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Heterogene Arbeit: Positive und Normative
Aspekte der Qualifikationsstruktur der Arbeit

Christian Lumpe
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Immigration Policy, Equilibrium Unemployment, and Underinvestment in Human Capital

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Immigration Policy, Equilibrium Unemployment, and Underinvestment in Human Capital?

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Abstract:

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Immigration Policy, Equilibrium Unemployment, and Underinvestment in Human Capital

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January 11, 2007

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1 Introduction

The impact of immigration on the labour market prospects of natives has been subject of a public and academic discussion. Most of the discussion is about the influence of actual immigration flows and how to choose immigration policy optimally. In fact, in some countries (e.g. Canada, Australia) immigration policy explicitly aims at augmenting the domestic labour supply of skilled workers by a certain quality (in terms of skills) and volume to support economic development of the host country. Moreover, in other countries like Germany, the UK and the US, the discussion is about reforming immigration policy in favour of a more selective immigration in terms of skills. Our subsequent analysis shows that such an immigration policy is indeed able to foster human capital acquisition of natives and we can also show that this kind of policy is Pareto-improving. If at the same time, domestic education is subsidised, even a Pareto-optimal investment level can be reached. Therefore, our analysis gives a theoretical underpinning for a skill selective immigration policy.

We present a search-theoretic model with endogenous human capital investment. As the human capital investment decision is taken before workers enter the labour force and because of the existence of search frictions, this class of models features underinvestment in human capital (cf. Acemoglu (1996); Moen (1998); Sato and Sugiura (2003)). Our model extends this literature by including immigration in terms of the total flows (amount of immigrants) and its characteristics (amount of human capital). This modelling approach of the labour market contrasts sharply with the existing literature on immigration because its major focus has been mostly on stocks and its composition in a static context (Borjas, 1995, 1999). One of the few articles which analyses immigration in a search-theoretic context is Ortega (2000). However, he analyses employment and wage effects of immigration in a two country model without considering human capital investment.

Our modelling approach accounts for the fact that immigrants return to their home country with a positive probability.[1] Introducing this positive probability of returning home leads to a higher job destruction rate (shorter employment spells) for immigrants than for natives. As a partial result, our model can explain two stylised facts of economies with immigration: first, immigrants with the same human capital endowment earn lower wages than natives. Second, the unemployment rate of immigrants is higher than the unemployment rate of natives. Immigrants are therefore discriminated ex-post against natives because of their higher probability to leave the match.[2] This has to be distinguished from ex-ante discrimination because in our model firms do not offer vacancies which are specific to immigrants or natives.

Our main result is that an immigration policy aiming at well educated immigrants increases the number of vacancies which in turn increases the wage paid by firms. Therefore high skilled immigration leads to rising educational attainment of natives. Furthermore, relying on education subsidies, the distortion leading to underinvestment in human capital can be removed such that a Pareto-optimal investment level is reached. As an additional result, we demonstrate that either the appropriate number of immigrants (the flows) or the appropriate educational attainment (its characteristics in terms of human capital) of immigrants can have the same effect as unemployment benefits proposed by Sato and Sugiura (2003).

The remainder of the paper is structured as follows: in section 2 we present the basic structure of the model. In section 3 we derive both the solution of the individual human capital investment decision problem and the market equilibrium. In section 4 we analyse

[1] For a detailed theoretical and empirical discussion on return migration see Dustmann (2003). Müller (2003) also introduces return migration in an efficiency-wage model.

[2] This kind of discrimination is similar to the analysis of Müller (2003) .

the efficiency of the market outcome and discuss different immigration policies which are appropriate to overcome the underinvestment. Section 5 presents an extension of the basic model by including a labour market of different skill groups. Section 6 concludes.

2 Basic Model

2.1 Households

We develop an equilibrium matching model of the Diamond-Mortensen-Pissarides type (cf. Pissarides (2000)) . The economy is populated by a mass one of identical risk-neutral native workers $N = 1$ and foreign workers (immigrants) $I \geq 0$ adding to a total population $L = 1 + I$.[3] All individuals and firms discount future payments at the common discount rate ρ. Native workers enter and exit the labour market at a constant rate $\delta_N > 0$ such that the number of native workers is constant over time.[4] The number of *potential* immigrants is normalised to one which simplifies the exposition of the model. Immigrants enter a country's labour market at rate $\mu > 0$ and leave the labour market due to retirement ($\delta_N > 0$) or migration back to the home country ($r > 0$). The total exit rate of immigrants adds to: $\delta_I = \delta_N + r$.[5] The net flow of immigrants can therefore be calculated as $\dot{I} = \mu - \delta_I I$. The steady state number of immigrants ($\dot{I} = 0$) in the host country is $I = \mu/\delta_I$. To simplify the exposition of the model we denote the immigrants share in total population by $\eta_L = I/(1 + I) = \mu/(\delta_I + \mu)$.

Both native and immigrant workers start their working life in the unemployment pool. Before entering the unemployment pool, native workers have to decide about their human

[3]Throughout the paper subscript N denotes natives and subscript I denotes immigrants.
[4]The rate δ_N is the birth and retirement rate in the economy.
[5]In our model the return rate r is assumed to be exogenous. Typically, the decision to return to the home country is taken by the immigrant. However, in most industrialised countries we observe a large return migration which justifies our assumption of $r > 0$.

capital investment $z_N > 0$. Once taken the educational decision is irreversible. The cost per unit of human capital z_N amounts to c and the total cost of education cz_N will be borne by workers.

Immigrants entering the labour market are assumed to be endowed with human capital z_I which they already acquired in their home country. We assume that there exists no principal difference between the quality of human capital of natives and immigrants.[6] The acquired human capital can be used by any firm meaning that firms make no differences between an immigrant and a native worker.[7] The difference of endowments of human capital between natives and immigrants will only be reflected in the wages paid by firms.

Natives and immigrants can be in two different states: they are either working or searching for a job. Hence we abstract from on-the-job search.

2.2 Matching

We denote the number of unemployed workers by u and the number of vacancies searching for a worker by v. The ratio $\theta = v/u$ is then called labour market tightness. The random process by which vacancies and unemployed workers find each other is represented by a matching function: $m(u, v) > 0$ with $u, v > 0$. The matching function denotes the number of matched vacancies and workers per unit of time.[8] The application arrival rate for vacant jobs $q(\theta)$ can then be written as: $q(\theta) = m(u, v)/v = m(1/\theta, 1)$ with $q'(\theta) < 0$ and $\lim_{\theta\to 0} q(\theta) = \infty$, $\lim_{\theta\to\infty} q(\theta) = 0$. An unemployed worker meets

[6] At least at the beginning, the human capital quality of immigrants will differ from the human capital of natives (e.g. by language proficiency) but including this assimilation process would only strengthen the results of the model.

[7] See Bowlus and Eckstein (2002) or Black (1995) for discrimination in search models.

[8] The matching function $m(u, v)$ is assumed to be twice continuously differentiable, homogeneous of degree one and exhibits the following properties: $m(0, v) = m(u, 0) = 0$, $\partial m/\partial u$, $\partial m/\partial v > 0$, $\partial^2 m/\partial u^2, \partial^2 m/\partial v^2 < 0$ and $\partial m/\partial u$, $\partial m/\partial v > 0$.

a vacant job at the rate $p(\theta) = m(u,v)/u = \theta q(\theta)$ with $p'(\theta) > 0$ and $\lim_{\theta\to 0} p(\theta) = 0$, $\lim_{\theta\to\infty} p(\theta) = \infty$. Native workers and immigrants meet a vacant job at the same rate. Note that potential firms cannot directly search either a native worker or an immigrant worker. Whether it is a native worker or an immigrant will be revealed when a firm and a worker meet.

