

Image Denoising using Wavelet Thresholding

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Abstract

*This paper proposes an adaptive threshold estimation method for image denoising in the wavelet domain based on the generalized Gaussian distribution (GGD) modeling of subband coefficients. The proposed method called **NormalShrink** is computationally more efficient and adaptive because the parameters required for estimating the threshold depend on subband data. The threshold is computed by $\beta\sigma^2/\sigma_y$, where σ and σ_y are the standard deviation of the noise and the subband data of noisy image respectively. β is the scale parameter, which depends upon the subband size and number of decompositions. Experimental results on several test images are compared with various denoising techniques like Wiener Filtering [2], BayesShrink [3] and SureShrink [4]. To benchmark against the best possible performance of a threshold estimate, the comparison also include OracleShrink. Experimental results show that the proposed threshold removes noise significantly and remains within 4% of OracleShrink and outperforms SureShrink, BayesShrink and Wiener filtering most of the time.*

Keywords: Wavelet Thresholding, Image Denoising, Discrete Wavelet Transform.

1. Introduction

An image is often corrupted by noise in its acquisition and transmission. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features. In the recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal de-noising [1], [3]-[10], [12], because wavelet provides an appropriate basis for separating noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficient are more likely due to noise and large coefficient due to important signal features [8]. These small coefficients can be thresholded without affecting the significant features of the image.

Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its

most basic form, each coefficient is thresholded by comparing against threshold, if the coefficient is smaller than threshold, set to zero; otherwise it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the result may lead to reconstruction with the essential signal characteristics and with less noise. Since the work of Donoho & Johnstone [1], [4], [9], [10], there has been much research on finding thresholds, however few are specifically designed for images. In this paper, a near optimal threshold estimation technique for image denoising is proposed which is subband dependent i.e. the parameters for computing the threshold are estimated from the observed data, one set for each subband.

The paper is organized as follows. Section 2 introduces the concept of wavelet thresholding. Section 3 explains the parameter estimation for *NormalShrink*. Section 4 describes the proposed denoising algorithm. Experimental results & discussions are given in section 5 for three test images at various noise levels. Finally the concluding remarks are given in section 6.

2. Wavelet Thresholding

Let $f = \{f_{ij}, i, j = 1, 2, \dots, M\}$ denote the $M \times M$ matrix of the original image to be recovered and M is some integer power of 2. During transmission the signal f is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussian Noise n_{ij} with standard deviation σ i.e. $n_{ij} \sim N(0, \sigma^2)$ and at the receiver end, the noisy observations $g_{ij} = f_{ij} + \sigma n_{ij}$ is obtained. The goal is to estimate the signal f from noisy observations g_{ij} such that Mean Squared error (MSE)[11] is minimum. Let W and W^{-1} denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then $Y = Wg$ represents the matrix of wavelet coefficients of g having four subbands (LL, LH, HL and HH) [7], [11]. The sub-bands HH_k , HL_k , LH_k are called *details*, where k is the scale varying from 1, 2, ..., J and J is the total number of decompositions. The size of the subband at scale k is $N/2^k \times N/2^k$. The subband LL_J is the low-resolution residue. The wavelet thresholding denoising method processes each coefficient of Y from the detail

subbands with a soft threshold function to obtain \hat{X} . The denoised estimate is inverse transformed to $\hat{f} = W^{-1}\hat{X}$.

In the experiments, soft thresholding has been used over hard thresholding because it gives more visually pleasant images as compared to hard thresholding; reason being the latter is discontinuous and yields abrupt artifacts in the recovered images especially when the noise energy is significant.

3. Estimation of Parameters for NormalShrink

This section describes the method for computing the various parameters used to calculate the threshold value (T_N), which is adaptive to different subband characteristics.

$$T_N = \frac{\beta \hat{\sigma}^2}{\hat{\sigma}_y} \quad (1)$$

Where, the scale parameter β is computed once for each scale using the following equation:

$$\beta = \sqrt{\log\left(\frac{L_k}{J}\right)}, \quad (2)$$

L_k is the length of the subband at k^{th} scale.

$\hat{\sigma}^2$ is the noise variance, which is estimated from the subband HH1, using the formula [7][13]:

$$\hat{\sigma}^2 = \left[\frac{\text{median}(|Y_{ij}|)}{0.6745} \right]^2, \quad Y_{ij} \in \text{subband HH}_1 \quad (3)$$

and $\hat{\sigma}_y$ is the standard deviation of the subband under consideration computed by using the standard MATLAB command.

To summarize, the proposed method is named as *NormalShrink* which performs soft thresholding with the data driven subband dependent threshold T_N .

