Journal of Information Engineering and Applications
ISSN 2224-5758 (print) ISSN 2224-896X (online)
Vol 1, No.2, 2011

Image Denoising Using Digital Image Curvelet

Aarti S.Gaikwad (Corresponding author)
Electronics Department
Rajarshri Shahu College of Engineering, Tathawade, Pune-33.
University of Pune
Tel: 020-22933423 E-mail: arati.g@rediffmail.com.

D.S.Bormane
Electronics Department
Rajarshri Shahu College of Engineering, Tathawade, Pune-33.
University of Pune
Tel: 020-22933423 E-mail: bdattatraya@yahoo.com

Abstract

Image reconstruction is one of the most important areas of image processing. As many scientific experiments result in datasets corrupted with noise, either because of the data acquisition process or because of environmental effects, denoising is necessary which a first pre-processing step in analyzing such datasets. There are several different approaches to denoise images. Despite similar visual effects, there are subtle differences between denoising, de-blurring, smoothing and restoration.

Although the discrete wavelet transform (DWT) is a powerful tool in image processing, it has three serious disadvantages: shift sensitivity, poor directionality and lack of phase information. To overcome these disadvantages, a method is proposed which is based on Curvelet transforms which has very high degree of directional specificity. Allows the transform to provide approximate shift invariance and directionally selective filters while preserving the usual properties of perfect reconstruction and computational efficiency with good well-balanced frequency responses where as these properties are lacking in the traditional wavelet transform. Curvelet reconstructions exhibit higher perceptual quality than Wavelet based reconstructions, offering visually sharper images and in particular higher quality recovery of edges and of faint linear and curve linear features. The Curvelet reconstruction does not contain the quantity of disturbing artifacts along edges that we see in wavelet reconstruction. Digital Implementations of newly developed multiscale representation systems namely Curvelets, Ridgelet and Contourlets transforms are used for denoising the image. We apply these digital transforms to the problem of restoring an image from noisy data and compare our results with those obtained from well established methods based on the thresholding of Wavelet Coefficients.

Keywords: Curvelets Transform, Discrete Wavelet Transform, Ridgelet Transform, Peak signal to Noise Ratio (PSNR), Mean Square Error (MSE).

1. Introduction

1.1 Wavelet Image Denoising

The need for image enhancement and restoration is encountered in many practical applications. For efficient image representation we should have multi resolution, localization, and directionality, critical sampling. In practice, any image may be degraded by various types and forms of noise and the most common one is additive noise.Intial efforts included very simple ideas like thresholding (D.L.Donoho 1995) of the orthogonal wavelet coefficients of noisy data, followed by reconstruction. Later efforts found that substail improvements in perceptual quality could be obtained by translation invariant methods based on thresholding (Coifman, R.R.and D.L.Donoho 1995) of an undecimated wavelet transform. More recently, ”tree-based” wavelet denoising methods were developed in the context of image denoising which exploit the tree structure of wavelet coefficients and the so called parent-child correlations which are present in the wavelet coefficients of images with edges. Also many investigators have experimented with variations on the basic schemes-modifications of thresholding functions, level-dependant thresholding and Bayesian conditional expectation nonlinearities (Er-hu Zhang, Shu-Ying Huang 2004) and so on.

1.2 Curvelet Image Denoising

Curvelets are based on multiscale ridgelets combined with a spatial bandpass filtering operation to isolate different scales (E.J.Candes and D.L.Dohono). Like ridgelets curvelets occur at all scales, locations and orientations. However while ridgelets have global length and so a variable widths, curvelets in addition to a variable width have variable length and so a variable anisotropy. The length and width at fine scales are related by a scaling law width=length^2 and so the anisotropy increases with decreasing scale like a power law, Recent work shows that thresholding of Discrete curvelet coefficients provide near optimal N-term representations of otherwise smooth objects with discontinuities along C^2 curves.Curvelet transform is the latest member of the evolving family of multiscale geometric transforms. In this transform the frame elements are indexed by scale, orientation and location parameters. It is designed to represent edges and singularities along curved paths more efficiently than the wavelet methods. For example to represent an edge to squared error 1/N, needs 1/N wavelets but curvelet needs 1/√N curvelets