2.2.1 The Beveridge curve

The flow equation of umployment $\dot{u}$ which characterises the labour market is the difference between the inflows into unemployment and the outflows from unemployment. With both natives and immigrants being in the pool of unemployed workers we have two different flow equations for each group: $\dot{u}_N$, $\dot{u}_I$. Inflows into unemployment occur if a job is closed or new workers enter the labour market. Any filled job can be destroyed due to two different reasons: either the job is hit by an exogenous negative productivity shock at rate s or the job is closed because the employee leaves the labour market completely which occurs at rate δ_i, $i = I, N$. Note that only the former increases the number of unemployed. The respective dynamics of unemployment are given by:

$$\dot{u}_N = \delta_N + s(1 - u_N) - p(\theta)u_N - \delta_N u_N, \tag{1}$$

$$\dot{u}_I = \mu + s(I - u_I) - p(\theta)u_I - \delta_I u_I. \tag{2}$$

In the steady state $\dot{u}_i = 0$ $i = I, N$ we obtain the following *number* of unemployed native[9] and immigrant workers:

$$u_N = \frac{\delta_N + s}{s + p(\theta) + \delta_N}, \quad u_I = \frac{\mu}{\delta_I}\frac{\delta_I + s}{s + p(\theta) + \delta_I} \tag{3}$$

[9]Because the number of natives is standardised to one, the number of unemployed natives is also the unemployment rate of natives.

with $u_N \in [0,1]$ and $u_I \in [0, \mu/\delta_I]$. The aggregated Beveridge curve of the economy is then given by the sum of unemployed natives and immigrants:

$$u = \frac{\delta_N + s}{s + p(\theta) + \delta_N} + \frac{\mu}{\delta_I}\frac{\delta_I + s}{s + p(\theta) + \delta_I}. \tag{4}$$

Comparing the unemployment *rates* of natives and immigrants we arrive at the following result:

Corollary 1. *The unemployment rate of immigrants is always higher than the unemployment rate of natives:* $u_N < u_I/I$.

Proof. Using equation (3) together with the definition of the unemployment rate it follows that $u_N < u_I/I$. □

Consequently, the immigrants' share in unemployment is always greater than the immigrants share in total population: $\eta_U(\theta) = u_I/(u_I + u_N) > I/(1+I) = \eta_L(\theta)$. Therefore our model features a well documented fact of labour markets in most industrialised countries (cf. Hatton and Williamson, 2005, pp.325 table 15.3).[10]

2.2.2 Match formation and wage setting

Let U_i, W_i, $i = I, N$, be the expected present value of unemployment and employment, respectively. Then the flow value (asset value) of unemployment is given by:

$$\rho U_i = b + p(\theta)(W_i - U_i) - \delta_i U_i, \quad i = I, N. \tag{5}$$

[10]Interestingly, most of the empirical literature concentrates on the explanation of wage differentials between natives and immigrants. There are very few papers analysing immigrants incidence of unemployment (cf. McDonald and Worswick (1997) for Canada, Arai and Vilhelmsson (2004) for Sweden).

An unemployed worker receives the instantaneous value of leisure b, and will meet a vacant job at rate $p(\theta)$, thereby swapping the value of unemployment U_i with the value of employment W_i. At the rate δ_i an unemployed worker is expected to leave the labour market and therefore loses the value of unemployment U_i.[11] By the same argument the flow value of an employed worker can be written as:

$$\rho W_i = w_i + s(U_i - W_i) - \delta_i W_i, \quad i = I, N. \tag{6}$$

While being employed a worker receives instantaneously the wage w_i. The job is expected to be closed at rate s and the worker enters the unemployment pool. Additionally, a job is randomly closed according to the retirement rate δ_i, $i = I, N$.

Now, we look at the expected present value of firms, which are either producing or searching for a worker. A firm searching for an applicant incurs search cost $k > 0$ at each instant of time. Note that a job can either be filled with a native worker or an immigrant worker. As mentioned before, apart from the differing retirement rates, the only potential difference between both types of workers is the endowment with human capital z_i, $i = I, N$.

The output of a job-worker pair is generated according to a general production function $f : \mathbb{R}^+ \to \mathbb{R}^+$ with human capital z being the only input of production. The production function has the following properties: $f'(z) > 0$, $f''(z) < 0$, $\lim_{z\to 0} f'(z) = \infty$ and $\lim_{z\to\infty} f'(z) = 0$. Furthermore, we assume that for any $z \geq 0$ the value of output is strictly greater than the value of leisure b: $f(z) > b$.[12]

Let V, J_i, $i = I, N$ be the expected present value of a vacant job and a filled job,

[11] For simplicity we assume that the value of returning to the home country is zero for immigrants . In any case, the value of returning home should be smaller than the value of unemployment.

[12] Without this requirement a situation can arise where no individual chooses to educate and work.

respectively. The flow value of a producing firm with worker $i = I, N$ is given by:

$$\rho J_i = f(z_i) - w_i + (s + \delta_i)(V - J_i) \quad i = I, N.$$

The flow value consists of the flow profits of a match $f(z_i) - w_i$ and the potential loss caused by either the destruction of the job (with rate s) or the retirement of the respective worker (with rate δ_i).

For the derivation of the flow value of a vacancy ρV it is important to bear in mind, that ex-ante a firm does not know whether it will produce with a native worker or an immigrant worker. The share of unemployed immigrants of the pool of unemployed workers $\eta_U(\theta)$ also reflects the conditional probability of meeting an immigrant job searcher. The effective rate of meeting an unemployed immigrant is $q(\theta)\eta_U(\theta)$ while the effective rate of meeting an unemployed natives is given by: $q(\theta)\,(1 - \eta_U(\theta))$. We assume that the effective rate for any group is negatively correlated with labour market tightness θ such that $dq(\theta)\eta_U(\theta)/d\theta < 0$ holds. Any firm offering a vacant job considers the expected present value of a filled job $J^e = \eta_U J_I + (1 - \eta_U)J_N$.[13] The flow value of a vacant job can then be written as:

$$\rho V = -k + q(\theta)(J^e - V).$$

and consists of the flow costs of searching k and the potential change from a vacant to a productive job $(J^e - V)$. Free entry of firms generates an asset value of a vacancy of zero: $V_i = 0$. Thus we can calculate the job creation condition of firms as:

$$J^e = \frac{k}{q(\theta)}. \tag{7}$$

Free entry leads to an expected present value of a filled job J^e which is equal to the expected costs of finding a worker. We also get the following expression for a filled job

[13]For notational convenience we drop the functional argument θ in $\eta_U(\theta)$ when this causes no confusion.

of type i:

$$J_i = \frac{f(z_i) - w_i}{\rho + \delta_i + s}, \quad i = I, N. \tag{8}$$

We assume that wages are negotiated between a matched worker-firm pair according to Nash-bargaining. This means that the wage for worker type i solves the following optimisation problem:

$$w_i = \arg\max \left(W_i(w_i) - U_i\right)^{\beta} \left(J(w_i) - V\right)^{1-\beta}, \quad i = I, N \tag{9}$$

where β is interpreted as the bargaining power of workers.[14] The wage setting function for each type of worker is given by:[15]

$$w_i = \beta f(z_i) + (1-\beta)\frac{b(\rho + \delta_i + s) + p(\theta)\beta f(z_i)}{\rho + \delta_i + s + p(\theta)\beta}, \quad i = I, N. \tag{10}$$

Comparing both the wage of natives and immigrants yields the following result:

Corollary 2. *Immigrant workers with human capital $z_I \leq z_N$ always earn a lower wage $w_I < w_N$ compared to a native worker.*

Proof. Taking the total differential of the native wage equation we get:

$$dw_N = -\frac{(1-\beta)\beta p(\theta)\left(f(z_N) - b\right)}{\left(\rho + \delta_N + s + p(\theta)\beta\right)^2} d\delta_N + \frac{(1-\beta)\beta p(\theta) f'(z_N)}{\rho + \delta_N + s + p(\theta)\beta} dz_N < 0$$

Evaluating the total differential at $d\delta_N = r$ and $dz_N \leq 0$ completes the proof. ☐

This result stems from the fact that immigrants have a higher risk of leaving the host

[14] By using this formulation we assume that there is no difference in the bargaining power of natives and immigrants. Presumably the bargaining power of immigrants is lower compared to natives at the beginning of their working life in the host country and the same in the long run. However, taking this into account would not alter the results of the model qualitatively.

