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DOI
10.1167/10.9.13

Publication date
2010

Document Version
Final published version

Published in
Journal of vision

Citation (APA)
Wijntjes, M. W. A., & Pont, S. C. (2010). Illusory gloss on Lambertian surfaces. *Journal of vision*, *10*(9), 1-12. [13]. https://doi.org/10.1167/10.9.13

Illusory gloss on Lambertian surfaces

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It has recently been shown that an increase of the relief height of a glossy surface positively correlates with the perceived level of gloss (Y.-H. Ho, M. S. Landy, & L. T. Maloney, 2008). In the study presented here we investigated whether this relation could be explained by the finding that glossiness perception correlates with the skewness of the luminance histogram (I. Motoyoshi, S. Nishida, L. Sharan, & E. H. Adelson, 2007). First, we formally derived a general relation between the depth range of a Lambertian surface, the illumination direction and the associated image intensity transformation. From this intensity transformation we could numerically simulate the relation between relief stretch and the skewness statistic. This relation predicts that skewness increases with increasing surface depth. Furthermore, it predicts that the correlation between skewness and illumination can be either positive or negative, depending on the depth range. We experimentally tested whether changes in the depth range and illumination direction alter the appearance. We indeed find a convincingly strong illusory gloss effect on stretched Lambertian surfaces. However, the results could not be fully explained by the skewness hypothesis. We reinterpreted our results in the context of the bas-relief ambiguity (P. N. Belhumeur, D. J. Kriegman, & L. Yuille, 1999) and show that this model qualitatively predicts illusory highlights on locations that differ from actual specular highlight locations with increasing illumination direction.

Keywords: 3D surface and shape perception, shading, ecological optics

Citation: Wijntjes, M. W. A., & Pont, S. C. (2010). Illusory gloss on Lambertian surfaces. *Journal of Vision, 10*(9):13, 1–12, http://www.journalofvision.org/content/10/9/13, doi:10.1167/10.9.13.

Introduction

An image contains too little information to reconstruct the shape, light field and (optical) material properties unambiguously. The reason behind this is the dimensional loss when projecting a scene that includes a 3D spatial layout, a 5D (neglecting temporal changes and color) light field and some unknown reflectance functions upon a 2D image. Nevertheless, humans do not seem to experience much problems in interpreting, for example, a photograph of a 3D scene. We perceive a stable solution, although not necessarily the veridical one. Inspired by computer vision, it is often thought that the human visual system uses an 'inverse optics' strategy to reconstruct the original scene: what optical conditions (the 'problem') could have resulted in a certain image (the 'solution'). This inverse problem is evidently underdetermined and the consequences of this are manifested through perceptual interactions. Perception of a single scene property (e.g. the shape) is affected by changes in other scene properties, e.g. light and reflectance (Adelson, 2001). For example, it was shown that a glossy object can appear matte by illuminating it by a diffuse instead of a collimated light source (Adelson & Pentland, 1996; Dror, Willsky, & Adelson, 2004; Pont & Te Pas, 2006). Also, the (meso) shape of 3D textures appears more rough for grazing illumination (Ho, Landy, & Maloney, 2006).

To decrease the ambiguity, assumptions (boundary conditions) can be used to solve the inverse optics problem. The observer can use assumptions that are based on mathematical constraints such as depth ordering by occlusion and shape inference by contours (Koenderink, 1984) but the observer can also base assumptions on regularities in nature such as global convexity (Langer & Bülthoff, 2001; Pentland, 1982) and light direction (Ramachandran, 1988; Sun & Perona, 1998). A class of assumptions that has recently received much attention are image statistics. It is thought that certain statistical properties may convey 'direct' information that supposedly bypasses the complex inverse optics scheme. For surface gloss, it has been proposed that the skewness of the luminance histogram can be diagnostic (Motoyoshi, Nishida, Sharan, & Adelson, 2007). According to this study, skewness is positively correlated with perceived glossiness. These types of mechanisms would greatly reduce the inverse optics scheme since direct knowledge of surface reflectance reduces the amount of ambiguity.

doi: 10.1167/10.9.13 Received March 15, 2010; published November 23, 2010 ISSN 1534-7362 © ARVO

Other research on the perception of glossiness has shown that there is an interaction between gloss and shape. Ho, Landy, and Maloney (2008) found that the 'bumpiness' of a stimulus increased the perceptual glossiness. In their study, bumpiness stands for the amount of stretch of the 3D surface in the viewing direction. Our study is motivated by the possible mediation of the skewness image statistic in the perceptual interaction of shape and gloss. As will be shown below, a stretch transformation will cause a transformation in the skewness of the luminance histogram such that the stretched, originally Lambertian (perfectly matte) stimulus should appear glossy. We wanted to measure whether this formal relationship indeed predicts a perceptual (illusory) gloss on Lambertian surfaces.

Model prediction for skew dependence on relief stretch and illumination angle

In Appendix A, we formally derived a relation between a geometric transformation of a surface and the pixel luminance transformation. Using this transformation we could simulate the relation between the geometric transformation and the luminance histogram distribution. More specifically we considered the effect of stretching a Lambertian surface in depth on the skewness of the luminance histogram. In case of perpendicular illumination this problem can be solved analytically. For a stretch

