\frac{1}{\sigma}} e^{\varepsilon_t^r} = r_t$$

For a given π_0 and Γ_t^K, the noise-to-signal ratio is increasing in the substitution elasticity, $\lim_{\sigma \to \infty} corr\left(r_t, e^{\varepsilon_t^r}\right) \to 1$, resulting in downward bias. In general, we would expect the FOC's to perform poorly as σ increases, and especially when above unity. Indeed, when $\sigma = 0.9$ the associated absolute percentage errors (for $\widehat{\sigma}$) for K_FOC (13) and N_FOC (14) are 86% and 4%, respectively; at $\sigma = 1.3$ they climb to 167% and 20%.[28]

However, in the wage/labor marginal-product condition,

$$\frac{\partial Y_t^*}{\partial N_t} = (1 - \pi_0) \left(\frac{Y_0^*}{N_0} \Gamma_t^N \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t^*}{N_t} \right)^{\frac{1}{\sigma}} e^{\varepsilon_t^w} = w_t$$

we have the additional apparent advantage that since output growth exceeds labor growth, productivity and the real wage are non-stationary. This trending aspect (compared to a largely stationary capital-output ratio and real interest rate) implies a generally more favorable noise-to-signal ratio.[29]

Moreover, since the data (recall Table 2) informs us that $se\left(\varepsilon_t^r\right) = 0.30 >> se\left(\varepsilon_t^w\right) = 0.05$, it is easy to appreciate why this measurement error problem is more severe in the capital FOC. Setting $se\left(\varepsilon_t^r\right) = se\left(\varepsilon_t^w\right)$ eliminated the asymmetry but is not an option for the econometrician. The only potential solution to this problem is the use of instrumental variables (IV) estimators. In our case, since we know the true characteristics of the data, we can make use of good instruments to estimate the FOCs. By construction, the first lag of factor payments will be strongly correlated with their contemporaneous value as it has been generated by (25) and (26) with labor and capital following a non-stationary process (22)-(23). However, the shocks to factor payments in t and $t-1$ are un-correlated. This implies that the first lags

bias with more than one regressor is generally unknown, and researchers would have to resort to simulation to understand how measurement error may be affecting their estimated parameters.

[28] We analyzed this argument further obtaining by simulation of what the *plim* of the estimated coefficient would be given our shock variances and the simulated data for r and w in the MC experiment. The results obtained yielded coefficient values very close to those obtained in estimation, reinforcing the case for this explanation of the OLS bias.

[29] Although, strictly speaking, this trending aspect will also be affected by the dynamics of Γ_t^N.

of $\log(r)$ and $\log(w)$ can be used as instruments for their contemporaneous values in (14) and (13) and estimate the equation using Two-Stages Least Squares (2SLS). The results (available on request) show that the IV estimator resolves the estimation bias problem, but only as the sample size increases. With $T = 30$ substantial biases persist, but for $T = 100$ the IV estimator correctly identifies technical progress and the substitution elasticity even for true values up to 1.3. The obvious problem with this approach is that, for practical purposes, the econometrician may not have good instruments and enough observations to eliminate this endogeneity/measurement error problem.[30] For instance, in practise, unlike our experiments, shocks to factor markets tend to be auto-correlated. If this is the case, one should have to use at least more complex lag structures for the instruments to achieve identification.[31]

The Kmenta approximation, as discussed earlier, cannot identify technical progress parameters and so we only report the results for σ. Results show that this estimation method performs poorly at identifying σ, which is consistently underestimated. It is noteworthy that as T increases, the Kmenta approximation does a better job at identifying the true value of the elasticity when it is close to unity.[32] This confirms our previous argument that the Kmenta approximation deteriorates especially when it is far from the supporting unitary value.

In terms of the non-linear direct estimate of the production function, its performance is close but inferior to the labor FOC in terms of estimating the substitution elasticity but it has of course the advantage of being able to identify the individual technical progress parameters.

Results from the normalized system identify it as the superior method.[33] Estimates of both the elasticity of substitution and technical change are very close to their true values. This is irrespective of whether we pre-fix the normalization constant to unity or not.[34] The system (as we shall see in section 8.2) performs well

[30]The FOCs equations were also estimated using Fully Modified OLS methods, but the results remained very close to those obtained via OLS.

[31]We repeated the IV estimation experiment assuming that the shocks to factor markets are autocorrelated with an autocorrelation coefficient of 0.5. The results showed that using the first lag as instrument did not resolve the problem of the OLS bias.

[32]For instance, for $T = 100$ and $\sigma = 0.9$, the median values obtained for the technical progress configurations shown in Tables 3a and 3b are 0.86 and 0.82 respectively.

