

Image Segmentation Based on the Integration of Markov Random Fields and Deformable Models

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Abstract. This paper proposes a new methodology for image segmentation based on the integration of deformable and Markov Random Field models. Our method makes use of Markov Random Field theory to build a Gibbs Prior model of medical images with arbitrary initial parameters to estimate the boundary of organs with low signal to noise ratio (SNR). Then we use a deformable model to fit the estimated boundary. The result of the deformable model fit is used to update the Gibbs prior model parameters, such as the gradient threshold of a boundary. Based on the updated parameters we restart the Gibbs prior models. By iteratively integrating these processes we achieve an automated segmentation of the initial images. By careful choice of the method used for the Gibbs prior models, and based on the above method of integration with deformable model our segmentation solution runs in close to real time. Results of the method are presented for several examples, including some MRI images with significant amount of noise.

1 Introduction

Segmentation is the process of assigning pixels in an image to distinct objects or the background. It is one of the fundamental processes for higher-level image analysis. However, it still remains an open research problem. Region-based and boundary-based methods are the two major classes of segmentation algorithms.

In region-based methods, e.g., region growing [1], image pixels are assigned to objects based on homogeneity statistics. If a pixel value is similar to that of its neighbors, then the two pixels will be assigned to the same object. The advantage of this method is that the image information inside the object is considered. The disadvantage is that it may lead to noisy boundaries and holes inside the object because the method does not consider the boundary information explicitly.

In the boundary-based method, e.g., snakes [2], Fourier-Parameterized models [3], a shape model is initialized close to the object boundary and fits to the boundary based on image features. To avoid local minima, most boundary-based methods require that the model be initialized near the solution and it is controlled via an interactive interface. A disadvantage of these methods is that they

do not consider pixel information inside the object. For images with low SNR, the boundary-based method may lead to incorrect assignment of pixels unless significant manual intervention is used.

To improve the segmentation results of either of the above two classes of methods a hybrid segmentation framework was proposed in [4] and [5] that combines region-based and boundary-based methods. The algorithm works as follows. In the first step the user selects a single initial pixel inside the object to be segmented and then an affinity operator is applied to the image to estimate the pixels that most likely lie on the boundary of the object. Since the data are usually sparse and/or noisy, in the second step a deformable model is fit to improve the segmentation from the first step that is then used to re-compute the parameters of the affinity operator. The above two steps are then applied recursively to further refine the segmentation results.

A limitation of the affinity operator is that it only considers the local characteristics in the direct neighborhood of every new pixel in order to make a decision as to whether the pixel should be included to the already segmented cluster of pixels or not. The affinity operator does not use any boundary shape or boundary continuity information.

Chan et al. apply the Gibbs prior model in [6] and [7] for image processing based on the work of Besag [8] and Geman and Geman[9]. Compared to the affinity operator, the Gibbs prior models can incorporate much more information. By using specific neighborhood information, the Gibbs prior model can incorporate both boundary and region information during segmentation.

In this paper we develop a new segmentation approach that integrates Gibbs prior models and deformable models for segmenting the boundaries of organs in medical images. The advantages of the method is that:

1. The user only selects a single pixel inside the object,
2. The method can deal with images with significant amount of noise and poorly defined boundaries, and
3. it is computationally efficient and runs in real time (5Hz on SGI R10000 workstation).

1.1 Related Work

As opposed to most previous work on segmentation that falls under either the boundary [14,22] or the region growing methods [1], our method follows a hybrid approach. Recently, there have been several hybrid methods such as the ones developed in [5, 6, 15, 16, 17]. In [18], Zhu et al. develop a unifying framework that generalizes the deformable models, region growing, and prior matching approaches. They define a new energy function that represents them all. By combining the region-based method and the boundary-based method, all these approaches achieve good results. However, they still need good initialization to avoid local minima and they use fixed prior models that may be difficult to compute accurately in advance.

Various successful approaches for object segmentation have been proposed that use the Markov Random Fields property of images in the past few years. [8, 9, 19] used MRF or Gibbsian models that include nearest neighbor correlation information, but do not use object boundary information. [6] and [7] defined higher order neighborhoods in MRF image models that model both region and boundary object information based on theoretical results from [20,21].