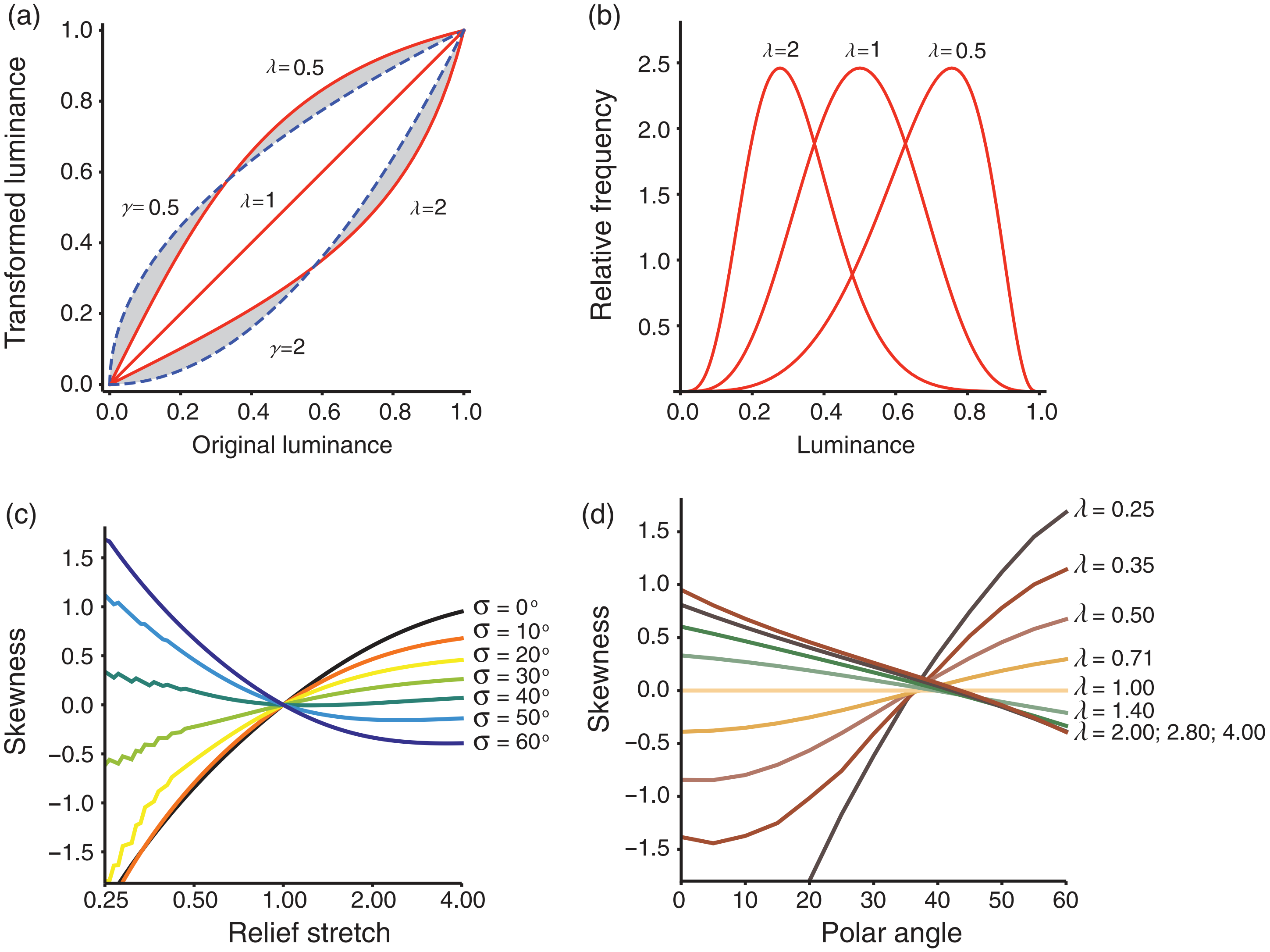


Figure 1. (a) Luminance transform for the lambda (red) and gamma (blue) transformation. (b) Effect of a lambda transformation on a zero-skew beta distribution. (c) The effect of relief stretch on luminance histogram skewness for various illumination directions. (d) The effect of illumination direction on skewness for various stretch values.

λ of the relief $\tilde{z}(x, y) = \lambda z(x, y)$, the associated image transform for each image pixel intensity $I(x, y)$ is

$$\tilde{I}_\lambda(I) = \sqrt{\frac{I^2}{I^2 + \lambda^2(1 - I^2)}}. \tag{1}$$

In Figure 1a, the luminance transform Equation 1 is plotted for three stretch parameters λ (red solid lines). This transformation, which we call the lambda transformation, resembles the gamma transformation ($\tilde{I}_\gamma(I) = I^\gamma$) that is used to linearize screen luminance but can also be used to alter the skewness of the histogram (see for example the supplementary notes of Motoyoshi et al., 2007). The resemblance between the two transformations can be seen in Figure 1a where the dashed blue lines denote the gamma transformation. The gray area between the gamma and lambda transformation denotes the difference. To quantify the effect of the lambda transformation on a luminance histogram, it was applied to a beta distribution, which can be seen in Figure 1b. While Equation 1 only deals with frontal illumination ($\sigma = 0$), we also derived a relation for non-zero polar angle. As can be read in Appendix A, this relation relies on a simplification of the 3D surface and is this less generic than Equation 1. We solved Equation A9 (see Appendix A) numerically to calculate the results of the non-zero polar angle model, using the same zero skew beta distribution as reference distribution. The results are presented in Figures 1c and 1d.

As can be seen from the y-axis of Figure 1c, a relief stretch (x-axis) in a range between 1/4 and 4 for frontal illumination ($\sigma = 0$) would result in a considerable skew change. However, when the illumination direction increases, the relation between skewness and relief stretch changes from positive to negative. In Figure 1d the same skewness data is plotted but now as a function of illumination direction for various relief stretches. Note that when an image has zero skew (in this case for $\lambda = 1$), it is invariant under illumination change. The range of skewness values in these figures is of comparable magnitude as the range that previously resulted in percepts ranging from matte to glossy (Motoyoshi et al., 2007). On the basis of those psychophysical results, it can be predicted that stretching a Lambertian (i.e. matte) surface about 8 times, would change the appearance substantially towards glossy. We wanted to test this hypothesis that a relief stretch transformation can change the material appearance from matte to glossy.

The model predictions are based on the relative changes in skewness with respect to an image with a zero skew beta distribution. This is why in this section we used a λ-range around 1. However, for the psychophysical stimuli that we will describe below, we could not predict the absolute skew. Therefore, the λ-ranges are relative and we attributed the shallowest relief the arbitrary value of 1. As will become clear in the Results section (Figure 3b), our stimuli were in fact approximately distributed around zero skewness for frontal illumination.

Methods

Stimuli

The stimuli were renderings of rectangular Brownian surfaces, 512 pixels wide and high. These surfaces have a power spectral density of approximately $1/f^2$, which defines a Brownian random surface (Saupe, 1988). We used Brownian surfaces because their geometry is close to natural surfaces (Mandelbrot, 1983). As such they can be used as generic natural (but random) surface stimuli.

We generated a total of 300 statistically independent surfaces, subdivided in four groups of different relief scales. Four values of rms height were used, 16, 32, 64 and 128 pixels, which relates to depth stretch values $\lambda = 1, 2, 4, 8$. Within each group the surfaces were rendered with a collimated light source that varied in polar angle from 0 (frontal illumination) to 60 degrees (grazing illumination). The rendering procedure was programmed in *C* and consisted of two phases. First, at each pixel location the inner product of the surface normal and illumination direction (Lambertian shading) was calculated. The illumination strength was set at one (white), so surface patches oriented in the direction of illumination were white. No tone mapping was applied. Second, the image was multiplied with a cast and body shadow map so that all pixels located in body or cast shadows were rendered black. Inter-reflections were discarded. The azimuthal angle was uniformly, randomly distributed between 1 and 360 degrees. The reflectance was Lambertian. A black circular aperture with a diameter of 488 pixels was superimposed on the rendering. Examples of the stimuli can be seen in Figure 2.