[15] The derivation of (10) can be found in the appendix.

countries' labour market. A higher risk of closing a productive job translates into a lower average job duration which reduces the potential surplus of the job. Therefore the wage rate, a share of the total surplus, has to be smaller to compensate for this lower duration.[16]

For future reference it will be convenient to derive closed form solutions for U_i and J_i, $i = N, I$. Together with the wage setting function we derive the expected present value of unemployment in terms of human capital z_i and labour market tightness θ:

$$U_i = \frac{b(\rho + \delta_i + s) + p(\theta)\beta f(z_i)}{(\rho + \delta_i)\,(\rho + \delta_i + s + p(\theta)\beta)} \quad i = I, N. \tag{11}$$

U_i is a weighted average of the value of unemployment b and the share β of the output $f(z_i)$. Note that z_N is endogenous and will be chosen by natives. Using the expression for the wage rate w_i together with the definition of the expected value of a filled job (8) of type i yields:

$$J_i = \frac{(1-\beta)(f(z_i) - b)}{\rho + \delta_i + s + \beta p(\theta)} \quad i = I, N. \tag{12}$$

This expression can then be used in the free entry condition (7) to yield the firms' job creation curve (JCC):

$$\eta_U \frac{q(\theta)(1-\beta)(f(z_I) - b)}{\rho + \delta_I + s + \beta p(\theta)} + (1 - \eta_U)\,\frac{q(\theta)(1-\beta)(f(z_N) - b)}{\rho + \delta_N + s + \beta p(\theta)} = k. \tag{13}$$

This job creation curve is equivalent to the standard formulation in search models except that we have two different types of filled jobs.

[16]There is a huge empirical literature analysing the evident wage differential between natives and immigrants: cf. Borjas (1999).

3 Educational Decisions and Equilibrium

3.1 Educational Decision

Before entering the labour market natives must decide how much to invest into education. After the investment decision is made, each new entrant will start as an unemployed worker searching for a job. As the expected present value of unemployment U_N already incorporates any future periods of employment and unemployment, it is the expected total lifetime income of a native worker. Consequently, an individual entering the labour market will seek to maximise U_N by choosing the level of human capital z_N appropriately. Therefore native workers' optimisation problem is to maximise the net expected value of unemployment:

$$\max_{z_N} U_N - cz_N.$$

Using the closed form of U_N the first order condition for a native worker is given by:

$$\frac{\beta p(\theta)}{(\rho + \delta_N)(\rho + \delta_N + s + \beta p(\theta))} f'(z_N) = c. \tag{14}$$

Any native workers chooses investment level z_N as to equalise the marginal return and the marginal cost c. For future reference we will refer to (14) as investment decision condition (IDC). Note that both, a higher retirement rate δ_N and higher destruction rate s decrease the level of human capital investment because the time period to recoup the investment will be shorter. Additionally and with the same line of reasoning, increased labour market tightness θ increases the investment level, because unemployment spells are shorter. It is important to note that immigration does not directly influence the individual investment decision. However, immigration influences the equilibrium outcome of the economy in terms of θ and z_N.

3.2 Competitive Equilibrium

A competitive equilibrium consists of a triple $\{z_N^E, \theta^E, u^E\}$ which simultaneously solves the job creation condition (JCC) of firms,

$$G_1^E(z_N, \theta) := q(\theta)\left[\eta_U(\theta)J_I(\theta) + (1-\eta_U(\theta))\, J_N(\theta, z_N)\right] = k, \tag{15}$$

the investment decision (IDC) of native workers:

$$G_2^E(z_N, \theta) := \frac{\beta p(\theta)}{(\rho+\delta_N)\,(\rho+\delta_N+s+\beta p(\theta))} f'(z_N) = c, \tag{16}$$

and the Beveridge curve:

$$u = \frac{\delta_N + s}{s + p(\theta) + \delta_N} + \frac{\mu}{\delta_I}\frac{\delta_I + s}{s + p(\theta) + \delta_I}. \tag{17}$$

Note that the system is block recursive so that equilibrium values of the labour market tightness θ^E and the human capital z_N^E are completely identified by (13) and (14). Using the resulting θ^E in (4) yields the equilibrium number of unemployed workers u^E. As shown in the appendix, both, the JCC and the IDC are positively sloped curves in the $z_N - \theta$ space. The IDC starts at the origin and z_N is bounded from above by $\bar{z}$ according to $\{\bar{z}_N : f'(\bar{z}) = (\rho + \delta_N)\, c\}$. In contrast, the JCC starts at a positive θ with no upper bound for z_N and θ. It can be shown, that both curves intersect at least once such that at least one equilibrium exists.[17] In order to discuss the comparative statics of the economy it is necessary to analyse stable equilibria only. To define a stable equilibrium we construct simple out-of-steady-state dynamics. Consider a triple $\{z_N^1, \theta^1, u^1\}$ in a sufficiently small neighbourhood of an equilibrium triple $\{z_N^E, \theta^E, u^E\}$. Assuming that labour market tightness θ will respond fastest to eliminate positive profits from open

[17] For a detailed proof, please consult the appendix.

vacancies, we get a new θ according to $\theta^2 = \theta_{G_1^E=k}(z_N^1)$. This new θ^2 will induce workers to revise their investment decision to $z_N^2 = \theta_{G_2^E=c}(\theta^2)$. The sketched dynamics creates a series $\{z_{iN}, \theta_i, u_i\}_{i=1,...}$ which is stable if it converges to $\{z_N^E, \theta^E, u^E\}$.

With this characterisation of a stable equilibrium we can show that a stable equilibrium is reached if at the intersection of both equilibrium conditions the slope of the IDC is steeper than the slope of the JCC: $\frac{d\theta}{dz_N} \mid_{G_1^E=k} < \frac{d\theta}{dz_N} \mid_{G_2^E=c}$.[18]

Figure 1 illustrates the JCC and the IDC in the $z_N - \theta$ space in a situation with a stable equilibrium.

