Impact Analysis and Post-Impact Motion Control Issues of a Free-Floating Space Robot Subject to a Force Impulse

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***Abstract*— This article presents impact dynamic analysis of a free-floating space robot, subject to a force impulse at the hand. We study the joint and the base reactions in terms of finite velocity changes and clarify their role for the post-impact motion behavior of the robot. The analysis makes use of a joint-space orthogonal decomposition procedure involving the so called reaction null space. The article focuses on the specific case of a nonredundant arm and a reaction null space in terms of base angular motion. We further show that with proper post-impact control it is possible to transfer the whole angular momentum from the base toward the manipulator, and in the same time to reduce the joint velocity.**

***Index Terms*— Impact dynamic analysis, post-impact control, reaction null space, space robot.**

I. Introduction

THE importance of capturing operations of free-floating objects by a space robot, supported either by a flexible structure or by a satellite, can be expected to increase in the near future. A capturing operation comprises three specific phases: the pre-impact phase, the impact phase, and the post-impact phase. The pre-impact phase determines the initial conditions. During the impact phase, contact between the manipulator hand and the object is established, and a *force impulse*[1] is generated. The magnitude of this force impulse is estimated in a straightforward manner by applying the classical theory of dynamics of systems of rigid bodies [1]. Information about the pre-impact configuration, the mass and inertia properties, and the pre-impact relative velocity between the manipulator hand and the object is required. In addition, the post-impact velocities of the hand and the object must be also estimated. The objective is to keep the magnitude of the force impulse as small as possible. There are two main reasons for this objective. First, the impulse could damage either the manipulator, or the object. Second, the impulse would load the space robot with additional momentum. The angular component of the this momentum especially might be quite harmful. For a satellite-based space robot, this component may lead to attitude destabilization. Or, when the robot is mounted on a flexible supporting structure, high-amplitude vibrations could be induced. Additional control efforts during the post-impact phase will be then needed to stop the motion and/or to stabilize the robot. To avoid such situations, in practice, e.g., with the space shuttle remote manipulator system, the magnitude of the impact impulse is kept always very low, mainly by ensuring small pre-impact relative velocity.

Till now, the impact problem has been discussed mainly with regard to ground-fixed robots [2]–[6], focusing on the force impulse occurring at the point of contact. In case of a space robot, however, the analysis is more complicated due to the presence of the free-floating or the flexible-structure base dynamics, and the respective coupling effects. There are only few studies on this problem. The effect of impacts upon a flexible-link free-floating space robot has been discussed by Cyril *et al.* [7]. In other works mostly a rigid multi-body system notation is employed. Wee and Walker [8] tackled the problem of force impulse minimization through a configuration-dependent scalar function. The minimization is achieved by proper trajectory planning in configuration space, based on the gradient projection technique. We note, however, that the motion along a specified trajectory introduces an additional constraint into the system. The combination of this constraint with the impulse minimization task yields a highly nonlinear system equation. Thus, the gradient projection approach might easily arrive at a local minimum.

The impact phenomenon for free-floating space robots has been also studied by Yoshida *et al.* [9]. They introduced the *extended-inverse inertia tensor (Ex-IIT)* notation and developed a comprehensive framework for the impact dynamics with regard to the force impulse acting at the hand of the manipulator. This framework includes some means to conveniently express force impulse characteristics, such as the *impulse index* and the *impulse ellipsoid*. The authors stressed on the necessity of proper joint resistance models. They proposed the so-called *virtual rotor inertia* model [10], [11] and verified its efficiency through experiments [12]. Yoshikawa and Yamada derived a joint resistance model which includes servo stiffness and damping effects [13], [14] and analyzed it using frequency domain analysis. The concept has been also experimentally verified [15]. It is apparent

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[1] Under "force impulse" we understand a force-time product, i.e., momentum.

that various models of the joint resistance during the impact can be used. Careful examination of the contact behavior is needed to determine which model is appropriate. For example, Yoshikawa and Yamada [15] pointed out two boundary cases of contact behavior: when the object inertia is much smaller than the effective inertia at the hand, and vice versa. In the former case, the object bounces back immediately after the impact; this is undesirable because the object will be lost. In the latter case, on the contrary, the hand of the manipulator can be controlled to "stick" to the object without any significant rebouncing.

Yoshikawa and Yamada's approach is feasible for tasks when the relative velocity is controllable, and "soft" approach to the object is possible, i.e., approach with small relative velocity. We note, however, that when the object is tumbling, such soft approach might not exist. The object cannot be grasped successfully until its angular momentum decreases. Some ideas to handle this type of task are presented in [16]. Another possible solution was proposed in [17]. A device with controllable momentum wheels (a so-called "space leech") is to be attached to the tumbling object. The way of attachment, which is obviously related to the impact problem, has not been discussed, however.

The aim of the present study is twofold:

1) to provide some further insight into the impact phenomenon of a free-floating serial rigid-body system;
2) to propose control laws for the post-impact phase;

More specifically, we intent to investigate the *joint reaction* and the *base reaction* to the force impulse, both defined in terms of a finite change of the respective velocity. Both, impact analysis and post-impact control law design will be based on our previous work [18] making use of a specific manipulator joint space decomposition technique[2] [19]–[21].

The paper is organized as follows. In Section II we present preliminaries and the main notation. Section III discusses the orthogonalization of joint acceleration and joint velocity via the reaction null space concept. Section IV analyzes the behavior of the space robot during the impact phase. In Section V two basic control approaches for the post-impact phase are discussed and respective simulation study is performed. The conclusions are given finally in Section VI.

