Image deblurring by sparsity constraint on the Fourier coefficients

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The problem of interest is the restoration of blurred and noisy images. The classical Tikhonov regularization method is often used since it leads to a linear system which can be easily solved by fast Fourier transforms (FFTs) or few iterations of a Krylov method. On the other hand, the provided solutions are usually oversmoothed. A way to overcome this drawback is to replace in the regularization term the 2-norm with the 1-norm. The resulting method is mainly suitable for sparse signals but leads to nonlinear systems. A possible compromise was proposed in [1] by the introduction of a diagonal regularization operator in the Tikhonov method. Unfortunately, such diagonal matrix prevents the use of FFT for solving the arising linear system.

Taking advantage of the well-known sparsity of the Fourier coefficients of a signal/image, we extend the effectiveness of the previous technique also to non sparse signals, preserving at the same time the computational efficiency of the FFT. More precisely, the Tikhonov regularization is applied to the Fourier coefficients instead of to the signal and we use a diagonal regularization operator, which contains the Fourier coefficients of an approximation of the true solution, in order to approximate the 1-norm in the regularization term. When the blurring operator can be diagonalized through FFTs, the resulting linear system is diagonal and then easy to solve.

We give an estimation of the optimal regularization parameter by the Generalized Cross Validation (GCV) and show that when the initial approximation is computed using the classical Tikhonov method, the proposed approach provides better reconstructions. An embedding of this method in an outer iteration is also possible and it yields to further improvement.

References

[1] Huckle T., Sedlacek M., (2013) Data based regularization for discrete deconvolution problems, *BIT Numerical Mathematics*, Vol. 53, pp. 459–473.