Y. Boykov et al. [24], Vittorio Murino and Andrea Trucco [25] use MRF models in their segmentation applications. But neither of them provides a self-adaptive algorithm to refine the prior models. So their methods need an exact prior model to begin with.

J. Dehmeshki et al. developed an automatic adaptive algorithm for MRF image segmentation [26]. In their recursive algorithm, they first segment the image with Gibbs prior models. Then they update the Gibbs prior model based on the segmentation result, and then they segment the image again. They do this repeatedly until a satisfactory result is achieved. However, when they updated the Gibbs prior model, they recalculate parameters by doing statistics on every pixel in the image, which makes the adaptive process computationally expensive. Our method combines the deformable model and the Markov random field model. The deformable model gives a better estimation for the prior used in the MRF since it gives an explicit object boundary estimation. The iteration step between the MRF and deformable model allow us to perform only a few steps for the MRF segmentation, which in turn results in a very efficient segmentation algorithm.

2 Method

2.1 Markov Random Fields and Gibbs Prior for Segmentation

Most medical images are Markov Random Field images, that is, the statistics of a pixel are related to the statistics of pixels in its neighborhood. According to the Equivalent Theorem proved by Hammersley and Clifford in [11], a Markov random field is equivalent to a Gibbs field. Thus for medical images which are MRF images, their joint distribution can be written in the Gibbsian form as follows.

$$\Pi(X) = Z^{-1} \exp(-H(X)) \quad Z = \sum_{z \in \bar{X}} (-H(z)) \quad (1)$$

where $H(X)$ models the energy function of image X , \bar{X} is the set of all possible configurations of the image X , Z is a normalizing factor or partition function in statistics terminology, and z is image. The local and global properties of images will be incorporated into the model by designing an appropriate energy function $H(X)$. The lower the value of the energy function, the higher the value of the image joint distribution, the better the image fits to the prior distribution.

We began the establishment of the Gibbs model by designing the energy function as

$$H(X) = H_1(X) + H_2(X) + H_3(X) \quad (2)$$

where $H_1(X)$ models the piecewise pixel homogeneity statistics, $H_2(X)$ models the continuity of the boundary and $H_3(X)$ is the noise model.

To model the piecewise homogeneity, we model $H_1(X)$ as:

$$H_1(X) = \vartheta_1 \sum_{s \in X} \sum_{t \in \partial s} (\Phi(\Delta_{s,t})(1 - \Psi_{st}) + \alpha \Psi_{st}) \quad \text{with } \Psi_{st} = \begin{cases} 0 & \Delta_{s,t} < 0 \\ 1 & \Delta_{s,t} \geq 0 \end{cases} \quad (3)$$

s and t are pixels, pixel t is in the neighborhood of pixel s , ϑ_1 is the weight for $H_1(X)$, α is the threshold for the object boundary, $\Delta_{s,t}$ is the variance between pixels s and t . Φ is a function based on the variance. For simplicity, let $\Phi(\Delta_{s,t}) = \Delta_{s,t}^2$. When we minimize $H_1(X)$, pixels that have similar gray values with their neighbors are considered to be in the same object ($\Delta_{s,t}$ assigned to 0) and will be further smoothed. But for pixels that have different gray values (the variance between two pixels beyond the threshold α) than their neighbors, will be assigned to 1. Thus the variance between these two pixels will remain. Therefore the term will smooth the pixels inside the object and will keep the boundary untouched.

The boundary continuity is modeled as follows:

$$H_2(X) = \vartheta_2 \sum_{s \in X} \sum_{i=1}^N W_i(x) \quad (4)$$

where x is a pixel, ϑ_2 is the weight term, N is the number of local configurations that may lie on the boundaries. $W_i(x)$ is the weight function (also called the potential function) for the local configuration of the boundary. For our purpose we consider the potential function on 9 pixels. In our model, the potential functions are defined on a neighborhood system in which the 3 by 3 cliques are used to model the boundary continuity. We depict the clique potential definition in Figure 1. All these local configurations can have different orientations. The vertical and horizontal configurations share the same potential function.

