Ignorance, Debt and Financial Crises

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Abstract

Debt is optimal for trading, and the optimal collateral backing that debt is also debt. Debt as collateral transfers the most value intertemporally. When that debt is used as collateral for another debt contract, the "debt-on-debt" preserves symmetric ignorance because it minimizes the incentive to produce private information about the payoffs, so debt is least information-sensitive, i.e., liquid. But, bad public news (a shock) about the value of the collateral that backs the debt can cause information-insensitive debt to become information-sensitive. To prevent endogenous adverse selection agents reduce the amount of trade below the expected value of the debt collateral. The shock is amplified, a financial crisis.

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1. Introduction

In funding markets investors trade hundreds of millions or even billions of dollars very quickly without the need to conduct due diligence about the value of the security. Prime examples are Treasuries, repos, asset backed commercial papers (ABCP), Agency mortgage-backed securities (MBS) and money market fund (MMF) shares. Investors trade these debt instruments so as to manage their cash balances and short term liquidity needs. For a long time period these short term debt funding markets had been working very well. Therefore, the sudden breakdown of several types of these markets during the recent financial crisis came as a big surprise and raises several questions about how debt funding markets are functioning. Understanding the nature of liquidity provisions for financial institutions and corporations is central for the regulation of the banking and financial system.

A key characteristic of debt trading in funding markets is that investors trade debt instruments which use other debt contracts as collateral. ABCP is debt that is backed by commercial papers which are debt. MBS is debt and backed by a pool of mortgages which is debt. Repo is a debt contract that uses other debt instruments as collateral. Institutional investors can write checks (a debt claim) backed by MMF shares (portfolio of highly rated debt). In this paper we provide a theory of funding markets that explains the optimality of debt-on-debt, which has not been the focus in the academic literature and policy discussions but it is a central aspect of trade in funding markets. Our theory also shows that a collapse of trade in debt funding markets (financial crisis) is a discontinuous event and occurs when public news about fundamentals make investors "suspicious" about the value of the debt collateral that backs the tradable debt.

We consider a model with three dates (t=0,1,2) and three agents {A,B,C}. Agent A owns a project that delivers some uncertain amount of consumption goods date 2. Agent B has goods at date 0 but wants to consume at date 1. So at date 0 agent B wants to buy a security from agent A to store his goods. At date 1 agent B uses this security as collateral to trade with agent C for agent C's goods. We address two interrelated questions. First, what is the optimal collateral security for agent B to buy from agent A at date 0? Second, what is the optimal security backed by the collateral for agent B to sell to agent C at date 1 when there is public information and agent C can acquire private information about the payoff of the project?

In order to solve this two layer optimal security design problem with endogenous private information acquisition as well as the exogenous arrival of public news we introduce a new measure of tail risks called "information sensitivity". This measure captures the value of private information and thus an agent's incentive to produce information. When trading a security with low information sensitivity, agents have no incentive to acquire information and there is no endogenous adverse selection. We use this concept to derive three main results.

First we show that debt is least information sensitive. In particular, we show that it is not the "flat" part of a standard debt contract that is relevant for minimizing the incentive to produce information. The key driver for the optimality of debt when there is endogenous information production and potential adverse selection is the 45 degree line of the debt contract, i.e., the seniority of repayment. Intuitively speaking, private information is valuable to a buyer (agent C) if it helps him to avoid a loss in low payoff states by not buying the security. With seniority where the holder gets paid back first and gets everything what is available in low payoff states, this expected loss is smallest and therefore, the value of information and thus the incentive to acquire information is minimized.

Second, we show that debt-on-debt is optimal and equilibrium has the following properties. At date 0, agent B buys debt from agent A that is backed by the project. Then agent B uses that debt as collateral to sell another debt security to agent C. Debt-on-debt is optimal for the following reasons. Given an arbitrary collateral that agent B owns, selling debt to agent C is optimal because debt is least information sensitive and thus minimizes agent C's incentive to produce private information. There are two reasons why debt is also the optimal collateral. The information sensitivity of the tradable debt is (further) minimized by the debt collateral. And debt collateral is also optimal because its value is least sensitive to the arrival of public information and thus maximizes the value of the collateral when there is bad public news.

We argue that trade in funding markets is characterized by "trust" or the absence of private information acquisition (due diligence) and adverse selection concerns. Since debt-on-debt is least information sensitive it is optimal in funding markets. We provide a theoretical foundation for the observations why instruments traded in funding markets (e.g. ABCP, MBS, MMF, repos) are debt instruments that use other debt contracts as collateral.[1] Another prominent example is demand deposit. It is a debt contract backed by the bank's assets, i.e. a portfolio of debt.

Finally, our theory shows how these markets can break down. A public shock about the fundamental value of the underlying project that backs the debt collateral which backs the

[1]. Interestingly, repo that uses MBS as collateral is debt-on-debt-on-debt.

tradable debt can create an incentive to produce private information. Bad public news about fundamentals (a shock) causes the market value of collateral debt to drop. But more severely, it can cause information-insensitive tradable debt to become information sensitive. Agents who are capable to produce information have an incentive to learn about tail risks. Other agents become "suspicious" in the sense of fearing about adverse selection. In our model there are two potential equilibrium responses. Agent B who is uninformed prevents endogenous adverse selection by reducing the amount of trade below the expected value of the debt collateral. Or he gives in to adverse selection and there is a positive probability that there is no trade. We show that in both cases a collapse of trade or a financial crisis is a discontinuous event.

Historically, systemic crises are associated with bank runs in the commercial banking system that creates money-like securities for households and firms in the form of demand deposits which are backed by banks assets. As mentioned, demand deposits are debt and demand deposit is backed by the bank's portfolio of debt. The recent financial crisis was caused by runs on different parts of the wholesale and shadow banking system where "private" money takes the form of various types of short term debt instruments backed by different types of debt. The liquidity provision for households (demand deposits) and firms and financial institutions (funding markets) are vital for the real economy. Therefore, a collapse of these markets causes a financial crisis.

Systemic financial crises have the common feature that they involve debt. Yet current theories of crisis assume the existence of debt, and current theories of debt do not explain the origins of crises. In this paper we provide a theory of the existence and optimality of debt-on-debt as private money, a theory that also shows that debt - while optimal - is vulnerable to a crisis in which trade collapses. The breakdown of these markets is then a manifestation of the tail risk that is endogenously created by agents in the economy who optimally use debt backed by debt collateral in order to trade for liquidity reasons, precisely because it is best in maintaining symmetric ignorance by design.

The recent financial crisis has been blamed in part on the complexity and opacity of financial instruments, leading to calls for more transparency. On the contrary, we show that symmetric ignorance creates liquidity in funding markets. Furthermore, we show that the public provision of information that is imperfect can trigger the production of private information and create endogenous adverse selection. Agents can most easily trade when it is common knowledge that no one knows anything privately about the value of the security used to transact and no one has an incentive to conduct due diligence about the value of the security. Debt backed by debt collateral has this property.

In the setting we explore there is a fixed cost of producing information. Debt minimizes the value of the private information that can be learned, so that this cost is not worth bearing. In fact, if it was possible to raise the cost of producing information, say by making the security less information sensitive that would be even better. A cost of infinity would be best. This contrasts starkly with many existing models of debt in a corporate finance setting. For example, in the model of Townsend (1979) a lender must pay a cost to determine the output of a borrower to see if the loan can be repaid. In that setting, the cost of producing information would be best if it were zero. The lender wants information. But, in the trading context is better if no party to the transaction engages in such due diligence.

The paper proceeds as follows. In Section 2 we very briefly review the relevant prior literature. In Section 3 we introduce and explain the model. In Section 4 we introduce a new measure of tail risks and characterize its properties. In Section 5 we analyze optimal security design and characterize the properties of equilibrium. In Section 6 we discuss extensions of the baseline model. Section 7 concludes.

2. Previous Literature

Our paper builds on several prior literatures. With regard to "liquidity," Diamond and Dybvig (1983) and Gorton and Pennacchi (1990) study liquidity provision but assume the existence of debt. Also important is Holmström (2008). Diamond and Dybvig (1983) associate "liquidity" with intertemporal consumption smoothing and argue that a banking system with demand deposits provides this type of liquidity. Gorton and Pennacchi (1990) argue that debt is an optimal trading security because it minimizes trading losses to informed traders when used by uniformed traders. Hence debt provides liquidity in that sense. In Gorton and Pennacchi (1990) the debt is riskless, and it is not formally shown that debt is an optimal contract. Since debt is riskless there is no crisis.

There is a large literature on the optimality of debt in firms' capital structures, based on agency issues in corporate finance. In DeMarzo and Duffie (1999) the problem is to design a security that maximizes the payoff of a seller who will exogenously become (privately) informed prior to actually selling the securities. Since there is adverse selection, the demand curve of the uninformed buyers is downward-sloping. Prior to obtaining private information but anticipating the competitive separating signaling market equilibrium at the trading stage, the seller designs a security that trades-off the price and quantity effects ex ante. The seller cannot redesign the security after obtaining the private signal. They show that under some conditions debt is the optimal security. The key driver for the optimality of debt for an informed seller is the “flat” part of the debt contract. The intuition is that the “flat” part excludes the smallest set of high type sellers and thus reduces the price sensitivity when the seller increases the quantity.[2]

Our design problem is very different. Rather than analyzing how security design can mitigate exogenous adverse selection problems, we analyze two layer optimal security design with endogenous information acquisition and ask which security is optimal as backing collateral in the first stage and which security is optimal to trade in the second stage that preserve symmetric information and minimize endogenous adverse selection concerns after observing a public signal. We design a security that maximizes the payoff of an uninformed agent who faces a potentially informed buyer when he needs to sell. We show that it is not the “flat” part of a standard debt contract that is relevant for minimizing the incentive to produce information. The key driver for the optimality of debt when there is endogenous information production and potential adverse selection is the 45 degree line of the debt contract, i.e., the seniority of repayment. But we also show that the “flat” part of the debt contract becomes relevant (and a standard debt contract is uniquely optimal) when there is public information or (endogenous) adverse selection in equilibrium.[3]

In our setting efficient trade is inhibited by “transparency.” There are a few papers that raise the issue of whether more information is better in the context of trading or banking. These include Andolfatto (2009), Kaplan (2006), and Pagano and Volpin (2009). Andolfatto (2009) considers an economy where agents need to trade, and shows that when there is news about the value of the “money” used to trade, some agents cannot achieve their desired consumption levels. Agents would prefer that the news be suppressed. Kaplan (2006) studies a Diamond and Dybvig-type model and in which the bank acquires information before depositors do. He derives conditions under which the optimal deposit contract is non-contingent. Pagano and

[2] See also Biais and Mariotti (2005) who extend DeMarzo and Duffie (1999) to a setting where buyers are strategic and derive an optimal screening mechanism at the trading stage rather than assuming a separating signaling equilibrium. DeMarzo (2005) shows that pooling reduces the adverse selection problem an uninformed agent faces when he sells to an informed intermediary while tranching increases the amount that the informed intermediary (seller) can sell to uninformed buyers subsequently. Innes (1990) shows that debt is optimal in setting with moral hazard where a manager needs to exert effort.

[3] For our main results, we impose no restrictions on the set of securities except limited liabilities, i.e. the security payoff cannot be larger than the project payoff that is used to back the security. Securities can be non-monotonic.

Volpin (2009) study the incentives a security issuer has to release information about a security, which may enhance primary market issuance profits, but harm secondary market trading. All these authors assume debt contracts.

