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Illusion Optics: The Optical Transformation of an Object into Another Object

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We propose to use transformation optics to generate a general illusion such that an arbitrary object appears to be like some other object of our choice. This is achieved by using a remote device that can transform the scattered light outside a virtual boundary into that of the object chosen for the illusion, irrespective of the profile and direction of the incident light. This type of illusion device also enables people to see through walls. Our work extends the concept of cloaking as a special form of illusion to the wider realm of illusion optics.

Recently, the concept of transformation optics [1–22] has enabled the design of new materials that can steer light along arbitrary curves, and the implementation is made possible by employing metamaterials [23–27]. Among various novel applications, the most fascinating is a cloaking device designed to bend light around a concealed region, rendering any object inside the region "invisible" [1–10]. Cloaking can be regarded as creating an illusion of free space. In this Letter, we discuss a more generalized concept of illusion: making an object of arbitrary shape and material properties appear exactly like another object of some other shape and material makeup. Using transformation optics, we design an illusion device consisting of two distinct pieces of metamaterials, which are called the "complementary medium" and the "restoring medium." The complementary medium, which was first proposed by Pendry and Ramakrishna to make focusing lenses [28,29], is applied here to "cancel" a piece of space optically, including the object [21,22]. Then the restoring medium restores the cancelled space with a piece of the illusion space that is embedded with the other object chosen for the illusion. Regardless of the profile and the direction of the incident light, the illusion device can transform the scattered light outside a virtual boundary as if the light is scattered from the second (illusion) object; it therefore creates a stereoscopic illusion for any observer outside the virtual boundary.

The principle behind this illusion device is not light bending but rather the exact cancellation and restoration of the optical path of light within the virtual boundary. As a consequence, unlike previous light-bending cloaking devices [1–10], the constitutive parameters of the illusion device need not have a complex spatial distribution or any singularities. More surprisingly, the illusion device works at a distance from the object. An interesting implication of this "remote" feature is the ability to open a virtual aperture in a wall so that one can peep through the wall in a noninvasive manner. For this purpose, monochromic functionality is sufficient.

A simple schematic diagram illustrating our idea is shown in Fig. 1. In Fig. 1(a), an illusion device is placed next to a domain that contains a man (the object). The passive device causes any observer outside the virtual boundary (the dashed curves) to see the image of a woman [the illusion, Fig. 1(b)] instead. In other words, such an illusion device makes the electromagnetic fields outside the virtual boundary in both the real [Fig. 1(a)] and illusion [Fig. 1(b)] spaces exactly the same, irrespective of the profile and direction of the incident waves. A blueprint
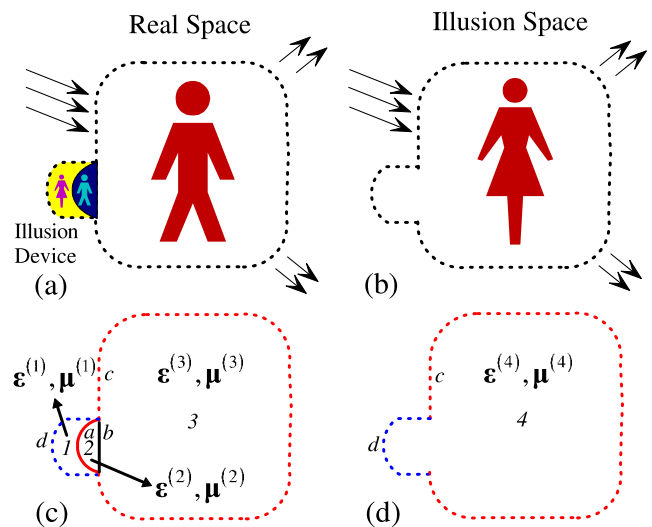

FIG. 1 (color online). The working principle of an illusion device that transforms the stereoscopic image of the object (a man) into that of the illusion (a woman). (a) The man (the object) and the illusion device in real space. (b) The woman (the illusion) in the illusion space. (c) The physical description of the system in real space. The illusion device is composed of two parts, the complementary medium (region 2) that optically "cancels" a piece of space including the man (region 3) and the restoring medium (region 1) that restores a piece of the illusion space including the illusion [region 4 in (d)]. Both real and illusion spaces share the same virtual boundary (dashed curves).