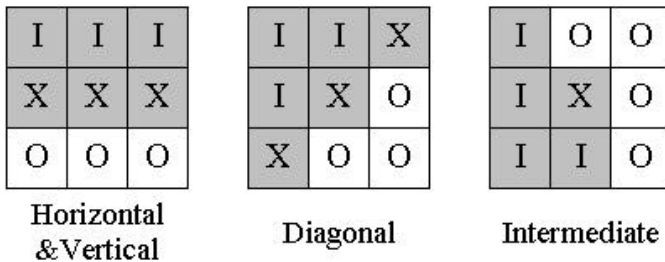


Fig. 1. Figure 1. Pixels labeled "I" are inside the object; pixels labeled "O" are outside the object; pixels labeled "X" all have gray values similar to either "I" or "O". The shading area is the boundary shape when assuming "X" is similar to "I"

Different values are assigned to different local configurations. All three local configurations depicted in Figure 1 most likely exist in an image with smooth and continuous boundary. In $H_2(X)$ we give weight to these local configurations. When we minimize $H_2(X)$, the pixels in the image (especially those near the boundary) will alter their gray values to achieve a smooth and continuous boundary.

To cope with noise in the data, we use as mentioned previously the term $H_3(X)$ in the energy function $H(X)$. For Gaussian Noise, it has the form:

$$H_3(X) = (2\sigma^2)^{-1} \sum_s (y_s - x_s)^2 \quad (5)$$

where y_s is the observed value of pixel s , x_s is the estimated value and σ is the standard deviation.

To minimize $H(x)$ we use a Bayesian framework. According to Bayes's theorem, the posterior distribution of an image is given by

$$P(X|Y) = \frac{\Pi(X)L(Y|X)}{P(Y)} \quad (6)$$

where $\Pi(X)$ is the prior distribution (see (1)) and $L(Y|X)$ is the probability of obtaining an image Y while the actual image is X . Under a Bayesian framework, the segmentation problem can be formulated as the maximization of

$$\Pi(X)L(Y|X) \quad (7)$$

The method requires choosing suitable ϑ_1 and ϑ_2 to achieve a balance between the piecewise homogeneity and boundary continuity. We also need to input the threshold of boundary α and the weight $W_i(x)$ for local characteristics to enable the Gibbs prior model:

We use the ICM method to minimize the energy function $H(X)$, as described in [11].

It is important to note that the result of the Gibbs prior model depends on the proper selection of the respective parameters. In addition, the solution based on the Gibbs prior model can be trapped in a local minimum. Based on our method, the use of the deformable model to fit the estimated boundary at every iteration serves as a way to push the Gibbs prior model solution to the global minimum. This is due to the fact that we re-compute at the end of every iteration the Gibbs prior model parameters based on the deformable model segmentation.

2.2 Deformable Model Framework

We summarize the physics-based deformable model framework developed in [10] that we use and its integration with the estimated boundary data produced by the Gibbs prior model we described above.

Our model is a superellipsoid with local deformations. Given the reference shape s (the superellipsoid) and the displacement d (the local deformations), the

position of points p on the model are defined by $p = s + d$. To keep the continuity of the model surface, we impose a continuous loaded membrane deformation strain energy.

$$\varepsilon_m(d) = \int \omega_{10} \left(\frac{\partial d}{\partial u} \right)^2 + \omega_{01} \left(\frac{\partial d}{\partial v} \right)^2 + \omega_{00} d^2 du \quad (8)$$

Where d is a node's local deformation, ω_{00} controls the local magnitude and ω_{10} , ω_{01} control the local variation of the deformation. We can calculate the stiffness matrix K of model based on the strain energy [10].

The model nodes move under the influence of external forces. The model dynamics can be described by the first order Lagrangian method:

$$\dot{d} + Kd = f \quad (9)$$

where f are external generalized forces.

