

Image Compression Using Modified Fast Haar Wavelet Transform

Anuj Bhardwaj and Rashid Ali

Department of Mathematics, Vishveshwarya Institute of Engineering and Technology,
Dadri, G. B. Nagar-203207, U.P. India

Abstract: Wavelets are mathematical tools for hierarchically decomposing functions. Wavelet Transform has been proved to be a very useful tool for image processing in recent years. It allows a function which may be described in terms of a coarse overall shape, plus details that range from broad to narrow. The most distinctive feature of Haar Transform lies in the fact that it lends itself easily to simple manual calculations. Modified Fast Haar Wavelet Transform (MFHWT), is one of the algorithms which can reduce the calculation work in Haar Transform (HT) and Fast Haar Transform (FHT). The present paper attempts to describe the algorithm for image compression using MFHWT and shows better results than those obtained by using any other method on an average. It includes a number of examples of different images to validate the utility and significance of algorithm's performance.

Key words: Haar Wavelet Transform • Fast Haar Wavelet Transform • Modified Fast Haar Wavelet Transform
• PSNR

INTRODUCTION

As computers have become more and more powerful, the temptation to use digital images has become irresistible. Image compression plays a vital role in several important and diverse applications, including televideoconferencing, remote sensing, medical imaging [1, 2] and magnetic resonance imaging [3] and many more [4]. These requirements are not fulfilled with old techniques of compression like Fourier Transform, Hadamard and Cosine Transform etc. due to large mean square error occurring between original and reconstructed images. The wavelet transform approach serves the purpose very efficiently. The wavelet transform, developed for signal and image processing, has been extended for use on relational data sets [5, 6].

The basic idea behind the image compression is that in most of the images we find that their neighbouring pixels are highly correlated and have redundant information [7]. It is, therefore, necessary to find a less correlated representation of the image and it can be done by removing redundancy and irrelevancy. Redundancy reduction removes duplication in image and irrelevancy reduction omits that part of the signal which is not noticed by Human Visual System (HVS)[8]. In context of an image, it produces a multiresolution

representation, which has been shown to be naturally suited for progressive transmission. The wavelet transform is often used for signal and /or image smoothing keeping in view of its "energy compaction" properties, i.e. large values tend to become larger and small values smaller, when the wavelet transform is applied.

Since the Haar Transform is memory efficient, exactly reversible without the edge effects, it is fast and simple. As such the Haar Transform technique is widely used these days in wavelet analysis. Fast Haar Transform is one of the algorithms which can reduce the tedious work of calculations. One of the earliest versions of FHT is included in HT [9]. FHT involves addition, subtraction and division by 2. Its application in atmospheric turbulence analysis, image analysis, signal and image compression has been discussed in [10].

The Modified Fast Haar Wavelet Transform (MFHWT) has been discussed in [11], in which the MFHWT is used for one-dimensional approach and FHT is used to find the $N/2$ detail coefficients at each level for a signal of length N . In this paper the author has used the same concept of finding averages and differences as in [11] but here that approach is extended for 2D images with the addition of considering the detail coefficients 0 for $N/2$ elements at each level.

In section 2, the Haar Transform and Fast Haar Transform have been explained. In section 3, Modified Fast Haar Wavelet Transform is presented with the proposed algorithm for 2D images. Results and discussion are given in section 4 followed by conclusion in 5.

Haar Transform and Fast Haar Transform: The Haar Transform (HT) is one of the simplest and basic transformations from the space domain to a local frequency domain. A HT decomposes each signal into two components, one is called average (approximation) or trend and the other is known as difference (detail) or fluctuation. A precise formula for the values of first average subsignal, $a^1 = (a_1, a_2, \dots, a_{N/2})$, at one level for a signal of length N i.e. $f = (f_1, f_2, \dots, f_N)$ is

$$a_n = \frac{f_{2n-1} + f_{2n}}{\sqrt{2}}, n = 1, 2, 3, \dots, N/2,$$

and the first detail subsignal, $d^1 = (d_1, d_2, \dots, d_{N/2})$, at the same level is given as

$$d_n = \frac{f_{2n-1} - f_{2n}}{\sqrt{2}}, n = 1, 2, 3, \dots, N/2.$$

In order to give an idea of its implementation in image compression, the procedure of its application may be explained with the help of a simple example as shown below. Apply 2D HT to the following finite 2D signal.