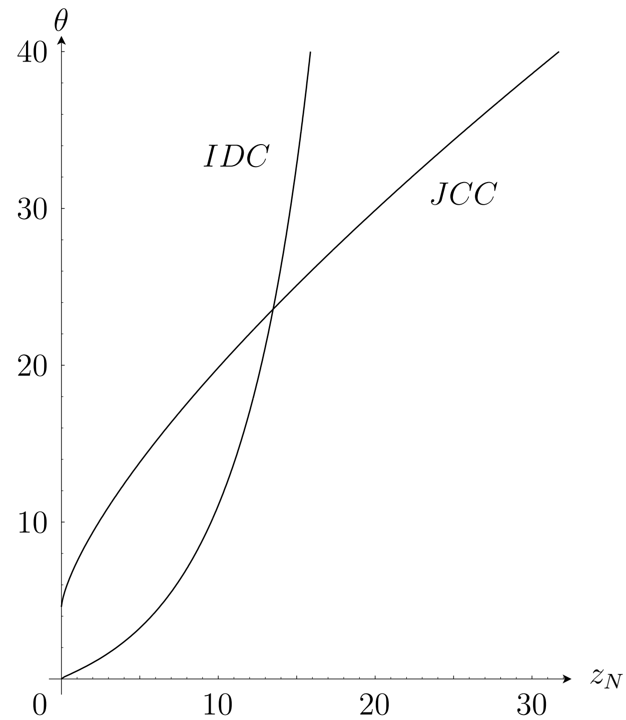

Figure 1: The IDC and the JCC determining a stable equilibrium.

[18]For the derivation of the slope of the JCC and IDC, please consult the appendix.

3.3 Comparative statics results

In this section we analyse the impact of immigration on the labour market equilibrium. We discuss two different scenarios: first time immigration into a formerly closed economy without any immigration ($\mu = 0$), and the case of sustained immigration into an economy with existing immigration ($\mu > 0$, $z_I > 0$). Throughout the following sections we assume that we are in a stable equilibrium. Note that immigration only affects the JCC, while the IDC is unaffected: $G^E_{2,\mu} = G^E_{2,z_I} = 0$.[19] Therefore, we can concentrate on the influence of immigration on the JCC only.

In the case of first time migration, the calculation of the partial effect of z_I on the JCC reveals that $G^E_{1,z_I}\big|_{\mu=0} = 0$. However, the influence of the migration rate μ on the JCC is nonzero such that we derive the following results:

$$G^E_{1,\mu}\big|_{\mu=0} = \eta_{U,\mu}\big|_{\mu=0}\, q(\theta)(J_I - J_N) \begin{cases} > 0 & \text{if} \quad (J_I - J_N) > 0 \\ < 0 & \text{if} \quad (J_I - J_N) < 0 \end{cases} \tag{18}$$

because $\eta_{U,\mu}\big|_{\mu=0} > 0$. Together with $G^E_{1,z_N}\big|_{\mu=0} > 0$ and $G^E_{1,\theta}\big|_{\mu=0} < 0$ we find that first time immigration leads to a clockwise rotation of the JCC at its intersection with the curve $J_I(\theta) = J_N(\theta, z_N)$ (see point A in figure 1).[20] With first time immigration, the human capital endowment of potential migration (or the minimum requirement in terms of human capital for immigration) only matters for the comparative statics. First time immigration will increase (decrease) the labour market tightness θ^E and native human capital z^E_N if the expected present value of migrant jobs J_I are more (less) valuable than comparative native jobs J_N. Due to a higher exit rate δ_I potential immigrants need

[19] To simplify the exposition of our results we use the following notation to denote partial derivatives: $G_{2,k} \equiv \frac{\partial G^E_2}{\partial k}$.

[20] The curve implicitly defined by $J_I(\theta; z_I) = J_N(\theta, z_N)$ is positively sloped in the $(z_N - \theta)$-space originating from $\tilde{z}_N$ which is the solution to $J_I(0, z_I) = J_N(0, \tilde{z}_N)$. For $\theta \to \infty$ we get $z_N \to z_I$.

more human capital than natives to offset this negative effect. Thus, even if potential immigrants are better educated than native workers $z_I > z_N$ the job value of natives can be higher $J_I < J_N$ and both θ^E and z_N^E decrease. To increase the labour market tightness θ^E and native human capital z_N^E immigration policy has to aim at immigrants who are very well educated compared to natives. The native wage rate w_N is positively correlated with θ and z_N such that immigration policy directly influences native labour income. The same is true for native employment which will increase if $J_I > J_N$. The effect on total unemployment $u_I + u_N$ is ambiguous if immigration is high skilled $(J_I > J_N)$ because the decrease of native unemployment is counteracted by an increasing number of unemployed immigrants.

First time immigration is rather unlikely, because today most industrialised countries experience sustained immigration and try to implement a specific immigration policy given a certain history of migration $\{z_I, \mu\}$. The equilibrium of an economy with existing immigration is depicted by point B of figure 1. A change of the immigration policy can be either a change in the number of immigrants by changing the inflow $d\mu$ or a change of the human capital standards dz_I. We start by assuming that the equilibrium human capital of natives z_N is high enough such that $J_I < J_N$ holds. Either changing the amount of the existing quality of immigration ($d\mu \gtreqless 0$) or changing the future quality of immigration ($dz_I \gtreqless 0$) leads to the following change of the equilibrium values z_N^E, θ:[21]

$$\frac{d\theta^E}{d\mu}, \frac{dz_N^E}{d\mu} < 0 \quad \frac{d\theta^E}{dz_I}, \frac{dz_N^E}{dz_I} > 0 \quad J_I < J_N. \tag{19}$$

Thus, we derive the same result as with first time immigration, that both increased unskilled immigration and decreasing human capital standards will reduce equilibrium labour market tightness and human capital investments. This is because the expected present value of a filled job is reduced with a lower educational attainment of immigrants

[21] For a detailed derivation, please consult the appendix.

or an increased number of unskilled immigrants. Therefore, offering a vacancy is less attractive for firms which reduces the number of vacancies and consequently the labour market tightness. Next, we assume that the existing immigration is sufficiently high skilled such that $J_I > J_N$ holds. In this situation, a change of the migration policy will result in the following change of the equilibrium values:

$$\frac{d\theta^E}{d\mu}, \frac{dz_N^E}{d\mu} > 0 \quad \frac{d\theta^E}{dz_I}, \frac{dz_N^E}{dz_I} > 0 \quad J_I > J_N \tag{20}$$

We get the result, that an increased number and higher quality of migrants will increase the labour market tightness and the human capital acquisition of natives. For later reference we summarise our findings in:

Proposition 1. *In the stable equilibrium, if $J_I < J_N$ an increase in the endowment of human capital of immigrants z_I^E increases θ^E and z_N^E and an increase in the flow of immigrants μ decreases θ^E and z_N^E . If $J_I > J_N$ an increase in the endowment of human capital of immigrants z_I^E and an increased inflow of immigrants μ increases θ^E and z_N^E .*

Our result shows that in the context of search frictions, higher minimum requirement of human capital for immigrants leads to skill upgrading of native workers, because this immigration increases the firms' incentive to supply jobs for workers in the host country. This contrasts with much of the literature which mostly focuses on competitive labour markets. In these models high skilled immigration reduces the incentive to invest into education (Fuest and Thum (2001)). Considering the different labour market institutions shown by search models reveals totally different immigration policy implications.