The external forces we apply in our method are the balloon forces proposed by Cohen[23], based on which the model nodes move outward in the direction of their normal vector. The deformable model is initialized at a pixel inside the object and expands under the influence of the balloon forces. Once the model reaches the estimated boundary based on the MRF segmentation, boundary forces are applied in the opposite direction of the balloon force and the associated nodes stop deforming. Thus the model is aligned with the boundary. The nodal deformations are computed based on the use of finite elements. When most model nodes stop moving, the model stops deforming.

2.3 Integration

Our method is based on the integration of Gibbs Prior models and deformable models that results in efficient and reliable segmentations.

The quality of the Gibbs Prior model segmentation depends on parameters, such as α (the threshold of boundary), the mean value of pixels in the object, etc. In our approach we integrate the deformable and the Gibbs prior models by using the deformable models to compute the parameter values for the Gibbs prior model.

First we have an initial prior model to begin with. This Gibbs prior model will be applied on the image and give the MAP boundary estimation (M step). Then a deformable model is fit to the estimated boundaries (Fit step). Then we recalculate the parameters (such as the mean value of pixels within the object and the boundary threshold) of the Gibbs prior models based on the statistics of pixels inside the boundary of the object that was segmented based on the deformable model. We term this the prior estimation step or E step. The updated prior model parameters based on the deformable model will be more accurate for the object in the given image, since we include both the information inside and on the surface of the object. This way we compute suitable parameters regardless of the chosen initial values.

The recursive approach can be summarized as follows.

1. First use the Gibbs prior model to get an initial segmentation of the object (M step).
2. Once the estimated boundary has been computed, a deformable model is fitted to it (F step).
3. Update the Gibbs parameters based on the property of pixels within and on the object boundary. Use the updated parameters to create a new Gibbs prior model (E step). Then go to step 1.

We apply this three-step procedure recursively until the segmentation results do not change from iteration to iteration.

The advantage of the method is that it is very fast (5Hz on an SGI workstation) and can be used to segment medical images even when there is not enough prior information and the boundaries are not well defined.

3 Experiments

We present several experiments to demonstrate the power of our algorithm. All the experiments run in close to real time (5Hz on an SGI R10000 workstation).

The first experiment shown in Figure 2 shows the various steps of integrating the Gibbs prior model and the deformable model to achieve correct object boundary segmentation. The boundary threshold $\alpha = 6$, the weights for the local configuration $W_i(x) = 5$, there are 100 nodes on the deformable model. A successful segmentation is achieved after one iteration between the deformable and the MRF models.

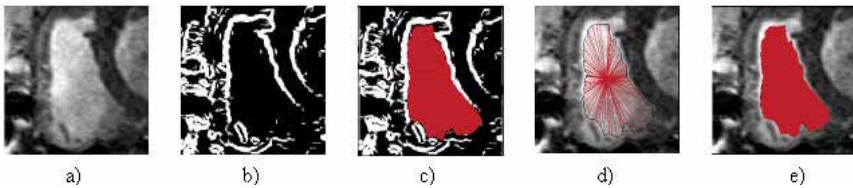


Fig. 2. a) is the MRI image of right ventricular; b) is the result of applying the Gibbs Prior models, the estimated boundary is assigned with white color; c) is the result of fitting a deformable model, the shading area is inside the object; d) shows the deformable model nodes; e) shows the final result after one iteration between the deformable model and the MRF model.

The second experiment shown in Figure 3 shows the effect of the recursive method between deformable model and the MRF model. The initial boundary threshold $\alpha = 6$, the weights for the local configuration $W_i(x) = 5$, there are 100 nodes on the deformable model. A successful segmentation is achieved after four iteration between the deformable and the MRF models.