II. Preliminaries and Main Notation

A. Impact Scenario and Assumptions

We assume a serial rigid-link manipulator attached to a floating base, as shown in Fig. 1. Points of interest include the system centroid (denoted with suffix "g"), the base centroid (with suffix "b"), and a point at the manipulator hand where the impact occurs (with suffix "h"). The external force input considered is an impulse generated through collision with a tumbling object. A tumbling object renders pre-impact end-effector motion synchronization (i.e., "soft approach" to the object) impossible. The following pre-impact strategy is then envisioned:

[2] Called *fixed-attitude-restricted (FAR) path planning.*

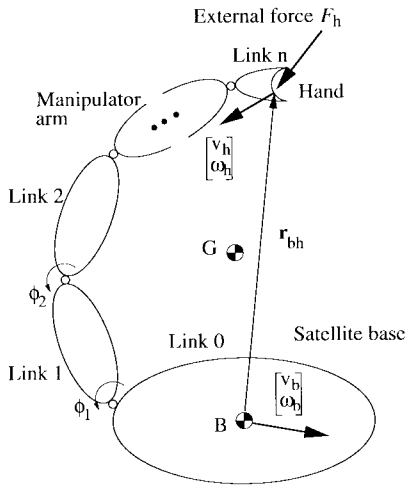


Fig. 1. General model of a space robot with external force input.

1) estimate the motion trajectory of the object through data from visual and/or other sensors;
2) determine the expected point of impact in inertial coordinates;
3) determine the respective pre-impact configuration of the robot, based upon the reasoning introduced below;
4) move to the point of impact with the desired pre-impact configuration and wait until an impact occurs.

Obviously, such a strategy implies a stationary initial (pre-impact) state of the robot, and conversely, zero initial momentum. We assume further that no external forces, other than the force impulse, act on the space robot. Hence, the momentum transferred to the robot during the impact will be conserved in the post-impact phase.

The main idea here is to use the dependence of the change of the two partial momenta, that of the base and the manipulator arm, upon the pre-impact configuration. With proper pre-impact configuration the change of base partial momentum can be minimized. This implies that a minimal part of the impact impulse will be transferred toward the satellite base. The partial momentum of the manipulator arm will be, however, maximized, yielding fast post-impact manipulator motion. This drawback can be remedied with a kinematically redundant arm, which would accommodate the same amount of momentum with less joint rate in the individual joints.

In this article, we focus on a nonredundant (in the conventional sense) manipulator. The impulse transmission minimization task described above can be redefined with respect to the change of base angular momentum only, thus yielding a sort of "artificial redundancy." We already pointed out that angular momentum is considered to be of greater importance than the linear one. In addition to this argument we note also that angular momentum conservation imposes a nonholonomic constraint for the space robot, which renders control more difficult. As far as the linear part of the momentum is concerned, we will implicitly assume that proper post-impact control is available through jet thrusters.

B. Conservation of Momenta During Pre-Impact and Post-Impact Phases

Generalized coordinates of the system under consideration are the manipulator joint variables $\boldsymbol{\phi} \in \Re^n$ along with six variables for position and attitude of the base with respect to the inertial frame. We assume that during the pre-impact and post-impact phases system momenta are conserved: only environmental forces (e.g., solar pressure, air drag, and microgravity) act. Such forces are orders of magnitude less than the driving forces acting at the manipulator joints, and hence, are negligeable. Further on, we choose a reference frame attached to the base centroid, and express vector quantities in this reference frame. The advantage is that expressions related to a fixed-base manipulator model will appear explicitly in the equations. The momenta will be then functions of both the base spatial velocity $\boldsymbol{\Omega} = [\boldsymbol{v}^T \boldsymbol{\omega}^T]^T \in \Re^6$ and the joint velocity $\dot{\boldsymbol{\phi}}$. Following a basic procedure (see [22, pp. 47–50]) one arrives at the following momentum equation [23], [24, pp. 172–175]

$$\begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{L}_g \end{bmatrix} = \boldsymbol{H}_b \boldsymbol{\Omega} + \boldsymbol{H}_{b\phi} \dot{\boldsymbol{\phi}} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{r}_{gb} \times \boldsymbol{P} \end{bmatrix} \tag{1}$$

where $\boldsymbol{P}$ and $\boldsymbol{L}_g$ denote momentum and central angular momentum, respectively, and $\boldsymbol{r}_{gb}(\boldsymbol{\phi})$ is the position of the base centroid with respect to the system centroid. Matrix

$$\boldsymbol{H}_b = \begin{bmatrix} \boldsymbol{H}_v & \boldsymbol{H}_{v\omega} \\ \boldsymbol{H}_{v\omega}^T & \boldsymbol{H}_\omega \end{bmatrix} \in \Re^{6\times 6}$$

is the base inertia matrix, $\boldsymbol{H}_{b\phi} = [\boldsymbol{H}_{v\phi}^T \ \boldsymbol{H}_{\omega\phi}^T]^T \in \Re^{6\times n}$ we call the *inertia coupling matrix*. The latter will be shown to play an important role in further derivations. We will assume that configurations rendering this matrix rank deficient will be avoided. Expressions for the submatrices can be found in [23] or [24, pp. 172–175]. We must note immediately that, in the general case when external forces are present, the two inertia matrices $\boldsymbol{H}_b$ and $\boldsymbol{H}_{b\phi}$ are functions of both joint variables and base attitude variables. Indeed, the inertial properties of the system with respect to an external force depend upon the manipulator configuration and the orientation of the system in inertial space. In our case, however, there are no external forces, and the above momenta are conserved. But on the other hand, the inertial properties with respect to internal forces (joint driving forces) do not depend upon position/orientation of the base in inertial space. Therefore, the above inertia matrices are functions of the joint variables only. Such functional independence can be also shown via a general argument from classical mechanics [25, pp. 125–129].[3]