for the device is shown in Fig. 1(c). The device itself can be divided into two regions (regions 1 and 2). Region 2 includes the complementary medium used to annihilate the optical signature of the man, while region 1 includes the restoring medium that creates the image of the woman. Both media are designed using transformation optics [1–4]. The complementary medium is formed by a coordinate transformation of folding region 3, which contains the man, into region 2. The restoring medium is formed by a coordinate transformation of compressing region 4 in Fig. 1(d), which contains the illusion, into region 1. The permittivity and permeability tensors of both media in the illusion device are obtained by $\varepsilon^{(2)} = \mathbf{A}\varepsilon^{(3)}\mathbf{A}^{\mathbf{T}}/\det\mathbf{A}$, $\mu^{(2)} = \mathbf{A}\mu^{(3)}\mathbf{A}^{\mathbf{T}}/\det\mathbf{A}$, $\varepsilon^{(1)} = \mathbf{B}\varepsilon^{(4)}\mathbf{B}^{\mathbf{T}}/\det\mathbf{B}$, and $\mu^{(1)} = \mathbf{B}\mu^{(4)}\mathbf{B}^{\mathbf{T}}/\det\mathbf{B}$, where $\varepsilon^{(i)}$ and $\mu^{(i)}$ are the permittivity and permeability tensors in region i, respectively. $\mathbf{A}$ and $\mathbf{B}$ are the Jacobian transformation tensors with components $A_{ij} = \partial x_i^{(2)}/\partial x_j^{(3)}$ and $B_{ij} = \partial x_i^{(1)}/\partial x_j^{(4)}$, corresponding to the coordinate transformations of folding region 3 into region 2 and compressing region 4 into region 1, respectively.

The electromagnetic fields in the complementary and the restoring media can also be obtained from transformation optics [1–4] as $\mathbf{E}^{(2)} = (\mathbf{A}^{\mathbf{T}})^{-1}\mathbf{E}^{(3)}$, $\mathbf{H}^{(2)} = (\mathbf{A}^{\mathbf{T}})^{-1}\mathbf{H}^{(3)}$, $\mathbf{E}^{(1)} = (\mathbf{B}^{\mathbf{T}})^{-1}\mathbf{E}^{(4)}$, and $\mathbf{H}^{(1)} = (\mathbf{B}^{\mathbf{T}})^{-1}\mathbf{H}^{(4)}$, where $\mathbf{E}^{(i)}$ and $\mathbf{H}^{(i)}$ are the electric and magnetic fields in region i, respectively. From the matching of the boundary conditions on surface a (the red solid curve) between the complementary medium and the restoring medium, we have $\mathbf{E}_t^{(2)}(a) = \mathbf{E}_t^{(1)}(a)$ and $\mathbf{H}_t^{(2)}(a) = \mathbf{H}_t^{(1)}(a)$, where subscript t indicates transverse components along the surface. Both the folding transformation $\mathbf{A}$ and compression transformation $\mathbf{B}$ map one part of the virtual boundary, i.e., surface c (the red dashed curves), to surface a. If this one-to-one mapping from c to a is the same for both $\mathbf{A}$ and $\mathbf{B}$, we can obtain from transformation optics that $\mathbf{E}_t^{(3)}(c) = \mathbf{E}_t^{(4)}(c)$ and $\mathbf{H}_t^{(3)}(c) = \mathbf{H}_t^{(4)}(c)$ on surface c. In addition, we also have $\mathbf{E}_t^{(1)}(d) = \mathbf{E}_t^{(4)}(d)$ and $\mathbf{H}_t^{(1)}(d) = \mathbf{H}_t^{(4)}(d)$ on the other part of the virtual boundary, i.e., surface d (the blue dashed curves), as long as d is mapped to itself during transformation $\mathbf{B}$. Therefore, the tangential components of the electromagnetic fields on the whole virtual boundary (including c and d) are exactly the same in the real and illusion spaces, and, consequently, by the uniqueness theorem, the electromagnetic fields outside are also exactly the same. Any observer outside the virtual boundary will see electromagnetic waves as if they were scattered from the illusion object (the woman and nothing else), and thus an illusion is created. A detailed proof is provided in the auxiliary material [30].