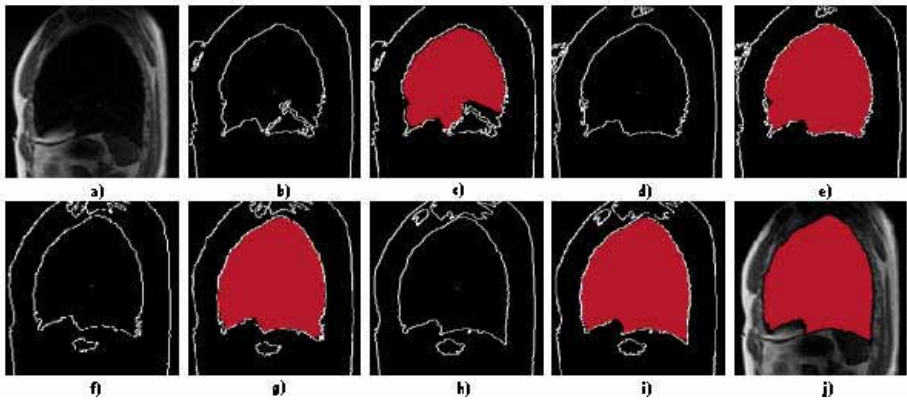


Fig. 3. shows how the recursive method works. a) is the original MRI image of lung; b) is the initial estimation of the boundary using Gibbs filter; c) shows the result of deformable model fitting; d) and f) show the Gibbs estimation after one iteration and two iterations; e) and g) show the result of deformable model fitting after one iteration and two iterations; h) shows the final Gibbs estimation and i) shows the final deformable model fitting; j) is the final segmentation result in the original image

Experiment three (Figure 4) and four (Figure 5) show the application of our method on visible human image data. In these two experiments, we apply the method to segment the eyeball and a muscle in human head respectively. The boundary threshold $\alpha = 6$. There are 100 model nodes. All the local configuration weights $W_i(x) = 5$. The program stops after one iterations between the deformable and the MRF models.

These two experiment show the effectness of our method on segmenting subtle humen body structures.

4 Conclusion

In this paper, we described a new approach to the segmentation problem by integrating a probabilistic model and a deformable model. By using the Gibbs prior models, we have a better estimation of the boundary to be used as an initialization for the deformable model. A recursive method was then developed to refine the segmentation results. The algorithm is very fast and is not sensitive to the selection of the initial parameters. The approach is robust to noise and can cope with low SNR. Methods have been developed to make the deformable models work well even in the presence of boundaries with sharp edges. Results of the method have been presented for several examples involving MRI images.

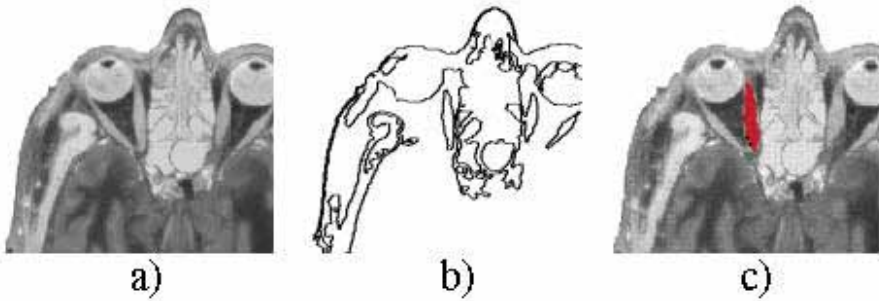


Fig. 4. a) is the original MRI image of human head, and we are going segment the muscle at the center of image; b) is the Gibbs estimation; c) is the final segment result.

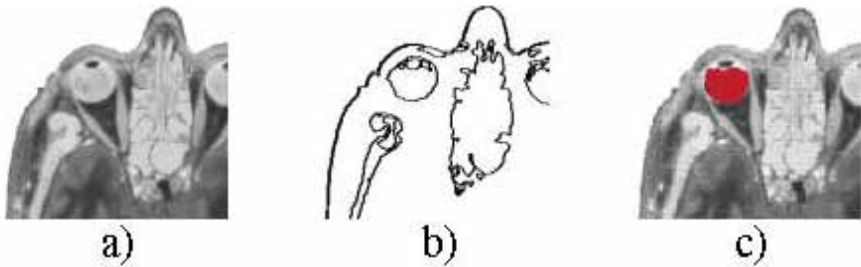


Fig. 5. a) is the original MRI image of human head, and we are going segment the eyeball at the center of image; b) is the Gibbs estimation; c) is the final segment result.

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