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IDENTIFYING THE POWER-IN REGION FOR VORTEX-INDUCED VIBRATIONS OF LONG FLEXIBLE CYLINDERS

Susan B. Swithenbank
Massachusetts Institute of Technology
& Center for Ships and Ocean Structures at NTNU

Prof. J. Kim Vandiver
Department of Mechanical Engineering
Massachusetts Institute of Technology

ABSTRACT

The primary objective of this research is to locate the source of the vortex-induced vibrations (VIV) for long flexible cylinders at high-mode number and to help determine the source region for future predictions. The two Gulf Stream tests were conducted to collect data on a scale-model pipe that was excited at high-mode numbers.

The high density of the sensors on the pipe allowed for analysis that had not previously been done. Two methodologies are presented to locate the area of the region that is the source of the vibration.

In VIV, the current which causes the vibrations is important, because the speed of the current will determine the frequency of the vibration. Therefore, one important question is which section of the pipe will be the source of the vibrations for a known current profile. This source region is known as the power-in region. Regions on the pipe that are not a source of power instead damp the structural vibrations.

Once the region where the vibration originated has been found, the different phenomena that effect the location of the power-in region that were discovered are shown. Four different factors are presented that effect the locations of the power-in region: the angle of the pipe with respect to the vertical, the gradient of the current direction, the current profile, and the end effects at high mode number.

A dimensionless parameter is presented which help in the prediction of VIV given a current profile. The power-in factor predicts the region where the source of the vibration occurs using a combination of the current velocity and the source region length.

EXPERIMENTS

The Gulf Stream tests, conducted in the fall of 2004 and the fall of 2006, focused on long pipes in sheared flow. Both tests were part of a testing program developed with DEEPSTAR, a joint industry technology development project. The goals of the overall test program were to understand the dynamics of a pipe undergoing VIV at high mode number. This included VIV suppression with strakes, drag coefficients of bare and straked pipes, in-line and cross-flow VIV, and damping factors.

The Gulf Stream tests were conducted on the Research Vessel F. G. Walton Smith from the University of Miami. In 2004, a carbon fiber composite pipe 484 feet long and 1.4 inches in diameter. In 2006, a fiberglass and epoxy pipe with a length of 500.4 ft and a diameter of 1.43 inches was used. The pipe was spooled on a drum that was mounted on the aft portion of the ship. The pipe was lowered directly from the drum into the water. A railroad wheel weighing 805 lbs (dry weight, 725 lbs in water), was attached to the bottom of the pipe to provide tension.

The top end of the pipe was attached to the stern of the boat. The boat steered on various headings relative to the Gulf Stream so as to produce a large variety of sheared currents, varying from nearly uniform to highly sheared in speed and direction. Eight optical fibers were embedded in the outer layers of the composite pipe. Each fiber contained thirty five strain gauges, which use the principle of Bragg diffraction to measure strain with a resolution of approximately 1 micro-strain. The measurements have a spacing of 7 ft. The experiment set-up can be seen in Figure 1.

The pipe from the 2004 test was made of a carbon fiber composite with an HDPE liner. The pipe properties from the 2004 test pipe are found in Figure 1.

Table 1 –Gulf Stream 2004 Pipe Properties

Inner Diameter	1.05 in. (0.0267 m)
Outer Diameter	1.40 in.(0.0356 m)
Optical Fiber Position	1.30 in.(0.033 m)
EI	1.7e5 lb.in^2 (488 Nm^2)
Modulus of Elasticity (E)	2.30e6 lb./in^2 (1.586e10 N/m^2)
EA	8.5e5 lb. (3.78e6 N)
Weight in Seawater	0.12 lb./ft. (flooded in Seawater) (1.75 N/m)
Weight in air, w/trapped water	0.83 lb./ft. (12.11 N/m)
Density	0.053 lb/in^3 (1.47 g./cc).
Effective Tension	725 lbs submg. bottom weight (3225N)
Material	Carbon fiber –epoxy
Length	485.3 ft (147.3 m) (U-joint to U-joint)

A fiberglass-epoxy pipe with similar properties was used in the 2006 Gulf Stream test; the properties of the 2006 pipe are in Table 2.