Procedure

The stimuli were presented on a calibrated CRT display with linear gamma. Viewing distance was 60 cm and the stimuli subtended an angle of 11.6 degrees. The psychophysical task was to rate the glossiness of each stimulus. Participants could adjust a linear slider that ranged from matte (−1) to glossy (+1). The linear slider was presented in a vertical position on the right side of the stimulus and could be controlled by the mouse. The experiment was programmed in PsychToolbox (Brainard, 1997; Pelli, 1997). Before the actual estimation experiment started, a random selection of 50 (of the 300) stimuli was presented each for 2 seconds. This was done to familiarize the

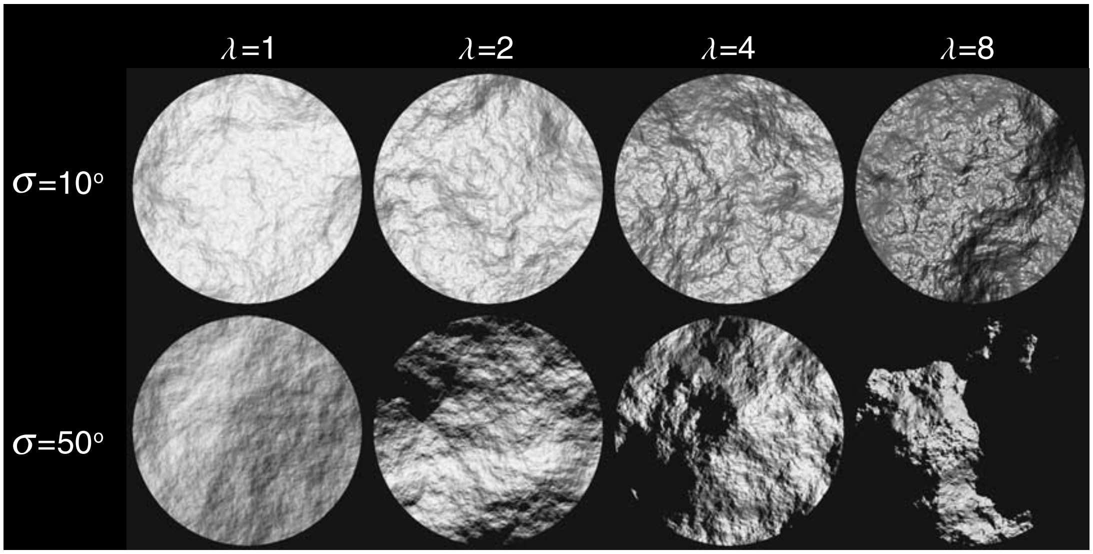


Figure 2. Stimulus examples for λ = 1, 2, 4, 8 and two illumination polar angles.

participants with the range of different stimuli. Immediately after the familiarization phase, the 300 stimuli were shown in random order. There was no time limit. After the participants estimated the glossiness on the slider, the spacebar could be hit to proceed to the next trial.

Participants

Six observers with normal or corrected-to-normal vision participated with the experiment. Four observers were naive to the purpose of the experiment and two were the authors. Participation was voluntary and no reimbursement was given.

Results

Skew analysis of the Brownian surface renderings

We first tested whether our model predicts the skewness of the rendered Brownian surface stimulus set. For each stimulus two skewness values were calculated: with and without the black pixels resulting from cast and body shadows. The results are presented in Figures 3a (with black pixels) and 3b (without black pixels). The circles denote individual stimuli, the colored line and region denote the moving average and moving standard deviation (window size = 12 data points), respectively. As can be seen, the difference between inclusion and exclusion of black pixels increases with increasing stretch, as can be expected. Furthermore, when black pixels are excluded, the results are in line with the prediction illustrated in Figure 1d. Apparently, the zero skewness relief has a stretch value a little below $\lambda = 4$. This is a free parameter since we did not know the skewness of the image of a Brownian surface in advance. The predictions were all relative to a zero skew image. When the stretch is lower, the relation between the polar angle and skewness is positive while for larger values the relation becomes negative. Lastly, we highlighted the near-frontal illumination results by the red symbols. This positive relation is (trivially) similar for with and without shadows calculations and is in line with our prediction.

Psychophysical experiments

The average glossiness estimations are plotted in Figure 4. The data are plotted in a similar way as the skewness data. On average the stimuli appear more glossy when the depth increases. Also, within each depth scale, a clear trend with respect to the illumination polar angle can be observed. The subjective gloss decreases with increasing polar angle, for all four depth scales. The influence of stretch and illumination direction on perceived glossiness (z) was confirmed by a multiple linear regression on the independent variables stretch (x) and illumination direction (y): $z = a_0 + a_{stretch}x + a_{illumination}y$ ($F(297, 2) = 957.934$, $p < 0.001$; $a_{stretch}$: $t(296) = 34.075$, $p < 0.001$; $a_{illumination}$: $t(296) = -27.881$, $p < 0.001$).

Discussion

The skewness analysis of the Brownian surfaces is in line with the model prediction. The derivation in the appendix does not take into account two factors: shadows and 3D surface variations (see Figure A1). The effects of