[33]The estimator used for the system is a non-linear Feasible Generalized Least Squares (FGLS) method which accounts for possible cross equation error correlation (much like a SUR model in linear contexts). The estimator, as implemented in the RATS programming language, performs NLLS on each individual equation and uses the estimated errors to build a variance-covariance (VCV) matrix and then estimates the system by GLS, completing one iteration. The estimated VCV matrix will be updated with each iteration until the system converges to a predetermined criterion.

[34]The reported results were obtained without pre-fixing ξ.

even for relatively small samples. Although our system estimation, unlike single-equation first order equations, were not sensitive with respect to simultaneous bias, we also checked the performance of the system method under different estimation techniques by using a 3SLS non-linear estimator (GMM) where we instrumentalized the variables with their first lag. The results did not change, yielding again very precise estimates of the true parameter values.

7.1 Normalization versus Non-Normalization

A legitimate question to ask is whether normalization makes a difference for estimation results with the system.[35] As we know, the interpretation of parameters with non-normalized production functions will in general be different depending on initial values of the DGP. The normalized system, however, is, by definition, invariant to initial values.

Accordingly, in addition to normalized system (19)-(21) we also estimate the non-normalized system:

$$\log(r) = \log(\pi) + \frac{1}{\sigma}\log\left(\frac{Y}{K}\right) + \frac{\sigma-1}{\sigma}\left(\log(C) + \gamma_K t\right) \tag{29}$$

$$\log(w) = \log(1-\pi) + \frac{1}{\sigma}\log\left(\frac{Y}{N}\right) + \frac{\sigma-1}{\sigma}\left(\log(C) + \gamma_N t\right) \tag{30}$$

$$\log(Y) = \log(C) + \frac{\sigma}{\sigma-1}\log\left[\pi\left(e^{\gamma_K t}K\right)^{\frac{\sigma-1}{\sigma}} + (1-\pi)\left(e^{\gamma_N t}N\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{31}$$

As discussed earlier, the major difference between the non-normalized system (29)-(31) and the normalized system (19)-(21) is that, in the former, parameters C and π are not "deep" but dependent on data values at the normalization point and the substitution elasticity (recall equations (11) and (12)).

Table 4 presents some consistent sets of (deterministic) initial values for generating data and the implied ranges of the true values of C and π and for $\sigma \in [0.2, 1.3]$. In all cases we assumed $\Gamma_0^K = \Gamma_0^N = 1$. The first row, with initial values of $\Gamma_0^K = \Gamma_0^N = Y_0 = 1$, represents a special case because indexing by the point of normalization equaling one is neutral implying that the true value of $C = 1$ and $\pi = \pi_0 = r_0 = 0.4\ \forall\sigma$. In this special case it does not matter if the same initial values of parameters are used, whether the system is estimated in normalized

[35] From the point of view of estimating the first order conditions and its relationship to normalization, the initial value does not matter, because they can be estimated by linear estimation methods and the estimated constant takes care of all variation in initial values in generating data.

or non-normalized form.

In all other cases, however, this is not so. To illustrate, in these other cases we have adjusted the initial conditions for output to make them consistent with an initial (and arguably reasonable) value for r equal to 5%. The sample average normalization insulates the normalized system from the effects of changes in initial values in generating the data but the true values of composite parameters C and π vary widely: $C \in [0.16, 0.49]$, $\pi \in [0.29, 0.99]$ (interestingly, the actual income distribution of the data appears unrelated to the true value of π). This illustrates the difficulty that a practitioner faces when trying to estimate non-normalized system (29)-(31); actual data scarcely gives any guidelines for appropriate choices for the initial parameter values of C and π and that results in serious estimation problems.

To examine how uncertainty relating the true values of C and π - and the resulting difficulty to define proper initial parameter values for these parameters - affect estimation results, we created data with starting values as presented in the last two rows of Table 4. Thereafter, we estimated the normalized and non-normalized systems. In the latter case the initial parameter values for C and π are selected randomly from their given range. In the first (normalized) case, the distribution parameter $\overline{\pi}$ and normalization constant ξ can be pre-set or (as here) freely estimated. In the estimated case, we have natural priors of the sample average of the capital income share and unity, respectively.

This comparison is presented in **Table 5** where, for brevity, we highlight the $\sigma = 0.5$ and $\sigma = 1.3$ cases (the remainder are available on request). One conjecture rationalizing this result is that to compensate large deviation in initial C from its true value, the estimation algorithm might minimize this discrepancy via a local maximum for $\widehat{\sigma}$, such that $\widehat{\sigma} \to 1$,hence $\frac{\widehat{\sigma}-1}{\widehat{\sigma}} \to 0$; as can be seen from (29) and (30), this diminishes the contribution of an incorrect C to overall fit.