Table 2 –Gulf Stream 2006 Pipe Properties

Inner Diameter	0.98 inch (0.0249 m)
Outer Diameter	1.43 inch (0.0363 m)
EI	1.483e3 lb ft^2 (613 N m^2)
EA	7.468e5 lb (3.322e6)
Weight in Seawater	0.1325 lb/ft (0.1972 kg/m)
Weight in air	0.511 lb/ft (0.760 kg/m)
Density	86.39 lb/ft^3 (1383 kg/m^3)
Effective Tension	725 lb
Material	Glass fiber epoxy composite
Length	500.4 ft

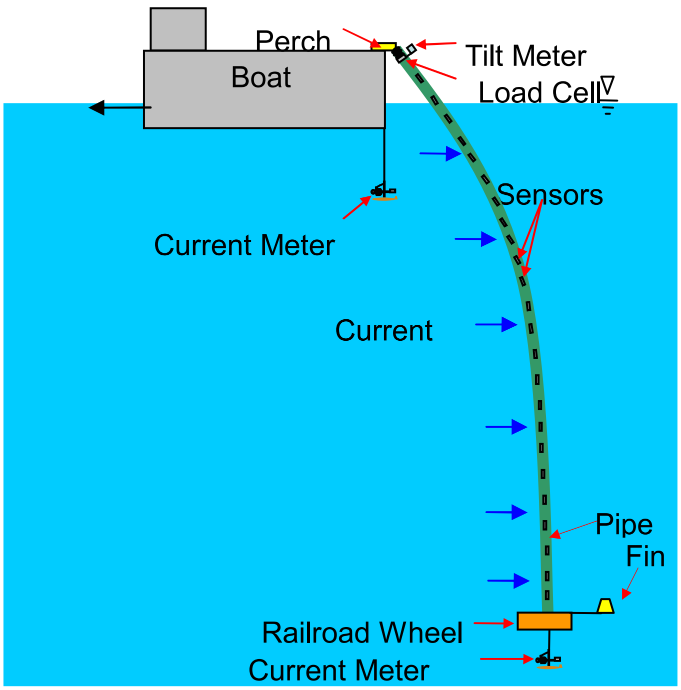

Figure 1 - Experimental Setup

For both tests, an Acoustic Doppler Current profiler (ADCP) recorded the current velocity and direction along the length of the pipe. On the R/V F. G. Walton Smith, there were two ADCPs. Each ADCP uses a different frequency to obtain currents at different depths. During the Gulf Stream tests both ADCPs were used to gather data.

Additional instrumentation during the tests included a tilt meter to measure the inclination at the top of the pipe, a load cell to measure the tension at the top of the pipe, two mechanical current meters to measure current at the top and the bottom of the pipe, and in the second Gulf Stream experiment a pressure gauge was used to measure the depth of the railroad wheel.

Significant wave induced vessel motion during the Gulf Stream test in 2004 caused tension variations and added low frequency components to the strain time series. An elliptical filter with a 1.5 Hz cut-off was used to remove this vessel motion from the data without interfering with the VIV frequencies. The filtering was done such that no phase shift was applied to the data. Tension fluctuations due to vessel motion varied from 10% to 25% of the mean. In the 2006, little variation was seen in the tension due to a calm sea state. A similar filter with a 1 Hz cut-off was applied to the 2006 data to remove the low-amplitude vessel motion.

REDUCED VELOCITY

The reduced velocity, V_r, is a dimensionless parameter used in the prediction of VIV on vibrating cylinders. The reduced velocity is defined as:

$$V_r = \frac{U_n}{f_v D} \qquad (1.1)$$

where D is the diameter, f_v is the frequency of vibration, and U_n is the normal incidence current.

For free vibration, the amplitude of vibration is dependant on the value of the reduced velocity. At reduced velocities of less than four, little vibration is seen. The largest amplitude of vibration is seen at a reduced velocity range of $5 < V_r < 7$ for sub-critical Reynolds numbers.

NORMAL INCIDENT CURRENT

At the top end, drag forces deflect the pipe from the vertical. An illustration of the shape of the pipe can be seen in Figure 2.

Since the pipe is not vertical, the current is not normal to the pipe. Only the component that is normal to the pipe is considered when calculating vibration of the pipe. Therefore:

$$U_n = U \cos\phi \qquad (1.2)$$

where Φ is the incidence angle of the pipe with the vertical.

For the Gulf Stream tests the inclination angle was as great as 60°. The normal component can reduced to 50% of the current measured by the ADCP.

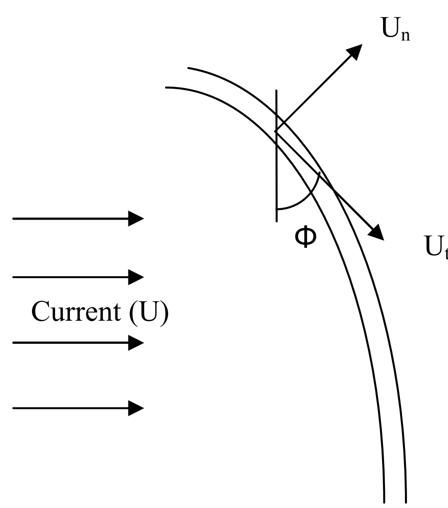

Figure 2 - An illustration of the inclined shape of the pipe.

FINDING THE POWER-IN REGION

On short pipes at low-mode number, standing wave response is frequently observed. In the Gulf Stream experiments, the length-to-diameter ratio was greater than 4000. Additionally the pipes responded at modes greater than the 10^{th}. At these mode numbers standing wave behavior over the entire pipe is not observed. Instead, short power-in regions are observed with traveling waves leaving the power-in region and propagating to other regions.