4. Image Denoising Algorithm

This section describes the image denoising algorithm, which achieves near optimal soft thresholding in the wavelet domain for recovering original signal from the noisy one. The algorithm is very simple to implement and computationally more efficient. It has following steps:

1. Perform multiscale decomposition [11] of the image corrupted by gaussian noise using wavelet transform.
2. Estimate the noise variance σ^2 using equation (3).
3. For each level, compute the scale parameter β using equation (2).
4. For each subband (except the lowpass residual)
 - a) Compute the standard deviation σ_y .
 - b) Compute threshold T_N using equation (1).
 - c) Apply soft thresholding to the noisy coefficients.
5. Invert the multiscale decomposition to reconstruct the denoised image \hat{f} .

5. Experimental Results and Discussions

The experiments are conducted on several natural gray scale test images like Lena, Barbara and Goldhill of size 512×512 at different noise levels $\sigma=10, 20, 30, 35$. The wavelet transform employs Daubechies' least asymmetric compactly supported wavelet with eight vanishing moments [14] at four scales of decomposition. To assess the performance of *NormalShrink*, it is compared with *SureShrink*, *BayesShrink*, *OracleThresh* and *Wiener*.

To benchmark against the best possible performance of a threshold estimate, the comparison include *OracleShrink*, the best soft thresholding estimate obtainable assuming the original image known. The PSNR from various methods are compared in Table I and the data are collected from an average of five runs. Since the main comparison is against *SureShrink* and *BayesShrink*, the better one among these is highlighted in bold font for each test set. *NormalShrink* outperforms *SureShrink* and *BayesShrink* most of the time in terms of PSNR as well as in terms of visual quality. Moreover *NormalShrink* is 4% faster than *BayesShrink*.

The choice of soft thresholding over hard thresholding is justified from the results of best possible performance of a hard threshold estimator, *OracleThresh*.

Comparisons are also made with the best possible linear filtering technique i.e. *Wiener filter* (from the MATLAB image processing toolbox, using 3×3 local window). The results in the table I show that PSNR are considerably worse than the nonlinear thresholding methods, especially when σ is large. The image quality is also not as good as those of the thresholding methods. Fig. 1 shows the noisy image and resulting images of *Wiener filter*, *BayesShrink* and *NormalShrink* for Lena at $\sigma = 30$.

Table I: PSNR results for various test images and σ values, of (1) OracleShrink, (2) SureShrink, (3) NormalShrink, (4) BayesShrink, (5) OracleThresh, and (6) Wiener

	OracleShrink	SureShrink	NormalShrink	BayesShrink	OracleThresh	Wiener
Lena						
$\sigma = 10$	33.6114	33.4755	33.5390	33.4106	32.6988	33.5793
$\sigma = 20$	30.3813	30.0724	30.3530	30.2258	29.5232	28.9868
$\sigma = 30$	28.6009	28.3935	28.5330	28.4901	27.7106	25.6915
$\sigma = 35$	27.9492	27.8293	27.8908	27.8593	27.0696	24.3901
Barbara						
$\sigma = 10$	31.5070	30.6327	31.3744	31.0322	30.4935	29.8159
$\sigma = 20$	27.4079	27.2961	27.3298	27.2843	26.3428	26.7916
$\sigma = 30$	25.3289	25.0969	25.2269	25.2842	24.0979	24.2973
$\sigma = 35$	24.5840	24.2202	24.5403	24.5200	23.3580	23.2381
Goldhill						
$\sigma = 10$	31.9734	31.8715	31.7108	31.9004	30.7980	31.8093
$\sigma = 20$	28.7682	28.4362	28.6590	28.6570	27.7837	28.2604
$\sigma = 30$	27.1687	27.0256	27.0963	27.1133	26.3061	25.3490
$\sigma = 35$	26.6525	26.3356	26.5098	26.6088	25.7732	24.1476

6. Conclusion

In this paper, a simple and subband adaptive threshold is proposed to address the issue of image recovery from its noisy counterpart. It is based on the generalized Gaussian distribution modeling of subband coefficients. The image denoising algorithm uses soft thresholding [1] to provide smoothness and better edge preservation at the same time. Experiments are conducted to assess the performance of *NormalShrink* in comparison with the *OracleShrink*, *SureShrink*, *BayesShrink*, *OracleThresh* and *Wiener*. The results show that *NormalShrink* removes noise significantly and remains within 4% of *OracleShrink* and outperforms *SureShrink*, *BayesShrink* and *Wiener* filtering most of the time. Moreover *NormalShrink* is 4% faster than *BayesShrink*. It is further suggested that the proposed threshold may be extended to the compression framework, which may further improve the denoising performance.

Acknowledgement

The work presented in this paper has been possible due to funding from the project sponsored by Ministry of Human Resources and Development; INDIA. This work has been carried out in research and development laboratory in Department of Computer Science and Engg. at SLIET. Authors would also like to thank Dr. S. Chang and Dr. Martin Vetterli for their kind support and timely help.

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(a)



(b)



(c)



(d)

Fig. 1. Comparing the performance of (a) Noisy Lena at $\sigma = 30$ with (c) Wiener filter (b) BayesShrink and (d) NormalShrink.