This bias increases the more initial conditions depart from their true values. The fact that both $\widehat{C}$ and $\widehat{\pi}$ substantially departs from their true, theoretical values, leads to biased estimates of the substitution elasticity and technical change. There are, hence, enormous advantages of normalization arising from the pre-fixing of the distribution parameter and a good initial guess for the normalization constant (which could further be fixed to unity). Normalization (when combined with a system approach) appears to be convenient not only for the theoretical interpretation of deep parameters of the economy, but also for estimation.

8 Some Robustness Exercises

Given that results strongly indicate the superiority of the normalized system, we proceed to investigate some key robustness concerns: namely, (i) residual auto-correlation, (ii) sample-size power and (iii) alternative forms of technical progress.

8.1 Alternative Shock Processes

We implemented the following auto-correlated shock processes:

(a) AR errors in the technology shocks ($\varepsilon_t^{\Gamma^K}$ and $\varepsilon_t^{\Gamma^N}$).

(b) AR errors in the FOC's for N and K (ε_t^r and ε_t^w).

(c) (a) and (b) together.

These innovation processes take the form: $\epsilon_t^i = \rho \epsilon_{t-1}^i + \vartheta_t$, $\vartheta_t \sim N\left(0, se\left(\vartheta_t\right)\right)$, $\epsilon_0^i = 0$ where two ρ values were used: 0.5 and 0.8. The latter represents a very high degree of persistence for annual data; caution is therefore warranted since this would imply that our variables are almost not co-integrated (especially for T=25-35). For brevity, we summarize the outcomes without detailing all the numbers:

1. Overall, when ρ= 0.5 there is no significant bias for any parameter regardless of the sample size. The results do not change if we consider technical shocks and FOC shocks as being both auto-correlated (case (c)).

2. When ρ= 0.8 there is only one case in which we have found some bias: for σ= 0.5 and T = 25 and 30 and option (c) implemented. Surprisingly, in the rest of cases there is only a very small bias in the technical progress coefficients, which almost disappears for T = 50 and 100.

8.2 Sample Size Robustness in the System

The **Graph** shows the performance of the normalized system (for brevity we concentrate on the $\sigma = 0.5$, $\gamma_N = 0.015$, $\gamma_K = 0.005$ case) when estimated over $T \in \{25, 100\}$. The system appears quite robust to sample-size variations. The main benefit of larger sample sizes relates to narrower confidence intervals although most of that benefit is achieved by $T = 40, 50$.

8.3 Alternative Forms of Technical Progress

So far, as in the bulk of empirical studies, we assumed linear (constant growth) technical progress. However, recent contributions as in Acemoglu (2002a, 2003), McAdam and Willman (2008) have highlighted the role of induced (or directed) innovations in shaping the dynamics of income distribution. Steady factor incomes can only be achieved if technical progress is purely labor-augmenting. However, in the transition towards that steady state, we might expect periods of capital-augmenting technical progress induced by endogenous changes in the direction of innovations. Thus, it is not unreasonable to think of non-constant rates of technical progress. The question then becomes how can this be done in a tractable manner. Klump et al. (2007) proposed the use of a more flexible specification for Γ_t^i based on the Box-Cox transformation. In the normalized CES function this implies that $\Gamma_t^i = e^{g_i\left(t,\bar{t}\right)}$ where $g_i\left(t,\bar{t}\right) = \frac{\gamma_i}{\lambda_i}\left(\left[\frac{t}{\bar{t}}\right]^{\lambda_i} - 1\right)$, $i = K, N$. Curvature parameter λ_i determines the shape of the technical progress function. $\lambda_i = 1$ yields the (textbook) linear specification; $\lambda_i = 0$ a log-linear specification; and $\lambda_i < 0$ a hyperbolic one for technical progress.

Accordingly, we analyzed the outcome of the Monte Carlo experiment for a system generated as in Section 5.2 but using the Box-Cox specification for $g_i\left(\cdot\right)$ as $\Gamma_t^i = e^{g_i\left(t,\bar{t}\right)+\varepsilon_t^{\Gamma i}}$. Together with values for $\sigma \in [0.2, 1.3]$, we used the three following parameterizations:

(a) $\gamma_N = 0.015,\ \gamma_K = 0.005,\ \lambda_N = \lambda_K = 1.0.$

(b) $\gamma_N = 0.015,\ \gamma_K = 0.030,\ \lambda_N = 0.75,\ \lambda_K = 0.5.$

(c) $\gamma_N = 0.030,\ \gamma_K = 0.015,\ \lambda_N = 1.00,\ \lambda_K = 0.2.$

The first case corresponds to the linear technological progress specification used in the previous experiments, which we analyze as a cross-check of earlier results. The second corresponds to a situation where the growth in both labor- and capital-augmenting technical progress continuously decelerates and converge asymptotically to zero (albeit faster for capital-augmenting technical progress). Case (c) implies that labor-augmenting technical progress is linear with capital-augmenting declining towards zero somewhat faster than in case (b). In all cases, the standard errors of the technology shocks were set to 0.01, as in several of these specifications technological progress continuously decelerates and the stochastic part would dominate. This is also the reason why we choose slightly higher values for γ_K and γ_N for cases (b) and

(c) than in the previous experiments as, in these cases, low and declining rates of technical progress are not economically distinguishable from zero.