Presented here are two methods to find the power-in region, the reduced velocity method and the coherence mesh. The first method uses a coherence calculation to find the range over which the vibrations are linearly dependant. The large amplitude waves that are generated in the power-in region are expected to be coherent over a large range.

The second method uses the local reduced velocity to determine whether regions that are power-in. Reduced velocities from 5 to 7 are traditionally associated with large VIV response at sub-critical Reynolds number. This is used to verify the coherence mesh.

COHERENCE MESH

The coherence mesh method involves calculating the coherence from one sensor in a quadrant to every other sensor in the same quadrant. Coherence is a commonly used signal processing tool that shows linear dependence between to signals and is defined as [Oppenheim et al 1999]:

$$C_{xy}(f) = \frac{\left| P_{xy}(f)^2 \right|}{P_{xx}(f) P_{yy}(f)} \qquad (1.3)$$

where P_{xy} is the cross-spectral density between the two signals, P_{xx} and P_{yy} are the power spectral density of the signals respectively.

The first step in creating the coherence mesh is to find the distance over which the waves are coherent from each point. Starting with one sensor, the coherence is calculated from that sensor to every other sensor point on that fiber. For each frequency, a distance range is calculated by summing the distance both up and down the pipe over which the coherence at that frequency is greater than 0.7. This process is then repeated for every other sensor, until a distance range has been calculated for every sensor for every frequency. Figure 3 shows an example of the distance ranges from each sensor for an example from the Gulf Stream test.

The blue colors represent sensors and frequencies that are coherent over less than 50 ft. Therefore, these waves have little energy and do not travel large distances. The yellow and red tones represent waves that are traveling from 140 ft to 200 ft. The red colors show waves that are traveling over a significant distance. The sensors located at 175 ft to 375 ft all show significant coherence over a large distance at a frequency of approximately 4.2 Hz.

A blurred yellow zone can be seen at approximately 8-9 Hz. This represents the power-in region for the in-line component. The orientation of this fiber is not known with respect to the flow, and the sensor may be oriented in the cross-flow or the in-line direction, but is likely oriented in between the two directions, which would result in both cross-flow and in-line components in the spectrum.

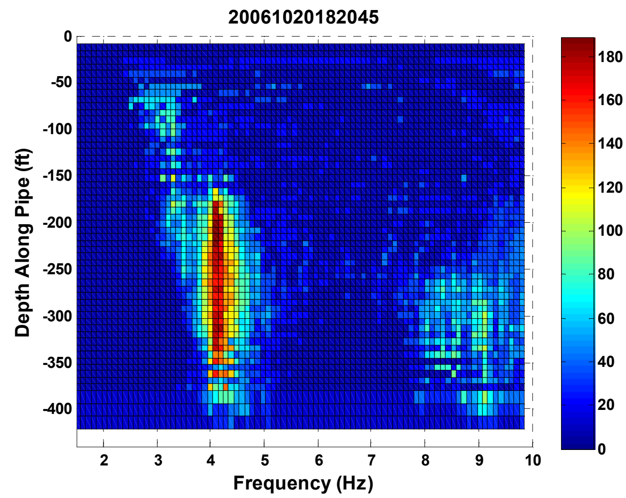

Figure 3 – Gulf Stream bare case coherence mesh showing the distance range from every point over which there is coherence of greater than 0.7. Distance in feet is given by the color bar at the right side.

More than one frequency can be seen to be coherent; this is because of the frequency changes over the test run. For the coherence calculation to be accurate, a number of averages must be taken. For the results shown here 12 to 15 overlapping averages were used. Each average contains 10 seconds of data with 50% overlap between averages. More averages in the coherence calculation can reduce the error from noise. Because response frequency varies over the course of the time record, increasing the amount of time used, and therefore the number of averages also has the effect of varying the dominant VIV frequency.

To define the power-in region, the point that has the largest total range, as calculated by the coherence mesh, is found. Then any point that has at least 70% of the range of the maximum point is considered part of the power-in region. Figure 4 shows the coherence mesh after the range cut-off has been applied. The red points represent the power-in region, where blue points represent the sensors and frequencies that are not adding power to the system.

The cut-off for coherence is based on signal processing literature that concludes that 0.7 indicates a high level of correlation between two signals. This cut-off is such that when the coherence is above 0.7 the signals are likely caused by the same source.

The 0.7 cut-off for the local range divided by the maximum range was set experimentally. This number is not exact, but gives a good estimate of the power-in region. The data is insensitive to this cut-off because the drop off in the range over a few sensors is significant. A cut-off of 0.5 to 0.9 can be used with similar results.

The coherence method is not meant to find the exact region of power-in; rather this method is an indication of where the vibration source is.