Example 1:

$$I = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 7 & 8 \\ 6 & 2 & 1 & 8 \\ 2 & 5 & 4 & 7 \end{pmatrix}$$

using 1D HT along first row, the approximation coefficients are

$$\frac{1}{\sqrt{2}}(1+2) \text{ and } \frac{1}{\sqrt{2}}(3+4)$$

and the detail coefficient are

$$\frac{1}{\sqrt{2}}(1-2) \text{ and } \frac{1}{\sqrt{2}}(3-4)$$

The same transform is applied to the other rows of I . By arranging the approximation parts of each row transform in the first two columns and the corresponding detail parts in the last two columns we get the following results:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 7 & 8 \\ 6 & 2 & 1 & 8 \\ 2 & 5 & 4 & 7 \end{pmatrix} \xrightarrow{1D HT \text{ on row}} \frac{1}{\sqrt{2}} \begin{pmatrix} 3 & 7 & : & -1 & -1 \\ 7 & 15 & : & 1 & -1 \\ 8 & 9 & : & 4 & -7 \\ 7 & 11 & : & -3 & -3 \end{pmatrix}$$

in which approximation and detail parts are separated by dots in each row. By applying the following step of 1D HT to the columns of the resultant matrix, we find that the resultant matrix at first level is

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 3 & 7 & : & -1 & -1 \\ 7 & 15 & : & 1 & -1 \\ 8 & 9 & : & 4 & -7 \\ 7 & 11 & : & -3 & -3 \end{pmatrix} \xrightarrow{1D HT \text{ on columns}} \frac{1}{2} \begin{pmatrix} 10 & 22 & : & 0 & -2 \\ 15 & 20 & : & 1 & -10 \\ \dots & \dots & : & \dots & \dots \\ -4 & -8 & : & -2 & 0 \\ 1 & -2 & : & 7 & -4 \end{pmatrix}$$

Thus we have

$$A = \begin{pmatrix} 10 & 22 \\ 15 & 20 \end{pmatrix}, H = \begin{pmatrix} 0 & -2 \\ 1 & -10 \end{pmatrix}, V = \begin{pmatrix} -4 & -8 \\ 1 & -2 \end{pmatrix} \text{ and } D = \begin{pmatrix} -2 & 0 \\ 7 & -4 \end{pmatrix}$$

Each piece shown in example 1 has a dimension (number of rows/2) × (number of columns/2) and is called A , H , V and D respectively. A (approximation area) includes information about the global properties of analysed image. Removal of spectral coefficients from this area leads to the biggest distortion in original image. H (horizontal area) includes information about the vertical lines hidden in image. Removal of spectral coefficients from this area excludes horizontal details from original image. V (vertical area) contains information about the horizontal lines hidden in image. Removal of spectral coefficients from this area eliminates vertical details from original image. D (diagonal area) embraces information about the diagonal details hidden in image. Removal of spectral coefficients from this area leads to minimum distortions in original image. To get the value at next level, again HT is applied row and column wise on the piece A , obtained earlier as in example 1. Thus the HT is suitable for application when the image matrix has number of rows and columns as a multiple of 2.

Fast Haar Transform (FHT) involves addition, subtraction and division by 2, due to which it becomes faster and reduces the calculation work in comparison to HT. For the decomposition of an image, we first apply 1D FHT to each row of pixel values of an input image matrix. These transformed rows are themselves an image and we apply the 1D FHT to each column. The resulting values are all detail coefficients except for a single overall average coefficient.