In the following, we show full wave simulations using a finite element solver (COMSOL MULTIPHYSICS) to demonstrate explicitly the effect of an illusion device that transforms a dielectric spoon of $\varepsilon_o = 2$ into a metallic cup of $\varepsilon_i = -1$ in two dimensions. In two dimensions, the electromagnetic waves can be decoupled into TE waves (E along the z direction) and TM waves (H along the z direction). We show only the TE results for brevity, and the parameters can be tuned to work for both TE and TM waves. Figures 2(a) and 2(c) plot, respectively, the scattering patterns of the dielectric spoon and the metallic cup, under the illumination of a TE plane wave (propagating from left to right) of wavelength $\lambda = 0.25$ unit. In Fig. 2(b), an illusion device is placed beside the spoon. The scattering pattern around the spoon and the illusion device is altered in such a way that it appears as if there is only a metallic cup. This can be clearly seen by comparing the field patterns of the spoon plus the illusion device shown in Fig. 2(b) with that of the metallic cup shown in Fig. 2(c). The field patterns are indeed identical outside the virtual boundary. Inside the virtual boundary, the field patterns in Figs. 2(a) and 2(c) are completely different. The fields between the spoon and the illusion device are strong due to the excitation of surface resonances induced by the multiple scattering of light between the spoon and the illusion device. This indicates that the illusion effect is a steady state phenomenon that takes some time to establish. More simulation results under
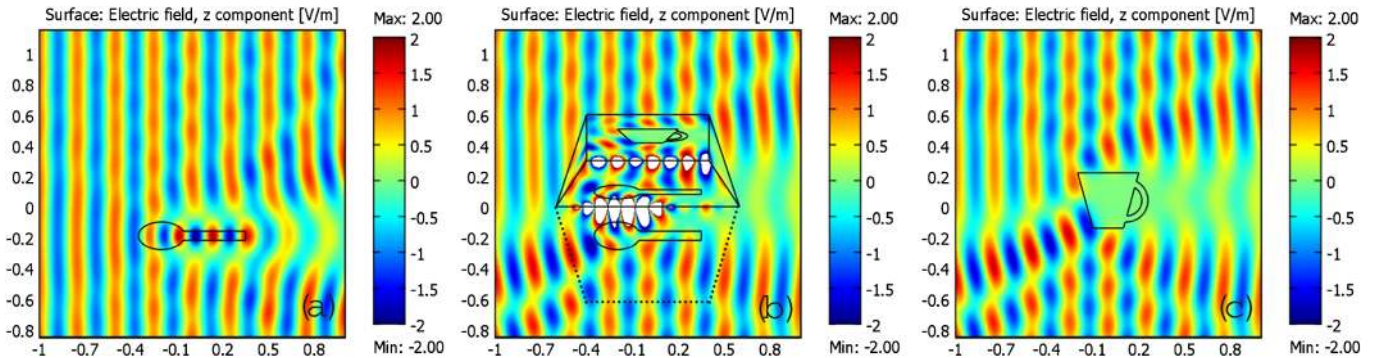

FIG. 2 (color online).   A numerical demonstration of transforming the stereoscopic image of a dielectric spoon of $\varepsilon_o = 2$ (the object) into that of a metallic cup of $\varepsilon_i = -1$ (the illusion) through an illusion device, under an incident TE plane wave from the left. (a) The scattering pattern of the dielectric spoon. (b) The scattering pattern of the dielectric spoon is changed by the illusion device. Outside the virtual boundary, the scattering pattern becomes the same as that of the metallic cup, which is shown in (c).

different kinds of incident waves can be found in the auxiliary material [30].