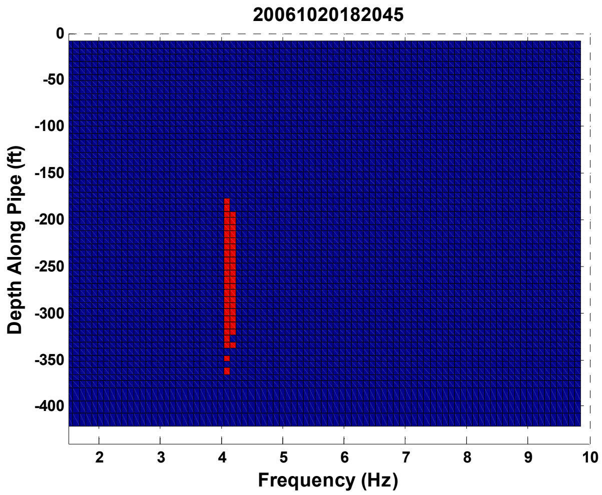

Figure 4 – Coherence mesh after the cut-off has been applied. The power-in region is shown in red.

REDUCED VELOCITY METHOD

In the Gulf Stream experiments, reduced velocities from approximately 3 to 7 were observed. Reduced velocities of 4.5 to 6.5 were observed in the regions with the largest Root Mean Square (RMS) strain response.

Figure 5(a) shows the normal incidence current profile for a run from the Gulf Stream test with both speed (blue) and direction (green). The current profile is sheared from 1.5 ft/s to 3.0 ft/s. The current direction is nearly uniform.

Figure 5(b) shows the total RMS strain (blue) and the RMS strain filtered to only contain the dominant VIV frequency (green).

Figure 6(a) shows the same current profile as Figure 5. Figure 6(b) shows the dominant VIV frequency for each location. The frequency that dominates for the most time is shown with the red dots, with the blue and green dots representing the maximum and minimum frequency observed. The maximum and minimum frequencies are due to variation of the dominant VIV frequency during the total record. This variation in frequency with a time invariant flow speed has been defined as time sharing [Swithenbank 2007].

Figure 6(c) shows the reduced velocity, calculated using the frequency of vibration. As the frequencies change with time sharing, the reduced velocities also vary. The reduced velocities are also show the maximum reduced velocity in green and the minimum reduced velocity in blue. These correspond to the change in reduced velocity caused by the shift in frequency over the entire time history.

At the top of the pipe, the current is smaller than lower in the pipe because of the normal incidence angle. The variation in reduced velocity is also larger here. The top of the pipe is an area with low RMS strain and is unlikely to be the power-in region. In this areas, traveling waves from the power-in region will sometimes dominate, which would cause the frequency to appear to be the same as in the power-in region. At other times, locally generated small amplitude waves that are caused by local currents, which are smaller than the current of the power-in region, will dominate. The difference in current speeds for the power-in region and the locally generated waves causes much greater variation in the frequency, because the local current speed and the current speed in the power-in region can be significantly different from each other.

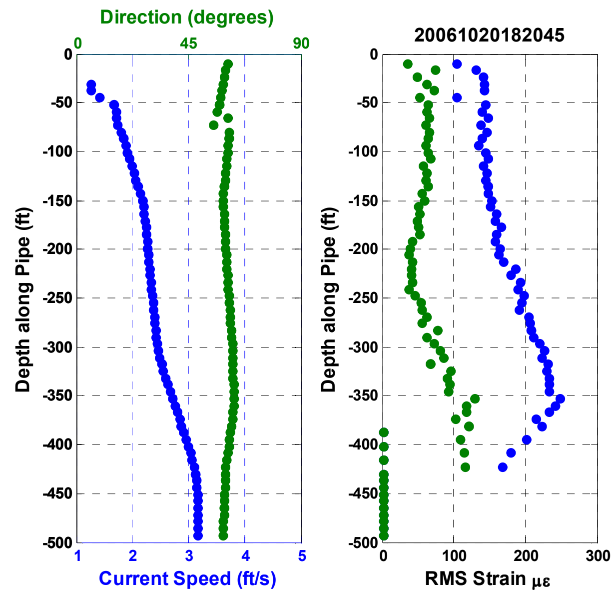

Figure 5 – Gulf Stream bare case (a) Current Speed (blue) and current direction (green); (b) RMS strain (blue) and RMS strain filtered to only show the contribution of the dominant VIV frequency.