Modified Fast Haar Wavelet Transform: In MFHWT, first average subsignal, $a^1 = (a_1, a_2, \dots, a_{N/2})$, at one level for a signal of length N i.e. $f = (f_1, f_2, \dots, f_N)$ is

$$a_m = \frac{f_{4m-3} + f_{4m-2} + f_{4m-1} + f_{4m}}{4}, m = 1, 2, 3, \dots, N/4,$$

and first detail subsignal, $d^1 = (d_1, d_2, \dots, d_{N/2})$, at the same level is given as

$$d_m = \begin{cases} \frac{(f_{4m-3} + f_{4m-2}) - (f_{4m-1} + f_{4m})}{4}, & m = 1, 2, 3, \dots, N/4, \\ 0 & , m = N/2, \dots, N. \end{cases}$$

Here four nodes are considered at a time instead of two nodes as in HT and FHT. The author has considered the values of $N/2$ detail coefficients zero in each step than to find the $N/2$ detail coefficients by FHT as in [11].

Proposed Algorithm of MFHWT in 2D: A 2D MFHWT can be done by performing the following steps

- Read the image as a matrix.
- Apply MFHWT, along row and column wise on entire matrix of the image.

- From (b) we get a transformed image matrix of one level of input image.
- For reconstruction process, FHT is used on the image matrix obtained in step (b).
- Calculate MSE and PSNR for reconstructed image.

RESULTS AND DISCUSSION

The MFHWT is faster in comparison to FHT and reduces the calculation work. In MFHWT, we get the values of approximation and detail coefficients one level ahead than the FHT and HT, which is shown in Figure 1.

In this Figure we see that at each level in MFHWT we need to store only half of the original data used in FHT, due to which it becomes much more memory efficient. Table 1 shows that the MSE and PSNR values of reconstructed images are as good as in HT and FHT. A number of examples have been presented in support of quality of reconstructed image. Table 2 shows that the number of non-zero coefficients is lesser in MFHWT than that in the other two transforms and it also preserves the energy of the original input image as in HT and FHT.

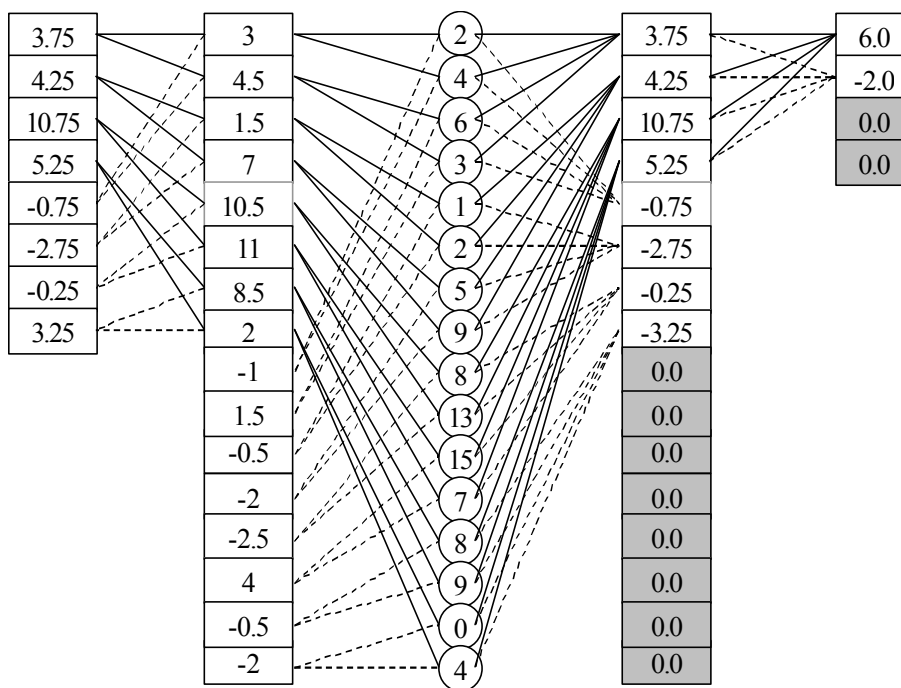
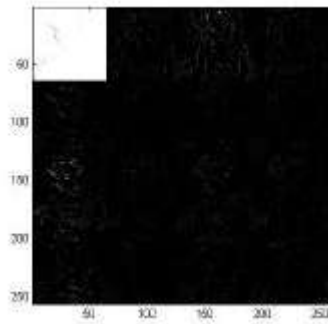