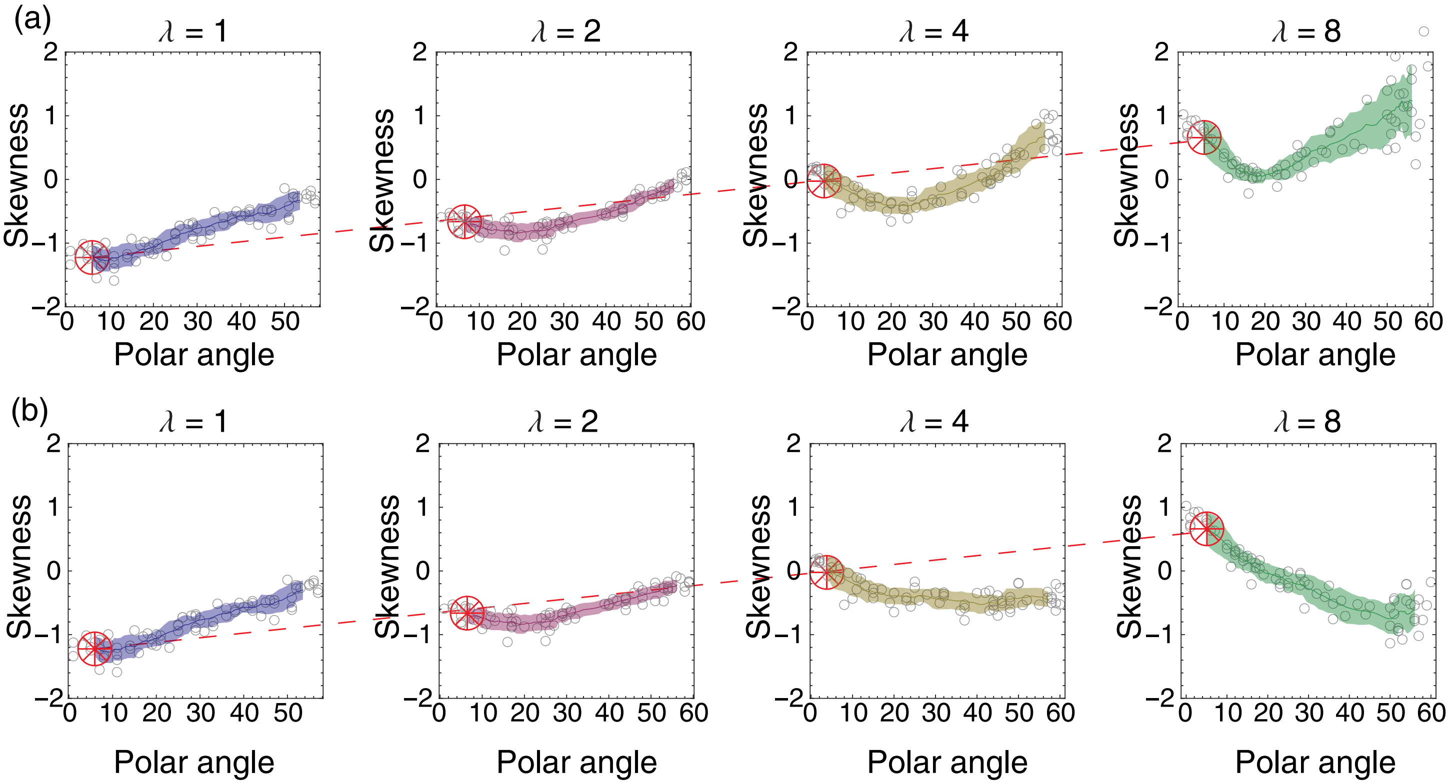


Figure 3. Skewness as a function of illumination direction for four stretch values. Above the black pixels were included, below excluded. The red symbol shows the skewness values for near-frontal illumination.

the first factor can be found in the difference between skewness values with and without taking into account black pixels as shown in Figure 3. The model predicts that for low reliefs there is a positive relation between skewness and polar angle and a negative relation for high reliefs (Figure 1d). This is indeed found for the Brownian surfaces when black pixels are not taken into account. However, black pixels are present in our stimuli and could have played a role in the psychophysical results. Furthermore, we did not take into account interreflections, which could have influenced both the skewness values and the psychophysical data. Secondly, the model did not take into account variations in the y-direction ($\frac{\partial z}{\partial y} = 0$). Since the model (Figure 1d) and data (Figure 3b) seem to agree qualitatively rather well, this assumption seems to be justified.

While our predictions about the dependence of skewness on stretch and illumination are in line with the stimulus data, the psychophysical data seems to partly contradict the relation between skewness and perceived gloss. For near-frontal illumination (indicated by the red symbols), the skewness hypothesis seems to hold: when the relief

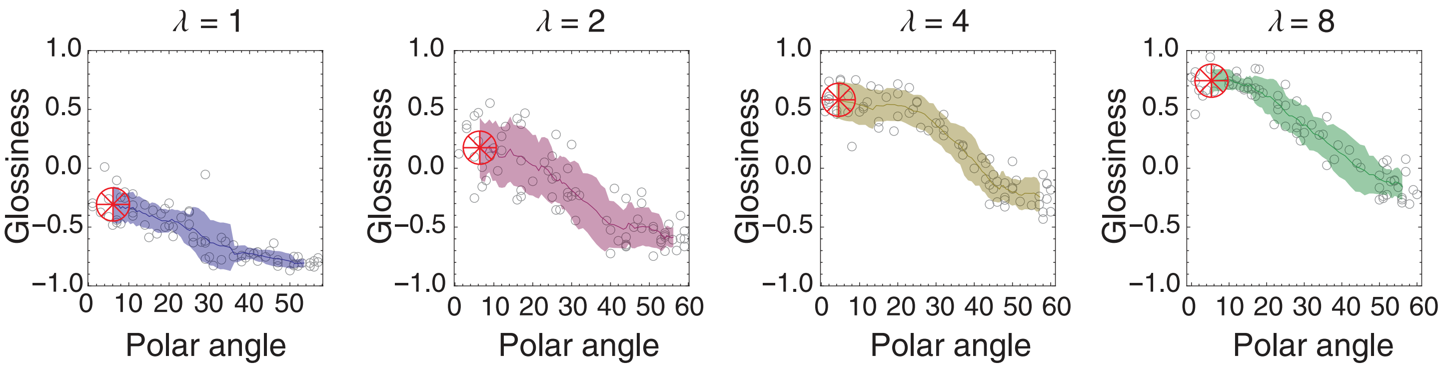


Figure 4. Perceived gloss as a function of illumination direction for four stretch values. On average the gloss increases with increasing relief but decreases with increasing illumination direction.

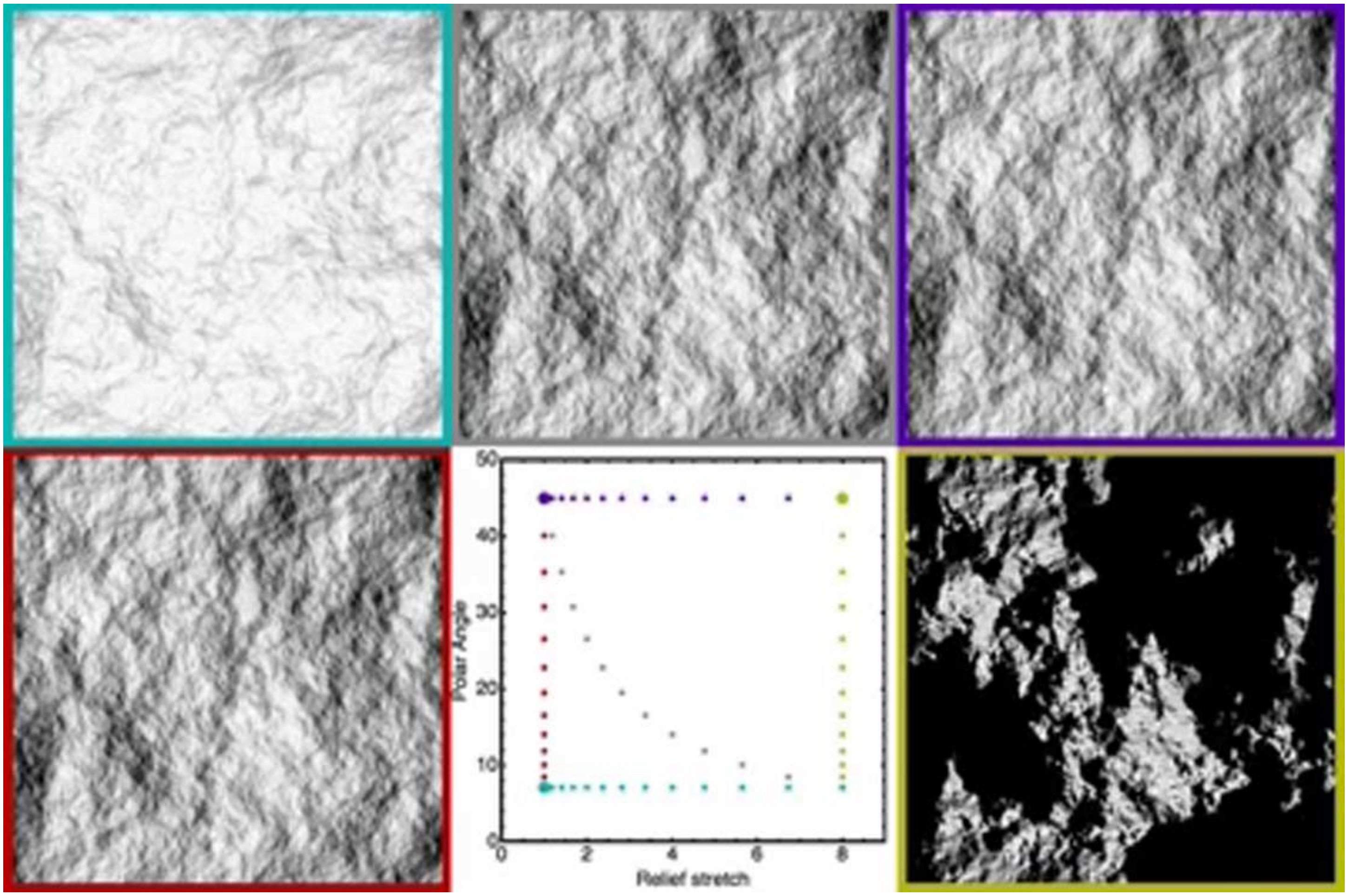
increases, the skewness increases and so does the perceived gloss. However, for more oblique illumination directions the skewness hypothesis does not hold anymore. This will be discussed further in the General discussion.