There is a very large literature on financial crises.[4] The concept of a "financial crisis" refers to a sort of "regime change" due to the simultaneous actions of a large number of agents, which causes real effects. The leading example is a banking panic, which occurs when a sufficiently large number of depositors choose to withdraw their deposits, relative to the cash available to the banks, forcing a suspension of convertibility. Broadly and briefly, there are various different theories of financial crisis. First, there are self-fulfilling expectations or sun spots theories, starting with Diamond and Dybvig (1983), and refined by Goldstein and Pauzner (2005) who apply the global games method of Carlsson and van Damme (1993). In these models, agents are unsure of other agents' actions or beliefs, and the crisis is an outcome of the coordination failure. Morris and Shin (1998) also use the global games modeling technique to model a coordination game in which each player's payoff depends on his own action and the actions of others, as well as unknown economic fundamentals. This view of crises focuses on a loss of confidence, which is related to beliefs about other agents.

In the second theory there is no coordination failure, but there is asymmetric information in that market participants do not know which institutions are most at risk. A shock can occur which is big enough to cause some banks to fail, but agents do not know which banks will fail. Risk averse agents rationally respond by, for example, seeking to withdraw their money from all banks even though only a few are actually insolvent. See Gorton (1985, 1988) and Gorton and Huang (2006). Again, there is a loss of confidence in the sense that agents are no longer sure of banks' solvency. The disruption can be large, although the overwhelming majority of banks are solvent.

The financial crisis in our economy comes from an entirely different source than the theories in the existing literature. Crises in the existing literature are not linked to the optimality of debt, while our theory follows naturally from the optimality of debt. Beliefs about the actions of other agents matter in our theory in that the fear of others producing private information when there is a shock is what makes debt information-sensitive. Like Kiyotaki and Moore (1997) the value of collateral is important in our theory because the debt which is backed by that collateral can become information-sensitive due to the shock to the collateral value. A

[4] See Allen, Babus, and Carletti (2009) for a survey.

“loss of confidence” also plays an important role in our theory. It corresponds to the debt becoming information-sensitive when there is a shock, resulting in the fear of adverse selection. In our theory, the crisis is linked to the underlying rationale for the existence of debt-on-debt as the optimal trading security and a crisis arises if such debt that is designed to be information-insensitive becomes information sensitive.

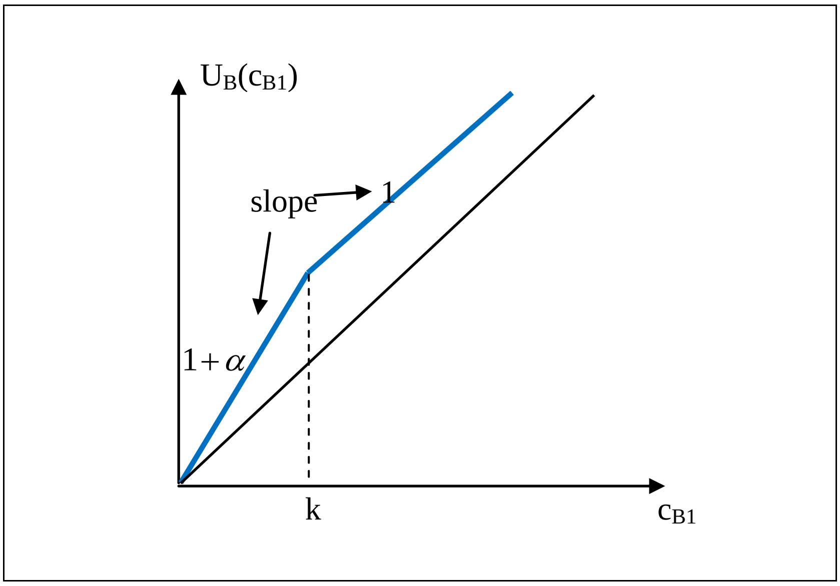
3. The Model

We consider an exchange economy with three dates ($t=0, 1, 2$) and three agents $\{A, B, C\}$ whose utility functions are given as follows:

$$U_A=C_{A0} + C_{A1} + C_{A2}$$
$$U_B=C_{B0} + C_{B1} + \alpha * min[C_{B1},k] + C_{B2}$$
$$U_C=C_{C0} + C_{C1} + C_{C2}$$

where $\alpha,k>0$ are constants. Agents A and C have linear utility and value consumption the same in all dates. Agent B also has linear utility but he values the first k units of date 1 consumption at the marginal rate $(1+\alpha)$. Figure 1 illustrates the utility function of agent B.

Figure 1



We interpret k as the desired amount of liquidity that agent B wishes to obtain at date 1. If agent B is a bank, then k can be interpreted as the amount of liabilities the bank has to repay. If agent B is a firm, then k could be the amount needed to continue a project at full scale. If B obtains less than k, the level of the continuation investment is inefficiently low. If agent B is a consumer, then he wishes to spend the amount k at date 1. If he obtains less than k, he is not able to buy the desired amount of consumption goods. If he obtains more than k, the marginal value is one for any extra amount he obtains. Agent B is indifferent between consuming more than k at date 1 and delaying that consumption to date 2, but he strictly prefers consumption at date 1 up to amount k.

The agents have the following endowments of goods:

$\varpi_A=(0,0,X)$

$\varpi_B=(w,0,0)$

$\varpi_C=(0,w_C,0)$

where w and w_C are constants and X is a random variable, the payoff on a project owned by agent A that is realized at date 2. The random variable X is described by a continuous distribution function, $F(x)$ and positive support on $[x_L, x_H]$. Agent A has no endowment of goods at dates 0 and 1, but receives x units of goods at date 2, where x is a verifiable realization of the random variable X, from a project. Agent B possesses w units of goods at date 0 and nothing at the other dates. Agent C has w_C units of goods at date 1. Goods are nonstorable.

The assumptions are made to create a demand for claims on x that will be traded over the two periods. The only reason for trade is that agent B's utility function gives him an extra benefit α from consuming the first k units at date 1. It is socially efficient for agent A to consume at date 0, for agent B to consume k units at date 1, and for agent C to consume at date 2. In order to make that problem interesting we assume that future endowments (i.e. w_C) are non-contractible.[5]

A. Securities

In order to trade, agents will need to write contracts which specify a price and a security. A security $s(x)$ maps the outcome of X to a repayment $s(x)$. At date 1, having purchased $s(x)$ from agent A, agent B can design a new security using $s(x)$ as collateral and trade the new security with agent C for agent C's t=1 goods.

[5] Otherwise, agent B and C can trade directly at date 0 and reach an efficient allocation. Alternatively, we could assume that agent C is not present at date 0 or has the utility function $U_C=\beta C_{C0}+C_{C1}+C_{C2}$ where $\beta<1$. In this case agent B and C will not trade at date 0. In funding markets investors typcially do not know in advance which counterparty has excess cash to lend out. Complete contracting at date 0, would need to specify for all contigencies who has excess cash and shortage of cash which is bascially too costly or even not feasible.

Date 0 securities: Let S_0 denote the set of all feasible date-0 securities, i.e., functions, $s_0(x)$, which satisfy the resource feasibility (or limited liability) constraint, $0 \leq s_0(x) \leq x$. So $S_0=\{s_0: s_0(x) \leq x\}$.Two examples are:

(i) Equity: $s_0(x)=\beta x$ where $\beta \in (0,1]$ is the share of x;

(ii) Debt: $s_0(x)=min[x, D]$ where D is the face value of the debt.

Date 1 securities: At date 1, agent B owns $s_0(x)$ which he can use as collateral for a new security $s_1(s_0(x))$. The set of feasible securities at date 1 that agent B can use to trade with agent C is given by $S_1 = \{s_1 : s_1(s_0(x)) \leq s_0(x)\}$.

B. Information

There are two types of information, public information z about the distribution $f(x)$ and private information (production) about the realization of x. We assume that at date 0 agents have symmetric information and the prior on X is given by the distribution $F(x,z_0)$ with density $f(x,z_0)$.

Public News: At date 1, before agent B and agent C interact, a public signal z is realized. The signal z is publicly observed, but is non-contractible ex ante. Signal z induces the posterior distribution $F(x,z) \equiv F_z$. z can be discrete or continuous. For z continuous, $g(z)$ is the density of z and the prior satisfies $f(x,z_0) = \int f(x|z)g(z)dz$ where $z \in [z_L, z_H]$. If there are Z possible signals and signal z occurs with probability λ_z, then the prior is: $f(x,z_0) = \sum_{z_1}^{z_N} \lambda_z \cdot f(x|z)$.

Private Information Production: We assume that agent C is more sophisticated; only he can produce private information. In the main analysis we assume that if agent C pays the cost γ (in terms of utility), he learns the true realization x.[6]

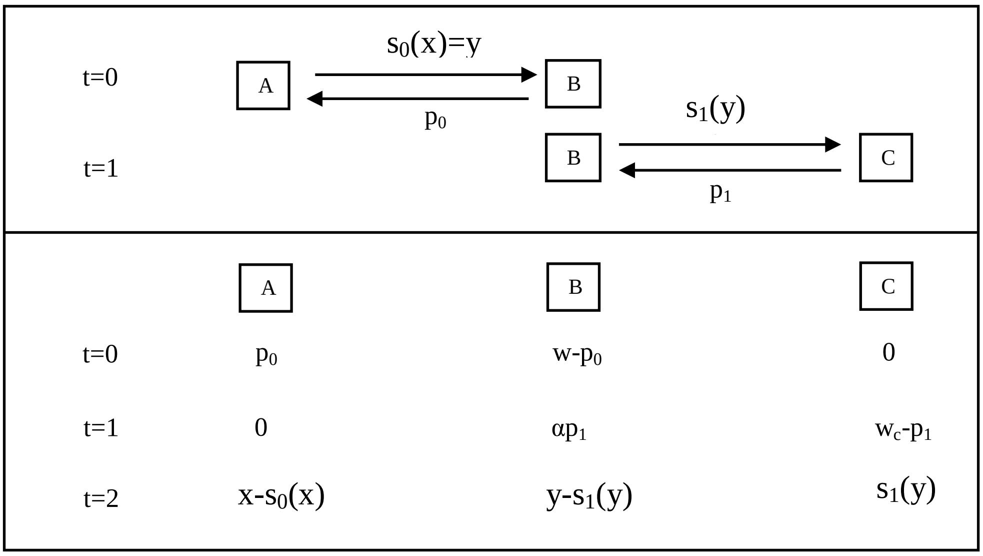
Agent C represents an investor type who has the financial technology to produce costly information about securities if it is profitable to do so. For example, in the case of asset-backed securities (ABS) we assume that all agents may have access to all documents but only agent C can build a data intensive simulation and valuation model of ABS while agent B has limited financial knowhow and cannot do this. In funding markets agent B represents pension funds, insurance companies, mutual funds, regional banks and corporate cash managers.

[6] Section 6.4 analyses noisy information acquisition.

C. Sequence of Moves

We write *(s(x), p)* for a contract which consists of two components, a security *s(x)* and its price *p*. The sequence of moves is shown in Figure 2. At date 1, agent B wants to buy a security to allow him to store some of his endowments until date 1. He makes a take-it-or-leave-it offer $(s_0(x), p_0)$ to agent A, the owner of the project *X*. The offer consists of a price p_0, i.e. the amount of goods that agent B intends to pay to agent A for $s_0(x)$, that promises the payment $s_0(x)$ at date 2 to the holder of the security. If agent A declines the offer, the game ends and parties just consume their endowments. At date 1, agent B makes a take-it-or-leave-it offer $(s_1(y), p_1)$ to agent C, where $y=s_0(x)$ is the collateral that backs B's promise to pay agent C $s_1(y)$ at date 2. If agent C accepts, he pays agent B the price p_1 at date 1.[7] The consumptions of the agents are described in Figure 2.