Fig. 1: Operations of MFHWT (right side) and FHT (left side)



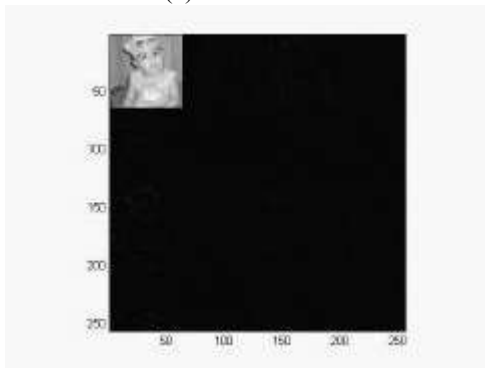
Original Image



(b) 2-level HT



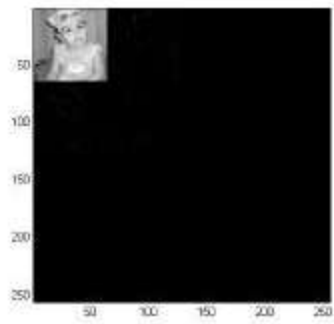
(c) Reconstructed image



(d) 2-level FHT



(e) Reconstructed image



(f) 1-level MFHWT

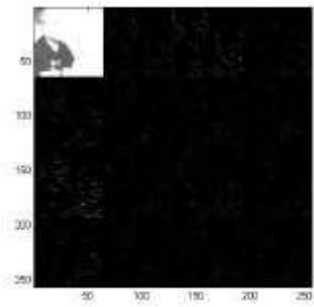


(g) Reconstructed image

Fig. 2: aditi.jpg



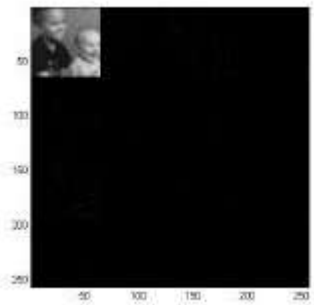
(a) Original Image



(b) 2-level HT



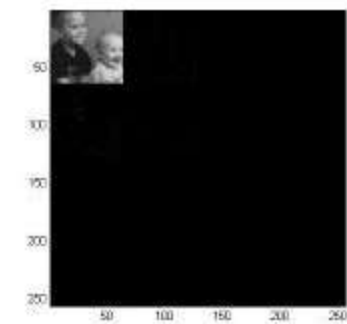
(c) Reconstructed image



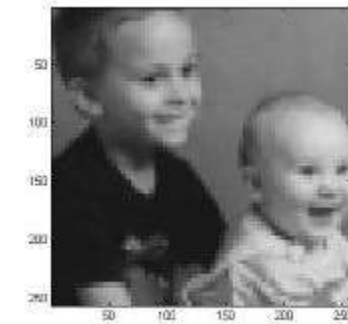
(d) 2-level FHT



(e) Reconstructed image



(f) 1-level MFHWT

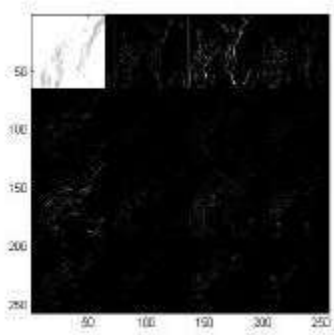


(g) Reconstructed image

Fig. 3: kids.jpg



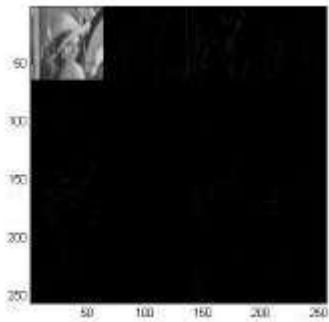
(a) Original Image



(b) 2-level HT



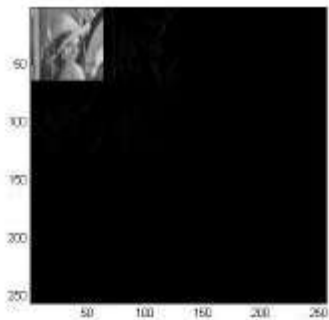
(c) Reconstructed image



(d) 2-level FHT



(e) Reconstructed image



(f) 1-level MFHWT



(g) Reconstructed image

Fig. 4: lena.jpg

Table1: Different types of error metrics for different images of size 256 × 256.

