Imaging dispersion curves of surface waves on multi-channel record

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Summary

Real and synthetic data verifies the wavefield transformation method described here converts surface waves on a shot gather directly into images of multi-mode dispersion curves. Pre-existing multi-channel processing methods require preparation of a shot gather with exceptionally large number of traces that cover wide range of source-to-receiver offsets for a reliable separation of different modes. This method constructs high-resolution images of dispersion curves with relatively small number of traces. The method is best suited for near-surface engineering project where surface coverage of a shot gather is often limited to near-source locations and higher-mode surface waves can be often generated with significant amount of energy.

Introduction

Dispersion properties of surface waves find many useful applications in geophysical (Park et al., 1996; 1998) and geotechnical (Stokoe et al., 1994) engineering projects. It is normally assumed in these applications that the fundamental mode of surface waves dominates the recorded wavefield and higher modes can be ignored.

In reality, however, higher modes are always generated and can sometimes possess significant amounts of energy (Gucunski and Woods, 1991; Stokoe et al., 1994). Tokimatsu et al. (1992) suggests that the actual contribution of each individual mode can be represented by a complicated function of layer earth model, frequency, and receiver locations. Failure to separate the different modes can lead to erroneous results using current surface-wave analysis methods which account for the fundamental mode only during analysis. Interference of different modes can alter the apparent dispersion characteristic of the fundamental mode or result in higher modes being misinterpreted as fundamental. Contribution of higher modes tends to become more significant at higher frequencies than normally analyzed in conventional application of surface waves (Tokimatsu et al., 1992). Near-surface application of surface waves deals with these higher frequencies.

Reliable separation of different modes is possible only through a multi-channel recording method combined with an appropriate multi-channel data-processing technique (Gucunski and Woods, 1991; Tokimatsu et al., 1992). Conventionally, two types of multi-channel processing method have been used for this purpose: frequency-wavenumber (*f-k*) spectrum and slowness-frequency (*p-w*) transformation (McMechan and Yedlin, 1981) methods. Successful application of these two methods, however, requires preparation of a record with an exceptionally large number of traces collected over a wide range of source-to-receiver offset (Gabriels et al., 1987; Mokhtar et al., 1988).

In many recent applications of surface waves, the optimum offset range is often limited to near-source locations. The closer offsets are due to several reasons: dominance of body wavefields at far offsets (Park et al., 1996), use of a seismograph with limited number of recording channels, and severe lateral heterogeneity commonly present at near surface. If only a limited number of traces is combined with limited offset range, the resolving power of the aforementioned two methods significantly degrades.

We develop a wavefield transformation method that provides images of dispersion curves directly from the recorded wavefields of a single shot gather. With this method, different modes are separated with higher resolution even if the shot gather consists of a relatively small number of traces collected over a limited offset range. It is a simple three-step transformation method. The resolving power of the presented method is illustrated by using both synthetic and real examples directly compared with the results obtained using the slowness-frequency (*p-w*) method of McMechan and Yedlin (1981).

Transformation Theory

Considering offset-time (*x-t*) domain representation $u(x,t)$ of a shot gather, the Fourier transformation can be applied to time axis of $u(x,t)$ to obtain $U(x,w)$:

$$U(x,w) = \int u(x,t) e^{iwt}\, dt\,. \qquad (1)$$

$U(x,w)$ can then be expressed as the multiplication of two separate terms:

$$U(x,w) = P(x,w)A(x,w), \qquad (2)$$

where $P(x,w)$ and $A(x,w)$ are phase and amplitude spectrum, respectively. In $U(x,w)$, each frequency component is completely separated from other frequencies and the arrival time information is preserved in the phase spectrum $P(x,w)$. Then, $P(x,w)$ contains all the information about dispersion properties, while $A(x,w)$ contains the information about all other properties such as attenuation and spherical divergence. Therefore, $U(x,w)$ can be expressed as follows:

$$U(x,w) = e^{-i\Phi x} A(x,w), \qquad (3)$$

where $\Phi = w / c_w$, w = frequency in radian, and c_w = phase velocity for frequency w.