Figure 2



One interpretation of what is happening in the model is as follows. Agent B is a (regional) bank that has excess cash at date 0. The bank wants to store the cash by using $s_0(x)$. At date 1, depositors of the bank want to withdraw the amount *k* so that the bank wants to sell $s_0(x)$ to agent C to raise cash. Or in the context of repo, we can interpret $s_0(x)$ as a long term bond that agent B buys and when he needs cash at date 1 he uses $s_0(x)$ as collateral for a repo trade with agent C. Our theoretical analysis is general and more abstract in the sense that we allow agent B to design a new security $s_1(y)$ that uses $y=s_0(x)$ as collateral and sell it to agent C. We want

[7] The notation $y=s_0(x)$ is intended to emphasize that the security that agent B offers agent C has $s_0(x)$ as collateral. We can analyze the trade between agents B and C in terms of ty $s_1(x)=s_1(s_0(x))$ or work with the representation $s_1(y)$.

to solve this two layer optimal security design problem without imposing any (unnecessary) restrictions on $s_0(x)$ and $s_1(y)$ except limited liabilities.[8] Our objective is to provide a theoretical foundation for the optimality of debt-on-debt, i.e. $s_1(\cdot)$ is debt and $s_0(\cdot)$ is also debt. In other words, tradable debt at date 1 is backed by debt collateral bought at date 0.

4. The Information Sensitivity of a Security

In this section we introduce a new measure of tail risks, called "information sensitivity" to solve the model. This measure captures the value of private information and the incentive of agents to acquire private information about the payoff of a security. We will use this concept to solve the B-C game where agent C can acquire private information.

At date 1 agent B owns an asset y with induced distribution $F(y)$. Agent B can use y as collateral for a contract $s_1(y)$ which will be sold to agent C. Agent B can choose any security from the set $\{s_1 : s_1(y) \le y\}$ and a price, p_1, to maximize his utility subject to the constraints that agent C is willing to buy and can produce information. To save on notation, in this section we use p and $s(y)$.

Suppose agent B proposes the contract $(s(y), p)$ to agent C, i.e. an agent C can buy the security $s(y)$ at price p. The value of information for agent C is defined as $\pi \equiv EU_C(I) - EU_C(NI)$, where $EU_C(I)$ is the expected utility based on the optimal transaction decision in each state under perfect information about x (I), and $EU_C(NI)$ denotes the expected utility of an optimal transaction decision based on the initial information only, i.e. no information about the true state (NI). We define

$$\pi_L(p) \equiv \int_{y_L}^{y_H} \max[p - s(y), 0] \cdot f(y)dy$$

and

$$\pi_R(p) \equiv \int_{y_L}^{y_H} \max[s(y) - p, 0] \cdot f(y)dy .$$

[8] Another difference between our model and DeMarzo and Duffie (2005) is that they restrict the set of date-1 securities that the exogenously informed seller can choose from to be $\{s_1(y): ay\}$, i.e. he decides what fraction a of the security y to sell. They show that under some conditions y is debt. We impose no restrictions on $\{s_1(y)\}$ *and on* $\{y\}$ except limited liability.

Lemma 1 (Value of Information): *Suppose agent C is offered a security s(y) at price p. The value of information to agent C or* $\pi(p)$*, of s(y), is given as follows: (i) If* $p \le E[s(y)]$*, then* $\pi(p) = \pi_L(p)$*. (ii) If* $p \ge E[s(y)]$*, then* $\pi(p) = \pi_R(p)$*. (iii) At* $p = E[s(y)]$*,* $\pi_L(p) = \pi_R(p)$*.*

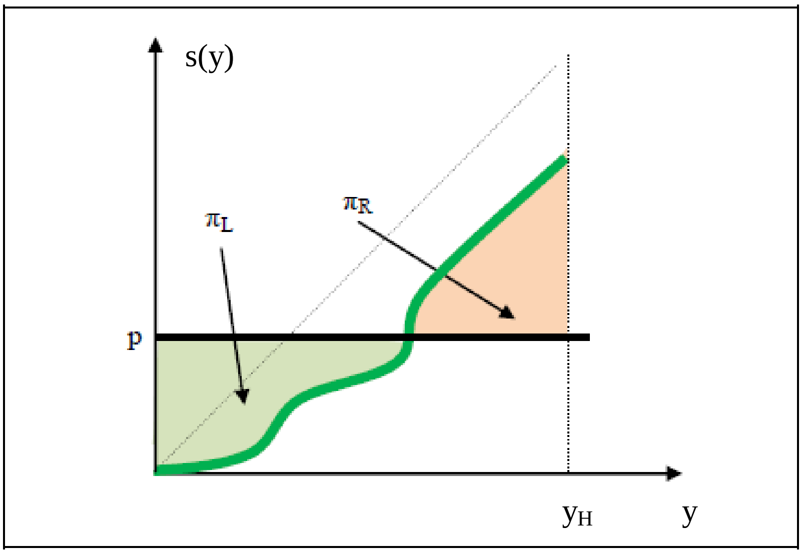
Proof: (i) For $p \le E[s(y)]$, without information agent C buys the security (because it is undervalued). If agent C is informed he will not buy the security in states where *s(y)<p*. The value of information is the amount he avoids over paying for the security in low states. Integrating over all *y* with *p-s(y)>0* gives $\pi(p) = \pi_L(p)$.

(ii) For $p > E[s(y)]$, without information, agent C does not buy the security (because it is overvalued) . If agent C is informed, he will buy the security in states where *s(y)>p*. The value of information is the amount of profit he makes in high states. Integrating over all *y* with *s(y)-p>0* gives $\pi(p) = \pi_R(p)$.

(iii) At $p = E[s(y)]$, the expected loss in low payoff equals the expected gains in high payoff states since *p=E[s(y)]*. So $\pi_L(p) = \pi_R(p)$. See Figure 3. **QED**

Definition: We call the value of information the *information sensitivity of a security*.

Figure 3



Now we consider which security, *s(y)*, minimizes both $\pi_L(p)$ and $\pi_R(p)$.

Proposition 1: *Consider the set of all securities* $\{s : s(y) \le y\}$ *with the same expected value V. For any f(y) and arbitrary price p, debt minimizes the value of information (i.e. is least information sensitive).*

Proof: We compare debt, $s^D(y)=\min[y,D]$ where D is the face value of debt, with a generic contract $s^g(y)$ where both contracts have the same expected value V, i.e. $E[s^D(y)]=E[s^g(y)]=V$ and price p. From *Lemma 1 (Value of information)*, for $p \le V$, the value of information of debt is $\pi_L^D = A$ where $A \equiv \int_{Q^D} (p-y)\cdot f(y)dy$ and $Q^D = \{y : y \le p\}$. See Figure 4. The value of information of $s^g(y)$ is $\pi_L^g = A + B$ where $A+B \equiv \int_{Q^g} (p - s(y))\cdot f(y)dy$ and $Q^g = \{y : s^g(y) \le p\}$. It is obvious that $\pi_L^D \le \pi_L^g$ for any $f(y)$. The inequality is strict if $s^g(y)$ is such that $s^g(y)<y$ for some $y<p$. For $p \ge V$, the value of information of debt is $\pi_R^D + D + E$ where $D+E = \int \max[s^D(y)-p,0]\cdot f(y)dy$. The value of information of $s^g(y)$ is $\pi_R^g = E + F$ where $E+F = \int \max[s(y)-p,0]\cdot f(y)dy$. Note,

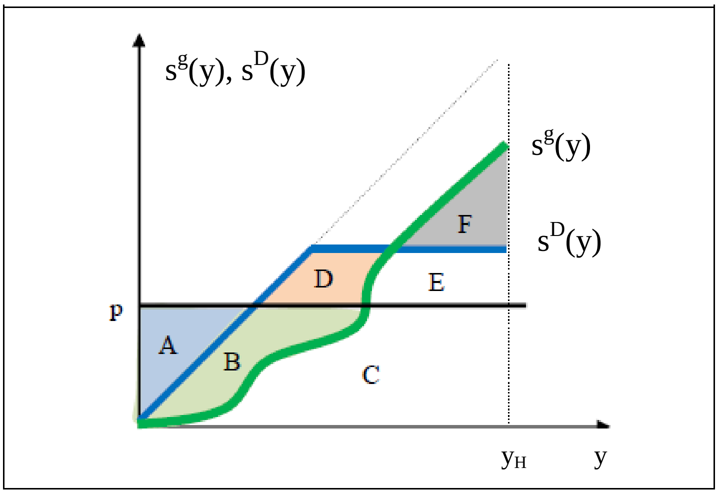
$$E[s^D(y)] = D+E+B+C = \int \max[s^D(y)-p,0]\cdot f(y)dy + \int \min[s^D(y),p]\cdot f(y)dy$$

and

$$E[s^g(y)] = E+F+C = \int \max[s^g(y)-p,0]\cdot f(y)dy + \int \min[s^g(y),p]\cdot f(y)dy$$

Finally, note that $E[s^g(y)]=E[s^D(y)]=V$ implies that $B+C+D+E=C+E+F$ and thus $D+B=F$. If $s^g(y)<y$ for some $y<p$, then $B>0$ and therefore, $\pi_R^D = D < F = \pi_R^g$. **QED**

Figure 4



Proposition 2: *If a debt contract triggers information acquisition by agent C, then so does any other contract with the same price and same expected payoff.*

Proof: Suppose agent C acquires information under a debt contract, i.e. $\gamma < \pi^D$. Then by Proposition 1, $\gamma < \pi^D \le \pi^g$, so agent C acquires information under the generic contract. **QED**

5. Optimal Security Design

In section 5.1 we analyze the B-C game and show that given an arbitrary collateral, debt is an optimal security for agent B to sell to agent C at date 1. Section 5.2 shows that debt-on-debt is optimal in the A-B-C game. Section 5.3 derives the equilibrium prices and amount of trade and characterizes the properties of the deb-on-debt equilibrium. Section 5.4 discusses the assumptions and results.

5.1. The Optimality of Debt in the B-C Game

At date 1, agent B owns a security $s_0(x)$ that he bought from agent A. Agent B can use $s_0(x)=y$ as collateral for a security $s_1(y)$ that he can sell to agent C. To save on notation we omit the subscript and use $s(y)$. In this section we analyze an optimization problem in the $(s(y), p)$-space where $p \in R$ and $s(y) \in \{s \mid s(y) \le y\}$ where $s(y)$ can be non-monotonic.

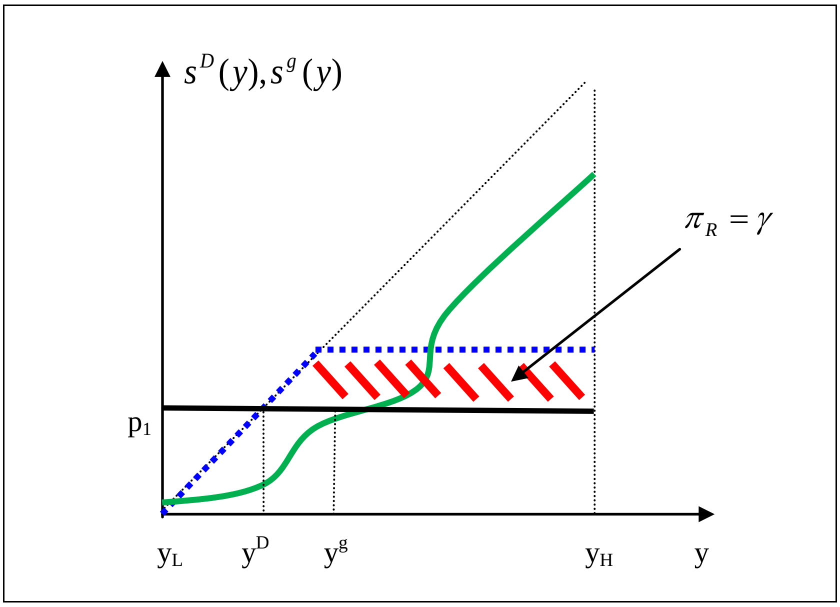
Proposition 3: *Debt is optimal in the B-C game.*

Proof: Let $(s^g(y), p^g)$ be a generic contract that is offered. We need to show that there exists a debt contract that gives agent B as high an expected utility as the generic contract. There are two cases to consider.