General discussion

We have shown that there is a formal, shape independent relationship between the skewness and stretch and illumination direction in case of Lambertian reflectance. On the basis of this relationship, we predicted that a Lambertian surface could be transformed and illuminated such that it appears glossy. The illusion is evidently strong and convincing and can be regarded as an extreme version of the interaction between shape and gloss reported by Ho et al. (2008). While in that study an interaction was revealed between relief stretch ('bumpiness') and gloss for stimuli that already possessed some specular components, our study shows that apparent gloss can be induced by geometrically transforming Lambertian surfaces.

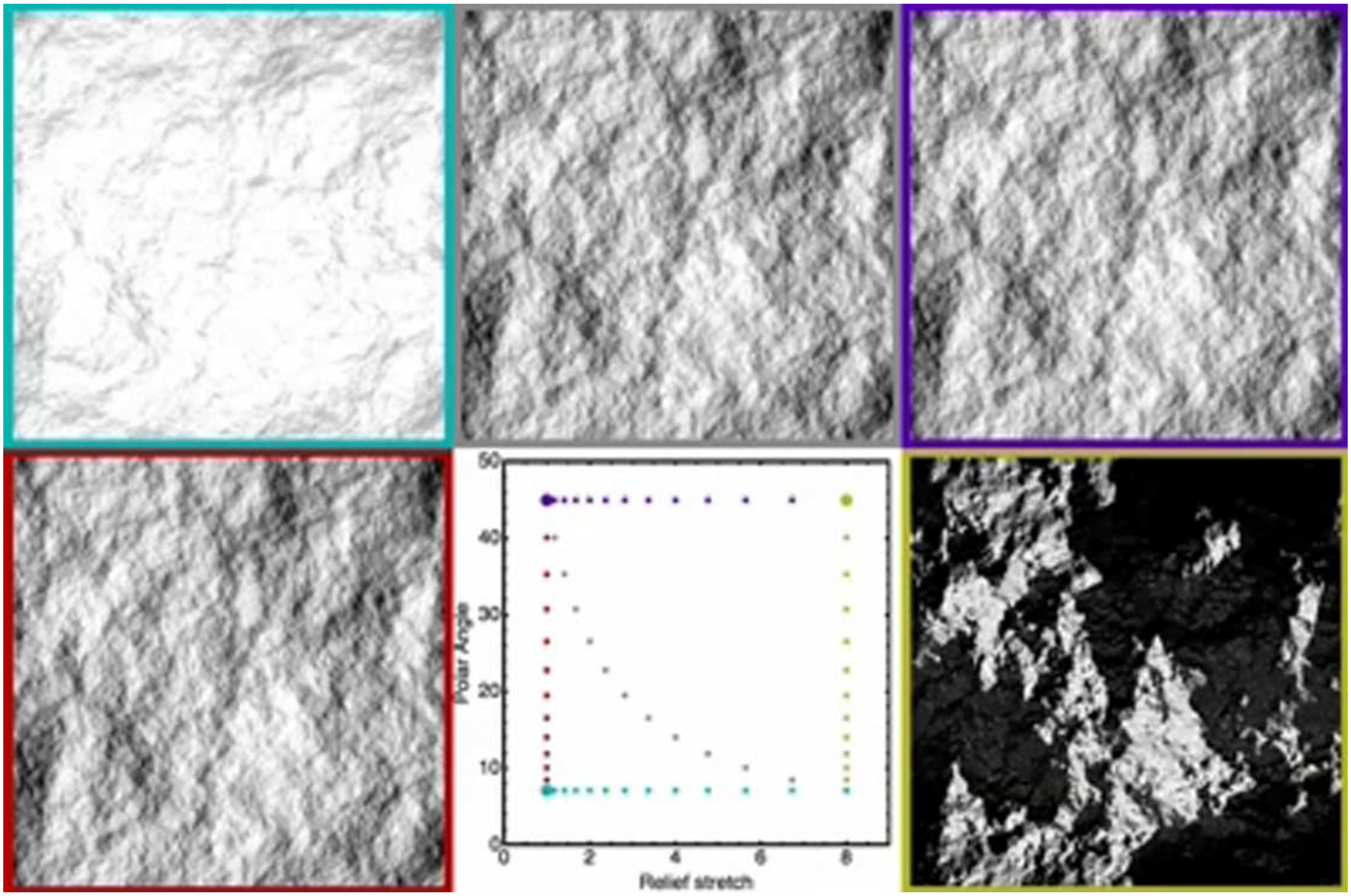
The generalizability of the gloss illusion

Before discussing possible explanations of the illusory gloss, we will first discuss the generalizability of the illusion. The main question is whether the illusion can also be found in the 'real world'. Our stimuli were surfaces of uniform albedo, Lambertian reflectance, illuminated by a collimated light source and rendered without inter-reflections. Although complete Lambertian reflectance does not occur in nature, it approximates matte appearance and the reason behind our use of a matte



Movie 1. During 13 frames, Brownian surfaces are rendered for five shape-light combinations. The red and yellow framed images show a of shallow and deep relief for decreasing illumination polar angle. As can be seen, the high relief shows large cast shadows when the polar angle is large (grazing illumination). The blue and purple framed images show reliefs illuminated near frontal (blue) and at a grazing angle (purple), while the relief height increases by a factor of eight. Finally, the gray framed image shows a surface along the bas-relief relation.