Table 6 reports the median value of the 5,000 draws for the relevant parameters using a sample size of T=50. In all the cases, the estimate of σ remains very close to its true value. The technical progress coefficients γ_K and γ_N are also captured well, although the bias is slightly larger than that obtained using the linear specification of previous sections. This is also the case for the curvature parameters λ_N and λ_K, where the estimated coefficients are very close to the true ones, but we can observe upward biases especially for values of $\sigma = 1.3$. This, however, is not surprising given the strong non-linearities introduced by the new terms and, in general, we see the system remains robust to the introduction of non-constant rates of technical progress.

9 Conclusions

The elasticity of substitution between capital and labor and the direction of technical change are pivotal parameters in many areas of economics. The received wisdom, in both theoretical and empirical literatures, suggests that their joint identification is infeasible. If so, this would render indeterminate a wide range of economic inquiries. However, given the vigor of recent debates on biased technical change (Acemoglu (2002a)); the shape of the local/global production function (Acemoglu (2003), Jones (2005)); the importance of normalization (La Grandville (1989), Klump and de La Grandville (2000)); and renewed interest in the estimated CES function itself (Klump et al. (2007)), disentangling these effects remains a key, unresolved matter.

We re-examined these issues using a comprehensive Monte Carlo exercise. We confirm that using many conventional approaches, identification problems can be substantial. In terms of the success of the FOCs, results depend on the relative shock processes of the measurement errors (implying that the labor FOC equation tends to work better). Although we derived some new identification results for the normalized (factor-augmenting) Kmenta approximation, identification of the substitution elasticity remains poor and that of technical change bleak. Also, direct estimation of the non-linear CES function remains highly problematic. However in contrast to the conventional approaches, our results suggested that the system approach of jointly estimating the FOCs and the production function worked extremely well and appeared robust to error mis-specification, sample-size variation and alternative forms of technical progress. Normalization adds considerably to these gains:

it allows the pre-setting of the capital income share; it provides a clear correspondence between theoretical and empirical production parameters; allows us ex-post validation of estimated parameters; and facilitates the setting of initial parameter conditions.

Accordingly, our results offer relief to the chronic identification concerns raised in the literature. Thus, we hope to have contributed towards better estimation practices, a better understanding of previous empirical findings, as well as to a more wide-spread appreciation of the properties of factor-augmenting (normalized) CES functions.

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Table 1. Empirical Studies of Aggregate Elasticity of Substitution and Technological Change in the US

Study	Sample	Assumption on Technological Change	Estimated Elasticity of Substitution $\hat{\sigma}$	Estimated Annual Rate Of Efficiency Change		
				Hicks Neutral: $\gamma_N = \gamma_K$	Labor-Augmenting: γ_N	Capital-Augmenting: γ_K
Arrow et al. (1961)	1909-1949	Hicks-Neutral	0.57	1.8	-	-
Kendrick and Sato (1963)	1919-1960	Hicks-Neutral	0.58	2.1	-	-
Brown and De Cani (1963)	1890-1918	Factor Augmenting	0.35	Labor saving ($\gamma_N - \gamma_K = 0.48$)		
	1919-1937		0.08	Labor saving ($\gamma_N - \gamma_K = 0.62$)		
	1938-1958		0.11	Labor saving ($\gamma_N - \gamma_K = 0.36$)		
	1890-1958		0.44	?		
David and van de Klundert (1965)	1899-1960	Factor Augmenting	0.32	-	2.2	1.5
Bodkin and Klein (1967)	1909-1949	Hicks-neutral	0.5-0.7	1.4-1.5		
Wilkinson (1968)	1899-1953	Factor Augmenting	0.5	Labor saving ($\gamma_N - \gamma_K = 0.51$)		
Sato (1970)	1909-1960	Factor Augmenting	0.5–0.7	-	2.0	1.0
Panik (1976)	1929-1966	Factor Augmenting	0.76	Labor saving ($\gamma_N - \gamma_K = 0.27$)		
Berndt (1976)	1929-1968	Hicks-neutral	0.96-1.25	?	-	-
Kalt (1978)	1929-1967	Factor Augmenting	0.76	-	2.2	0.01
Antràs (2004)	1948-1998	Hicks-neutral	0.94-1.02	1.14	-	-
		Factor-augmenting	0.80	Labor saving ($\gamma_N - \gamma_K = 3.15$)		
Klump, McAdam and Willman (2007)	1953-1998	Factor-augmenting	0.56	-	1.5	0.4

Table 2. Monte Carlo Parameter Values.