<u>Case A:</u> Information acquisition is not triggered under the generic contract. Let $(s^g(y), p^g)$ be a generic contract that agent C finds acceptable. We need to show that there exists a debt contract that agent C is willing to accept and that gives agent B as high an expected utility as the generic contract $(s^g(y), p^g)$. Trade with no information acquisition implies that $p^g \le E[s^g(y)] \equiv V^g$, because otherwise agent C would not buy the contract. Since the generic contract does not trigger information acquisition neither does a debt contract $(s^D(y), p^D)$ with $p^D = p^g = V^D \le V^g$ by Proposition 2. So the debt contract is as good as the generic contract.

Case B: Information acquisition is triggered by the generic contract with price p^g and value V^g. In this case $\pi_L^g < \gamma \le \pi_R^g$. Without loss of generality, we assume that the generic contract has $\pi_L^g < \gamma = \pi_H^g$, i.e. the (best) generic with price p^g that triggers information acquisition but offers agent C minimal rents (i.e. just γ as a compensation for information costs). Consider debt with price $p^D = p^g$ and value $V^D = V^g$.[9] This debt contract may not trigger information acquisition in which case it strictly dominates the generic contract because it sells for the same price and trades with probability one. If it triggers information acquisition, then reduce the face value of the debt to $D_1 < D$ such that $\pi_R^{D1}(p^g) = \gamma$. This dominates the generic contract since both trade at the same price and triggers information acquisition and have $\pi_R^{D1}(p^g) = \pi_R^g(p^g) = \gamma$ but debt is accepted by agent C with (weakly) higher probability, i.e. $prob(s^{D1}(y) \ge p^g) \ge prob(s^g(y) \ge p^g)$, since $y^{D1} \le y^g$ where $s^{D1}(y^{D1}) = p^g$ and $s^g(y^g) = p^g$. In other words, debt has a weakly steeper slope up to the price than any generic contract. See Figure 5. **QED**

Figure 5



5.2. The Optimality of Debt-on-Debt in the A-B-C Game

In this section we solve the full game, i.e. an optimization problem in the $\{s_0(\cdot), p_0, s_1(\cdot), p_1\}$-space where $p_0, p_1 \in R$, $s_0(\cdot) \in \{s_0 \mid s_0(x) \le x\}$ and $s_1(\cdot) \in \{s_1 \mid s_1(s_0(x)) \le s_0(x)\}$ where $s_0(\cdot)$

[9] This debt contract exists because if the face value of $D=y_H$ then $s(y)=y$.

and $s_1(\cdot)$ can be non-monotonic functions. We impose no assumptions on the set of posterior distributions *{f(x|z)}* induced by the public signal *z*. We denote the value function of $s_0(x)$ at date 1 as

$$V(z) \equiv E[s_0(x) \mid z] = \int s_0(x) f(x \mid z) dx \ .$$

The key assumption we make is that $V^D(z) \geq k$ for all *z*. In other words, agent B has enough resources and can afford to buy a debt collateral at date 0 that maintains value above *k* for all realizations of the public signal *z*. Note, if agent B is a bank, *k* can be interpreted as the amount that depositors will withdraw at date 1. This assumption implies that the bank is always solvent at date 1 but we will show that it might become illiquid yet and depositors cannot withdraw *k*. Furthermore, we think this case is an important case to elaborate. If *k* represents the liabilities of a bank, then the bank is insolvent when $V^D(z)<k$. The bank will be under bankruptcy protection.[10]

We will elaborate more on the economics of this assumption in the context of capital regulation and its effectiveness for avoiding a financial crisis in Section 5.4. We will analyze the case where $V^D(z) < k$ for some *z* and provide a sufficient condition for debt-on-debt to remain optimal in section 6. We will also show that it is least costly for agent B to buy debt collateral in order to fulfill capital requirement, i.e. have the expected collateral value stay above *k* and always remain solvent at date 1.

Proposition 4: *Debt-on-debt is optimal in the A-B-C game if* $V^D(z) \geq k$ *for every z.*

Proof: Consider a generic contract $\{(s_0^g(x), w), (s_1^g(x,z), p_1^g(z)\}$. We will construct a pure debt contract $\{(s_0^D(x), w), (s_1^D(x,z), p_1^D(z)\}$ which has the same initial price *w* as the generic contract and for which the continuation contract, also debt, dominates the generic contract for each realization of *z*. In particular, we will show that for each *z*:

(i) Agent C's expected rent, defined as his expected consumption over the two periods t=1,2 in excess of w_C, is no larger with debt-on-debt contract than with the generic contract, and

[10] In such a case bankruptcy lawyers and structuring bankers produce a lot of public information so that private information acquisition is not profitable and assets are typically traded under symmetric information.

(ii) Agent B's expected consumption premium, defined as $E(\alpha \cdot \min\{c_{B1}, k\})$ is no smaller with debt-on-debt contract than with the generic contract. Here the expectation is both over *z* and over consumption outcomes at date 1.

The optimality of debt follows from the two conditions above, because given a contract $\{(s_0^g(x), w), (s_1^g(x,z), p_1^g(z)\}$, agent B's expected utility over the two periods t=1,2 is

$$w + E[\alpha \cdot \min\{c_{B1}, k\} - E[s_1(x,z) - p_1(z)]]$$

The first term is the total market value of the contract B purchases from agent A. The second term is the expected consumption premium at date 1. The third term is the expected rent paid to agent C.

Because *z* is the posterior probability, we have that $E(z) = z_0$, the prior probability. Also, $E_z V^D(z) = E_x[s_0^D(x)] = w = E_x[s_0^g(x)] = E_z V^g(z)$ since the debt and the generic contract were both worth *w* at date 0.

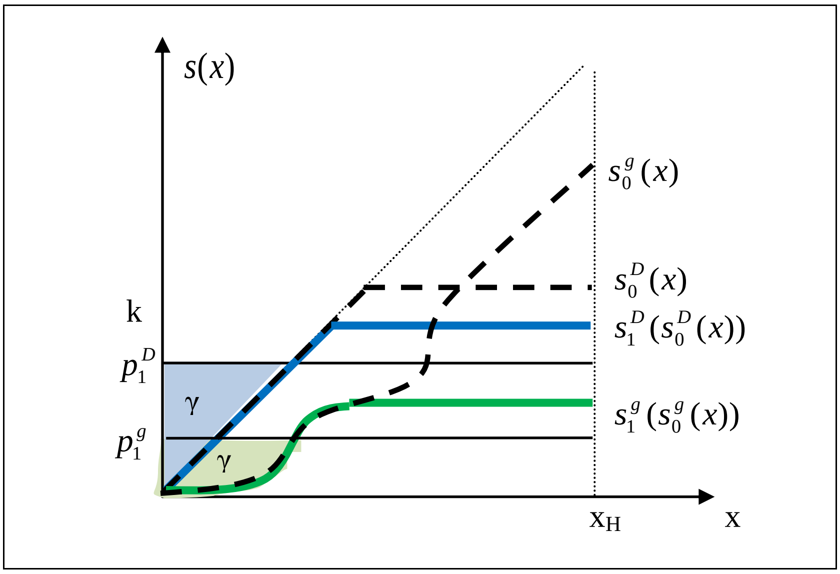
Our strategy is to show that for each *z*, we can do as well with a modified debt contract as with the generic contract.[11] Sometimes the modification entails lowering the face value *D*, while adjusting the price $p_1^D(z)$ so that it is fair (i.e. equals the market value of the modified debt); we refer to this as "writing down debt". In other cases we just lower the face value D without changing the initial price, we refer to this as "tightening debt". There are two cases to consider. Let *z* be an arbitrary realization of the public signal at date 1. Define $p_1^D(z) = \min\{k, p_1^g(z)\}$.

<u>Case 1:</u> The generic continuation contract does not trigger information acquisition. We choose the face value $D_1(z)$ so that $V^{D1}(z) = p_1^{D1}(z) = \min\{k, p_1^g(z)\}$. This is possible since the collateral debt has $V^D(z) \geq k$. Because $p_1^{D1}(z) \leq p_1^g(z)$, the debt contract with face value $D_1(z)$ will not trigger information acquisition because the generic does not. Both contracts will trade with certainty. The consumption premium of each contract will therefore be the same. Either prices are equal, or $p_1^{D1}(z) < p_1^g(z)$, we have $p_1^{D1}(z) = k$, which will give agent B the maximum consumption premium. Also, the rent is zero for both debt and the generic (assuming w.l.o.g. that the generic sells at a fair price, otherwise the generic is even worse).

[11] We are using point-wise dominance in the *z*-space to show that debt is optimal.

Consequently, the debt contract dominates the generic contract weakly (if $p_1^g(z) < k$, debt is strictly better). See Figure 6.

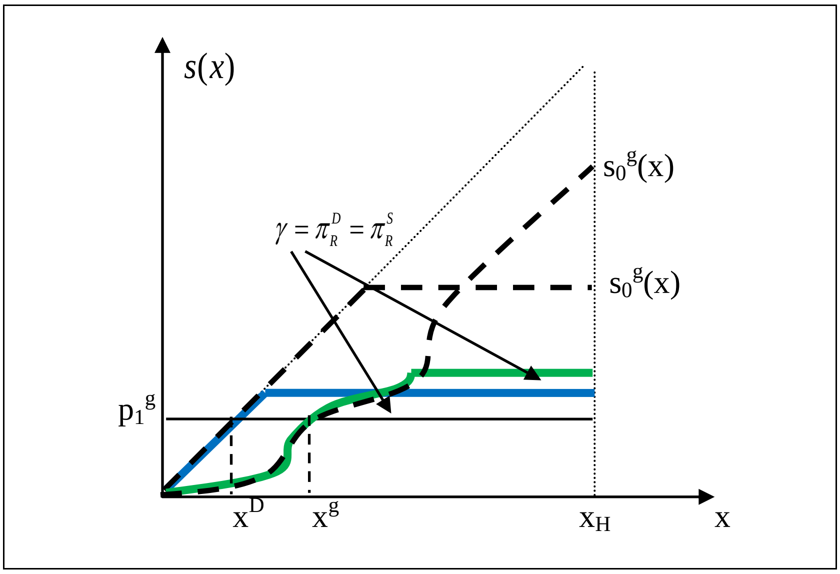
Figure 6



<u>Case 2:</u> The generic continuation contract triggers information acquisition. Define $D_1(z)$ as in case 1. If the debt contract $(s_1^{D1}(x,z), p_1^{D1}(z))$ does not trigger information acquisition, then debt is traded with probability 1 at a price that is either as high as the generic price $p_1^g(z)$ or at a price equal to *k*. In either case, the consumption premium is strictly higher than the consumption premium of the generic, which trades with probability strictly less than 1.

If the debt contract $(s_1^{D1}(x,z), p_1^{D1}(z))$ triggers information acquisition, "tighten" this debt contract, that is, lower its face value to $\tilde{D}_1(z)$ (without altering the price) so that the rent going to agent C is just sufficient to make agent C acquire information, $\tilde{\pi}_R^{D1}(x, p_1^{D1}(z)) = \gamma$. If $p_1^{D1}(z) = p_1^g(z)$ the tighten debt contract trades with a (weakly) higher probability as the generic (since the underlying collateral for the tighten debt is debt). $p_1^{D1}(z) = k < p_1^g(z)$ then the debt contract trades with a strictly higher probability than the generic. In either case, the rent is minimal γ for information acquisition, hence no larger than with the generic. See Figure 7. **QED**

Figure 7



5.3. Characterization of the Debt-on-Debt Equilibrium

Proposition 4 shows that debt-on-debt is optimal. Now we show how to construct a debt-on-debt equilibrium and characterize its properties.