BRDF was to show that it could be perceived as glossy. A collimated light source may be less natural than for example HDR environment maps. High order (in terms of spherical harmonics) light fields are important for the appearance of actual glossy stimuli (Fleming, Dror, & Adelson, 2003). However, they are of lesser importance for the appearance of Lambertian stimuli (Basri & Jacobs, 2003). Since our stimuli are Lambertian, it is reasonable to use a simple light field such as collimated illumination. Lastly, the absence of inter-reflections in our stimuli is an important difference with 'reality'. Especially because the scattering of light in high reliefs may decrease the local contrast by enlightening the dark slopes and thus decreasing the glossy appearance. To qualitatively assess the potential effect of interreflections we produced renderings with *Radiance* (Ward, 1994). To increase the understanding of how the appearance of a Brownian surface changes under the shape and light transformations we used in our experiments, we present a movie that shows five paths through this shape-light parameter space. Movie 1 is without inter-reflections and Movie 2 is rendered with 2 'ambient bounces'. The colors of the frames relate to the five parameter space paths. The gray path indicates the bas-relief ambiguity relation, which will be discussed later. The other paths denote changes in either the illumination or the relief stretch, keeping the other constant. In the last frame of the movie, the blue, gray, and yellow framed images should appear optimally glossy: high relief in combination with near-frontal illumination. As can be seen, it seems that there is no pronounced difference in gloss appearance between the with and without inter-reflections images. Besides renderings, we also photographed two real surfaces in our lab. Two Gaussian surfaces that only differed with respect to a stretch transformation were photographed with a collimated light source (a theatre spot light). The surfaces were computer milled and were approximately matte. As can be seen in Figure 5, the high relief surface (on the right) shows similar illusory gloss as we found in our experiments. Thus, from these illustrations it does not appear that inter-reflections play an important role in the gloss illusion.



Movie 2. Similarly organized as Movie 1 but the renderings are now performed with two inter-reflections (ambient bounces). The inter-reflections are most pronounced in the cast and body shadows in the yellow and purple framed images.

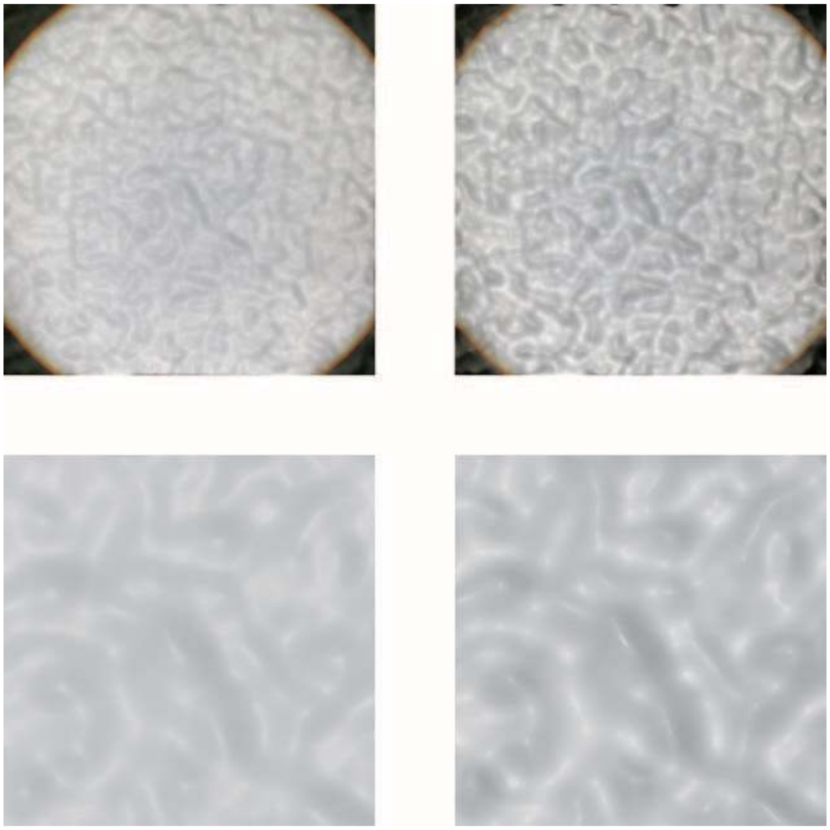


Figure 5. Photographs of real Gaussian surfaces. On top the whole surfaces are shown, at the bottom a magnification of the middle is shown. The left relief is has a three times shallower relief compared to the right surface. Both surfaces were 40 cm wide.

Besides differences between renderings and realistic images, our stimuli may be deprived from cues that help to disambiguate between gloss and depth. Moving surfaces, such as used by Nishida and Shinya (1998), may increase gloss constancy with respect to static images, both from the movement of specular highlights (specular flow) and structure-from-motion. It is likely that these additional cues will disambiguate the depth-gloss ambiguity but the strength of this disambiguation should be quantified experimentally and is beyond the scope of the current investigation.

The relation between skewness and perceived glossiness

Although our results are in line with the study of Ho et al. (2008), it is yet unclear whether we can explain both outcomes with the skewness hypothesis. Our results show that for near-frontal illumination, the relation between relief stretch and perceptual gloss can be explained by the skewness hypothesis. However, the relation of perceived gloss with increasing illumination polar angle is not in line with the skewness hypothesis. Recently, Anderson and Kim (2009) have argued against the validity of the skewness hypothesis. They found that if the highlights are artificially rotated or translated in the image (keeping the skewness constant), the perceived gloss decreases. Since the highlights should be in the 'correct' position with respect to the geometry of the stimulus, they argue that perceived gloss is mediated by a photo-geometric process instead of the luminance histogram skewness which is a purely photometric statistic. Furthermore, they argue that the stimulus set used by Motoyoshi et al. (2007) was rather restricted: only one illumination direction was used. Anderson and Kim (2009) note that altering the illumination direction may have an important effect on the skewness. Our study shows that this is indeed the case. Although the study presented here was motivated by the hypothesis that the shape-gloss interaction reported by Ho et al. (2008) could be explained by a shape-skewness interaction, our results seem to contradict the skewness hypothesis (Motoyoshi et al., 2007) and are more in line with the photo-geometric hypothesis by Anderson and Kim (2009). However, these two hypotheses are both extremes of possible mechanisms underlying gloss perception. On the one hand, the luminance histogram skewness is both an image based *and* non-spatial statistic. On the other hand, the photo-geometric hypothesis is based on a complex inverse-optics scheme. According to Anderson and Kim (2009), the actual geometry of the surface should be known after which the visual system can check whether the highlights are in the 'correct' positions. However, this requires surface geometry knowledge which can only be attained by having assumptions about the reflectance and illumination. As Anderson and Kim (2009) write, the shape, reflectance and illumination are all conflated in the 2D image. Precisely this difficulty would be solved if a 'short-cut' existed that is purely based on image statistics. An intermediary hypothesis could involve a *spatial* image statistic that depends on the geometry of the image instead of the geometry of the imaged scene, for example histograms of (multiscale) image structure curvatures or the statistics related to illuminance flow (Pont & Koenderink, 2003).