4 Efficiency and labour policies

4.1 Social planner

We assume that a social planner seeks to maximise native welfare only. The social welfare function used by the social planner is defined by:[22]

$$\Omega = \int_0^\infty e^{-\rho\tau}(y_N + bu_N - k\theta u_N - \delta_N c z_N)d\tau. \tag{21}$$

The first additive term is the total income of natives with y_N denoting the average output per native worker and bu_N denoting the leisure income of natives. The second term summarises total cost: the search cost of firms for native workers $k\theta u_N$ and the cost of education borne by natives entering the labour market $\delta_N c z_N$. While taking his choice of u_N and z_N the social planner has to obey the evolution of native unemployment $\dot{u}_N$:

$$\dot{u}_N = \delta_N + s(1 - u_N) - p(\theta)u_N - \delta_N u_N, \tag{22}$$

as well as the evolution of average output $\dot{y}_N$:

$$\dot{y}_N = p(\theta)u_N f(z_N) - (s + \delta_N)y_N. \tag{23}$$

The first term represents the new jobs which are producing with a native worker and the second term represents the fraction of mature jobs which are destroyed at each instant of time. Maximising (21) subject to (22) and (23) yields the following optimality conditions:[23]

$$G_1^o(z_N, \theta) \equiv \frac{p'(\theta)\left(f(z_N) - b\right)}{\rho + s + \delta_N + p(\theta) - p'(\theta)\theta} = k, \tag{24}$$

[22] For this formulation see Pissarides (2000).
[23] For a detailed derivation of the solution of the optimisation problem, please consult the appendix.

$$G_2^o(z_N, \theta) \equiv \frac{(\delta_N + s)\, p(\theta)}{\delta_N \,(s + \delta_N + p(\theta))\,(s + \delta_N + \rho)} f'(z_N) = c. \tag{25}$$

The solution of the optimisation problem is given by a triple $\{\theta^o, z_N^o, u_N^o\}$ solving the optimality conditions and the steady state condition for native unemployment, where the superscript o denotes the social optimum. Both optimality conditions are comparable to that of the market outcome. Equation (25) corresponds to the IDC and equation (24) corresponds to the JCC. Because the social planner is only interested in the welfare of natives these conditions do not reflect the fact that immigrants are active in the economy.

Comparison of the IDC of the market outcome (14) with the choice of the social planner (25) reveals, that the amount of individual human capital investment z_N in the competitive environment is biased downwards and generates underinvestment:

Lemma 1. *The IDC of native workers in the competitive environment generates underinvestment:* $z_N(\theta)|_{G_2^E=c} < z_N(\theta)|_{G_2^o=c}$. *(For a detailed proof, please consult the appendix.)*

In (z_N, θ)-space, the IDC of the social planner is shifted to the right compared to the IDC of the competitive situation. In our model underinvestment is due to the timing of the investment decision: bargaining after the decision of education leads to hold-ups of native workers. Analysing the loci defined by (13) and (24) shows, that the bargaining power of workers β is crucial in determining whether the labour market tightness is higher or lower in the competitive environment compared to the choice of a social planner. In equilibrium matching models with free entry condition, a certain bargaining power β^o will generate an efficient labour market tightness $\theta^E \mid_{z=z^o} = \theta^o \mid_{z=z^o}$ (cf. Hosios (1990)). Any other value of β will result in an inefficient labour market tightness. First, we analyse the efficiency of the market generated θ^E at the optimal investment level $\theta^E \mid_{z=z^o}$ if the expected present value of filled jobs is the same for both natives and immigrants: $J_N = J_I$. In this case, the efficient value β^o coincides with that of an economy without immigration: $\beta^o = \theta q'(\theta)/q(\theta)$, namely if the bargaining power is equivalent to the elasticity of the application rate with respect to the labour market tightness (cf. Pissarides 2000). With a β larger (smaller) than the efficient value β^o we have too small (large) labour market tightness.

Second, we have to differentiate the cases of immigration resulting in different expected present values of filled jobs of immigrant and natives $J_N \neq J_I$. In this case, the existing immigration in the host country plays a significant role for the threshold value β^o. The efficient value does not only depend now on the elasticity of the application rate $q(\theta)$, but also on the human capital endowment of natives z_N and immigrants z_I. In fact, the educational attainment of immigrants in the host country compared to the natives ($J^e \gtrless J_N$) is decisive for the efficient bargaining power β^o. The expected value of a filled job may be lower than the filled job of an economy without immigrants ($J^e < J_N, \quad z_I < z_N$) whereby the new efficient bargaining power in the case of low skilled immigration will be lower than the efficient bargaining power of an economy without immigration ($\beta^o_I < \beta^o$). Note that the benchmark case always is the the bargaining power of an economy without immigrants because the social planner only maximises native welfare. Again, in the case of low skilled immigrants, we get too high (small) labour market tightness if the bargaining power is too small (high). The respective analysis applies for the case of high skilled immigrants. Interestingly, an economy with low skilled immigrants would result in a lower efficient bargaining power than an economy without immigrants.

Lemma 2. *When the bargaining power of workers β is large enough:$\beta > \beta^o_I$ (small enough: $\beta < \beta^o_I$), the JCC in the competitive environment generates too small market tightness: $\theta(z_N)|_{G^E_1=k} < \theta(z_N)|_{G^o_1=k}$ (too large market tightness: $\theta^E \mid_{z=z^o} > \theta^o$). If $\beta = \beta^o_I$, it generates the optimal market tightness $\theta^E \mid_{z=z^o} = \theta^o$.(For a detailed proof,* *please consult the appendix.)*

The lemmas 1 and 2 reflect the inefficiency of the competitive equilibrium. The underinvestment in human capital always exists irrespective of the actual value of θ^E. However, the labour market tightness is either too small or too high depending on the bargaining power of workers as much as on the educational attainment of immigrants in the host country. We get therefore the same result as Acemoglu and Shimer (1999) and Sato and Sugiura (2003) with the same mechanism at work.

4.2 Education subsidies

As shown by the first proposition, directed immigration policy - regarding human capital characteristics and/or flows of immigrants - influences directly the JCC. To reach the social optimum, we need a policy tool which allows us to influence the IDC. By introducing education subsidies per invested unit h for native workers the IDC is changed according to:

$$\frac{\beta p(\theta)}{(\rho+\delta_N)(\rho+\delta_N+s+\beta p(\theta))} f'(z_N) = c - h.$$

At a given labour market tightness, the introduction of a subsidy leads to increased investment in human capital. In the $z_N - \theta$ space this results in a shift to the right (c.f figure 1). However, increased investment in human capital makes it more profitable for a firm to open a vacancy which in turn increases the labour market tightness. The equilibrium outcome of an education subsidy is described in

Proposition 2. *In the stable equilibrium, an increase in a subsidy to a unit investment in human capital h increases θ^E and z_N^E and lowers the unemployment rate.(For a detailed proof, please consult the appendix.)*

4.3 Pareto-optimal immigration and labour policy

We have shown that education subsidies affect the incentives to train and lead to more human capital investments by natives. Immigration policies - either by changing the human-capital composition of immigrants or by changing the inflow of immigrants into the host country - will have direct effect on the job creation of firms. Combining these policies, we can reach the Pareto-optimal human capital investment of natives and labour market tightness thereby removing the hold-up problem.

The starting point of our analysis are the properties of the competitive equilibrium. The investment of native workers always has to be subsidised to remove the underinvestment (see lemma 1). Therefore $h^* > 0$ should be the appropriate labour market policy which leads to a shift of the IDC towards the social planner equilibrium (see proposition 2).

With a change in the flows of immigrants μ and of the characteristics of immigration z_I on the JCC, we can influence the job creation of firms and therefore we correct for an either too high or too small labour market tightness (lemma 2). The effect of an increase of z_I always increases z_N and θ - independently of the human capital endowment of existing immigrants ($J_I \gtrless J_N$). But the effects of an increase in the flow of immigrants μ depends on the existing human capital endowment of immigrants (see proposition 1). Thus, the Pareto-optimal immigration policy has to be a combination of policies (h^*, z_I^*, μ^*) which increases z_N^E and either increases or decreases θ^E. For example, in the case of an economy with low skilled immigration and a too high bargaining power, the labour market tightness would be too low and underinvestment in human capital exists. Therefore, we augment education subsidies h to increase z_N^E and θ^E and we could increase the human capital endowment of immigrants z_I or decrease the inflow of low skilled immigrants μ.