<u>Proposition 5</u>*: At date 0, agent B buys debt with face value D_0 and price p_0 from agent A where $E[s_0^D(x)] = w$ and $p_0=w$. At date 1, the public signal z realizes. Define $M \equiv \min\{w_C, k\}$. Under signal z, denote $\tilde{s}_1^D(x) = \min\{s_0^D(x), \tilde{D}\} = \min\{x, \tilde{D}\}$ as a (new) debt contract with face value $\tilde{D} \le D_0$ that has an expected value $E[\tilde{s}_1^D(x) \mid z] = M$ and price $p_1=M$. Note that $\tilde{\pi}_L^D \equiv \int_{x_L}^{x_H} \max[M - x, 0] \cdot f(x \mid z) dx$ is the value of information of that debt contract. Agent B proposes the following debt contract (p_1, $s_1^D(x)$) to agent C.*

(i) If $\gamma \ge \tilde{\pi}_L^D$, then agent B sells the above debt contract. Agent C buys without information acquisition.

(ii) If $\gamma < \pi_L^D$, then depending on $\{\alpha, \gamma, z\}$ agent B chooses either:

Strategy I (Avoid information acquisition): Offer a debt contract with price p_I such that $\pi_L^{DI}(p_I) = \gamma$ and face value $D_I < \tilde{D}$ such that its value is $V^{DI}(z) = p_I$. Agent C buys without information acquisition; or

Strategy II (Induce information acquisition): Offer a debt contract with price $p_{II}=\min[k, \tilde{p}]$ where $\tilde{p}$ maximizes $p \cdot prob(x \geq p)$; and face value $D_{II} \in (D_I, \tilde{D}]$ such that $\pi_R^{D_{II}}(p_{II}) = \gamma$. Agent C acquires information and only buys if $x \geq p_{II}$.

Proof: By Proposition 4, debt-on-debt is optimal. Now we determine the equilibrium prices and amount of debt traded at date 0 and 1. It is easy to see that at date 0 it is optimal for agent B to spend his whole endowment w to buy a debt contract with face value D_0 for a price p_0 such that $p_0 = E[s_0^D(x)] = w$. Agent A accepts this contract. Note, agent B is indifferent between consumption at date 0 and 2. So agent B is not worst off by having no consumption at date 0 but consume the expected amount w at date 2. But he can do strictly better by consuming up to k units of goods at date 1 by using $s_0^D(x)$ as collateral to trade for agent C' goods.

At date 1, in an efficient allocation agent B consumes k units or if $w_C<k$, the maximum what agent C owns, i.e. w_C. We define $M = \min\{w_C, k\}$.[12] Consider such a debt contract, i.e. $\tilde{s}_1^D(x) = \min\{s_0^D(x), \tilde{D}\} = \min\{x, \tilde{D}\}$ *with face value* $\tilde{D} \leq D_0$ *that has an expected value* $E[\tilde{s}_1^D(x) \mid z] = M$ *and price* $p_1=M$. This contract has information sensitivity $\tilde{\pi}_L^D \equiv \int_{x_L}^{x_H} \max[M - x, 0] \cdot f(x \mid z) dx$.[13] There are two cases.

<u>Case 1:</u> Suppose the public signal z is such that $\tilde{\pi}_L^D \leq \gamma$. Agent B proposes above debt and C buys without information acquisition. Agent B's utility is maximized since agent C does not get any rents and agent B consumes the efficient amount M at date 1 and has expected consumption of $E[s_0^D(x) \mid z] - M$ at t=2.

<u>Case 2:</u> Suppose the public signal z is such that $\tilde{\pi}_L^D > \gamma$. If agent B proposes the above debt with face value $\tilde{D}$ for $p_1 = E[\tilde{s}_1^D(x) \mid z]$, then agent C acquires information. So agent B considers two alternative best response strategies:

<u>Strategy I:</u> The highest price that agent B can ask for without triggering information acquisition is p_I such that $\pi_L^D(p_I) = \gamma$, i.e. $\int_{x_L}^{x_H} \max[p_I - x, 0] \cdot f(x \mid z) dz = \gamma$. For that

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[12] Note, we assume that $E[s_0^D(x) \mid z] \geq k$ for all z. Otherwise, we would define $M = \min\{w_C, E[s_0^D(x) \mid z], k\}$.

[13] This is the "smallest" debt contract that implements efficient consumption if agent C buys. A debt contract with higher face value and $E[\tilde{s}_1^D(x) \mid z] = p_1 > M$ has higher information sensitivity.

price agent B sells debt with value $E[s_1^D(x)\mid z]=p_I$, and thus agent C gets no rents. The face value D_I associated with that debt contract solves: $\int_{x_L}^{x_H}\min[x,D_I]\cdot f(x\mid z)dx=p_I$. See Figure 8. At date 1 under signal z, agent B's expected utility is thus

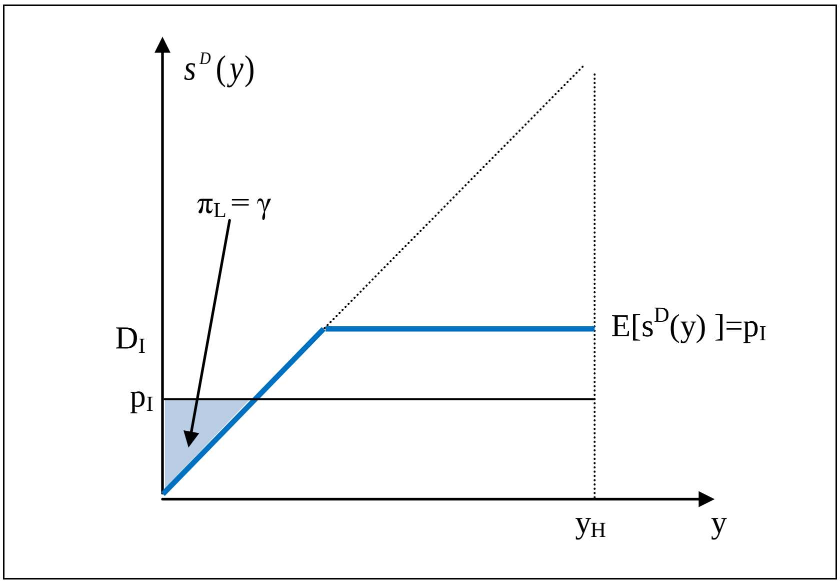
$$EU_B=E[s_0^D(x)\mid z]+(1+\alpha)p_I-E[s_1^D(x)\mid z].$$

Note, the first term is the expected amount he can consume at date 2 if there is no trade at date 1. If there is trade at date 1 the first k units gives an extra utility of α, thus $(1+\alpha)p_I$ and he sells $E[s_1^D(x)]=p_I$. So $EU_B=E[s_0^D(x)\mid z]+(1+\alpha)p_I-p_I$, i.e.

$$EU_B(I)=E[s_0^D(x)\mid z]+\alpha p_I.$$

Given the no-information acquisition constraint ($\pi_L^D(p_I)=\gamma$), if γ is small, then p_I is small and agent B can consume little at date 1. Whenever agent B proposes a contract with a higher price, i.e. $p_1=E[s_1^D(x)\mid z]>p_I$ then $\pi_L=\pi_R>\gamma$ and agent C acquires information.

Figure 8



Strategy II: Agent B proposes an offer that induces agent C to acquire information. Agent C does not always buy, but if trade occurs agent B can get a higher price and consume more at date 1. Any price $p_1>p_I$ (of Strategy I) implies $\pi_L>\gamma$. In order to induce agent C to acquire information he must obtain $\pi_R(p_1)=\gamma$. So agent B chooses a price p_1 to maximize

$$EU_B = E[s_0^D(x)\,|\,z] + \left(1-F(p_1)\right)\left((1+\alpha)p_1 - E[s_1^D(x)\,|\,trade]\right)$$

s.t. $\pi_R = \int_{x_L}^{x_H} \max[s_1^D(x)-p_1,0]\cdot f(x\,|\,z)dx = \gamma$. Note, the first term in EU_B is his expected consumption without trade. If there is trade agent B obtains p_1 and gives away $E[s_1^D(x)\,|\,trade]$ and trade only occurs if agent C observes $x \geq p_1$. The information acquisition constraint ($\pi_R=\gamma$) implies that $E[s_1^D(x)\,|\,trade] = E[s_1^D(x)\,|\,x\geq p_1] = p_1+\gamma$. See Figure 5. Therefore, $EU_B = E[s_0^D(x)\,|\,z] + \left(1-F(p_1)\right)\left((1+\alpha)p_1 - p_1 - \gamma\right)$ i.e.

$$EU_B(II) = E[s_0^D(x)\,|\,z] + (1-F(p_1))(\alpha p_1 - \gamma).$$

Since agent B only wants to consume k units of goods at date 1, the optimal debt contract with information acquisition has a price $p_{II}=\min[\tilde{p},k]$ where $\tilde{p}\in \arg\max(1-F(p))p$. Note that proposing $p_{II}>k$, reduces the probability of trade but the extra units $(p_{II}-k)$ only have a marginal consumption value of 1. So consuming that amount at date 2 is equally good but increases the probability of trade at date 1. The face value D_{II} solves: $\int_{p_{II}}^{x_H}(\min\{x,D_{II}\}-p_{II})\cdot f(x\,|\,z)dx = \gamma$.

Consequently, agent B chooses Strategy I if $\alpha p_I \geq (1-F(p_{II}))(\alpha p_{II}-\gamma)$. Otherwise he chooses Strategy II. **QED**

We can also use Proposition 5 to illustrate the optimality of debt-on-debt. Note, under signal z, if agent B chooses Strategy I or II, then $EU_B(I) = E[s_0^D(x)\,|\,z] + \alpha p_I(z)$ and $EU_B(II) = E[s_0^D(x)\,|\,z] + (1-F_Z(p_1(z)))(\alpha p_1(z)-\gamma)$, respectively. After observing signal z, agent B's expected utility is

$$EU_B = A(z) + \max[B(z),C(z)]$$

where $A(z)\equiv E[s_0^D(x)\,|\,z]$, $B(z)\equiv \alpha p_I(z)$ and $C(z)\equiv(1-F_Z(p_{II}(z)))(\alpha p_{II}(z)-\gamma)$. From the perspective of date 0,

$$EU_B = w + E_Z[\max\{B(z),C(z)\}]$$

since any security $s_0(x)$ has $E_Z[A(z)] = w$. We have shown that debt dominates any other security point-wise in *z*, i.e. $B^D(z) \geq B^g(z)$ and $C^D(z) \geq C^g(z)$. Therefore, at date 0, $EU_B(D) = w + E_Z[\max\{B^D(z), C^D(z)\}] \geq EU_B(g) = w + E_Z[\max\{B^g(z), C^g(z)\}]$. So debt-on-debt is optimal. Proposition 5 is perhaps best understood with an example.

Numerical Example (Debt-on-debt)

Suppose $F_1 \sim u[0,0.8]$, $F_2 \sim u[0.2, 1.2]$, $F_3 \sim u[1.2, 2]$ and $\lambda_1 = \lambda_2 = \varepsilon$, and $\lambda_3 = 1 - 2\varepsilon$. Suppose ε=0.00001, $w = 1$, k=0.3, $\gamma = 0.001$, and $\alpha = 1.003$. The subsequent numbers are exact up to the fourth decimal. Note, $E[x]=1.6$. In this example, at date 0 agent B buys debt with face value $D_0=1$ and price $p_0=1$. Equilibrium outcomes at t=1 are as follows.

(i) If F_3 is the true distribution, then $V^D(z_3) = 1$ and $\pi_1^D(z_3) = 0$. Agent B sells debt with $D_1(z_3)=0.3$ for $p_1(z_3) = E[s_1^D(\cdot)] = 0.3$. Note, he can also sell the original debt (i.e. $D_1=D_0=1$) for price $p_1=1$ but his utility is the same.

(ii) If F_2 is the true distribution, then $V^D(z_2) = 0.68$ and $\pi_1^D(z_2) = 0.1152 > \gamma$. Agent B sells debt with face value $D_1(z_2)=0.245$ and $p_1(z_2) = E[s_1^D(\cdot)] = 0.244$.

(iii) If F_1 is the true distribution, then $V^D(z_1) = 0.4$ and $\pi_1^D(z_1) = 0.1 > \gamma$. Agent B sells debt with face value $D_1(z_1) = 0.0411$ and $p_1(z_1) = E[s_1^D(\cdot)] = 0.04$.