Name of image	Error metrics					
	HT		FHT		MFHWT	
	MSE	PSNR (db)	MSE	PSNR (db)	MSE	PSNR (db)
aditi.jpg	167.469	25.8915	167.469	25.8915	167.469	25.8915
kids.jpg	125.777	27.1348	125.777	27.1348	125.777	27.1348
lena.jpg	264.772	23.9021	264.772	23.9021	264.772	23.9021
rice.jpg	285.738	23.5711	285.738	23.5711	285.738	23.5711
cameraman.jpg	437.862	21.7174	437.862	21.7174	437.862	21.7174

Table 2: Percentage of zeros and energy retained in different transforms

Name of image	HT		FHT		MFHWT	
	Percentage of zeros	Percentage of energy retained	Percentage of zeros	Percentage of energy retained	Percentage of zeros	Percentage of energy retained
aditi.jpg	1.20	99.98	2.20	99.97	75.2	99.97
Kids.jpg	7.70	99.97	8.70	99.97	76.5	99.97
Lena.jpg	22.69	100.00	24.56	99.99	78.8	99.99
Rice.jpg	0.58	99.99	1.52	99.99	75.2	99.99
cameraman.jpg	9.45	99.98	11.00	99.98	76.0	99.98

CONCLUSION

The main benefit of MFHWT is sparse representation and fast transformation and possibility of implementation of fast algorithms. From test images we find that the reconstructed images are as good as in FHT and HT. Thus in the light of the above discussion it may be concluded that reasonably accurate numerical results can be obtained by using the MFHWT. This approach has the potentiality of application in colour images.

REFERENCES

- Menegaz, G., L. Grewe and J.P. Thiran, 2000. Multirate Coding of 3D Medical Data. In proceedings of the 2000 International Conference on Image Processing, IEEE, 3: 656-659.
- Wang, J. and H.K. Huang, 1996. Medical Image Compression by using Three-Dimensional Wavelet Transform. IEEE Transactions on Medical Imaging, 15(4): 547-554.
- Bullmore, E., J. Fadili, V. Maxim, L. Sendur, J. Suckling, B. Whitcher, M. Brammer and M. Breakspear, 2004. Wavelets and Functional Magnetic Resonance Imaging of the Human Brain. NeuroImage, 23(1): 234-249.
- Galli, A.W., G.T. Heydt and P.F. Ribeiro, 1996. Exploring the Power of Wavelet Analysis. IEEE Computer Application in Power, pp: 37-41.

- Joe, M.J., K.Y. Whang and S.W. Kim, 2001. Wavelet Transformation Based Management of Integrated Summary Data for Distributed Query Processing. Data and Knowledge Engg., 39(3): 293-312.
- Vitter, J.S. and M. Wang, 1999. Approximate Computation of Multidimensional Aggregates of Sparse Data using Wavelets. In proceedings of the ACM SIGMOD International Conference on Management of Data, pp: 193-204.
- Saha S., 2000. Image Compression-from DCT to Wavelets: A Review. ACM Cross Words Students Magazine, 6(3).
- Vandendrope, L., B. Maison and F. Labeau, 1997. An Adaptive Transform Approach for Image Compression. IEEE Digital Signal Processing Workshop, pp: 41-44.
- Roeser, P.R. and M.E. Jernigan, 1982. Fast Haar Transform Algorithm. IEEE Transactions on Computer, C-31: 175-177.
- Kaiser, G., 1998. The Fast Haar Transform: Gateway to Wavelet. Potentials, IEEE, 17(2): 34-37.
- Chang, P. and P. Piau, 2007. Modified Fast and Exact Algorithm for Fast Haar Transform. Proceedings of World Academy of Science, Engineering and Technology, 26: 509-512.