The bas-relief ambiguity

We found that two variables influence illusory gloss: relief stretch and illumination direction. The highest gloss was found for a high relief and small polar angle while the lowest gloss was found for a low relief and large polar angle. This relation resembles the relation between light and shape known as the generalized bas-relief ambiguity Belhumeur, Kriegman, and Yuille (1999). They prove that for any Lambertian shape illuminated by a collimated light source, an equivalence class exists of affine transformed shapes and illumination directions that result in a similar image. Thus, an image of a Lambertian shape is unique up to an affine transformation if the illumination direction is unknown. However, the proof also includes a local albedo transformation that depends on the surface attitude. For small shape transformations, the albedo transformation is small and thus negligible, but for larger transformations, e.g. a stretch difference of a factor 4, the albedo transformation becomes substantial. In Figure 6 (see also Movie 3, and the gray framed images in Movies 1 and 2) we have rendered a single Brownian surface for four different stretch values and illumination directions according to the generalized bas-relief transformation, as indicated by the plot. No albedo correction has been applied.

As can be seen, the reflectance of the surfaces appears to be increasingly glossy for increasing stretch magnitude. Thus, the bas-relief ambiguity seems to introduce an extra ambiguity, that of surface appearance. Belhumeur et al. (1999) showed that the appropriate albedo correction can be written as

$$\tilde{a} = \frac{a}{\lambda}\left(\frac{(\lambda f_x+\mu)^2+(\lambda f_y+\nu)^2+1}{f_x^2+f_y^2+1}\right)^{\frac{1}{2}}, \qquad (2)$$

where μ and ν are the affine shear components in the x and y direction, respectively. Since we only considered stretch in the viewing direction, these two values are zero.

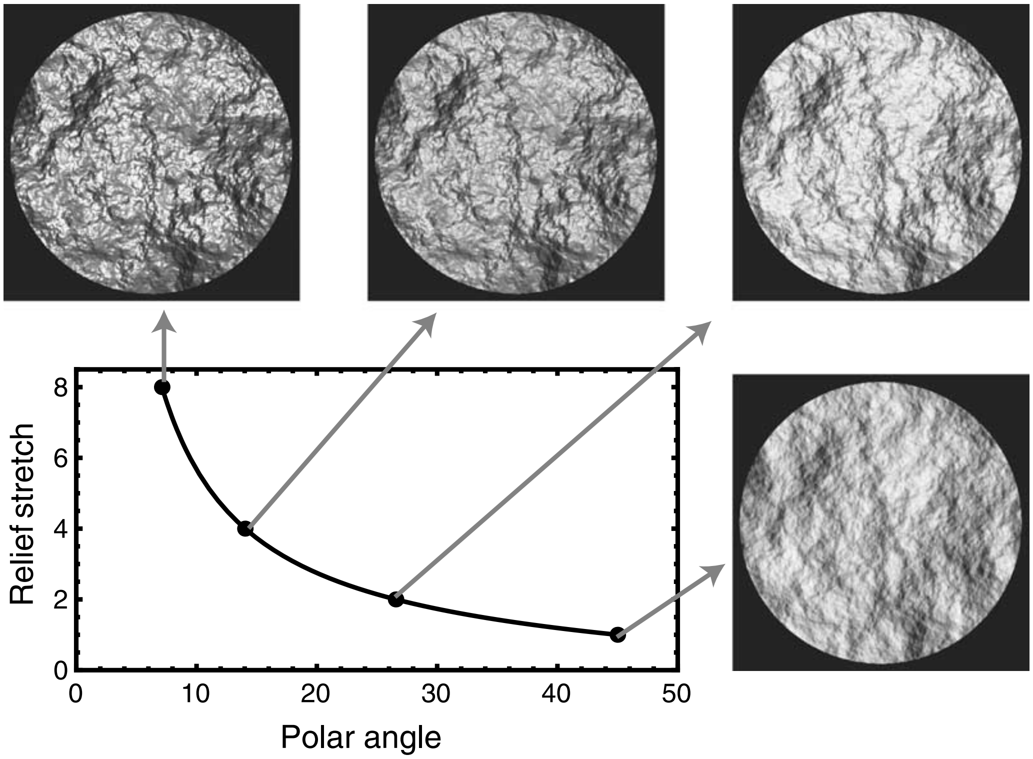
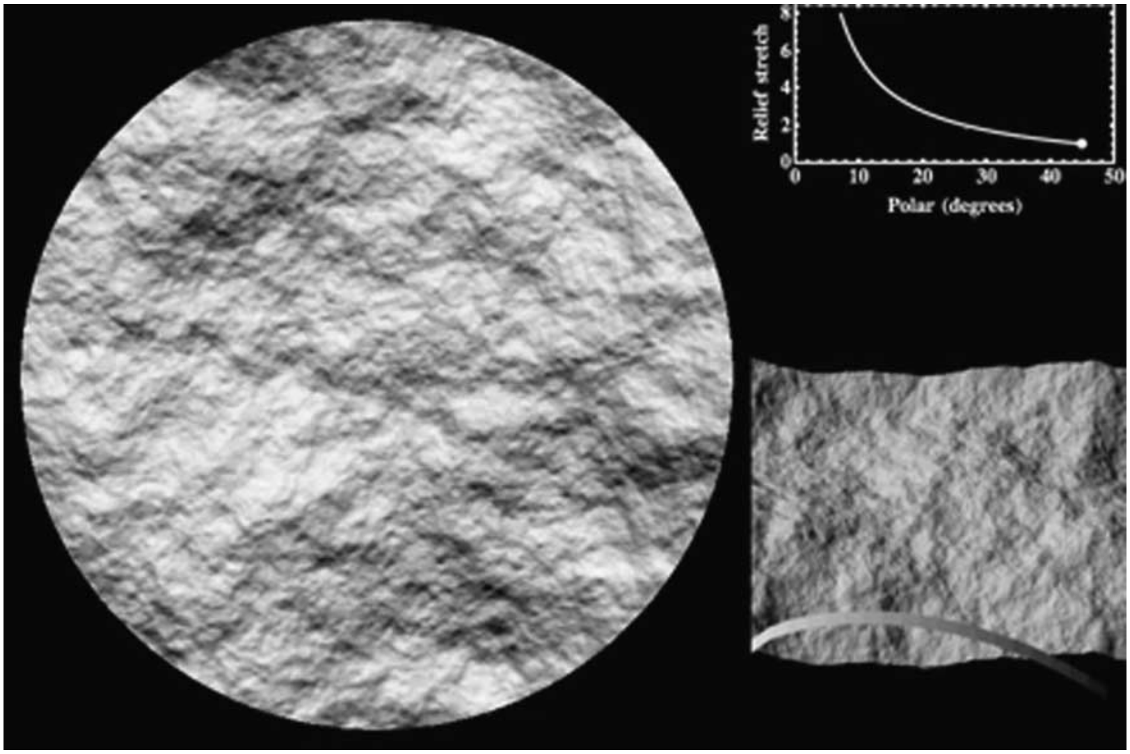


Figure 6. Four renderings on the bas-relief characteristic which should leave the image intact up to an albedo correction. No albedo correction was applied which resulted in illusory gloss for the high reliefs.



Movie 3. Another illustration of the changing surface appearance when light and shape change according to the bas-relief transformation. The side view of the surface gives a good idea of the amount of stretch that was used in our experiments.