Agent C does not acquire information in all three cases.

To summarize this example, at date 0, agent B buys debt from agent A. At date 1, agent B uses the date 0-debt as collateral for a (new) debt contract that he sells to agent C. In normal times (i.e., $F=F_3$), there is efficient trade between agents B and C at date 1. If fundamental is deteriorating (i.e., $F=F_2$), then the market value of debt collateral drops from 1 to 0.68 and agent B sells a (new) debt with face value 0.245 for price 0.244. Thus he consumes less than *k*. If there is crisis news (i.e., $F=F_1$), then the market value of the debt collateral drops to 0.4. Agent B offers to sell debt with face value 0.0411 for price 0.04. There is inefficient low consumption in equilibrium but this is best thing agent B can achieve.

In the above example, $\alpha = 1.003$ and is small so the best response is Strategy I if there is adverse selection concerns. If α is larger, say $\alpha = 1.4$, then when the true distribution is F_1, the best response of agent B is to induce agent C to acquire information. Agent B offers to sell debt with face value $D_1(z_1) = 0.34$ and $p_1(z_1) = 0.3$. In equilibrium agent C acquires information and only buys if $x \geq 0.3$. With probability 0.3875 there is no trade.[14]

Corollary 5.1: *Suppose agent C is privately informed (γ =0). The (uniquely) optimal contract that agent B offers to sell to agent C is a debt contract with face value* $D_{II} = p_{II} = \min[k, \tilde{p}]$ *where* $\tilde{p}$ *maximizes (1-F(p))p.*

Proof: Strategy II is optimal and the respective face value solves $\pi_R = \int_{x_L}^{x_H} \max\left[\min[x, D_{II}] - p_{II}, 0\right] f(y) = \gamma = 0$. Therefore, D_{II}=p_{II}. **QED**

5.4. Discussion of Debt-on-Debt Equilibrium

The numerical example illustrates a number of interesting insights. In that example, the three posterior distributions induced by the public signal can be ordered by (first order) stochastic dominance. The value of the collateral drops and information sensitivity increases if there is bad news in the sense of stochastic dominance (which we formalize in the next section). Although Propositions 4 and 5 hold without imposing any structure on the set of posterior distributions, we use the notion of stochastic dominance to discuss the economic implications of our model (Proposition 5) for the "dynamics" of a financial crisis and regulation.

Suppose posterior distributions induced by the public signal *z* are ordered by first order stochastic dominance (FOSD) such that $f(x \mid z_j) \overset{FOSD}{\succ} f(x \mid z_k)$ for $z_j > z_k$. There exists a signal *z'* such that $\tilde{\pi}_L^D(z') = \int_{x_L}^{x_H} \max[M - x, 0] \cdot f(x \mid z')dx = \gamma$ or just larger than γ (where *M* is the efficient amount of trade described in Proposition 5). Then $\tilde{\pi}_L^D(z) > \gamma$, for all $z < z'$. In other words, if agent B proposes to trade the efficient amount *M*, agent C acquires information under signal *z*. In such states, there is a discontinuous drop of trading volume. Although agent B owns a collateral with expected value $V^D(z)$ larger than *k*, he can only consume strictly less than *k*.

If market participants observe *f(x|z)* but not the econometrician or the regulator then it is not possible for them to "predict" the (equilibrium) outcome. Even though the market value of the

[14] Note, if agent B buys equity with $E[s^E(x)] = w$ at date 0. Then at date 1 agent B sell debt backed by that equity. As a consequence there is adverse selection even when there is good news. For all three public signals agent B consumes strictly less at date 1 than under the debt collateral.

collateral is higher than k and publicly observable the amount of trade or the drop in trading volume is not predictable. Furthermore, suppose the amount of trade under Strategy I is monotonic in z and the expected amount of trade under Strategy II is also monotonic in z (given some restrictions on the signal structure). Yet in equilibrium the realized amount of trade might not be monotonic in the public signal since it could be optimal for agent B to switch back and forth between the two strategies. The (realized) amount of trade under strategy II is strictly larger than the amount of trade under Strategy I. In addition, if agent B chooses Strategy II, there is a positive probability that no trade occurs.

Therefore, our model captures an interesting feature of a financial crisis. The reduction of trade or even the collapse of trade is not necessarily a monotonic function of the drop in collateral value triggered by public news. This implication is consistent with historical experiences of financial crises. A slight decrease in collateral value can sometimes cause a large drop in trading volume while a large drop in collateral value is not necessarily followed by a large reduction of trades.

Our model also offers an interesting implication for capital regulation. Proponents of capital requirement argue that by imposing a high enough capital buffer, a financial crisis can be avoided. Our model is able to speak to that issue. In the context of our model, this means $E[s(x)\,|\,z] \geq k$ for all public signal z, i.e. the bank is always solvent. We show that even if the bank is solvent by having enough assets the bank can be illiquid because the amount of cash it can raise by selling assets can be much lower than the market value of its assets. “Liquidity” or “illiquidity” of a security is an equilibrium object and depends on the outcome of strategic interactions and the best responses of agents rather than a purely statistical property. Suppose debt becomes information sensitive. If the gains from trade (i.e. α) are large relative to information costs (i.e. γ) then Strategy II is more “likely” to be played. Otherwise Strategy I is more “likely” to be played. Since $\{\alpha, \gamma, z\}$ are typically private information of market participants the “dynamics” of a financial crisis is not easy to predict by the regulator.

6. Extensions

In this section we analyze different extensions of the basic model and discuss uniqueness issues. In order to analyze these cases, we need to impose some structure on the set of posterior distributions. Section 6.1 provides preliminary results for analyzing these extensions. Section 6.2 shows when debt is the uniquely optimal collateral at date 0. Section 6.3 provides sufficient conditions for debt-on-debt to remain optimal if $E[s(x)\,|\,z] < k$ for some z. Section 6.4 shows that debt-on-debt is also optimal if agent C can acquire noisy private information. Section 6.5 shows that tradable debt is uniquely optimal at date 1 if information acquisition is noisy.

6.1. The Value Function of the Collateral Security

We provide a characterization of the value function of the collateral security $s_0(x)$, i.e. how $V(z) \equiv E[s_0(x) \mid z] = \int_{x_L}^{x_H} s_0(x) f(x \mid z) dx$ changes with the public signal z. We make two additional assumptions.

<u>Assumption 1:</u> $S_0 = \{s_0 : s_0(x) \leq x$ and $s_0(x)$ monotonic$\}$

<u>Assumption 2:</u> $\{f(x \mid z)\}$ satisfies Strict Monotone Likelihood Ratio Property (SMLRP)

In particular, we assume spanning, i.e. that there are two possible distributions F_L and F_H from which the final outcome x is drawn. The probability that x is drawn from F_H is denoted $z=Pr(F_H)$. Without loss of generality, we can assume that the public signal that the agents observe is the posterior z. The posterior is distributed $z \sim G$. The prior is denoted z_0.and satisfied $z_0=E[z]$. Given any posterior z the distribution of x is given by

$$x \sim F(x \mid z) \equiv zF_H(x) + (1-z)F_L(x).$$

We assume that F_L and F_H satisfy SMLRP (the Strict Monotone Likelihood Ratio Property), that is, the likelihood ratio

$$l(x) \equiv \frac{f_H(x)}{f_L(x)} \text{ is strictly increasing in } x.$$

We say that the parametrized family of x distributions $F(x|z)$ satisfies the SMLRP if for every $z_2 > z_1; z_1, z_2 \in [0,1]$, the likelihood ratio

$$l(x) \equiv \frac{z_2 f_H(x) + (1-z_2) f_L(x)}{z_1 f_H(x) + (1-z_1) f_L(x)} \text{ is strictly increasing in } x.$$

<u>Lemma 2:</u> *Suppose F_L and F_H satisfy SMLRP, then the family parametrized by $F(x|z)$ satisfies SMLRP.*

Proof: Let $z_2 > z_1; z_1, z_2 \in [0,1]$. We need to show that *l(x)* is strictly increasing in *x*. Differentiating we get:

$$\frac{dl(x)}{dx} \equiv \frac{z_2(z_1 l(x)+(1-z_1)) - z_1(z_2 l(x)+(1-z_2))}{(z_1 l(x)+(1-z_1))^2} l'(x) > 0$$

where the strict inequality follows because the numerator equals z_2-z_1 >0 and *l(x)* is strictly increasing by assumption. **QED**

Definition: A function $s_1(x)$ is said to cross the function $s_2(x)$ from below if there exists an x^* such that $s_1(x) \le s_2(x)$ for $x < x^*$ and $s_1(x) > s_2(x)$ for $x \ge x^*$ and $s_1(x) \ne s_2(x)$. If in addition the two functions are equal only at a single point, then $s_1(x)$ is said to cross the $s_2(x)$ strictly from below.

Lemma 3: *Let h(x|z) be the conditional density function of x given a continuous signal z. Assume h(x|z) is differentiable in z for every x and the family {h(x|z)} satisfies SMLRP. Let $s_1(x)$ and $s_2(x)$ be two nondecreasing functions such that $s_1(x)$ crosses $s_2(x)$ from below. If the functions $V_1(z) \equiv E[s_1(x) | z]$ and $V_2(z) \equiv E[s_2(x) | z]$ cross, then they cross strictly from below.*

Proof: Let $\Delta s(x) = s_1(x) - s_2(x)$. Assume there is a z_0 such that $E[\Delta s(x) | z_0] = 0$, i.e. $V_1(z_0) - V_2(z_0)$. Then:

$$\begin{aligned} \tfrac{d}{dz} E[\Delta s(x) | z_0] &= \int \Delta s(x) h_z(x | z_0) dz \\ &= \int \Delta s(x) \tfrac{h_Z(x|z_0)}{h(x|z_0)} h_z(x | z_0) dz \\ &= \int \Delta s(x) \left[\tfrac{h_Z(x|z_0)}{h(x|z_0)} - \tfrac{h_Z(x^*|z_0)}{h(x^*|z_0)}\right] h_z(x | z_0) dz \end{aligned}$$

where $h_z(x | z_0) \equiv \frac{d}{dz} h(x | z_0)$. The last equality follows from the definition of z_0, i.e. $E[\Delta s(x) | z_0] = 0$. From SMLRP, $\frac{h_Z(x|z_0)}{h(x|z_0)}$ is strictly increasing in x. Therefore,

$$\int \Delta s(x) \left[\tfrac{h_Z(x|z_0)}{h(x|z_0)} - \tfrac{h_Z(x^*|z_0)}{h(x^*|z_0)}\right] h_z(x | z_0) dz \ge 0, \text{ for every x,}$$

with strict inequality on the set where $s_1(x) \ne s_2(x)$. Therefore,

$$\tfrac{d}{dz} E[\Delta s(x) | z_0] = \tfrac{d}{dz}[V_1(z_0) - V_2(z_0)] > 0.$$

In other words, V_1 crosses V_2 strictly from below. **QED**

Lemma 3 is a variant of a Lemma 1 in DeMarzo et al (2010). They use that Lemma to show that debt is the worst security as a mean of payments in a private value auction context. We will use this Lemma to show that debt is the optimal collateral in our trading context.

6.2. Debt is the uniquely optimal collateral at date 0

In this section we derive conditions under which debt is the uniquely optimal collateral security (in the set of non-decreasing securities).