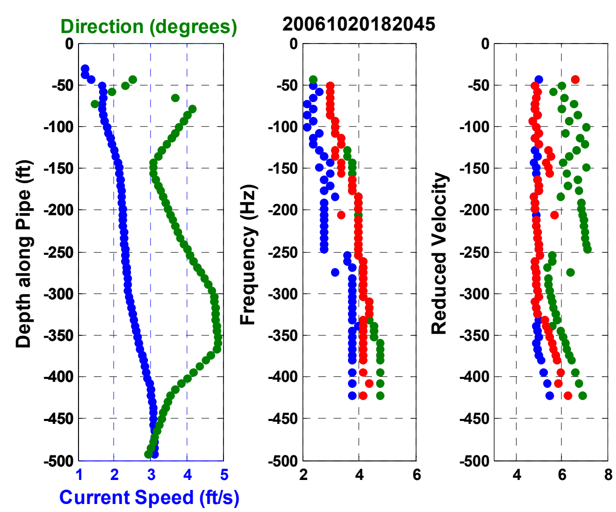

Figure 6 – Gulf Stream bare case (a) Current speed (blue) and the current direction (green); (b) Dominant VIV frequency, showing red, with the maximum frequency shown in green and the minimum in blue, the varying frequencies are due to time sharing; (c) the reduced velocity, using dominant VIV frequency with the same colors as (b), the variance is due to the variation in frequency with time shifting.

FACTORS INFLUENCEING THE POWER-IN REGION

In addition to the reduced velocity, three factors were found to effect the location of the power-in region: the angle of the pipe from the vertical, the gradient of the current direction, and boundary conditions.

THE ANGLE OF THE PIPE FROM THE VERTICAL

For the bare pipe cases from the Gulf Stream test, the maximum angle of the pipe with respect to vertical where a dominant power-in region existed was 47 degrees. This number was derived by looking at all the cases from the Gulf Stream to find the maximum incidence angle sustainable in a power-in region. Figure 7 and Figure 8 show an example from the Gulf Stream.

Figure 7 shows a case from the Gulf Stream test from 2004. On the left is the current profile in blue, with the power-in region superimposed on the current profile in green.

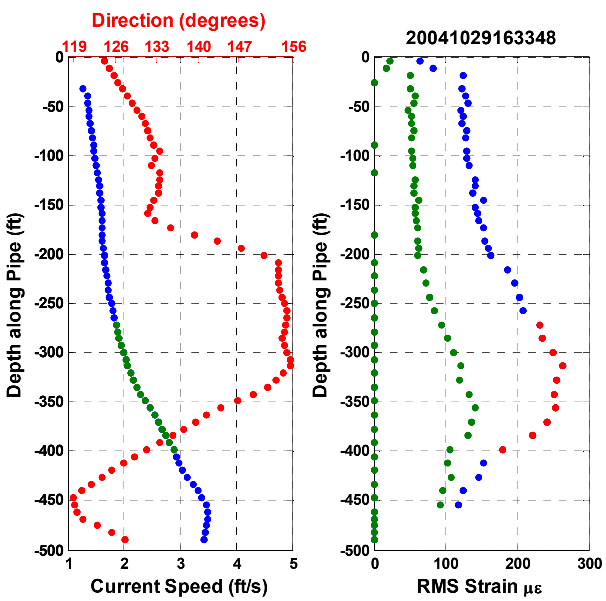

Figure 7 – Gulf Stream bare case (a) Current speed (blue), current direction (red), and power-in region (green); (b) Total RMS strain (blue) and RMS strain filtered (green) to only show the contribution of the dominant VIV frequency with power-in (red).

The direction of the current is shown in red. On the right is the RMS Strain, with the RMS strain for the dominant VIV frequency in green and the total RMS strain for the case shown in blue. The power-in region found using the methods shown above is superimposed on the total RMS strain in red.

Figure 8 shows the angle of the pipe with respect to the vertical versus depth. Note that this is not the shape of the pipe, but rather the angle of tilt at each location, where zero degrees is a vertical pipe.

The dominant power-in region is shown in green. For this case the dominant power-in region has a tilt angle with respect to the vertical which does not exceed 40 degrees. .

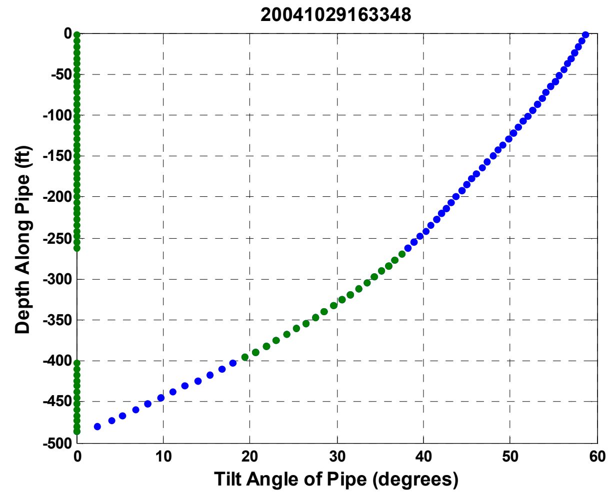

Figure 8 - Angle of the pipe with respect to vertical shown versus depth for the case of two power-in regions, showing the angle of incidence (blue) and the angle of incidence in the power-in region (green). A second power-in region is from 50 ft to 100 ft, but in this power-in region no large amplitude vibrations are seen.