Parameter	Values
π : Distribution parameter	0.4
σ : Substitution elasticity	0.2, 0.5, 0.9, 1.3
γ_K : Growth Rate of capital-augmenting technical progress*	0.00, 0.005, 0.01, 0.015, 0.02
γ_N : Growth Rate of labor-augmenting technical progress*	0.02, 0.015, 0.01, 0.005, 0.00
η : Labor Force growth rate	0.015
κ : Capital Stock growth rate	$\eta + \gamma_N$
$se(\varepsilon_t^N), se(\varepsilon_t^K)$: Standard Error in the Labor and Capital DGP shock	0.10
$se(\varepsilon_t^{\Gamma^K})$: Standard Error in Capital-Augmenting Technical Progress shock	0.01 for $\gamma_K = 0$; 0.05 for $\gamma_K \neq 0$
$se(\varepsilon_t^{\Gamma^N})$: Standard Error in Labor-Augmenting Technical Progress shock	0.01 for $\gamma_N = 0$; 0.05 for $\gamma_N \neq 0$
$se(\varepsilon_t^w)$: Standard Error of Real Wage shock	0.05
$se(\varepsilon_t^r)$: Standard Error of Real Interest Rate shock	0.30
$se(\vartheta_t)$: Standard Error of AR(1) error shock	0.10
T: Sample Size (annual)	25 – 100
M: Monte Carlo Draws	5,000

Note: *, $\gamma_K + \gamma_N = 0.02$

Table 3a. $T = 50, \gamma_K = 0.005, \gamma_N = 0.015$

	Single Equation FOCs				Kmenta	Non-Linear CES	System
	K_FOC	N_FOC	Factor Prices	Factor Shares			
$\sigma = 0.2$							
$\hat{\sigma}$	0.224 [0.185 : 0.602]	0.211 [0.107 : 0.452]	0.170 [0.145 : 0.188]	0.164 [0.136 : 0.184]	0.180 [-0.3662 : 0.726]	0.261 [0.078 : 1.000]	0.232 [0.204 : 0.287]
$\hat{\gamma}_K$	0.004 [-0.011 : 0.008]	—	—	—	—	0.004 [-0.016 : 0.030]	0.004 [0.000 : 0.007]
$\hat{\gamma}_N$	—	0.014 [0.009 : 0.018]	—	—	—	0.014 [-0.001 : 0.024]	0.015 [0.010 : 0.019]
$\hat{\gamma}_K - \hat{\gamma}_N$	—	—	-0.011 [-0.017 :- 0.006]	-0.011 [-0.018 :- 0.005]	—	-0.011 [0.037 : 0.034]	-0.010 [-0.015 : -0.007]
$\hat{\zeta}$	—					1.026 [0.996 : 1.098]	1.028 [1.007 : 1.077]
$\sigma = 0.5$							
$\hat{\sigma}$	0.436 [0.357 : 0.539]	0.537 [0.369 : 0.718]	0.291 [0.195 : 0.382]	0.204 [0.113 : 0.306]	0.3470 [-0.603 : 1.173]	0.422 [0.109 : 1.110]	0.541 [0.502 : 0.601]
$\hat{\gamma}_K$	0.003 [-0.005 : 0.006]	—	—	—	—	0.003 [-0.019 : 0.028]	0.005 [0.002 : 0.01]
$\hat{\gamma}_N$	—	0.014 [0.005 : 0.020]	—	—	—	0.015 [-0.002 : 0.025]	0.015 [0.011 : 0.019]
$\hat{\gamma}_K - \hat{\gamma}_N$	—	—	-0.013 [-0.029 : 0.003]	-0.013 [-0.034 : 0.006]	—	-0.012 [-0.043 : 0.030]	-0.009 [-0.016 : -0.002]
$\hat{\zeta}$	—					1.015 [0.981 : 1.055]	1.008 [0.999 : 1.034]
$\sigma = 0.9$							
$\hat{\sigma}$	0.481 [0.426 : 0.551]	0.870 [0.624 : 1.108]	0.277 [0.153 : 0.456]	0.041 [-0.074 : 0.147]	0.4196 [-1.21 : 1.98]	0.612 [0.107 : 2E5]	0.873 [0.804 : 0.951]
$\hat{\gamma}_K$	0.002 [-0.014 : 0.018]	—	—	—	—	0.003 [-0.038 : 0.078]	0.005 [-0.005 : 0.025]
$\hat{\gamma}_N$	—	0.017 [-0.008 : 0.043]	—	—	—	0.015 [-0.022 : 0.044]	0.016 [-0.001 : 0.031]
$\hat{\gamma}_K - \hat{\gamma}_N$	—	—	-0.015 [-0.042 : 0.013]	-0.015 [-0.043 : 0.014]	—	-0.012 [-0.079 : 0.113]	-0.011 [-0.046 : 0.025]
$\hat{\zeta}$	—					1.002 [0.963 : 1.036]	0.997 [0.995 : 1.005]
$\sigma = 1.3$							
$\hat{\sigma}$	0.483 [0.417 : 0.561]	1.049 [0.726 : 1.345]	0.228 [0.098 : 0.419]	-0.095 [-0.306 : 0.024]	0.386 [-1.487 : 2.06]	1.018 [-1E5 : 3E6]	1.237 [0.930 : 1.355]
$\hat{\gamma}_K$	0.002 [-0.018 : 0.022]	—	—	—	—	0.005 [-0.032 : 0.107]	0.006 [-0.006: 0.068]
$\hat{\gamma}_N$	—	0.016 [-0.015 : 0.049]	—	—	—	0.014 [-0.055 : 0.039]	0.014 [-0.028 : 0.066]
$\hat{\gamma}_K - \hat{\gamma}_N$	—	—	-0.015 [-0.045 : 0.015]	-0.014 [-0.042 : 0.013]	—	-0.009 [-0.069 : 0.167]	-0.008 [-0.134 : 0.095]
$\hat{\zeta}$	—					0.976 [0.941 : 1.021]	0.990 [0.975 : 1.003]