Lemma 4: *Consider a debt contract $s^D(x)=\min[x,D]$ where D is face value of debt and a generic contract $s^g(x)$ that intersects debt from below and where $E[s^g(x)|z_0]=E[s^D(x)|z_0]=w$. At date 1 debt maintains the maximum value for all $z<z_0$.*

Proof: $s(x)$ intersects $s^D(x)$ from below. Lemmas 2 and 3 imply $V^g(z)<V^D(z)$ for all $z<z_0$. See Figure 9. **QED**

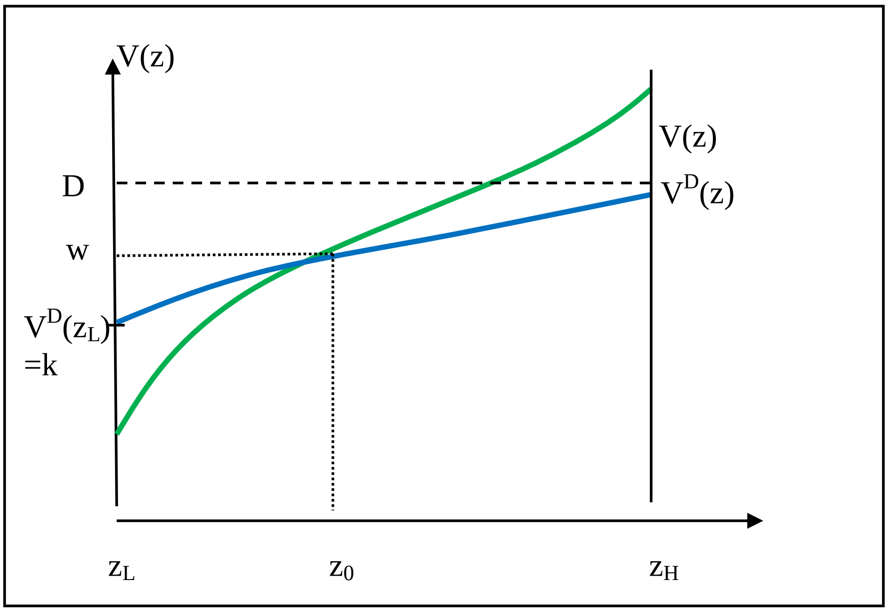
Lemma 5: *Define z_L as the signal such that $f(x\,|\,z) \overset{FOSD}{\succ} f(x\,|\,z_L)$ for all z. Consider a debt collateral that has $V^D(z_L)=k$. Suppose the (fair) price of this debt is p_D at date 0. (i) Then any other collateral security with price p_D has $V(z_L)<k$. (ii) Any other collateral security with $V(z_L)=k$ has a (fair) price $p>p_D$ at date 0.*

Proof: All monotonic security intersects debt from below. By Proposition 6, $V^g(z)$ intersects $V^D(z)$ strictly from below. See Figure 9. **QED**

We assume that there is capital regulation, i.e. agent B must be solvent, i.e. $V(z)\geq k$ for all z. In the main analysis we assume that agent B values consumption at date 0 and date 2 the same. Suppose agent B values consumption at date 0 more than consumption at date 2 by $\varepsilon>0$ (i.e. $U_B=(1+\varepsilon)c_0+c_1+\alpha\cdot\min[c_1,k]+c_2$) where ε is sufficiently small.[15] If agent B is a bank, he can provide additional credits at date 0 with the extra cash.

[15] Alternatively, we could also assume that $U_B=C_{B0}+C_{B1}+\alpha{*}\min[C_{B1},k]+C_{B2}$ *and there is a zero probability event where agent B has no value for consumption at date 2.*

Figure 9



Proposition 6: *Date-0 debt is the uniquely optimal collateral security provided the following assumptions hold:*

(i) $V(z) \geq k$ for all *z* (capital requirement that agent B be solvent);

(ii) $U_B = (1+\varepsilon)c_0 + c_1 + \alpha \cdot \min[c_1, k] + c_2$ with $\varepsilon > 0$ sufficiently small.

Proof: Suppose agent B buys debt with face value *D* from agent A at date 0 with $V^D(z_L)=k$ for (fair) price p_D. From the perspective of date 0, $EU_B(D) = (1+\varepsilon)(w - p_D) + p_D$ $+ E_Z[\max\{B^D(z), C^D(z)\}]$ where $B(z) \equiv \alpha p_I(z)$ and $C(z) \equiv (1 - F_Z(p_{II}(z)))(\alpha p_{II}(z) - \gamma)$ as in Proposition 5. Note, the first term is the amount agent B can consume at date 0. The second term is the expected amount he can consume at date 2 (if there is no trade). The third term is the utility gain by consuming something at date 1 if there is trade. We show that any other debt contract with higher value and any other (non-debt) contract gives agent B strictly lower utility. There are two cases.

<u>Case A:</u> Consider a debt contract with face value $\tilde{D} > D$ and thus $V^{\tilde{D}}(z_L) > k$. This debt has price $\tilde{p}_D > p_D$ and $EU_B(\tilde{D}) = (1+\varepsilon)(w - \tilde{p}_D) + \tilde{p}_D + E_Z[\max\{B^{\tilde{D}}(z), C^{\tilde{D}}(z)\}]$. Since $B^{\tilde{D}}(z) = B^D(z)$ and $C^{\tilde{D}}(z) = C^D(z)$ for all z and $\tilde{p}_D > p_D$, therefore $EU_B(D) > EU_B(\tilde{D})$.

<u>Case B:</u> Any other security with $V^g(z_L) = k$, has $p_g > p_D$; $B^g(z) \leq B^D(z)$ and $C^g(z) \leq C^D(z)$ for any z. Therefore, $EU_B(D) > EU_B(g)$. **QED**

6.3. Optimal Security Design When Agent B Can Be Insolvent

In this section we provide conditions for debt-on-debt to be optimal even if the market value of the collateral asset can drop below *k* induced by some public signal. In a banking context, this case corresponds to the case where the bank is insolvent since the value of its assets is smaller than liabilities.

<u>Proposition 7</u>: *Debt-on-debt is optimal also if* $V^D(z) < k$ *for some z, provided that the following sufficient conditions hold:*

A1: $\{s_0(x) \mid s_0(x) \le x \text{ and } s_0(x) \text{ monotonic}\}$.

A2: Spanning: $f(x \mid z) = (1-z) f_L(x) + z f_H(x)$ and strict MLRP.

A3: $\alpha k \le \gamma$.

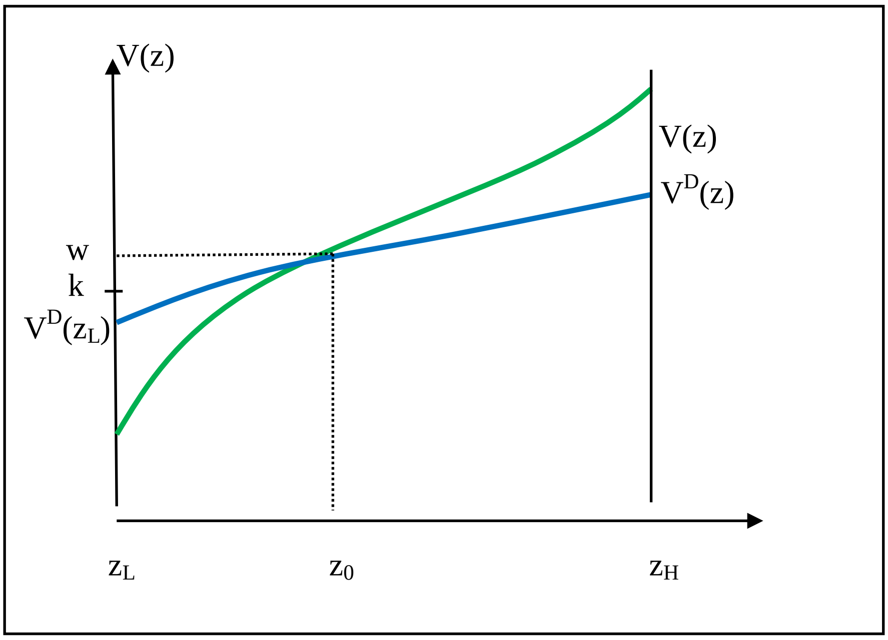
<u>Proof:</u> Assumption A3 implies that it is a strictly dominated strategy for agent B to induce information acquisition by agent C since the gains from trade is smaller than information costs to be paid to agent C. Therefore, we only consider a generic contract that does not trigger information acquisition. There are two subcases. We can assume w.l.o.g. that the generic contract in either cases is priced fairly (i.e. $p_1^g(z) = V^g(z)$ and agent C gets no rents).

<u>Case 1:</u> The generic contract has $V^g(z) \ge k.$ We choose a face value $D_1 \le D$, such that $V^{D1}(z) = k.$ This is possible since $V^g(z)>k$ implies $V^D(z) \ge k.$ Note, Proposition 1 shows that $V^D(z) \ge V^g(z)$ for all $z \le z_0$ and $V^D(z_0) = V^g(z_0) = w_0$ and $k \le w_0$. See Figure 10. The (amended) debt contract with price $p_1^{D1}(z) = V^{D1}(z) = p_1^g(z)$ does not trigger information acquisition because the generic does not and debt is minimally information sensitive. It performs as well as the generic contract since both result in the same consumption premium and there is no rent to agent C. The debt-on-debt contract is therefore as good as the generic.

<u>Case 2:</u> The generic contract has $V^g(z)<k$. We write down the face value of debt to $D_1 < D$, such that the amended debt contract equals the generic $V^{D1}(z) = V^g(z)$. As shown above, this is possible. The amended debt contract does not trigger information as well. (Note the original debt contract could trigger information but it is written down to the same price as the generic which by assumption does not trigger information. So the amended debt does not trigger information acquisition since it has minimal information sensitivity.) Since both contracts are traded with probability one the debt-on-debt contract is therefore as good as the generic

contract. Debt-on-debt is strictly better if at z, there exists a $D_1 < D$ such $V^g(z) < V^{D1}(z)$ and the amended debt traded at $p_1^{D1}(z) = V^{D1}(z) \in (p_1^g(z), k]$ does not trigger information acquisition.

Figure 10



When the generic contract triggers information acquisition, there are two subcases to consider. We assume w.l.o.g. that the generic has a price (i) $p_1^g \leq k$ (since agent B has no consumption premium for $c_{B1}>k$ and a higher price reduces the probability that an informed agent C is buying the security) and (ii) $p_1^g > V^g(z)$ such that the rent is minimal: $\pi_R^g(z, p_1^g(z)) = \gamma$.**QED**

6.4. Optimal Security Design with Noisy Information Acquisition

In this section we discuss another extension. In the baseline model we assume that agent C can produce private information about the true realization of the payoff x of the project. Now we show that debt-on-debt is also optimal if agent C can only acquire noisy private information. If agent C pays the information cost γ he obtains a private signal ϕ which induces the distribution $F(x \mid \phi)$. We make the following assumptions in this section.

(i) Date 0 and date 1 securities are monotonic (i.e. non-decreasing).

(ii) Spanning: The set of posteriors $\{F(x \mid z)\}$ induced by the public signal z satisfies SMLRP.

(iii) The set of posteriors $\{F(x \mid \phi)\}$ induced by the private signal ϕ satisfies SMLRP.

(iv) Agent C's private signal ϕ is conditionally independent of the public signal *z*.

The value of information (i.e. information sensitivity) can be defined analogously if private information is noisy. Note, if agent C learns the true realization he know *s(x)*. The value of information is $\pi_L = \int \min\{p - s(x), 0\} f(x) dx$. In the case of noisy information, agent C does not observe *x* but obtains a signal ϕ that induces $F(x \mid \phi)$. So given a signal ϕ, he knows that $E[s(x) \mid \phi] = \int_{x_L}^{x_H} s(x) f(x \mid \phi) dx$. We denote $V(\phi) \equiv E[s(x) \mid \phi]$. So the value of information is $\Pi_L = \int \min\{p - V(\phi), 0\} g(\phi) d\phi$ where $g(\phi)$ is the density of ϕ. Note, after observing a signal ϕ, agent C does not buy the security for price *p* if $V(\phi) = E[s(x) \mid \phi] < p$. (If the signal is perfect and he observes $\phi = x$, he does not buy if *s(x)<p*.). Similarly, we define $\Pi_R = \int \max\{p - V(\theta), 0\} g(\phi) d\phi$. We can use Figure 10 to illustrate this. If we replace $V(z)$ by $V(\phi)$ in Figure 10, the probability weighted area between the price line *p=w* and $V(\phi)$ is Π_L. The probability weighted area between $V(\phi)$ and the price line is Π_R.