It is possible to cancel out the linear part $\boldsymbol{P}$ and the base velocity $\boldsymbol{v}$, to obtain

$$\boldsymbol{L}_g = \tilde{\boldsymbol{H}}_\omega \boldsymbol{\omega} + \tilde{\boldsymbol{H}}_{\omega\phi} \dot{\boldsymbol{\phi}} \tag{2}$$

where $\tilde{\boldsymbol{H}}_\omega = \boldsymbol{H}_\omega + wS^2(\boldsymbol{r}_{gb})$ and $\tilde{\boldsymbol{H}}_{\omega\phi} = \boldsymbol{H}_{\omega\phi} + S(\boldsymbol{r}_{gb})\boldsymbol{H}_{v\phi}$. The notation $S(\circ)$ stands for the 3×3 skew-symmetric matrix of a three-dimensional vector and w denotes the total mass. When deriving the above equation, we used the relations $\boldsymbol{H}_v = w\boldsymbol{E}$, $\boldsymbol{E}$ denoting a unit matrix of proper dimension, $\boldsymbol{H}_{vw} = wS(\boldsymbol{r}_{gb})$, and $\boldsymbol{H}_{v\phi} = w(\partial \boldsymbol{r}_{gb}/\partial \boldsymbol{\phi})$. Each of the two components on the right-hand-side of (2) defines a *partial angular momentum* of the space robot. $\tilde{\boldsymbol{H}}_\omega \boldsymbol{\omega}$ will be called the *angular momentum of the base*. The other partial momentum, $\tilde{\boldsymbol{H}}_{\omega\phi}\dot{\boldsymbol{\phi}}$, is related to manipulator motion, and will be referred to as the *coupling angular momentum*, or shortly, the *coupling momentum*.

The derivation so far is valid for both pre-impact and post-impact phases. Their characteristics are distinguished via the initial states. We will assume that the initial state for the pre-impact phase is zero. Then, the momenta in (1) are conserved at zero. On the other hand, note that post-impact initial momenta cannot be assumed zero since the impact has changed the state.

C. The Impact Phase

The distinguishing characteristics of the impact phase is the presence of an external force. We will consider first a general form of the equation of motion.

Manipulator joint forces are generated by joint motors; these are internal forces and will be denoted as $\boldsymbol{\tau} \in \Re^n$. Forces generated during the contact of the manipulator hand with an object, are external. They will be denoted by $\mathcal{F}_h = [\boldsymbol{f}_h^T \boldsymbol{t}_h^T]^T \in \Re^6$. Further on, in general, there are forces acting directly on the satellite base: $\mathcal{F}_b = [\boldsymbol{f}_b^T \boldsymbol{t}_b^T]^T \in \Re^6$. Such forces can be regarded either internal, if generated by attitude control actuators such as reaction/momentum wheels, or external, if generated by jet thrusters.[4] The general form of the equation of motion is

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{b\phi} \\ \boldsymbol{H}_{b\phi}^T & \boldsymbol{H}_\phi \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{\Omega}} \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_b \\ \boldsymbol{c}_\phi \end{bmatrix} = \begin{bmatrix} \mathcal{F}_b \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \boldsymbol{R}_{bh}^T \\ \boldsymbol{J}_\phi^T \end{bmatrix} \mathcal{F}_h \tag{3}$$

where matrix

$$\boldsymbol{R}_{bh}^T = \begin{bmatrix} \boldsymbol{R}_{bhv}^T \\ \boldsymbol{R}_{bh\omega}^T \end{bmatrix} = \begin{bmatrix} \boldsymbol{E} & \boldsymbol{0} \\ S(\boldsymbol{r}_{bh}) & \boldsymbol{E} \end{bmatrix} \in \Re^{6\times 6} \quad \text{and}$$
$$\boldsymbol{c}_b = [\boldsymbol{c}_v^T \quad \boldsymbol{c}_\omega^T]^T$$

denotes nonlinear Coriolis and centrifugal forces acting at the base. The above representation makes use of quantities from the fixed-base manipulator: $\boldsymbol{J}_\phi \in \Re^{6\times n}$ and $\boldsymbol{H}_\phi \in \Re^{n\times n}$ denote the Jacobian matrix and the inertia matrix, respectively, while $\boldsymbol{c}_\phi$ represents the manipulator's Coriolis and centrifugal forces. We must emphasize that there is some abuse in the above notation: the inertia matrices $\boldsymbol{H}_b$ and $\boldsymbol{H}_{b\phi}$ depend here upon both base attitude and joint angle variables. But it can be easily verified that, when no forces are present, the upper part of the equation of motion is the differential of the momentum conservation equation (1), and there will be no dependency upon the base attitude variables, as already explained. We will show shortly hereafter that the same is valid also in the special case of impact.

[3] It is well known that whenever there is a constant momentum component, the respective generalized coordinate is *cyclic* or *ignorable*. This means that partial derivatives of the system Lagrangian with respect to this coordinate must be zero. In our case we can say that, due to the momentum conservation condition, the partial derivatives of the kinetic energy with respect to the base variables must be zero, and hence, all base variables are ignorable variables.

[4] Jet thrusters are capable of altering the total momentum of the space robot, and hence, cannot be considered to generate internal forces.