Proposition 3. *Human capital investments are subsidised with a labour policy $h^* > 0$.*

If $\beta < \beta_I^o$ and, the educational attainment of immigrants has to be higher or the inflows of unskilled immigrants have to be lower. If $\beta > \beta_I^o$, the educational attainment of immigrants has to be lower or the inflows of unskilled immigrants have to be lower. A combination of these policies (h^*, z_I^*, μ^*) induce the Pareto-optimal equilibrium of the social planner. (For a detailed proof, please consult the appendix.)

4.4 Simulation results

The following simulation serves to illustrate the theoretical discussion. For the parameters we take the following values:

Parameter	δ_N	δ_I	s	ρ	b	μ	k	c	β
Value	0.025	0.05	0.2	0.1	15	0.01	25	10	0.5

Table 1: Parameters of the numerical example

Figure 1 illustrates the case of the competitive equilibrium for a human capital endowment of immigrants of $z_I = 10$ and an inflow rate $\mu = 0.01$. With a $\delta_N = 0.025$, we get an average working life time of 40 years. With these parameters we obtain a steady-state stock of immigrants of $I = \mu/\delta_I = 0.2$ meaning that $\sim 18\%$ of the total population are immigrants. The matching function is assumed to be of Cobb-Douglas type $m(u, v) = 5\sqrt{uv}$ which gives an arrival rate of $p(\theta) = 5\sqrt{\theta}$. The production function takes the following form $f(z) = 5z^{0.7} + b$. Using these parameters and solving the social planner's problem results in an efficient labour market tightness $\theta^o = 38.6$ and an efficient level of human capital of natives of $z_N^o = 1895$. The value of unemployment for natives U_N takes the value of 509. The value for a filled native job J_N is equal to 6.12, which is much higher then the assumed search costs. The respective values of unemployment and filled jobs for immigrants are lower than for natives: $U_I = 249$ and

$J_I = 3.9$. Because the social planner's problem is independent of immigration, any of the following simulation results can be compared to the outcome of the planner's problem. In this example economy, we get an optimal bargaining power of $\beta_I^o = 0.45$. Therefore the labour market tightness is too low (because the existing bargaining power $\beta = 0.5$) and we would have to admit higher skilled immigrants or more skilled immigrants.

First we discuss the case of an economy with low skilled immigration (compared to the level of native human capital) and a too small labour market tightness. Now, we solely increase the human capital endowment of immigrants, $\theta^E$and z_N^E change as follows:

	$z_I = 10$	$z_I = 20$	$z_I = 30$
θ^E	1.45	1.6	1.67
z_N^E	22	22.3	22.5

Table 2: Increasing the human capital requirement for immigrants

With rising human capital of immigrants, the labour market tightness increases as well as the human capital endowment of natives. If we leave the human capital of immigrants constant and change the inflow of immigrants we get the following results:

	$\mu = 0.01$	$\mu = 0.02$	$\mu = 0.03$
θ^E	1.45	1.35	1.28
z_N^E	22	21.7	21.5

Table 3: Increasing the inflow of immigrants

In the case of existing low skilled immigration, an increase in the inflows of this kind of immigrants leads to a decreasing labour market tightness and drop of human capital investment of natives. In the case of existing high skilled immigration, increasing the inflow μ leads to an increasing labour market tightness and human capital investment.

The case of a too high labour market tightness (remember that the bargaining power has to be lower than the efficient bargaining power) is rather unlikely:

	$\beta = 0.6$	$\beta = 0.5$	$\beta = 0.4$
θ^E	0.96	1.45	2.16
z_N^E	21.8	22	21.8

Table 4: Bargaining power

A combination of labour immigration policy (h^*, z_I^*, μ^*) leads to the following optimal values. First we calculate the optimal education subsidy through the IDC which gives $h^* = 7.15$. The optimal human capital endowment of immigrants and the optimal inflow of immigrants lead to the following values: $\mu^* = 0.05$ and $z_I^* = 1899.41$.

5 Extension

In the US and the UK, the distribution of educational attainment of immigrants is rather bimodal with both a large number of highly skilled immigrants and a large number of low skilled immigrants. For example Chiswick and Sullivan (2005) report for the US, that immigrants from Asia, Europe and Canada mostly embody at least the same human capital as US natives of the respective group. But immigrants from Mexico and Latin America have significantly lower educational attainment than their native counterparts in the group of unskilled workers. We can discuss this kind of immigration if we consider perfectly segmented labour markets between skill groups in the spirit of Mortensen and Pissarides (1999).

Suppose that an economy consists of two different labour markets, one for high skilled workers and one for low skilled workers. Both labour markets are perfectly separated meaning that a high skilled worker can not switch to the low skilled labour market and vice versa. Assume further that individuals differ with respect to their abilities $a \in [0, \infty)$ distributed according to some general distribution function $g(a)$. High skilled workers acquire the skills needed on their respective labour market at university as discussed in the basic model. If access to universities requires a certain ability $\bar{a}$ individuals with abilities $a \leq \bar{a}$ work as low skilled workers and those with $a > \bar{a}$ work as high skilled workers.[24] Using this simple setup we end up with two segmented labour markets instead of N segmented labour markets as modelled by Mortensen and Pissarides (1999).

Considering the bimodal immigration of e.g. the US or the UK, our analysis of the impact of immigration applies separately for both labour markets. We have immigration of high skilled workers on the labour market of high skilled natives which is comparatively better skilled than their native counterparts. Simultaneously, immigration of low skilled workers takes place on the labour market of low skilled natives. First, entrance of high skilled immigrants (resulting from a higher μ) or higher skilled immigrants (resulting from a higher z_I) lead to increasing job creation of firms and higher wages of high skilled natives. Therefore, native workers have a higher incentive to invest more into education. Second, the same analysis applies for the impact of immigration of low skilled workers. If immigrants in the low skilled sector are comparatively less skilled than native low skilled workers, firms in the low skilled sector will react by opening less vacancies in this sector and the wage rate will decline for low skilled workers. The total effect will be higher investments in education by native high skilled workers due to their wage increases. The labour market prospects of low skilled workers deteriorates due to decreased wages. This summarises a possible impact of the bimodality of US immigration on the existing wage inequality (cf. Borjas et al. (1997)).

[24]The individual with ability $\bar{a}$ is indifferent between going to university or working as low skilled worker.

6 Conclusion

We introduce immigration into a search model of equilibrium unemployment. This allows us to model immigration in terms of flows and its characteristics in terms of human capital. Because of a positive probability of returning to their home countries, immigrants receive lower wages and have a higher unemployment rate compared to natives. We can show that an immigration policy which is concerned about the human capital endowment of immigrants and/or the number of immigrants has a decisive impact on the educational decision of natives. Immigration policy which favours higher skilled immigrants will increase the wage rate for the group of high skilled workers because firms have incentives to increase the number of vacancies. This induces natives to invest more in education. Furthermore, we can show that a combination of education subsidies and directed immigration policy can remove underinvestment in human capital. Education subsidies foster the investment decision of natives and the appropriate immigration policy generates Pareto-improving job creation by firms. The model can be extended to introduce bimodal immigration concerning the educational attainment of immigrants. Applying perfectly segmented labour markets in combination with an immigration of high and low skilled workers results in higher native investment in human capital by high skilled natives and lower wages of low skilled natives.