Table 3b.: $T = 50, \gamma_K = 0.015, \gamma_N = 0.005$

	Single Equation FOCs				Kmenta	Non-Linear CES	System
	K_FOC	N_FOC	Factor Prices	Factor Shares			
$\sigma = 0.2$							
$\hat{\sigma}$	0.196 [0.165 : 0.422]	0.226 [0.114 : 0.428]	0.170 [0.145 : 0.189]	0.163 [0.136 : 0.184]	0.161 [-0.279 : 0.569]	0.283 [0.095 : 0.999]	0.238 [0.206 : 0.303]
$\hat{\gamma}_K$	0.013 [0.005 : 0.017]	—	—	—		0.013 [-0.010 : 0.077]	0.014 [0.010 : 0.017]
$\hat{\gamma}_N$	—	0.004 [0.001 : 0.007]	—	—		0.004 [-0.010 : 0.009]	0.004 [0.000 : 0.007]
$\hat{\gamma}_K - \hat{\gamma}_N$	—	—	0.008 [0.001 : 0.013]	0.007 [0.001: 0.013]		0.009 [-0.018 : 0.089]	0.010 [0.006 : 0.014]
$\hat{\zeta}$	—					1.022 [0.999 : 1.085]	1.029 [1.009 : 1.083]
$\sigma = 0.5$							
$\hat{\sigma}$	0.405 [0.325 : 0.498]	0.519 [0.360 : 0.691]	0.293 [0.201 : 0.383]	0.206 [0.116 : 0.307]	0.335 [-0.455 : 1.009]	0.418 [0.126 : 1.020]	0.545 [0.498 : 0.616]
$\hat{\gamma}_K$	0.011 [0.000 : 0.016]	—	—	—		0.010 [-0.012 : 0.046]	0.016 [0.011 : 0.020]
$\hat{\gamma}_N$	—	0.004 [-0.003 : 0.009]	—	—		0.006 [-0.010 : 0.013]	0.004 [0.000 : 0.009]
$\hat{\gamma}_K - \hat{\gamma}_N$	—		0.002 [-0.015 : 0.018]	-0.001 [-0.021 : 0.020]		0.004 [-0.024 : 0.057]	0.012 [0.004 : 0.020]
$\hat{\zeta}$	—					1.016 [0.986 : 1.054]	1.012 [1.001 : 1.041]
$\sigma = 0.9$							
$\hat{\sigma}$	0.475 [0.416 : 0.544]	0.853 [0.598 : 1.095]	0.280 [0.148 : 0.451]	0.041 [-0.075 : 0.147]	0.385 [-1.141 : 1.728]	0.608 [0.108 : 3.2E5]	0.881 [0.805: 0.952]
$\hat{\gamma}_K$	0.006 [-0.009 : 0.022]	—	—	—		0.009 [-0.025 : 0.073]	0.014 [-0.007 : 0.035]
$\hat{\gamma}_N$	—	0.009 [-0.014 : 0.034]	—	—		0.008 [-0.027 : 0.030]	0.006 [-0.010 : 0.021]
$\hat{\gamma}_K - \hat{\gamma}_N$	—		-0.005 [-0.031 : 0.022]	-0.005 [-0.033 : 0.022]		0.002 [-0.053 : 0.109]	0.008 [-0.028 : 0.045]
$\hat{\zeta}$	—					1.003 [0.964 : 1.036]	0.998 [0.996 : 1.006]
$\sigma = 1.3$							
$\hat{\sigma}$	0.493 [0.426 : 0.570]	1.106 [0.749 : 1.427]	0.232 [0.102: 0.429]	-0.089 [-0.312 : 0.021]	0.332 [-1.476 : 1.850]	1.017 [-1.8E5 : 2.8E6]	1.242 [0.985 : 1.363]
$\hat{\gamma}_K$	0.005 [-0.014 : 0.024]	—	—	—		0.015 [-0.016 : 0.311]	0.016 [-0.062 : 0.073]
$\hat{\gamma}_N$	—	0.006 [-0.023 : 0.041]	—	—		0.003 [-0.217 : 0.026]	0.004 [-0.038 : 0.060]
$\hat{\gamma}_K - \hat{\gamma}_N$	—		-0.006 [-0.037 : 0.024]	-0.004 [-0.032 : 0.023]		0.013 [-0.042 : 0.540]	0.012 [-0.121 : 0.111]
$\hat{\zeta}$	—					0.976 [0.941 : 1.020]	0.990 [0.974 : 1.002]