Further on, the following reduced form of the equation of motion is useful:

$$\hat{\boldsymbol{H}}_{\phi}\ddot{\boldsymbol{\phi}}+\hat{\boldsymbol{c}}=\hat{\boldsymbol{\tau}}+\hat{\boldsymbol{J}}_{\phi}^{T}\mathcal{F}_{h} \tag{4}$$

where $\hat{\boldsymbol{H}}_{\phi}=\boldsymbol{H}_{\phi}-\boldsymbol{H}_{b\phi}^{T}\boldsymbol{H}_{b}^{-1}\boldsymbol{H}_{b\phi}$, $\hat{\boldsymbol{c}}=\boldsymbol{c}_{\phi}-\boldsymbol{H}_{b\phi}^{T}\boldsymbol{H}_{b}^{-1}\boldsymbol{c}_{b}$, $\hat{\boldsymbol{\tau}}=\boldsymbol{\tau}-\boldsymbol{H}_{b\phi}^{T}\boldsymbol{H}_{b}^{-1}\mathcal{F}_{b}$ and $\hat{\boldsymbol{J}}_{\phi}^{T}=\boldsymbol{J}_{\phi}^{T}-\boldsymbol{H}_{b\phi}^{T}\boldsymbol{H}_{b}^{-1}\boldsymbol{R}_{bh}^{T}$. The last equation was obtained by canceling out base acceleration $\dot{\boldsymbol{\Omega}}$ from (3). Note that the reduced form can be considered as a generalization of the fixed-base manipulator equation of motion. Indeed, matrix $\hat{\boldsymbol{J}}_{\phi}$ has been called the generalized Jacobian of a free-flying space robot [26]. By analogy, we can refer to $\hat{\boldsymbol{H}}_{\phi}$ and $\hat{\boldsymbol{c}}$ as the *generalized inertia matrix* and the *generalized Coriolis and centrifugal force*, respectively.

Now we proceed with impact modeling. As usual in impact studies [1], [2] it is assumed that the time interval of the impact is infinitesimal: $\Delta t\rightarrow 0$. The nature of $\mathcal{F}_h$ is then that of an impulse force, with an infinite amplitude. The integral

$$\overline{\mathcal{F}}=\begin{bmatrix}\overline{\boldsymbol{f}}\\ \overline{\boldsymbol{t}}\end{bmatrix}=\lim_{\Delta t\rightarrow 0}\int_{t_i}^{t_i+\Delta t}\mathcal{F}_h dt \tag{5}$$

converges to a finite value; it represents the force impulse acting at the hand. Further on, when integrating (3) over an infinitesimally small time period, we can cancel velocity-dependent terms and internal forces, and replace all accelerations with respective finite changes of velocity. The change of any velocity will be denoted as $\Delta(\circ)$. From the equation of motion (3) we obtain then

$$\begin{bmatrix}\boldsymbol{H}_{b} & \boldsymbol{H}_{b\phi}\\ \boldsymbol{H}_{b\phi}^{T} & \boldsymbol{H}_{\phi}\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{\Omega}\\ \Delta\dot{\boldsymbol{\phi}}\end{bmatrix}=\begin{bmatrix}\boldsymbol{R}_{bh}^{T}\\ \boldsymbol{J}_{\phi}^{T}\end{bmatrix}\overline{\mathcal{F}}. \tag{6}$$

Two remarks are due. First, the last equation shows that during an impact with infinitesimal time duration, the manipulator joints are considered to be free. Second, since zero initial conditions in the pre-impact phase are assumed, we can fix the reference frame to the satellite base. Then, for the infinitesimal time interval Δt, the inertia matrices become functionally independent of the base variables, similarly to the case of momentum conservation. This justifies the use of identical notation for $\boldsymbol{H}_{b}$ and $\boldsymbol{H}_{b\phi}$ in the basic (1) and (6).

III. The Reaction Null Space

In our previous work [19], [20] we have introduced a zero-base-disturbance path planning technique for free-floating space robots which makes use of the fact that the inertia coupling matrix $\boldsymbol{H}_{b\phi}$ is not necessarily a square matrix. The null space of this matrix, denoted as $\aleph(\boldsymbol{H}_{b\phi})$, we called *the reaction null space of free-floating space robots* [18]. The reaction null space is useful in analysis, path planning and control of space robots, or more generally, of moving base robots [27]. We shall briefly introduce the concept below.

A. The Case of a Kinematically Redundant Manipulator

Kinematic redundancy is defined here with respect to the base variables. In other words, we require the number of joints n to be larger than six.[5] This provides a necessary condition for an underdetermined upper part of the equation of motion (3), with joint acceleration being the unknown variable. The general solution can be presented as

$$\ddot{\boldsymbol{\phi}}=\boldsymbol{H}_{b\phi}^{+}(\mathcal{F}_{b}+\boldsymbol{R}_{bh}^{T}\mathcal{F}_{h}-\boldsymbol{H}_{b}\dot{\boldsymbol{\Omega}}-\boldsymbol{c}_{b})+\boldsymbol{P}_{\mathrm{RNS}}\boldsymbol{\xi} \tag{7}$$

where $\boldsymbol{\xi}\in\Re^{n}$ is arbitrary, $\boldsymbol{H}_{b\phi}^{+}$ denotes the pseudoinverse of the inertia coupling matrix, and $\boldsymbol{P}_{\mathrm{RNS}}\equiv(\boldsymbol{E}-\boldsymbol{H}_{b\phi}^{+}\boldsymbol{H}_{b\phi})$ is a projector onto its null space. This solution shows that there is a set of joint accelerations $\boldsymbol{P}_{\mathrm{RNS}}\boldsymbol{\xi}$ that do not affect the base motion at all. The set belongs to the reaction null space $\aleph(\boldsymbol{H}_{b\phi})$. When the inertia coupling matrix $\boldsymbol{H}_{b\phi}$ is full rank ($\mathrm{rank}(\boldsymbol{H}_{b\phi})=6$), the reaction null space exists if and only if the manipulator arm is redundant in the above sense.