Applying the following integral transformation to $U(x,w)$ in (3) we obtain $V(w,\phi)$:

$$\begin{aligned} V(w,\phi) &= \int e^{i\phi x} \left[U(x,w) / |U(x,w)| \right] dx \\ &= \int e^{-i(\Phi-\phi)x} \left[A(x,w) / |A(x,w)| \right] dx. \end{aligned} \qquad (4)$$

The integral transformation in (4) can be thought of as the summing over offset of wavefields of a frequency after applying offset-dependent phase shift determined for an assumed phase velocity c_w ($=w/\phi$) to the wavefields in (3). This process is identical to applying a slant stack to the equivalent time-domain expression of $U(x,w)/|U(x,w)|$ for a single frequency. To insure equal weighting during analysis of wavefields from different offsets $U(x,w)$ is normalized with respect to offset compensating for the effects of attenuation and spherical divergence. Therefore, for a given w, $V(w,\phi)$ will have a maximum if

$$\phi = \Phi = w / c_w \qquad (5)$$

because $A(x,w)$ is both real and positive. For a value of ϕ where a peak of $V(w,\phi)$ occurs, the phase velocity c_w can be determined. If higher modes get appreciable amount of energy, there will be more than one peaks

Dispersion curves result from transforming of $V(w,\phi)$ to obtain $I(w,c_w)$ through changing the variables such that $c_w = w/\phi$. In the $I(w,c_w)$ wavefields, there will be peaks along the c_w-axis that satisfy (5) for a given w. The locus along these peaks over different values of w permits the images of dispersion curves to be constructed.

Examples — Synthetic Data

The previously described method was applied to a synthetic 24-channel shot gather (Figure 1a) containing three different modes of surface waves whose dispersion relations and amplitude spectrum are displayed in Figures 1b and 1c, respectively. Figure 2a displays images of dispersion curves obtained by applying this method to the synthetic data set (Figure 1a). Each individual dispersion curve is distinguishable at frequencies higher than 40 Hz. At frequencies lower than 40 Hz the first higher mode lacks separation from the fundamental mode. This apparent overlap is due to the similarities in phase velocities in that frequency range.

To illustrate the resolving power of the presented method, the slowness-frequency (p-w) transformation method by McMechan and Yedlin (1981) was also applied to the same synthetic data set in Figure 1a and the results are shown in Figure 3a. Comparing this image to Figure 2a, it is clear that overall resolution of the image for the method presented here is much higher.

To investigate the importance of including far-offset traces in both methods, two additional shot gathers were modeled (not shown in this paper) with sufficient channels to expand the offset coverage. The results of applying this and p-w methods to these shot gathers are displayed in Figures 2b, 2c, and Figures 3b, 3c, respectively. It is noticeable that resolution of the images generally increases in both methods as the number of traces increases to include further offsets. In any case the resolving power of the presented method is superior to that of the other method. The superiority becomes more significant when the total number of traces is fairly small.

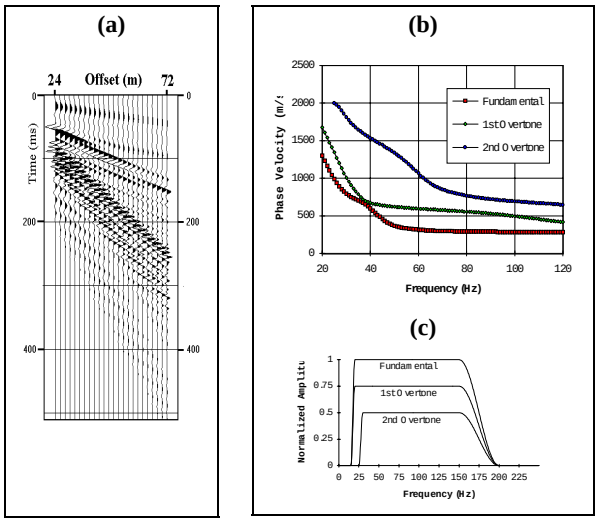

Figure 1. (a) A synthetic 24-channel shot gather that contains surface waves only. Three different modes of surface waves are modeled. (b) Dispersion relations and (c) amplitude spectra for each mode.