7 Appendix

Derivation of the wage setting equation (10)

Maximisation of the Nash Product (9) yields

$$w_i = \beta f(z_i) + (1-\beta)(\delta_i + \rho)U_i \quad i = I, N. \tag{A.1}$$

Substitution of (A.1) in(6) gives:

$$W_i = \frac{\beta f(z_i) + ((1-\beta)(\rho + \delta_i) + s)\, U_i}{\rho + \delta_i + s} \quad i = I, N. \tag{A.2}$$

Substitution of (A.2) in (5) we end up with reservation wage:

$$(\rho + \delta_i)\, U_i = \frac{b(\rho + \delta_i + s) + p(\theta)\beta f(z_i)}{\rho + \delta_i + s + p(\theta)\beta} \quad i = I, N. \tag{A.3}$$

Substitution of the reservation wage in (A.1) yields the wage setting equation (10).

Existence of the equilibrium

Proof. It is to show that the equilibrium $\{z_N^E, \theta^E, u^E\}$ exists. The functions $G_1^E(\theta, z_N)$ and $G_2^E(\theta, z_N)$ are continuous and $G_{i,z_N}^E \neq 0$ $i = 1, 2$ on the open interval $(0, \infty)$. Therefore, we can apply the implicit function theorem and express z_N as a function of θ denoted by: $z_{1N}(\theta)$, $z_{2N}(\theta)$. Because $\lim_{\theta \to 0} \lim_{z_N \to 0} G_1^E(\theta, z_N) = \infty > k$ the domain of $z_{1N}(\theta)$ is the open interval $(\bar{\theta}_1, \infty)$ with $\bar{\theta}_1 > 0$ and the domain of $z_{2N}(\theta)$ is the open interval $(0, \infty)$. Analysing these functions at their respective domain limits reveals: $\lim_{\theta \to \bar{\theta}_1} z_{1N}(\theta) = 0$ and $\lim_{\theta \to \infty} z_{1N}(\theta) = \infty$. Given that $\lim_{\theta \to \infty} p(\theta) = \infty$, we get $\lim_{\theta \to \infty} z_{2N}(\theta) = \bar{z}_2$ where $\bar{z}_2$ is defined by: $\bar{z}_2 := \{z_2 : f'(z_2) = (\rho + \delta_N)\, c\}$. At the lower boundary we get $\lim_{\theta\to 0} z_{2N}(\theta) = 0$. Next we define the function $\Gamma(\theta) = z_{2N}(\theta) - z_{1N}(\theta)$. Using the previous results we get $\lim_{\theta\to\bar{\theta}_1} \Gamma(\theta) > 0$ because $z_{2N}(\theta)$ is strictly increasing. Furthermore we get$\lim_{\theta\to\infty} \Gamma(\theta) = -\infty$. Thus, the intermediate value theorem guarantees at least one θ' such that $\Gamma(\theta') = 0$. This concludes the proof that at least one equilibrium exists. □

Slope of the IDC and JCC

Differentiation of $G_1^E(\theta, z_N)$ and $G_2^E(\theta, z_N)$ with respect to θ and z_N gives:

$$G^E_{1,\theta} = \frac{dq(\theta)\eta_U}{d\theta} J_I + \frac{dq(\theta)\left(_U\right)}{d\theta} J_I + q(\theta)\eta_U J_{I,\theta} + q(\theta)\left(1-\eta_U\right) J_{N,\theta} < 0, \quad \text{(A.4)}$$

$$G^E_{1,z_N} = q(\theta)\left(1-\eta_U(\theta)\right) J_{N,z_N} > 0, \quad \text{(A.5)}$$

$$G^E_{2,\theta} = \frac{\beta p'(\theta)\left(\rho+\delta_N+s\right)}{\left(\rho+\delta_N\right)\left(\rho+\delta_N+s+\beta p(\theta)\right)^2} f'(z_N) > 0, \quad \text{(A.6)}$$

$$G^E_{2,z_N} = \frac{\beta p(\theta)}{\left(\rho+\delta_N\right)\left(\rho+\delta_N+s+\beta p(\theta)\right)} f''(z_N) < 0, \quad \text{(A.7)}$$

The slope of the JCC and the IDC can then be calculated as:

$$\left.\frac{d\theta}{dz_N}\right|_{G_1^E(\cdot)=k} > 0, \quad \left.\frac{d\theta}{dz_N}\right|_{G_2^E(\cdot)=c} > 0.$$

Comparative statics

The first two derivatives are needed for the further analysis:

$$\frac{\partial\eta_U}{\partial\mu} = (1-\eta_U)\frac{1}{\delta_I}\frac{\delta_I+s}{\delta_I+s+p(\theta)} > 0,$$

$$\frac{\partial\eta_U}{\partial\theta} = p'(\theta)\eta_U(1-\eta_U)\left(\frac{1}{\delta_I+s+p(\theta)} - \frac{1}{\delta_N+s+p(\theta)}\right) > 0.$$

The derivatives of the JCC and the IDC in the competitive equilibrium look as follows:

$$G^E_{1\mu} = \frac{d\eta_U}{d\mu}(J_I - J_N)\begin{cases} > 0 & (J_I - J_N) > 0 \\ < 0 & (J_I - J_N) < 0 \end{cases},$$

$$G^E_{1z_I} = \eta_U(\theta)\frac{q(\theta)(1-\beta)f'(z_I)}{\rho + \delta_I + s + \beta p(\theta)} > 0.$$

Proof Proposition 1

Proof. We are in a stable equilibrium: $G^E_{1\theta}G^E_{2z_N} - G^E_{2\theta}G^E_{1z_N} > 0$:

$$\frac{d\theta^E}{d\mu} = -\frac{G^E_{1\mu}G^E_{2z_N}}{G^E_{1\theta}G^E_{2z_N} - G^E_{2\theta}G^E_{1z_N}}\begin{cases} > 0 & iff \quad (J_I - J_N) > 0 \\ < 0 & iff \quad (J_I - J_N) < 0 \end{cases},$$

$$\frac{dz^E_N}{d\mu} = -\frac{-G^E_{2\theta}G^E_{1\mu}}{G^E_{1\theta}G^E_{2z_N} - G^E_{2\theta}G^E_{1z_N}}\begin{cases} > 0 & iff \quad (J_I - J_N) > 0 \\ < 0 & iff \quad (J_I - J_N) < 0 \end{cases}.$$

$$\frac{d\theta^E}{dz_I} = -\frac{G^E_{1z_I}G^E_{2z_N}}{G^E_{1\theta}G^E_{2z_N} - G^E_{2\theta}G^E_{1z_N}} > 0,$$

$$\frac{dz^E_N}{dz_I} = -\frac{-G^E_{2\theta}G^E_{1z_I}}{G^E_{1\theta}G^E_{2z_N} - G^E_{2\theta}G^E_{1z_N}} > 0.$$

The number of unemployed workers changes as follows:

$$du = \left.\frac{\partial u}{\partial \theta}\right|_{\mu=0} d\theta + \left.\frac{\partial u}{\partial \mu}\right|_{\mu=0} d\mu,$$

$$du = -\frac{(s+\delta_N)\,p'(\theta)}{(s+\delta_N+p(\theta))^2}d\theta + \frac{1}{\delta_I}\frac{(s+\delta_I)}{(s+\delta_I+p(\theta))}d\mu.$$