B. The Case of a Nonredundant Manipulator

Recall the note we made previously on the significance of satellite-base attitude motion as compared to satellite-base translational motion. We reformulate now the reaction null space with respect to base attitude only. First, eliminate the base acceleration $\dot{\boldsymbol{v}}$ from the equation of motion (3), to obtain

$$\begin{bmatrix}\tilde{\boldsymbol{H}}_{\omega} & \tilde{\boldsymbol{H}}_{\omega\phi}\\ \tilde{\boldsymbol{H}}_{\omega\phi}^{T} & \tilde{\boldsymbol{H}}_{\phi}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\omega}}\\ \ddot{\boldsymbol{\phi}}\end{bmatrix}+\begin{bmatrix}\tilde{\boldsymbol{c}}_{\omega}\\ \tilde{\boldsymbol{c}}_{\phi}\end{bmatrix}=\begin{bmatrix}\tilde{\boldsymbol{t}}_{b}\\ \tilde{\boldsymbol{\tau}}\end{bmatrix}+\begin{bmatrix}\tilde{\boldsymbol{R}}_{bh\omega}^{T}\\ \tilde{\boldsymbol{J}}_{\phi}^{T}\end{bmatrix}\mathcal{F}_{h}. \tag{8}$$

The tilde operator modifies the respective matrix in such a way that linear motion of the base is implicitly accounted for, see the expressions of $\tilde{\boldsymbol{H}}_{\omega}$ and $\tilde{\boldsymbol{H}}_{\omega\phi}$ appearing in (2). Also, $\tilde{\boldsymbol{R}}_{bh\omega}^{T}=\boldsymbol{R}_{gh\omega}^{T}$, where matrix $\boldsymbol{R}_{gh\omega}$ has the same structure as $\boldsymbol{R}_{bh\omega}$, with submatrix $S(\boldsymbol{r}_{bh})$ replaced by $S(\boldsymbol{r}_{gh})$. Matrix $\tilde{\boldsymbol{H}}_{\omega\phi}\in\Re^{3\times n}$ plays now the role of the inertia (base-attitude) coupling matrix. The set of joint accelerations derived from the upper part of the last equation, is

$$\ddot{\boldsymbol{\phi}}=\tilde{\boldsymbol{H}}_{\omega\phi}^{+}(\tilde{\boldsymbol{t}}_{b}+\boldsymbol{R}_{gh\omega}^{T}\mathcal{F}_{h}-\tilde{\boldsymbol{H}}_{\omega}\dot{\boldsymbol{\omega}}-\tilde{\boldsymbol{c}}_{\omega})+\boldsymbol{P}_{\mathrm{RNS}\omega}\boldsymbol{\xi} \tag{9}$$

where $\boldsymbol{P}_{\mathrm{RNS}\omega}$ stands for the projector onto the *angular reaction null space* $\aleph(\tilde{\boldsymbol{H}}_{\omega\phi})$. The redundancy condition will be now met with $n>3$.

The above derivations show that joint acceleration decomposition via the reaction null space is possible either with a kinematically redundant arm, with respect to the total base motion, or with a kinematically nonredundant arm, with respect to the angular base motion. It would be straightforward to formulate a similar framework for a kinematically nonredundant arm, with respect to the translational base motion. Without loosing generality, henceforth we assume a kinematically nonredundant manipulator, and focus on the angular motion component of the base.

C. Angular Momentum Decomposition

The (angular) reaction null space notation yields in fact a decomposition not only in terms of acceleration, but of the entire joint space. Consider the angular momentum of the robot as in (2). Due to the existence of the angular reaction null space, the joint velocity can be decomposed into two components: $\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}$ from the angular reaction null space, and

[5] Some further comments on redundancy of free-flying space robots are given in [20].

a component $\dot{\boldsymbol{\phi}}_{\perp}$, orthogonal to the previous one. Now, the angular momentum can be represented as

$$\boldsymbol{L}_g = \tilde{\boldsymbol{H}}_{\omega}\boldsymbol{\omega} + \tilde{\boldsymbol{H}}_{\omega\phi}\dot{\boldsymbol{\phi}}_{\perp} + \tilde{\boldsymbol{H}}_{\omega\phi}\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}. \tag{10}$$

Note that $\tilde{\boldsymbol{H}}_{\omega\phi}\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega} = \tilde{\boldsymbol{H}}_{\omega\phi}\boldsymbol{P}_{\mathrm{RNS}\omega}\boldsymbol{\xi} = \boldsymbol{0}$ for any $\boldsymbol{\xi}$, since $\tilde{\boldsymbol{H}}_{\omega\phi}\boldsymbol{P}_{\mathrm{RNS}\omega} = \boldsymbol{0}$. Thus we have proven the following

Proposition 1: A component of the joint velocity exists (the component $\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}$), which does not contribute to the angular momentum of the space robot.

Corollary 1: The minimum representation of the coupling momentum is $\tilde{\boldsymbol{H}}_{\omega\phi}\dot{\boldsymbol{\phi}}_{\perp}$.

IV. Analysis of the Space Robot During the Impact Phase

This analysis focuses on two main topics:

1) reactions of the system in terms of finite changes of the velocity;
2) change of angular momentum during the impact.

A. Impact Reactions of the Space Robot

We consider the following two reactions generated through the force impulse $\overline{\mathcal{F}}$ during the collision: the *joint reaction* $\Delta\dot{\boldsymbol{\phi}}$, and the *base reaction* $\Delta\boldsymbol{\Omega}$. These reaction can be uniquely determined from Equation (6).