Examples— Real Data

Both methods were also applied to a real 32-channel shot gather obtained by using 10-kg sledge hammer as source at a test site near Kansas Geological Survey (KGS), Lawrence, Kansas (Figure 4). Most of the wavefield visible on the record represent ground roll with no significant amount of body wave present.

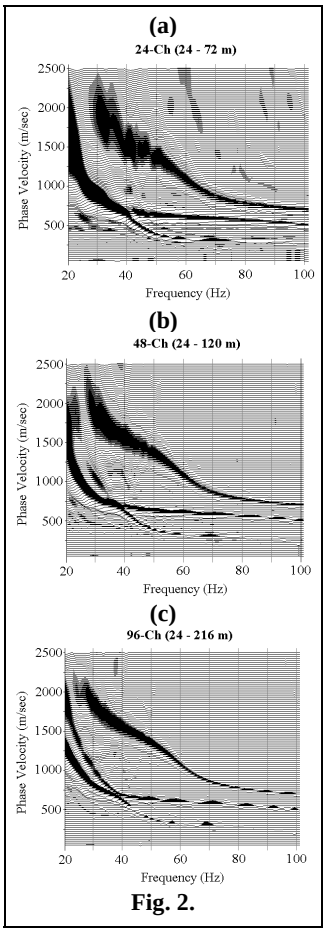

Fig. 2.

Figure 2. Images of dispersion curves obtained by applying presented method to synthetic shot gather of (a) 24-channel (Figure 1a), (b) 48-channel, and (c) 96-channel. Source-to-receiver offset ranges covered in each case are also indicated in title of each figure.

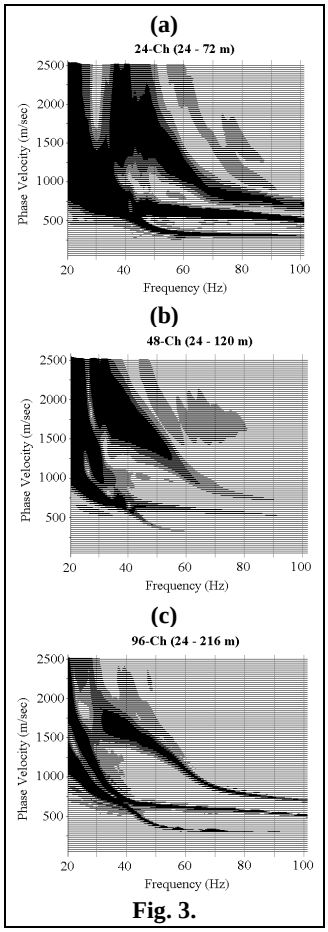

Fig. 3.

Figure 3. Same images obtained by applying *p-w* method.

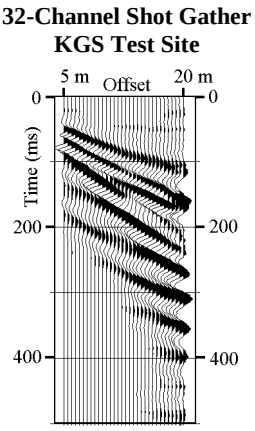

Figure 4. A 32-channel shot gather collected at a test site near Kansas Geological Survey, Lawrence, Kansas.

Images of dispersion curves obtained by applying the presented and *p-w* methods to this field record are shown in Figures 5a and 5b, respectively. Dispersion curves of three different modes are clearly identified in the frequency range of 20-100 Hz using the presented method. Only two dispersion curves of low resolution can be identified in a much narrower frequency band on section obtained using the *p-w* transformation method.