1.3 Proposed Work

In this paper efforts are taken for denoising of image based on a recently introduced family of transforms-the ridgelets and curvelet transforms which have been proposed as alternatives to wavelet representation of image data. In this paper we exhibit higher PSNR on standard images such as Barbara and Lenna, across the range of underlying noise levels. Which predicts that, in recovering images which are smooth away from edges, curvelets will obtain dramatically smaller asymptotic mean square error of reconstruction than wavelet methods. We implement image denoising using Wavelet Transform, Curvelet and Ridgelet transforms. Four types of additive noise Gaussian noise, Salt & Pepper noise and Speckle noise are chosen for mixing with noise. For each type of noise the extent of mixing corresponds to the standard deviations of 0.1, 0.15, 0.2, 0.25, 0.3 and 0.35.As well efforts are taken to denoise the multiplicative noise which is signal dependant. The quality of reconstructed image is usually specified in terms of Peak Signal to Noise Ratio (PSNR) and Mean Square Error (MSE).The Performance parameters are calculated as follows

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}[I(i,j) - \hat{I}(i,j)]^2$$

$$PSNR = 10\log_{10}\frac{255^2}{MSE}$$

where, *I(i,j)* is an original image and $\hat{I}(i,j)$ is an estimate of *I(i,j)* after Reconstruction.

1.4 Steps to denoise the Image

1. Read Input Image.
2. Add noise to Input Image.
3. Apply Wavelet and Curvelet Transform.
4. Compute the performance parameters of reconstructed image.
5. Compare Wavelet Transform Results with Curvelet and Ridgelet Transform.

2. Transforms

The new ridgelet and curvelet transforms were developed over several years in an attempt to break an inherent limit plaguing wavelet denoising of images. This limit arises from the well-known and frequently depicted fact that the two-dimensional (2-D) wavelet transform of images exhibits large wavelet coefficients even at fine scales, all along the important edges in the image, so that in a map of the large wavelet coefficients one sees the edges of the images repeated at scale after scale. While this effect is visually interesting, it means that many wavelet coefficients are required in order to reconstruct the edges in an image properly. With so many coefficients to estimate, denoising faces certain difficulties. There is, owing to well-known statistical principles, an imposing tradeoff between parsimony and accuracy which even in the best balancing leads to a relatively high mean squared error (MSE).To approach ideal MSE one should develop new expansions which accurately represent smooth functions using only a few nonzero coefficients and which also accurately represent edges using only few nonzero coefficients.Beacause so few coefficients are required either for smooth parts or the edge parts, the balance between parsimony and accuracy will be much more favorable and lower MSE results. In image processing, edges are typically curved rather than straight and ridgelets alone cannot yield efficient representations. However at sufficiently fine Scales, a curved edge is almost straight, and so to capture curved edges, one ought to be able to deploy ridgelets in a localized manner, at sufficiently fine scales. Two approaches to localization of ridgelets are possible.

- Monoscale ridgelets*:* Here one thinks of the plane as partitioned into congruent squares of a fixed side-length and constructs a system of renormalized ridgelets smoothly localized near each such square.
- Multiscale ridgelets:Here, one thinks of the plane as subjected to an infinite series of partitions, based on dyadic scales, where each partition, like in the monoscale case, consists of squares of the given dyadic side length.

Curvelets are based on multiscale ridgelets combined with a spatial band pass filtering operation to isolate different scales. Like ridgelets, curvelets occur at all scales, locations and orientation. While ridgelets all have global length and variable widths, curvelets in addition to a variable width have variable length and so variable anisotropy. The length and width at fine scales are related by a scaling law width=length^2 and so the anisotropy increases with decreasing scale like power law.