1) Joint Reaction: The joint reaction is obtained by eliminating the base reaction from (6). Alternatively, from the following impact form representation of (4):

$$\hat{\boldsymbol{H}}_{\phi}\Delta\dot{\boldsymbol{\phi}} = \hat{\boldsymbol{J}}_{\phi}^{T}\overline{\mathcal{F}} \tag{11}$$

we obtain

$$\Delta\dot{\boldsymbol{\phi}} = \hat{\boldsymbol{H}}_{\phi}^{-1}\hat{\boldsymbol{J}}_{\phi}^{T}\overline{\mathcal{F}}. \tag{12}$$

Using the notation of angular reaction null space, the joint reaction can be decomposed into two orthogonal components

$$\Delta\dot{\boldsymbol{\phi}} = \Delta\dot{\boldsymbol{\phi}}_{\perp} + \Delta\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}$$

where

$$\Delta\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega} = \boldsymbol{P}_{\mathrm{RNS}\omega}\Delta\dot{\boldsymbol{\phi}}.$$

The angular reaction null space component $\Delta\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}$ is "invisible" in the upper part of (6). The "invisibility" does not necessarily mean that $\Delta\dot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}$ is zero. On the other hand, we note that the norm of the orthogonal component does not exceed the norm of the joint reaction. This means that the velocity which the manipulator gains due to the impact is not less than the velocity which contributes to the change of the robot's angular momentum.

2) Base Reactions: The total base reaction is obtained from (6) as

$$\Delta\boldsymbol{\Omega} = \boldsymbol{H}_{b}^{-1}(\boldsymbol{R}_{bh}^{T} - \boldsymbol{H}_{b\phi}\hat{\boldsymbol{H}}_{\phi}^{-1}\hat{\boldsymbol{J}}_{\phi}^{T})\overline{\mathcal{F}}. \tag{13}$$

Similarly, using the impact form of (8), we obtain the base angular reaction as

$$\Delta\boldsymbol{\omega} = \boldsymbol{B}\overline{\mathcal{F}} \tag{14}$$

where

$$\boldsymbol{B} \equiv \tilde{\boldsymbol{H}}_{\omega}^{-1}(\boldsymbol{R}_{gh\omega}^{T} - \tilde{\boldsymbol{H}}_{\omega\phi}\hat{\boldsymbol{H}}_{\phi}^{-1}\hat{\boldsymbol{J}}_{\phi}^{T}) \tag{15}$$

is a 3×6 matrix that transforms the force impulse acting at the hand, into base angular reaction in terms of a finite change of base angular velocity.

B. Change of Angular Momentum

The change of angular momentum of the robot during the impact is expressed as

$$\Delta\boldsymbol{L}_g \equiv \tilde{\boldsymbol{H}}_{\omega}\Delta\boldsymbol{\omega} + \tilde{\boldsymbol{H}}_{\omega\phi}\Delta\dot{\boldsymbol{\phi}}_{\perp} = \boldsymbol{R}_{gh\omega}^{T}\overline{\mathcal{F}}. \tag{16}$$

$\Delta\boldsymbol{L}_g$ comprises two components representing the change of the two partial momenta. Equation (16) plays an important role when analyzing the behavior during the impact phase, and also in view of the post-impact behavior of the space robot. The following two main cases can be distinguished.

$\mathbf{G}$ $\Delta\boldsymbol{L}_g \neq \boldsymbol{0}$ (impacts that change the angular momentum).
$\mathbf{C}$ $\Delta\boldsymbol{L}_g = \boldsymbol{0}$ (impacts that do not change it).

For a fixed impulse direction, one can invoke either $\mathbf{G}$ or $\mathbf{C}$ by selecting a proper pre-impact configuration, since $\boldsymbol{R}_{gh\omega}^{T}$ is configuration dependent.

1) Change of Partial Angular Momenta: Case $\mathbf{G}$ above will be referred to as the general case of impact. We can further distinguish the following subcases.

$\mathbf{G1}$ $\tilde{\boldsymbol{H}}_{\omega}\Delta\boldsymbol{\omega} \neq \boldsymbol{0}, \tilde{\boldsymbol{H}}_{\omega\phi}\Delta\dot{\boldsymbol{\phi}}_{\perp} \neq \boldsymbol{0} (\tilde{\boldsymbol{H}}_{\omega}\Delta\boldsymbol{\omega} \neq \tilde{\boldsymbol{H}}_{\omega\phi}\Delta\dot{\boldsymbol{\phi}}_{\perp})$.
$\mathbf{G2}$ $\tilde{\boldsymbol{H}}_{\omega}\Delta\boldsymbol{\omega} = \boldsymbol{0}, \tilde{\boldsymbol{H}}_{\omega\phi}\Delta\dot{\boldsymbol{\phi}}_{\perp} \neq \boldsymbol{0}$.
$\mathbf{G3}$ $\tilde{\boldsymbol{H}}_{\omega}\Delta\boldsymbol{\omega} \neq \boldsymbol{0}, \tilde{\boldsymbol{H}}_{\omega\phi}\Delta\dot{\boldsymbol{\phi}}_{\perp} = \boldsymbol{0}$.

Subcase $\mathbf{G1}$ is the most general one. It does not require *a priori* knowledge of the impulse direction. This subcase may most probably occur in practice. On the other hand, note that the special subcases $\mathbf{G2}$ and $\mathbf{G3}$, as well as case $\mathbf{C}$, may be invoked only if the the direction of the impulse is precisely known in advance. Such a knowledge, however, might be very difficult to obtain in practice. Nevertheless, we will pay attention to the cases here to give some additional insight into the problem.