Phase velocities picked from the fundamental-mode dispersion curve in Figure 5a were used for an inversion process (Xia et al., 1997) to generate a preliminary velocity model. The velocity model was updated by comparing the dispersion curves of first three modes with experimental dispersion curves (Figure 5a) until a satisfactory match was achieved. The velocity profile in Figure 5c represents the model obtained in this manner. Because the overall inversion process was based upon examination of multi-mode dispersion curves, we believe the model should represent more realistic results than the

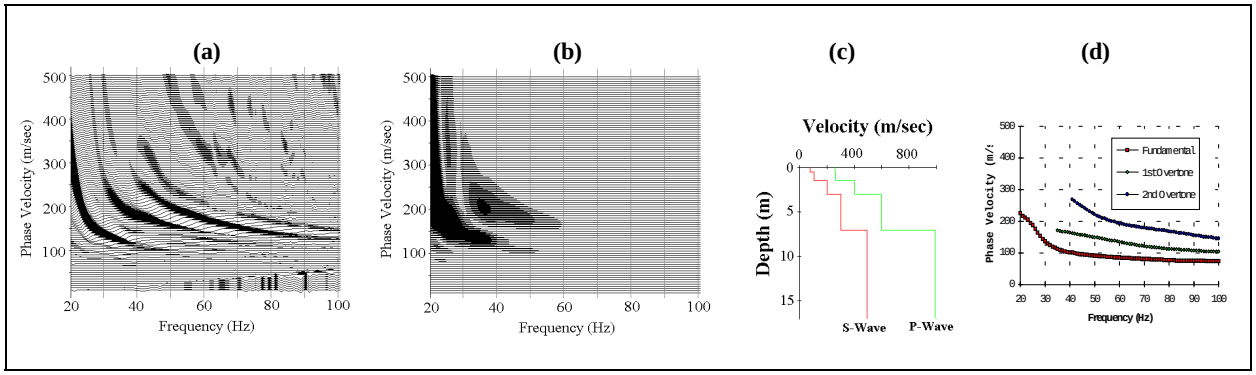

Figure 5. Images of dispersion curves obtained from the shot gather in Figure 4 by using (a) this, and (b) *p-w* methods. (c) Velocity model obtained through an inversion process accounting for all the three dispersion curves identified in (a), and (d) dispersion curves of first three modes corresponding to this velocity model.

preliminary velocity model obtained using only one dispersion curve.

Discussions

Resolution of the imaged dispersion curves in frequency-phase velocity ($w - c_w$) space can be separated into two independent terms: resolution along the frequency (w) axis and resolution along the phase velocity (c_w) axis. Resolution along the w-axis depends upon a method's ability to discriminate one frequency from all other frequencies for a given phase velocity. Resolution along the c_w-axis depends upon a method's ability to discriminate one phase velocity from all other phase velocity values for a given frequency.

Slant stacking in the p-w method applied to shot gathers in offset-time (x-t) space breaks down in resolution along the both axes when surface waves with different frequencies and phase velocities are superimposed. With the method presented here all the frequencies are completely separated by the Fourier transformation applied to time axis at the beginning of process. This allows the imaged dispersion curves to achieve a perfect resolution along the frequency (w)-axis. This explains why the overall resolution of the presented method is always superior.

In both methods the phase velocity for a particular frequency is found by slant stacking with different values of slowness [this is implied by the integral transformation (4) in the presented method] and determining the slowness that resulted at a maximum stacked amplitude. The presented method encounters the same degree of imperfectness that p-w method does as long as c_w-axis resolution is concerned.

Conclusions

Dispersion curves of different modes can be imaged directly from the full wavefield on a shot gather using a simple three-step transformation method as presented in this paper. Resolution of the image is superior to that obtained by using a pre-existing transformation method especially when the shot gather contains a relatively small number of traces over relatively narrow range of source-to-receiver offset. The method is best suited for geophysical or geotechnical engineering projects with near-surface application.

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