Table 4. Consistent Normalization Values

N_0	π_0	r_0	K_0	$Y_0^* = Y_0 = \frac{r_0}{\pi_0} K_0$	$w_0 = \frac{(1-\pi_0)Y_0}{N_0}$	C		π	
						Max	Min	Max	Min
1	0.4	0.4	1	1	0.6	1		0.4	
1	0.4	0.05	5	0.625	0.375	0.352	0.157	0.998	0.315
1	0.4	0.05	8	1	0.6	0.488	0.157	0.999	0.292

Notes: C and π in the final two columns are calculated according to 4.4a and 4.4b for $\sigma \in [0.2, 1.3]$; outside of the "special case" note the following partial derivatives showing how ceteris paribus changes in initial values change these last two parameters: , $C_\sigma, C_{Y_0}, C_{K_0}, C_{w_0} > 0, C_{N_0} < 0; \pi_\sigma, \pi_{N_0} < 0, \pi_{K_0}, \pi_{w_0} > 0$.

Table 5. Normalized .vs. Non-Normalized System Results.

Case: $T = 50, \gamma_K = 0.005, \gamma_N = 0.015$

	Normalized	Non-Normalized	Normalized	Non-Normalized	Normalized	Non-Normalized	Normalized	Non-Normalized
	$N_0 = 1, K_0 = 5, Y_0 = 0.625$				$N_0 = 1, K_0 = 8, Y_0 = 1$			
	$\sigma = 0.5$		$\sigma = 1.3$		$\sigma = 0.5$		$\sigma = 1.3$	
$\hat{\sigma}$	0.541	0.808	1.237	1.090	0.541	0.870	1.237	1.039
$\hat{\gamma}_K$	0.005	-0.005	0.006	-0.032	0.005	-0.012	0.006	-0.042
$\hat{\gamma}_N$	0.015	0.030	0.014	0.036	0.015	0.028	0.014	-0.044
$\hat{\gamma}_K - \hat{\gamma}_N$	-0.009	-0.042	-0.008	-0.068	-0.009	-0.041	-0.008	-0.087
$\bar{\pi}$	Prefixed to Sample average	—	Prefixed to Sample average	—	Prefixed to Sample average	—	Prefixed to Sample average	—
$\hat{\pi}$	–	0.421	—	0.412	–	0.422	—	0.413
$\hat{\xi}$	1.008	–	0.990	–	1.008	–	0.990	–
$\hat{C}$	–	0.343	–	0.298	–	0.493	–	0.324

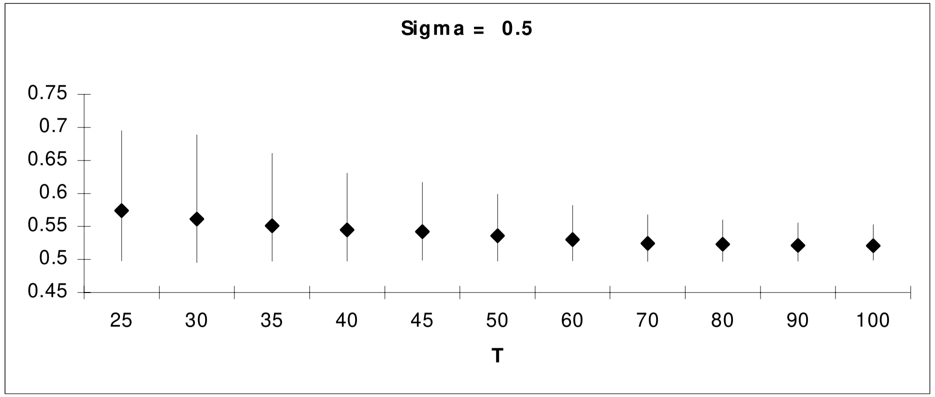
Note: Median values reported.