□

Optimality Conditions

Given the following Hamiltonian:

$$H = e^{-\rho\tau}[y_N + bu_N - k\theta u_N - \delta_N c z_N] + \lambda_1\{\delta_N + s(1-u_N) - [p(\theta)+\delta_N]u_N\}$$
$$+\lambda_2\{p(\theta)u_N f(z_N) - (s+\delta_N)y_N\},$$

we get the following first order conditions:

$$\frac{\partial H}{\partial u_N} = e^{-\rho\tau}(b - k\theta) - \lambda_1[s + p(\theta) + \delta_N] + \lambda_2 p(\theta) f(z_N) + \dot{\lambda}_1 = 0, \tag{A.8}$$

$$\frac{\partial H}{\partial y_N} = e^{-\rho\tau} - \lambda_2(s+\delta_N) + \dot{\lambda}_2 = 0, \tag{A.9}$$

$$\frac{\partial H}{\partial \theta} = -e^{-\rho\tau}ku_N - \lambda_1 p'(\theta)u_N + \lambda_2 p'(\theta)u_N f(z_N) = 0, \tag{A.10}$$

$$\frac{\partial H}{\partial z_N} = -e^{-\rho\tau}\delta_N c + \lambda_2 p(\theta)u_N f'(z_N) = 0. \tag{A.11}$$

Solving the differential equation (A.9) and equating the solution at the steady state $\dot{\lambda}_2/\lambda_2 = -\rho$ we yield the steady state value of λ_2:

$$\lambda_2 = \frac{e^{-\rho\tau}}{s + \delta_N + \rho}.$$

Replacing λ_2 in (A.8), solving the differential equation and equating the solution at the steady state $\dot{\lambda}_1/\lambda_1 = -\rho$ gives:

$$\lambda_1 = \frac{e^{-\rho\tau}}{\rho + s + p(\theta) + \delta_N}\left((b - k\theta) + \frac{p(\theta) f(z_N)}{s + \delta_N + \rho}\right).$$

Using λ_2 in (A.11) and solving for c yields (25):

$$\frac{p(\theta) u_N}{\delta_N (s + \delta_N + \rho)} f'(z_N) = c.$$

Replacing $u_N$with the steady state value yields (24):

$$\frac{(\delta_N + s)\, p(\theta)}{\delta_N\,(s + \delta_N + p(\theta))\,(s + \delta_N + \rho)} f'(z_N) = c.$$

Using λ_1 and λ_2 in (A.10) and solving for k gives:

$$\frac{p'(\theta)\,(f(z_N) - b)}{\rho + s + \delta_N + p(\theta) - p'(\theta)\theta} = k.$$

Proof of Lemma 1

Proof. We need to compare the loci defined by (14) and (25). Assume that $z_N(\theta)|_{G_2^E = c} < z_N(\theta)|_{G_2^o = c}$ holds for any $\theta = \bar{\theta}$. This implies the following inequality:

$$\frac{\beta p(\bar{\theta})}{(\rho + \delta_N)\left(\rho + A + \beta p(\bar{\theta})\right)} < \frac{A p(\bar{\theta})}{\delta_N \left(p(\bar{\theta}) + A\right)(A + \rho)},$$

with $A \equiv \delta_N + s$. First note that if the inequality holds for $\beta = 1$ it will also hold for $\beta < 1$ because the LHS increases in β. Therefore we set $\beta = 1$ and check whether this is true or not. Reorganising terms yields:

$$\begin{aligned}\delta_N \left(p(\bar{\theta}) + A\right)(A+\rho) &< A(\rho+\delta_N)\left(\rho + A + p(\bar{\theta})\right), \\ \delta_N p(\bar{\theta}) &< A\rho + A^2 + Ap(\bar{\theta}).\end{aligned}$$

which by using the definition of A is true for any value of $\bar{\theta}$ and completes the proof. $\square$

Proof Lemma 2

Proof. Assume that $z_N^E = z_N^o$. First we consider an economy without immigration $\mu = 0$. Evaluating (13) and (24) at $z_N^E = z_N^o$ and comparing both terms yields:

$$\frac{(1-\beta)p(\theta^E)/\theta^E}{\rho + \delta_N + s + \beta p(\theta^E)} = \frac{p'(\theta^o)}{s + \delta_N + \rho + p(\theta^o) - \theta^o p'(\theta^o)}$$

$\theta^E = \theta^o$ holds if $\beta = \frac{p(\theta^E) - \theta^E p'(\theta^E)}{p(\theta^E)} = \theta^E \frac{q'(\theta^E)}{q(\theta^E)} \equiv \tilde{\beta}$. This is the well known Hosios-condition for an efficient bargaining power of workers (Hosios, 1990). Note that, because $G_{1,\theta}^E < 0$ and $G_{1,\beta}^E < 0$ we can conclude that for any $\beta \gtreqless \tilde{\beta}$ $\theta^E \lesseqgtr \theta^o$.

Next we are considering an economy with immigration. With immigration we can not analytically find an efficient $\tilde{\beta}$. However, an efficient $\tilde{\beta}$ solves the following equation:

$$\tilde{\beta} := \{\beta : q(\theta^o) J^e(\theta^o, z_N^o; \beta) = G_1^o(z_N^o, \theta^o)\}$$

Note, that we have to differentiate the two possible cases $J_I > J^e > J_N$ and $J_N > J^e > J_I$. Furthermore, because $G_{1,\beta}^E < 0$ and $G_{1,\theta}^E < 0$ with same line of reasoning as before we can conclude that for any $\beta \gtreqless \tilde{\beta}_I$ $\theta^E \lesseqgtr \theta^o$. $\square$

Proof Proposition 3

Proof. For the stable equilibrium $G_{1\theta}^E < 0$ and $G_{1\theta}^E G_{2z_N}^E - G_{2\theta}^E G_{1z_N}^E > 0$ hold:

$$\frac{d\theta^E}{dh} = \frac{G_{1z_N}^E}{G_{1\theta}^E G_{2z_N}^E - G_{2\theta}^E G_{1z_N}^E} > 0,$$
$$\frac{dz_N^E}{dh} = -\frac{G_{1\theta}^E}{G_{1\theta}^E G_{2z_N}^E - G_{2\theta}^E G_{1z_N}^E} > 0.$$

□

Proof Proposition 4

Proof. We will compare the JCC and the IDC in the competitive equilibrium with their counterparts of the social planner. Education subsidies shall be positive: $h^* > 0$ which can be shown by the following expression:

$$h^* = f'(z_N)\left[\frac{(\delta_N + s)\,p(\theta)}{\delta_N\,(\delta_N + s + \delta_N)\,(s + p(\theta) + \rho)} - \frac{\beta p(\theta)}{(\rho + \delta_N)\,(\rho + \delta_N + s + \beta p(\theta))}\right] > 0,$$

where the first expression is IDC of the social planner (25) and the second expression is the IDC of the competitive equilibrium (14).

The comparison of the JCC cannot not be made in the same way. We have to rely on the comparative static results for changes in $z_I$and μ. First, we consider a change in z_I: $\frac{d\theta^E}{dz_I}, \frac{dz_N^E}{dz_I} > 0$ irrespectively of the expected present value of a filled job. If $\beta \in [0, \beta^E]$, we have too high labour market tightness and therefore we need lower educational attainment of immigrants $z_I^* < z_I$. For a $\beta \in [\beta^E, 1]$, $z_I^* > z_I$.

Second we consider a change in the inflow of immigrants μ: if $J_I < J_N$ then $\frac{d\theta^E}{d\mu}, \frac{dz_N^E}{d\mu} < 0$. With a $\beta > \beta_I^o$, we have too small labour market tightness and we have to decrease the inflow of immigrants $\mu^* < \mu$. If $J_I > J_N$ then $\frac{d\theta^E}{d\mu}, \frac{dz_N^E}{d\mu} > 0$. With a $\beta < \beta_I^o$, we have too high labour market tightness and we have to decrease again the inflow of immigrants $\mu^* < \mu$. $\square$

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