Subcase $\mathbf{G2}$ can be easily recognized to imply zero base reaction. The impact momentum is accommodated entirely by the manipulator arm. Subcase $\mathbf{G3}$ is that of joint reaction from the angular reaction null space. The whole momentum will be then transferred to the base. As far as case $\mathbf{C}$ is concerned, we can distinguish the following two subcases.

$\mathbf{C1}$ $\tilde{\boldsymbol{H}}_{\omega}\Delta\boldsymbol{\omega} = -\tilde{\boldsymbol{H}}_{\omega\phi}\Delta\dot{\boldsymbol{\phi}}_{\perp} \neq \boldsymbol{0}$.
$\mathbf{C2}$ $\tilde{\boldsymbol{H}}_{\omega}\Delta\boldsymbol{\omega} = \tilde{\boldsymbol{H}}_{\omega\phi}\Delta\dot{\boldsymbol{\phi}}_{\perp} = \boldsymbol{0}$.

Subcase $\mathbf{C1}$ is important in light of simplifying post-impact control. Since the two partial momenta are of equal magnitude and opposite sign, it would be straightforward to obtain a pure translational motion of the robot after the impact, by simply stopping the motion of the manipulator. There will be no need of activating the attitude control system for this purpose.

Subcase $\mathbf{C2}$ yields the most favorable condition from our point of view: there will be no angular disturbance neither during the impact nor after it. It is easy to imagine the configuration when all link centroid and the base centroid are aligned, and the force impulse direction is along that line.

Obviously, the angular momentum of the space robot will not change; the whole momentum will be accommodated as translational momentum of the base.

2) Proper Initial Post-Impact Joint Velocity: An interesting problem is to find out the initial post-impact joint velocity which will ensure zero base attitude motion disturbance. This problem is related to the changes of partial momenta during the impact.

Proposition 2: Under momentum conservation, zero base-angular disturbance can be achieved if and only if the *joint velocity* is derived from the angular reaction null space.

Proof: First, assume that the joint velocity comes from the angular reaction null space. According to Proposition 1, this joint velocity has no influence on angular momentum. Hence, it cannot disturb the angular motion of the satellite base. Next, suppose there exists a joint velocity $\dot{\boldsymbol{\phi}}^*$ out of the angular reaction null space and which would not disturb the base. Since $\dot{\boldsymbol{\phi}}^*$ does not belong the angular reaction null space, it must be in its orthogonal complement, and hence $\dot{\boldsymbol{\phi}}^* = \tilde{\boldsymbol{H}}^+_{\omega\phi}\tilde{\boldsymbol{H}}_{\omega\phi}\boldsymbol{\eta}$ for some nonzero $\boldsymbol{\eta}$. The condition for zero base disturbance implies $\tilde{\boldsymbol{H}}_{\omega\phi}\dot{\boldsymbol{\phi}}^* = \mathbf{0}$. But $\tilde{\boldsymbol{H}}_{\omega\phi}\tilde{\boldsymbol{H}}^+_{\omega\phi}\tilde{\boldsymbol{H}}_{\omega\phi}\boldsymbol{\eta}$ cannot be zero with $\boldsymbol{\eta} \neq \mathbf{0}$. We arrived at a contradiction which is due to the wrong assumption. This proves the second part. □

Corollary 2: The necessary initial condition[6] for zero base-angular disturbance during the post-impact phase is

$$\Delta\dot{\boldsymbol{\phi}} \in \aleph(\tilde{\boldsymbol{H}}_{\omega\phi}). \tag{17}$$

From the corollary it is apparent that only in the particular impact subcases **G3** and **C2** the necessary initial condition will be met. With any other type of impact there will be an additional base disturbance due to the failure in meeting the condition. It is interesting to note that in case **G2**, although there is no base disturbance during the impact, immediately after the impact the base will be disturbed from the joint reaction which does not comply with condition (17).

C. Example

We can pick up an illustrative case study among a variety of space manipulator projects. Most of the manipulators comprise a distinctive lower/upper arm structure connected by a rotational "elbow" joint, thus forming a plane which we shall refer to as the manipulator plane. From the viewpoint of the present formulation, we can neglect the contribution of the other links of the system that do not constitute the manipulator plane. Fig. 2 depicts as an example the configuration of the ETS VII space robot [28], comprising a six-DOF manipulator mounted on a free-flying satellite base. We simplify the example by assuming that the shoulder yaw joint is fixed in a position such that the base centroid lies within the manipulator plane, and by ignoring the wrist. Hence, we consider only the planar subsystem consisting of the free-floating base and a two-link manipulator arm with two rotational joints: the shoulder and elbow pitch joints (cf., Fig. 3). Generally, this is a nonredundant case. With regard to the angular reaction

[6] Recall that zero pre-impact conditions have been assumed.

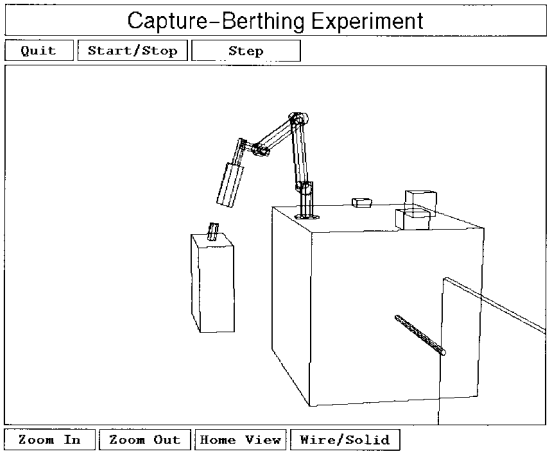


Fig. 2. Manipulator capturing and berthing operation with a floating sub-satellite in the Japanese ETS-VII mission.

perpendicular to the manipulator plane, however, the arm has one degree of redundancy, and the framework of the angular reaction null space can be applied with respect to the component of the force impulse acting in the manipulator plane. We should note also that the equation of motion of such a planar model is relatively simple, e.g., inertia submatrices being functionally independent of the base variables [24, pp. 202–204].