Lemma 6: *Debt is the least information sensitive security among all securities with the same expected value and price.*

Proof: Define ϕ_0 such that $V^D(\phi_0) = V^D(\phi_0) = p$. Proposition 6 implies (i) $\Pi_L^D < \Pi_L^g$ because $V^D(\phi) > V^g(\phi)$ for all $\phi < \phi_0$ and (ii) $\Pi_R^D < \Pi_R^g$ because $V^D(\phi) < V^g(\phi)$ for all $\phi > \phi_0$.

Proposition 8: *Debt is optimal in the B-C game with noisy information acquisition.*

Proof: Suppose agent B owns a collateral *y* at date 1. Let $(s^g(y), p^g)$ be a generic contract that agent B offers agent C. We need to show that there exists a debt contract that gives agent B as high an expected utility as the generic contract. There are two cases to consider.

Case A. The generic contract does not trigger information acquisition. Consider a debt contract with the same expected payoff and the same price as the generic contract $E(s^D(y)) = E(s^g(y)) = w$ and $p^D = p^g$. Let $V^D(\phi)$ and $V^g(\phi)$ be the corresponding value function conditional on the private signal ϕ. Both value functions are strictly increasing by

SMLRP. By Lemma 2, $V^g(\phi)$ intersects $V^D(\phi)$ strictly from below and $V^g(\phi_0) = V^D(\phi_0)$. This implies that $\pi_L^D(p^g) \le \pi_L^g(p^g)$. So the generic contract does not trigger information acquisition neither does the debt contract. Both contracts offer identical expected consumption, so debt is as good as the generic contract.

<u>Case B.</u> Information acquisition is triggered by the generic contract. There are two subcases to consider.

<u>Case B1.</u> $p^g = p_L < w$. Since p^g is below w, $V^g(\phi)$ intersects the line $p^g = p_L$ to the left of ϕ_0, at $\phi = \bar{\phi}$. Consider the debt contract in Case A above, with value function $V^D(\phi)$ but with $p^D = p_L$. Because $V^D(\bar{\phi}) > V^g(\bar{\phi})$, we can reduce the face value of debt to a level $\tilde{D} < D$ such that $V^{\tilde{D}}(\phi)$ intersects the line at the same point $\bar{\phi}$ as the value function $V^g(\phi)$. Similarly $V^g(\phi)$ crosses $V^{\tilde{D}}(\phi)$ strictly from below. Suppose the amended debt contract triggers information acquisition. Then the amended debt contract provides agent B with higher expected utility because (i) when $\phi < \bar{\phi}$, agent B consumes in expectation more at date 1 with the amended debt than the generic (by virtue of $V^{\tilde{D}}(\phi) > V^g(\phi)$; (ii) When agent B gives away less rent to agent C with amended debt $\Pi_R^{\tilde{D}}(p_L) < \Pi_R^g(p_L)$ as $V^{\tilde{D}}(\phi) \le V^g(\phi)$. Finally, if the amended debt contract does not trigger information acquisition, agent B is even better off.

<u>Case B2.</u> $p^g = p_H \ge w$. Now Π_R is the relevant measure of value of information. We construct a debt contract that is as good as the generic as follows. We start by setting, $D = y_H$, i.e. sells the whole collateral. Recall that we assume that generic contract $s^g(y)$ triggers information acquisition so does selling the whole collateral s(y)=y. Lower D to $\tilde{D}$ while keeping the price of debt at fair value, i.e. $\tilde{p} = E(s^{\tilde{D}}(y))$. Continue until either (a) information is no longer triggered ($\Pi_R^{\tilde{D}}(p_H) = \gamma$) or (b) $\tilde{p} = p_H$ while information is still being triggered ($\Pi_R^{\tilde{D}}(p_H) > \gamma$).

In case (a) agent B can sell the debt contract at price $\tilde{p} > p_H$ without agent C acquiring information. This provides agent B with higher date 1 consumption since debt is sold with probability 1 at a price above the generic. Moreover, agent B does not give agent C any rent to compensate for information acquisition so the total expected consumption over the two dates is higher with debt contract, so it dominates the generic.

In case (b), we continue lowering $\tilde{D}$ while keeping the price fixed at p_H. The intersection of $V^{\tilde{D}}(\phi)$ and $p = p_H$ moves to the right. We stop lowering $\tilde{D}$ until either (b1) information is just no longer triggered; or (b2) we reach the intersection between $V^{\tilde{D}}(\phi)$ and the value $V^g(\phi)$ at the value $\phi = \phi_0$. In Case (b1) we have $\pi_R^{\tilde{D}}(p_H) = \gamma$, so this induce agent C to acquire information (since he is indifferent). The debt contract results in a higher probability of trade than the generic, trade takes place at the same price p_H as the generic and debt gives away the smallest possible rent (γ) to agent C. In case (b2) agent B consumes in expectation the same amount at date 1 under both contracts, but strictly more at date 2 because $\pi_R^{\tilde{D}}(p_H) < \pi_R^g(p_H)$. Again, debt dominates the generic contract. **QED**

Proposition 9: *Debt-on-debt is optimal in the A-B-C game with noisy information acquisition under the following assumptions:*

(a) $V^D(z) \geq k$ *For every z.*

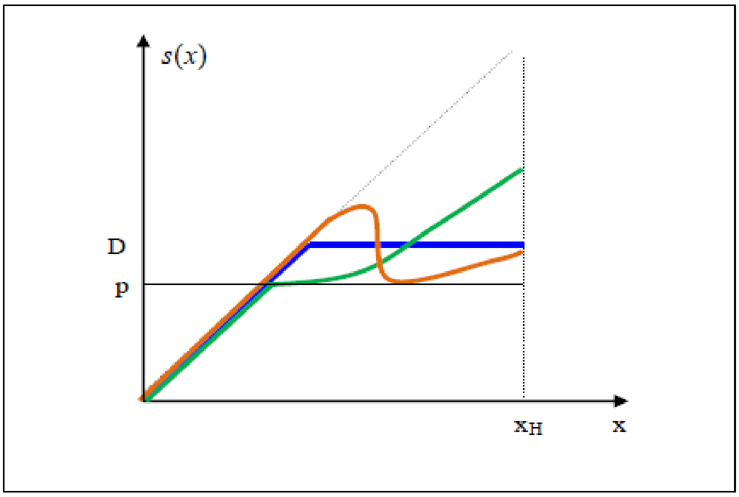
(b) Agent C's private signal ϕ *is conditionally independent of the public signal z.*

Proof: This proof is similar to the proof of Proposition 4 and thus omitted.

6.5. Debt is the uniquely optimal trading security at date 1

Proposition 3 shows that debt is an optimal trading security for agent B to sell it to agent C at date 1. Under the assumption that agent C can acquire perfect information acquisition about *x*, that debt is not uniquely optimal. Any security with slope 1 up to the price has minimal information sensitivity. So any security with the same price and same expected payoff as the respective optimal debt contract and the following properties $s(x) = x$ for all $x \leq p$ and $s(x) \geq p$ for $x > p$ is optimal. We call this type of securities quasi-debt. (See Figure 11). This multiplicity result is driven by the fact that π_L only depends on the density *f(x)* in the interval [0,*x'*] where *s(x')=p*. The shape of the security above *p* does not affect the information sensitivity $\pi_L = \int \max[p - s(x), 0] f(x) dx$.

Figure 11



When information is noisy, then agent C observes ϕ and knows $V(\phi) = E[s(x) | \phi]$. The value of information is $\Pi_L = \int \min\{p - V(\phi), 0\} g(\phi) d\phi$. Since $E[s(x) | \phi]$ depends on the "full" shape of s(x), information sensitivity depends on the "full" security. We obtain uniqueness.

Proposition 10: *Suppose the private signal is noisy and satisfies SMLPR. Debt is the uniquely optimal trading security at date 1 (among the set of non-decreasing securities).*

Proof: Lemma 6 shows that $\Pi_L^D < \Pi_L^g$ for all non-decreasing securities. In other words, any security intersects debt strictly from below. Proposition 5 shows that if there is adverse selection concern when trading the efficient amount, agent B chooses either Strategy I or Strategy II. Under Strategy I, debt maximizes the price without triggering information acquisition, i.e. any other security that does not trigger information acquisition has a strictly smaller price by Lemma 6. Under Strategy II, given any price, debt maximizes the probability of trade, i.e. if agent B proposes any other security with the same price, trade occurs with strictly smaller probability by Lemma 6. **QED**

7. Conclusion

In contrast to the literature on trading in stock markets which is extensive, there is little theoretical work on debt funding markets. In funding markets firms and financial institutions trade so as to manage their cash balances and short term liquidity needs. Examples include repos, asset-backed commercial papers, Agency mortgage-backed securities and money market fund shares. A key characteristic of these securities is that they are debt instruments that use other debt contracts as collateral. In this paper we provide a theory of funding markets that explains the optimality of debt-on-debt.

In our model an agent wants to buy a security to store his wealth at date 0. Then he uses this collateral to back a security that he wants to sell so as to raise cash at date 1. We address two interrelated questions. What is the optimal collateral and what is the optimal trading security? In order to solve this two layer optimal security design problem with private information acquisition and the arrival of public news we introduce a new measure of tail risks, called "information sensitivity". This measure captures an agent's incentive to produce private information. We show that debt-on-debt is optimal because debt is least information sensitive. In addition, the information sensitivity of tradable debt is further minimized by debt collateral. Our theory also shows that a collapse of trade in debt funding markets (financial crisis) is a discontinuous event and occurs when public news about fundamentals makes information insensitive debt to become information sensitive.

Financial crises have been difficult to explain. Systemic crises concern debt. In such a crisis, agents holding debt somehow "lose confidence," usually modeled as a coordination failure. But, the coordination failure requires some mechanism other than debt per se, e.g., a sequential service constraint or a lack of common knowledge. We show that crises and the optimality of debt for liquidity provision are inextricably intertwined. The crisis that can occur with debt is due to the fact that the debt is not riskless. But, it is not the risk per se that is the problem. Debt is designed so that no agent has an incentive to produce information about the states of the world where the risk will cause a low pay-out.

The crisis is not just the bad shock about fundamentals that back debts. Instead, the crisis is a bad enough shock to cause information-insensitive debt to become information sensitive. Agents who are capable to produce information have an incentive to learn about tail risks. Other agents become "suspicious" in the sense of fearing about adverse selection. There are two potential best responses. There is information acquisition and adverse selection and there is positive probability that no trade occurs. Another potential equilibrium outcome is that agents avoid private information production by trading at a price that is less than the fundamental value of the debt conditional on the public news. Such a "write-down" of debt, to "fire sale" prices, can be preferred because it recovers information-insensitivity and where no agent has an incentive to produce information, but an inefficiently low amount is traded. A financial crisis is a manifestation of the "tail risk" that is endogenously created by agents in the economy in order to trade.

If maintaining symmetric ignorance is central for liquidity provisions in debt funding markets, then this has implications for the regulation of the banking and financial system. For example, should money market funds reveal their net asset value in a timely fashion? Should banks that create short term liabilities for trade, provide more information about the value of their assets on the balance sheet? Should the regulator announce the outcome of stress test of banks so that investors have better information about individual banks and can run their own valuation models? Public provision of imperfect information can reduce liquidity because it can make information insensitive debt become information sensitive and triggers endogenous adverse selection concerns. When agents have an incentive and need to conduct due diligence about the value of money-like instruments, these financial instruments will lose their money-like property.

In this paper we provide a theoretical foundation of debt funding markets. Since funding markets are vital for the real economy and a collapse of debt funding markets can bankrupt firms and financial institutions, more theoretical and empirical research about these markets is needed.

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