THE GRADIENT OF THE DIRECTION OF THE CURRENT

The gradient of the direction of the current is important to the length of the power-in region. A rapid change in the direction of the current was found to prevent coherent vortex shedding.

Figure 9(a) shows the current profile (blue) and direction (red). The power-in region is overlaid on the current profile (green). The current direction varies 35 degrees over 150 ft with a gradient of 0.24 degree/ft. At the bottom of the power-in region, the direction gradient goes to 0.75 degree/ft. After analyzing each of the steady state runs, it was observed that the power-in region could not be sustained in an area with a large directional gradient.

Using the same current profile shown in Figure 9, Figure 10 shows the current direction (blue) overlaid with the power-in region (green). Additionally, the gradient of the current is shown in red. Over the power-in region, the gradient of the direction can be seen as between -0.5 degree/ft and 0 degree/ft.

Over all the runs analyzed from the Gulf Stream tests, the maximum allowable direction gradient within a power in region is ±0.5 degree/ft. Further investigation is needed to explain this observation.

Likely, the gradient of the current that is allowed in the power-in region will reduce to a dimensionless parameter, which depends on either diameter or wavelength.

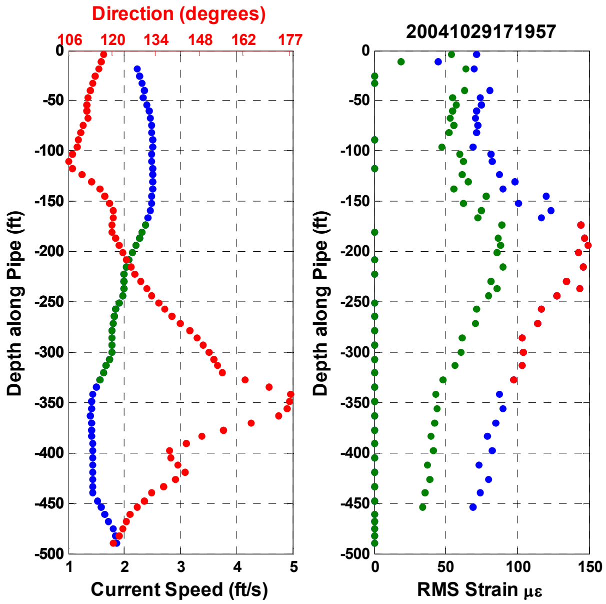

Figure 9 – Gulf Stream bare case (a) Current speed (blue), current direction (red), and power-in region (green); (b) Total RMS strain (blue) and RMS strain filtered to only show the contribution of the dominant VIV frequency (green). The power-in region found using the coherence method is shown on both graphs; (a) green and (b) red.

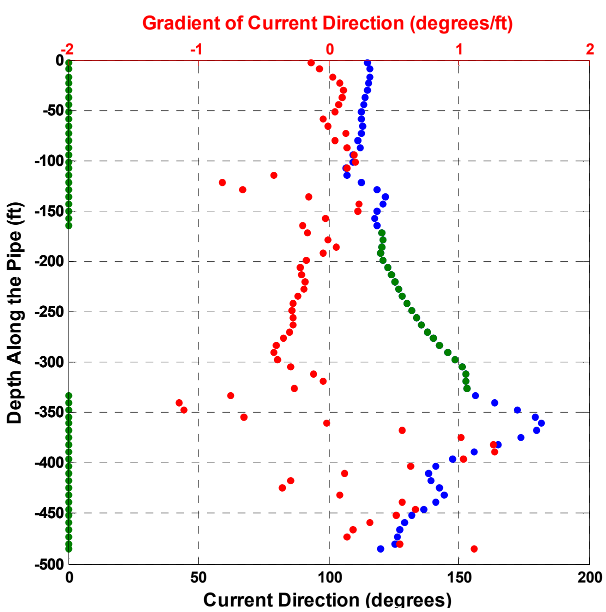

Figure 10 – Gulf Stream bare case; Current direction (blue) overlaid with the power-in region (green) found with the coherence mesh; The gradient of the current direction shown in red

REDUCED VELOCITY BANDWIDTH

The section of the pipe over which the wake is correlated is known as the power-in region. The correlated wake equates to a correlated input force, thus over this section of the pipe power is entering the system. The length of this power-in region is defined as L_{in}. The wake in this region is assumed to be correlated with a single frequency of input at any moment in time.

The current speed may change over the length of the power-in region; since the frequency is constant across the entire length, the reduced velocity must change in proportion to the change in current speed.