The illusion device in Fig. 2(b) is composed of four parts of homogeneous media. The lower trapezoidal part is the complementary medium formed by a simple coordinate transformation of $y^{(2)} = -y^{(3)}/2$. It is composed of a negative index homogeneous medium of $\varepsilon_z^{(2)} = -2$, $\mu_x^{(2)} = -2$, and $\mu_y^{(2)} = -0.5$, with an embedded "anti-object" of the dielectric spoon with $\varepsilon_{oz}^{(2)} = -4$ and $\mu_o^{(2)} = \mu^{(2)}$. The upper left triangular part, the upper right triangular part, and the upper middle rectangular part collectively constitute the restoring medium. The upper left and right triangular parts are composed of an homogeneous medium with $\varepsilon_z^{(1)} = 4$, $\mu_{xx}^{(1)} = 4$, $\mu_{yy}^{(1)} = 20.5$, and $\mu_{xy}^{(1)} = \pm 9$, formed by the coordinate transformations of $y^{(1)} \mp 3(x \pm 0.6) = 1/4[y^{(4)} \mp 3(x \pm 0.6)]$, respectively. The upper middle rectangular part is composed of an homogeneous medium of $\varepsilon_z^{(1)} = 4$, $\mu_x^{(1)} = 4$, and $\mu_y^{(1)} = 0.25$, with an embedded compressed version of the metallic cup illusion of $\varepsilon_{iz}^{(1)} = -4$ and $\mu_i^{(1)} = \mu^{(1)}$, formed by the coordinate transformation of $y^{(1)} - 0.6 = 1/4(y^{(4)} - 0.6)$. It is important to note that the homogeneity in the permittivity and permeability of the illusion device is a consequence of the simple coordinate transformations applied here. They do not bend straight light paths into curved ones as in light-bending cloaking devices [1–10].

The complementary medium is obtained from the transformation optics of folded geometry (see, for example, Leonhardt and Philbin [10]). It is composed of left-handed metamaterials with simultaneously negative permittivity and permeability. The medium can be isotropic if we apply a transformation of $y^{(2)} = -y^{(3)}$ instead of $y^{(2)} = -y^{(3)}/2$. This kind of metamaterial has been extensively studied in the application of the superlens [28], and it has been fabricated by resonant structures at various frequencies [23–27]. The other key component of the illusion device is the restoring medium, which projects the optical illusion of the metallic cup. It is composed of the homogeneous medium with positive but anisotropic permeability. This kind of medium can be designed from layer-structured metamaterials [15].

We note that some special illusion tricks by image projection using transformation optics have been discovered, such as the shifted-position image of an object inside a metamaterial shell [16], the cylindrical superlens [17], the "superscatterer" [18], the "reshaper" [19], and the "super absorber" [20]. Recently, we proposed an approach to realize "cloaking at a distance" by using an antiobject [21,22]. Here, by combining the antiobject cloaking functionality and the image projection functionality, we achieve a general form of illusion optics such that an object can be disguised into something else and the illusion device itself is invisible. This general form of illusion optics with arbitrary shape and generalized topology is proved mathematically as it is designed with transformation optics. The functionality is also demonstrated numerically. From a multiple scattering point of view, the illusion optics is in fact rather remarkable as it is by no means obvious that the antiobject cancelling and the image projection functionality do not obstruct or interfere with each other.