2.1 One-dimensional Wavelet Transform

To complete the ridgelet transform, we must take a one-dimensional wavelet transform along the radial variable in Radon space. We will discuss the choice of digital one-dimensional wavelet transform. Experience has shown that compactly-supported wavelets can lead to many visual artifacts when used in conjunction with nonlinear processing- such a hard-thresholding of individual wavelet coefficients- particular largely for decimated wavelet schemes used at critical sampling. Also because of the lack of localization of such compactly-supported wavelets in the frequency domain fluctuations in coarse-scale wavelet coefficients can introduce fine-scale fluctuations. This is undesirable in our setting. Here we take a frequency-domain approach, where the discrete Fourier transform is reconstructed from the inverse Radon transform. These considerations lead us to use band limited wavelet – whose support is compact in Fourier Domain rather than the time domain. Other implementations have made a choice of compact support in the frequency domain as well. The wavelet transform algorithm is based on a scaling function Ø such that Ø vanishes outside of interval [$-v_c$, v_c]. We define the scaling function Ø as the renormalized B3-spline

$$Ø (v) = 3/2\, B3 (4 v),$$

And ψ as the difference between two consecutive resolutions

$$\Psi (2v) = Ø (v) – Ø (2v).$$

Because ψ is compactly supported, the sampling theorem shows that one can easily build a pyramid of n + n/2 + …. +1= 2n elements. This transform enjoys the following features - The wavelet coefficients are directly calculated in the Fourier space. In the context of the ridgelet transform, this allows avoiding the computation of one dimensional inverse Fourier transform along each radial line. Each sub band is sample above the Nyquist rate hence avoiding aliasing – a phenomenon typically encountered by critically sampled orthogonal wavelet transforms.

The reconstruction is trivial. The wavelet coefficients simply need to be co-added to reconstruct the input signal at any given point. In our application, this implies that the ridgelet coefficients simply need to be co-added to reconstruct Fourier Coefficients. This wavelet transform introduces an extra redundancy factor, which might be viewed as an objection by advocates of orthogonally and critical sampling. However we note that our goal in this implementation is not data compression efficient coding – for which critical sampling might be relevant – but instead noise removal, for which it well known that over completeness can provided substantial advantages. The following (Note 1). Shows the flow graph of the ridgelet transform. The ridgelet transform of an image of size n x n is an image of size 2n x 2n, introducing a redundancy factor. We note that, because our transform is made of a chain of steps, each one of which is invertible, the whole transform is invertible, and so has the exact reconstruction property. For the same reason, the reconstruction is stable under perturbations of the coefficients. Last but not least, our discrete transform is computationally attractive. Indeed, the algorithm we presented here has low complexity since it runs in O (n^2log(n))flops for an n x n image

2.2 Radon Transform

A basic tool for calculating ridgelet coefficients is to view ridgelet analysis as a form of wavelet analysis in the Radon domain. We recall that the Radon transform of an object f is the collection of line integrals indexed by (θ, t) Є [0, 2π) X R given by

$Rf(\theta, t) = \int f(x_1, x2)\ \delta\ (x_1\cos\theta + x_2\sin\theta - t)\ dx1dx2$

Where δ is Dirac distribution. The ridgelet coefficients R_f (a, b, θ) of an object f is given by analysis of Radon transform via

$R_f(a, b, \theta) = \int Rf(\theta, t)\ a^{-1/2}\ \varphi\ ((t - b)/a)\ dt.$

Hence the ridgelet transform is precisely the application of a one-dimensional wavelet transform to the slices of the Radon transform where the angular variable θ is constant and t is varying.

2.3 Ridgelet Transform

Refer Note1

Ridgelet are an orthogonal set {ρ λ} for L2 ((ℜ2).

Developed by Cande's and Donoho in 1998.Divides the frequency domain to dyadic coronae

$|\xi| \in [2s, 2s+1]$.

In the angular direction, samples the s-th corona at least 2s times.

In radial direction, samples using local wavelets.

The ridgelet element has a formula in the frequency domain, where, ωi, l are periodic wavelets for ($-\pi, \pi$)

i is the angular scale and $l \epsilon [0, 2i - 1]$ is the angular location.

Ψj, k are Meyer Wavelets for ℜ.

J is the ridgelet scale and k is the ridgelet location.