The percentage change in reduced velocity over the power-in region is known as the reduced velocity bandwidth, and is defined as:

$$dV_r = \frac{V_{r,\max} - V_{r,\min}}{V_{r,mean}} = \frac{U_{\max} - U_{\min}}{U_{mean}} \qquad (1.4)$$

For each of the bare Gulf Stream cases, the reduced velocity bandwidth can be calculated for the power-in region found by the coherence mesh. The reduced velocity bandwidths varied from 0.15 to 0.44 for all the cases. The cases with lower reduced velocity bandwidths had power-in regions that were limited by outside factors such as the angle of incidence of the current or the gradient of the direction of the current. When the power-in region was not limited by incidence angle or direction of boundaries, the reduced velocity bandwidth in the power-in region found by the coherence mesh was approximately 0.4.

END EFFECTS

While analyzing all the Gulf Stream data, an unexpected anomaly was that the power-in region was never found in the bottom 75-100 ft. One potential reason for this is the presence of a fixed boundary at which waves propagating down the pipe are reflected at the end. A large drop in the RMS strain can be seen in the bottom 75 feet.

Figure 11 shows a typical strain measurement from the second Gulf Stream experiment. Figure 11(a) shows the RMS strain from each quadrant, and Figure 11(b) is the normal incidence current profile. The power-in region is from approximately 300 to 400 ft. Below the power-in region, there is a significant drop off in the RMS strain measurement. The current is maximum at the bottom. Therefore previous understanding would have predicted this to be the power-in region.

One possible explanation for this phenomenon is hydrodynamic. The reflections of the waves at the boundary may interfere with the formation of the wake in this region. Computational fluid dynamics models have shown that in low-mode standing-wave cases the vortex sheets separate from the cylinder in phase. In longer high-mode number cases, the vortex sheets separate from a moving point which travels along the pipe at the same speed at the traveling waves of the pipe. This leads to vortex sheets which are diagonal to the cylinder.

Tests were conducted during both Gulf Stream experiments to determine whether the bottom universal joint and the railroad wheel assembly had undesired motion. The bottom assembly was found not to be the cause of the absence of a power-in region measured at the ends. When a test was done on a shorter length of pipe without traveling waves, the power-in region extended to the end.

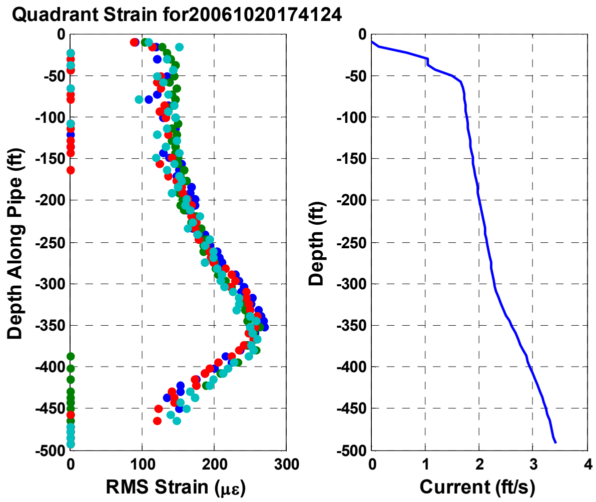

Figure 11 – (a) The quadrant strain for a bare test from the second Gulf Stream experiment. (b) the normal incidence current profile.

PREDICTION OF THE POWER-IN REGION

One of the problems with predicting VIV is finding the power-in region. Using a known current profile, a power-in factor, α, can be calculated for each point on the riser. The point with the largest power-in factor is assumed to be the approximate center velocity for the power-in region. Once the center velocity is found, using a reduced velocity bandwidth of 0.4, the power-in region can be predicted.

The power-in factor is the product of the cubed ratio of the local normal incidence current velocity to the maximum normal incidence current velocity and the ratio of the length of the power-in region available to the length of the pipe. (See Equation (1.5))

The region which dominates the VIV shedding response occurs at the location where the maximum power is available which is dependant on the current speed and the length of the power-in region. In [Vandiver 2002], Vandiver gave a modal explanation for the significance of the current cubed as the controlling parameter. Current ratio cubed is combined here with a length ratio to define the power-factor:

$$\alpha = \left(\frac{U_c}{U_{\max}}\right)^3 \left(\frac{L_{in}}{L}\right) \qquad (1.5)$$

Other factors, discussed above, can influence the determination of the power-in region. If the gradient of the current direction is greater than 0.5 degrees/ft, (for the Gulf Stream and Lake Seneca pipes), then alpha is zero. Additionally, if the incidence angle between the flow and the pipe is less than 45º, then alpha is zero. Lastly at the bottom of the pipe for the Gulf Stream tests, no power-in region was found in the bottom 75 ft. This may be caused by reflections interfering with the formation of a correlated vortex wake or other end effects. Therefore for the Gulf Stream tests, alpha for the bottom 75 ft was set to zero. This effect should be further investigated.