Another interesting application of our illusion device is that it enables people to open a virtual hole in a wall or obstacle. As our illusion device works at a distance from the object, it is capable of transforming only one part of an object into an illusion of free space, thus rendering that part invisible while leaving the rest of the object unaffected. By making one part of the wall invisible (i.e., making an illusion of a "hole"), we can see through the wall and obtain information from the other side. In Fig. 3(a), we see that a wall of $\varepsilon_o = -1$ with a width of 0.2 units is capable of blocking most of the energy of the TE electromagnetic waves of $\lambda = 0.25$ unit radiating from a point source placed at $(-0.7, 0)$. When the illusion device is placed on the right side of the wall, as shown in Fig. 3(b), the electromagnetic waves can penetrate through the wall as well as the illusion device and arrive on the right side. This effect can also be understood as the tunneling of electro-
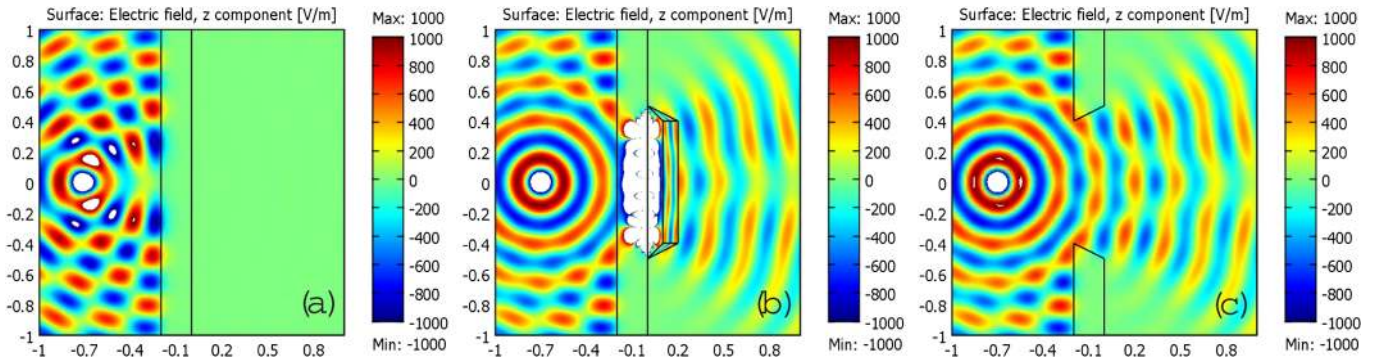

FIG. 3 (color online). The illusion device can create the illusion of a hole so that people can see through a wall at a selected frequency. (a) The electromagnetic radiation from a TE point source on the left side is blocked by a slab of $\varepsilon_o = -1$. (b) When an illusion device is attached to the wall, the electromagnetic radiation can now tunnel through the wall to the right side. The far field radiation pattern is exactly the same as that of the radiation through a real hole, which is shown in (c).

magnetic waves via the high-intensity surface waves localized at the interface between the wall and the complementary medium. The phase information is accurately corrected by the restoring medium in the illusion device, such that the transmitted field patterns on the right side become the same as those of the electromagnetic waves penetrating through a real hole, as shown in Fig. 3(c). Thus, an observer on the right side can peep through the virtual hole as if he or she is peeping through a real hole at the working frequency of the illusion device. The constitutive parts of the illusion device are similar to those in Fig. 2(b) and described in detail in the auxiliary material [30]. Similarly, an object hidden in a container can be completely revealed by using the illusion optics to change the container into an illusion of free space. This is also demonstrated in the auxiliary material [30].

In principle, the illusion optics allows us to remotely change the optical response of an object into that of any other object chosen for illusion at a selected frequency, without the need to change the constituents and shape of the true object or even cover its surface. This opens up interesting possibilities. For instance, an illusion waveguide or photonic crystal would allow the control of light propagation in actual free space; an illusion tip might perform near-field scanning optical microscopy without physically approaching a surface. However, the theoretical foundation of the illusion device is transformation optics, and, as such, our theory relies on the validity and accuracy of a linear continuous medium that describes the homogenized electromagnetic fields in metamaterials. This requirement is crucial in the interface between the complementary medium and the "cancelled" object due to the high-intensity local fields as well as rapid oscillations there. The range of the virtual boundary also plays an important role. When it is large, the field at the boundary will be large as well. Another issue that we have not considered is loss, which will compromise the illusion effect unless the object is close to the device. If these issues and challenges can be solved with advances in metamaterial technologies, we should be able to harness the power of transformation optics to create illusions.

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[30] See EPAPS Document No. E-PRLTAO-103-041927 for a detailed mathematical proof of illusion optics, more simulations of illusion optics under different incident waves as in Fig. 2(b), and a detailed description of the illusion device in Fig. 3(b). For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.