If we now assume a (locally) symmetric shape, i.e. $f_x = f_y$, the equation simplifies to

$$\tilde{a} = \frac{a}{\lambda}\left(\frac{2(\lambda f_x)^2 + 1}{2f_x^2 + 1}\right)^{\frac{1}{2}}. \tag{3}$$

For stretch values $\lambda > 1$ this correction will darken the points on the surface which are directed in the viewing direction ($f_x = 0$). This means that if this correction is not performed, these points in the direction of the viewing direction will be highlighted with respect to the unstretched version ($\lambda = 1$). Thus, if the illumination is in the same direction as the viewing direction, highlights will appear that would otherwise appear if the surface were glossy instead of Lambertian. If the illumination direction changes away from frontal, the specular highlights of an actual glossy surface would shift accordingly while the illusory highlights on the stretched Lambertian surface will be unaffected since there is no illumination term in the albedo correction Equation 2. In other words, the congruence of illusory (matte) highlights and actual (specular) highlights decreases with increasing illumination direction. Hence, the illusory gloss highlights are in the 'wrong' position which, as Anderson and Kim (2009) have shown, should lead to a decrease in apparent gloss (although off-specular reflection is physically possible (Torrance & Sparrow, 1967)). Indeed, this is what our data show. On the basis of our experiments we cannot completely discard the skewness hypothesis but we can conclude that there is more to perceived gloss than this statistic. The appearance of illusory highlights when stretching a relief could be an important factor that qualitatively explains the relation of apparent gloss with illumination direction.

While the explanation in terms of illusory highlights is still tentative, the main result of our study is that a simple geometric transformation can change the appearance of a matte surface into glossy. This is a novel visual illusion that exemplifies the perceptual interaction between shape and material appearance. This perceptual interaction is a key factor in understanding how the visual system resolves the problem of reconstructing the 3D scene from a 2D image.

Appendix A

Frontal illumination

For a relief $z(x, y)$ with Lambertian reflectance and unit albedo under collimated illumination, the orthographically projected image as seen from the positive z-direction equals:

$$I(x, y) = \mathbf{L} \cdot \mathbf{n}(x, y). \tag{A1}$$

The (unit) illumination vector can be written as a function of the polar angle σ and the azimuthal angle θ:

$$\mathbf{L} = (\cos\theta\sin\sigma, \sin\theta\sin\sigma, \cos\sigma). \tag{A2}$$

The surface normal vectors can be written as a function of the partial derivatives of the surface relief $z(x, y)$:

$$\mathbf{n} = \frac{(-z_x, -z_y, 1)}{\sqrt{z_x^2 + z_y^2 + 1}}, \tag{A3}$$

where z_x and z_y denote the partial derivatives $(\frac{\partial z}{\partial x})$ and $(\frac{\partial z}{\partial y})$, respectively.

We want to understand the influence of a relief stretch on the image, i.e. given a surface transformation $\tilde{z}_\lambda(z) = \lambda z$, where λ is a positive scalar, what is the associated image transformation $\tilde{I}_\lambda(I)$? Importantly, we are looking for a solution in which the shape information ($z(x, y)$ and its derivatives) can be eliminated. If this is possible the image transform will be a generic, shape independent transform. Since the surface is isotropic we can set $\theta = 0$ without loss of generality. The original and transformed image can now be written according to Equation A1:

$$I(x, y) = \frac{\cos\sigma - z_x\sin\sigma}{\sqrt{z_x^2 + z_y^2 + 1}}, \tag{A4}$$

$$\tilde{I}_\lambda(x, y) = \frac{\cos\sigma - \lambda z_x\sin\sigma}{\sqrt{\lambda^2(z_x^2 + z_y^2) + 1}}. \tag{A5}$$

A simple case for which Equations A4–A5 can lead directly to the desired $\tilde{z}_\lambda(z) = \lambda z$ relation is to take the illumination in frontal direction, i.e. $\sigma = 0$. This leads to

$$I(x, y) = \frac{1}{\sqrt{f(x, y) + 1}}, \tag{A6}$$

$$\tilde{I}(x, y) = \frac{1}{\sqrt{\lambda^2 f(x, y) + 1}}, \tag{A7}$$

with $f(x, y) = z_x^2 + z_y^2$, which contains the shape information of the surface. The shape information can be eliminated, which leads to the desired image transform

$$\tilde{I}_\lambda(I) = \sqrt{\frac{I^2}{I^2 + \lambda^2(1 - I^2)}}. \tag{A8}$$

Nonzero polar angle

The more generic case for arbitrary illumination polar angle σ is more complicated, and can only be found numerically. The only way to eliminate the shape information from Equations A4–A5 is to set either z_x or z_y to zero. This means that in one of these directions the shape is constant. Since we chose the illumination in the x-direction ($\theta = 0$), the variation in this direction should be non-zero. Therefore, we set $z_y = 0$. This resulting shape is some (irregular) grating. We have illustrated this for the case of Brownian surfaces in Figure A1. As can be seen in the axes of the figure, the illumination is directed perpendicular with respect to the grating direction.

Now it becomes possible to eliminate z_x from both Equations A4–A5 as follows:

$$z_x = \frac{\sin\sigma\cos\sigma \pm \tilde{I}\sqrt{1 - \tilde{I}^2}}{\lambda(\sin^2\sigma - \tilde{I}^2)} = \frac{\sin\sigma\cos\sigma \pm I\sqrt{1 - I^2}}{\sin^2\sigma - I^2}. \tag{A9}$$

This equation cannot be solved analytically for the transformed image intensity, $\tilde{I}_{\lambda,\sigma}(I)$. Nevertheless, the equation can be numerically solved. The results of this solution are presented in the main text.

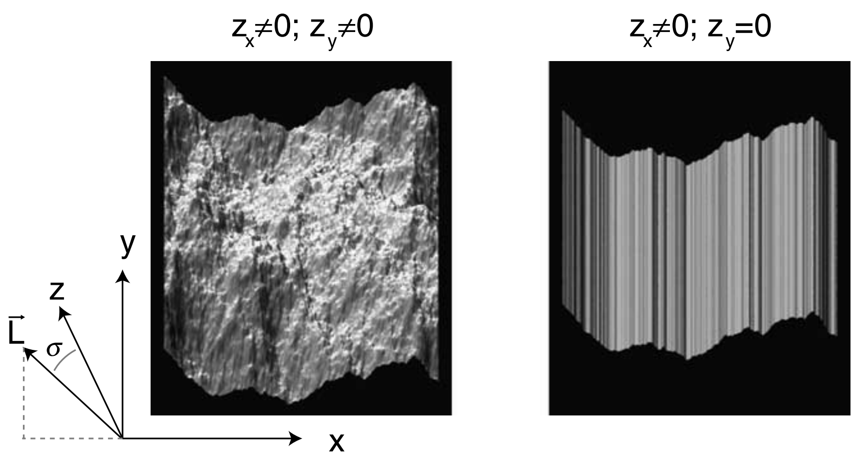


Figure A1. Illustration of $z_y = 0$.

Acknowledgments

This work was supported by a grant from the Netherlands Organisation of Scientific Research (NWO).

Commercial relationships: none.

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