After calculating the power-in factor for each point, and taking in to account the other factors mentioned above, the location of the largest power-in factor is used to predict the center of the power-in region. Therefore the power-in factor is:

$$\alpha = \left(\frac{U_c}{U_{\max}}\right)^3 \left(\frac{L_{in}}{L}\right) f(\theta) g(\phi) h(z) \qquad (1.6)$$

where f(θ) is a factor that is 0 or 1 depending on the inclination angle at that point; g(Φ) is a factor that is 0 or 1 depending on the local current direction gradient; h(z) is a factor that is 0 or 1 depending on the proximity to the boundary for high mode number cases.

Figure 12 shows an example from the Gulf Stream test. In the figure, (a) is the RMS strain for each quadrant shown; (b) is a plot of the normal incidence current; (c) is the power-in factor, α including the effects of the current gradient, the incidence angle, and the boundary effects; and (d) is the length of the power-in region assuming that the location is the center of the power-in region. A reduced-velocity bandwidth of 0.4 is used to determine the length of the region about the each point.

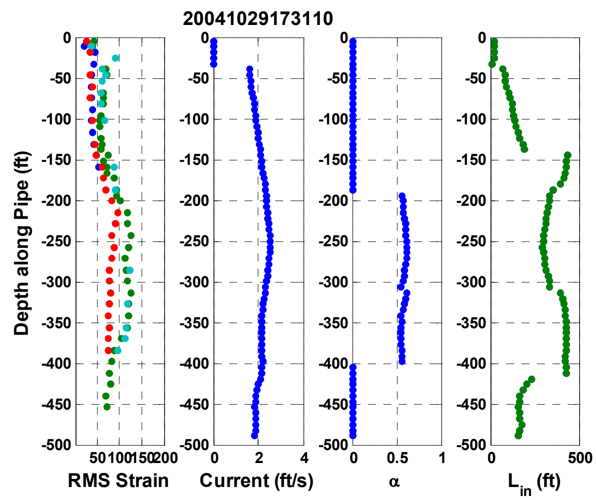

Figure 12 – (a) RMS Strain in each of the four quadrants; (b) The normal incidence current profile (ft/) (c) The Power-in Factor, α, (d) The Length of the power-in region assuming that each point is at the center velocity of the power-in region for test case 20041029173110

In Figure 12, the area over which the power-in factor is highest coincides with the region of highest RMS strain. The ends of the pipe can be seen to have little power-in available because the length of the power-in region is limited by the inclination at the top and unexplained boundary effects at the bottom.

CONCLUSIONS

Two valuable tools in locating the power-in region in data are shown here, the coherence mesh and the reduced velocity. The coherence mesh does have limitations. It can only be used on long cylinders at high mode number. When standing wave behavior is dominant in the test data, the coherence mesh does not work to locate the power-in region.

Using these tools, four influences to the location of the power-in region are shown:

- The power-in region will be dictated by the maximum current, unless either the gradient of the current direction is greater than 0.5 degree/ft or the pipes angle with respect to the vertical is greater than 47°.
- The angle of the pipe with respect to vertical in the power-in region can not be greater than 47°.
- No power-in regions were found in the bottom 75 ft because of the boundary conditions that was potentially caused by hydrodynamic effects.
- When the power-in region was not limited by incidence angle or direction of the current or other outside factors, the reduced velocity bandwidth in the power-in region was 0.40.

All of these conclusions are based on the data from the Gulf Stream tests. Various sets of tests could be conducted to help reduce the error from these tests and to more fully understand the different factors that affect the location and length of the power-in region. These results should be verified at otherl Reynolds number before they are used for riser design.

Lastly using the Gulf Stream data, the power-in factor was derived as a tool to locate the center velocity of the power-in region for a given current profile. Using a reduced velocity bandwidth of 0.4, the power-in region can be found.

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NOMENCLATURE

C_{xy}	Coherence Function
D	Diameter [in]
dV_r	Reduced Velocity Bandwidth
f	Frequency [Hz]
f_v	Frequency of Vibration [Hz]
$f(\theta)$	Function of the Local Current Direction Gradient
$g(\Phi)$	Function of the Local Angle of the Pipe
$h(z)$	Function of the Location w.r.t. the Boundary
L	Length of Pipe [ft]
L_{in}	Length of the Power-In Region [ft]
P_{xx}	Power Spectral Density
P_{xy}	Cross Spectral Density
U	Current Speed [ft/s]
U_c	Center Velocity in the Power-In Region [ft/s]
U_{max}	Maximum Current Speed in the Power-In [ft/s]
U_{mean}	Mean Current Speed in the Power-In [ft/s]
U_{min}	Minimum Current Speed in the Power-In [ft/s]
U_n	Normal Incidence Current Speed [ft/s]
V_r	Reduced Velocity [-]
α	Current Weight Factor [-]
Φ	Top tilt angle [-]

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