2.4 Curvelet Transform

Refer Note2

In image processing edges are curved rather than straight lines and ridgelets are not able to efficiently represent such images. However, one can still deploy the ridgelet machinery in localized way, at fine scales, where curved edges are almost straight lines. The Curvelet transform, opens the possibility to analyze an image with different block sizes, but with a single transform. The idea is to first decompose the image into a set of wavelet bands, and so to analyze each band by a local ridgelet transform. The block size can be changed at each scale level. Roughly speaking, different levels of the multiscale ridgelet pyramid are used to represent different sub-bands of a filter bank output. At the same time, this sub-band decomposition imposes a relationship between the width and length of the important frame elements so that are anisotropic and obey a parabolic scaling law width ≈ $length^2$.

An important ingredient of the curvelet transform is to restore sparsity by reducing redundancy across scales. In detail, one introduces interscale orthogonality by means of subband filtering. Roughly speaking, different levels of the multiscale ridgelet pyramid are used to represent different subbands of a filter bank output. At the same time, this subband decomposition imposes a relationship between the width and length of the important frame elements so that they are anisotropic and obey width = $length^2$.

The discrete curvelet transform of a continuum function $f(x_1, x_2)$ makes use of a dyadic sequence of scales, and a bank of filters (p_0f, $\Delta_1 f$, $\Delta_2 f$, …) with the property that the passband filter is concentrated near the frequencies $[2^{2s}, 2^{2s+2}]$, e.g.,

$\Delta_s = \psi_{2s} * f, \qquad \psi_{2s}(\xi) = \psi(2^{-2s}\xi).$

In wavelet theory, one uses decomposition into dyadic subbands $[2^s, 2^{s+1}]$. In contrast, the subbands used in the discrete curvelet transform of continuum functions have the nonstandard form $[2^{2s}, 2^{2s+2}]$. This is nonstandard feature of the discrete curvelet transform well worth remembering.

The curvelet decomposition is the sequence of the following steps.

- Subband Decomposition : The object 'f' is decomposed into subbands

  $f \rightarrow (P_o f, \Delta_1 f, \Delta_2 f, \ldots)$.

- Smooth Partitioning: Each subband is smoothly windowed into "squares" of an appropriate scale (of side length ~ 2^{-s})

  $\Delta_s f \rightarrow (w_Q \Delta_s f)_{Q \in Q_s}$.

- Renormalization: Each resulting square is renormalized to unit scale

  $I_Q = (T_Q)^{-1} (w_Q \Delta_s f), \quad Q \in Q_s$.

- Ridgelete Analysis: Each square is analyzed via the discrete ridgelet transform.

In this definition, the two dyadic subbands [2^{2s}, 2^{2s+1}] and [2^{2s+1}, 2^{2s+2}] are merged before applying the ridgelet transform.

5. Conclusion

Thus the Curvelet Transform is a new multi-scale representation most suitable for objects with curves. The multiscale transform with frame elements indexed by location, scale and orientation. It integrates the concept of directionality, optimally sparse representation of images containing objects with edges which obeys Parabolic scalling law $width = length^2$.

This shows that the Curvelet and Ridgelet Transforms are more suitable for the image data to represent the

Singularities over geometric structures in the image, than the Wavelet counterpart. Curvelet is designed to

handle the singularities on curves and Ridgelet handles it for the lines. Wavelets are effective for point singularities.

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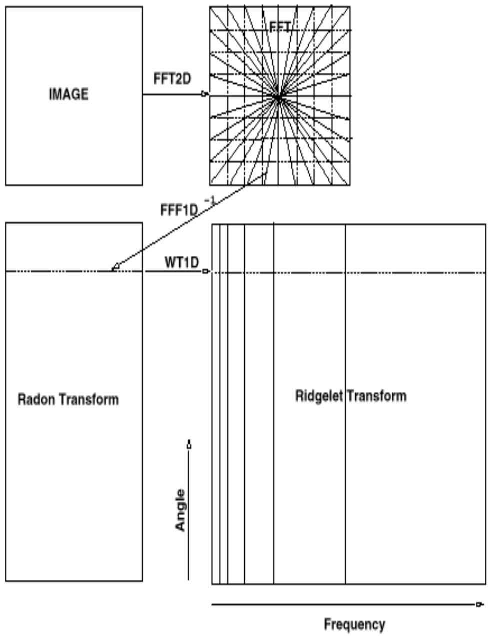
Notes

Note 1 .:Ridgelet transform



Note 2 : Curvelet Transform