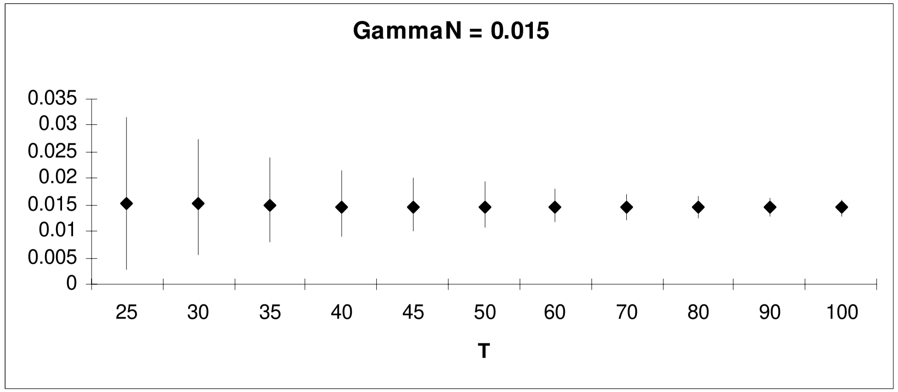
Table 6. System Results with Box-Cox technical progress functions.

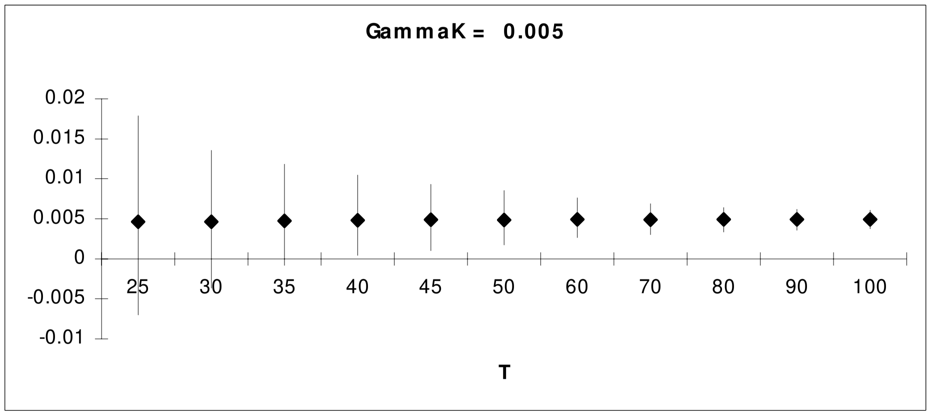
	$\sigma = 0.2$	$\sigma = 0.5$	$\sigma = 0.9$	$\sigma = 1.3$
	$\gamma_N = 0.015; \gamma_K = 0.005; \lambda_N = 1.0; \lambda_K = 1.0$			
$\hat{\sigma}$	0.205	0.513	0.893	1.282
$\hat{\gamma}_K$	0.005	0.005	0.007	0.008
$\hat{\gamma}_N$	0.016	0.016	0.015	0.015
$\hat{\lambda}_K$	1.050	1.061	1.030	1.050
$\hat{\lambda}_N$	1.014	1.050	1.046	1.050
	$\gamma_N = 0.015; \gamma_K = 0.03; \lambda_N = 0.75; \lambda_K = 0.5$			
$\hat{\sigma}$	0.204	0.512	0.893	1.273
$\hat{\gamma}_K$	0.032	0.030	0.033	0.035
$\hat{\gamma}_N$	0.016	0.015	0.013	0.014
$\hat{\lambda}_K$	0.477	0.513	0.537	0.529
$\hat{\lambda}_N$	0.698	0.751	0.785	0.699
	$\gamma_N = 0.03; \gamma_K = 0.015; \lambda_N = 1.0; \lambda_K = 0.2$			
$\hat{\sigma}$	0.203	0.513	0.893	1.279
$\hat{\gamma}_K$	0.013	0.012	0.018	0.013
$\hat{\gamma}_N$	0.029	0.032	0.026	0.030
$\hat{\lambda}_K$	0.232	0.267	0.322	0.471
$\hat{\lambda}_N$	1.015	0.983	1.035	0.990

Note: Median values reported.

Graph: Evolution of Estimated Substitution Elasticity and Technical Parameters (System Estimation).







Note: 10%, 90% percentiles lines reported with the 50% point indicated by a diamond.

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