The parameters of the planar model are shown in Table I.[7] The target object is represented by a point mass of 100 kg. It approaches the space robot always with the same constant velocity of 1 m/s. The impact impulse is calculated under the condition of pure elastic impact.

We examine the changes of the angular momentum and its components, as well as the norm of the joint reaction and its components, as a function of the impulse direction. First, the initial configuration is selected as (−80°, 160°) [cf., Fig. 4(a)]. The results are shown in Fig. 4(b) and (c), respectively. From Fig. 4(b) it is apparent that there is a preferable impulse direction (a line with a slope of about 30°) such that all momenta changes are relatively small. This direction points approximately to the base centroid and robot centroid. From the figure one can clearly distinguish the subcases **G2**, **G3**, and **C1** as the zero line crossings. As far as subcase **C2** is concerned, it turns out that for this robot configuration no impulse direction invokes the case. The reason is that the base and robot centroids cannot be aligned. This can be done with the configuration shown in Fig. 5(a). Fig. 5(b) and (c) display the respective data. The three curves in Fig. 5(b) cross the zero line now exactly at the same point.

An interesting observation from both cases studied is that for most impulse angles the null space component of the joint reaction norm is prevailing. This will be shown to play an important role for deriving a specific post-impact motion control strategy for base attitude motion damping.

V. Post-Impact Motion Control Issues

The objective of this section is to propose two basic control strategies and respective control laws that would guarantee

[7] The parameters are different from those of ETS VII.

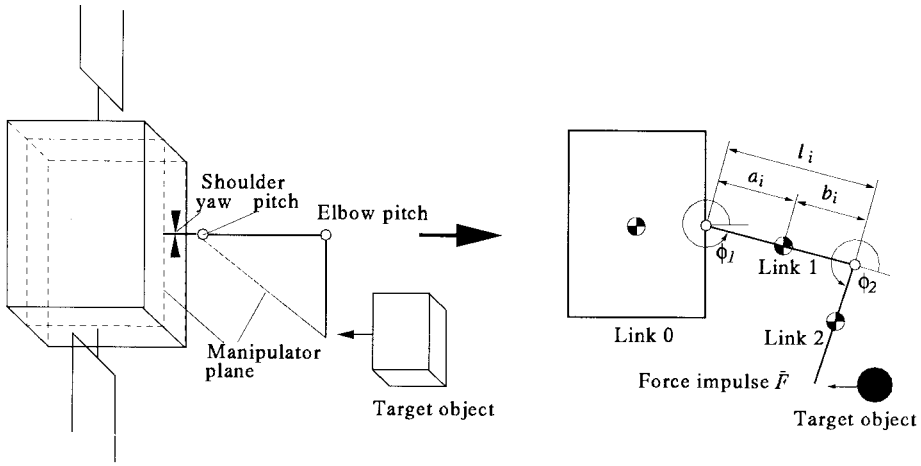


Fig. 3. Derivation of the planar simulation model.

TABLE I
MODEL PARAMETERS

	mass [kg]	length [m]			inertia [kgm^2]
	m_i	l_i	a_i	b_i	I_i
base	1000.0	2.6	1.2	1.4	563.3
link 1	100.0	2.0	1.0	1.0	33.3
link 2	100.0	2.0	1.0	1.0	33.3

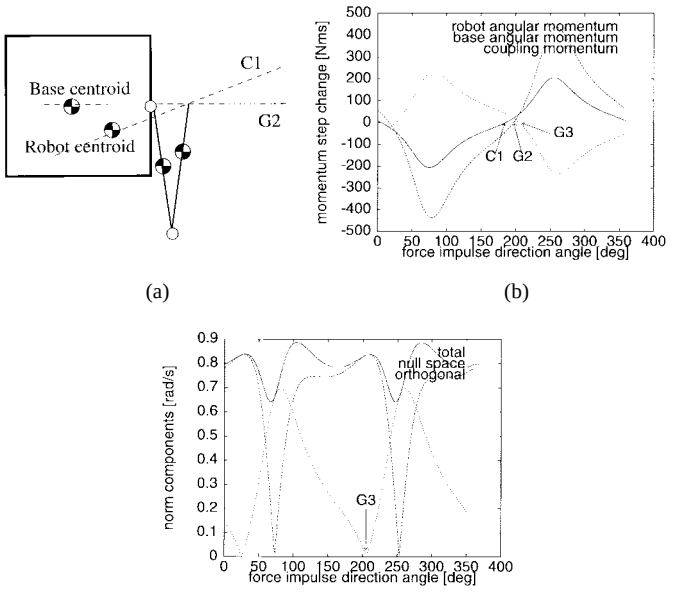


Fig. 4. Impact analysis data demonstrating impact case G and subcase $C1$: (a) pre-impact configuration, (b) step changes of the momenta, and (c) joint reaction norm and its components.

stable and smooth motion in the post-impact phase.

The distinguishing feature of the post-impact phase is momentum conservation, with nonzero initial condition. We will discuss here only the most general case of impact (case $\mathbf{G1}$) which, as explained, is practically meaningful. The initial condition is then such that both partial momenta, the coupling momentum and the base angular momentum, have different directions and magnitudes. Since we focus on coordination between manipulator motion and base angular motion, we will not discuss here the control of base motion through additional actuators such as reaction wheels or jet thrusters, i.e., assume $\mathcal{F}_b = \mathbf{0}$.

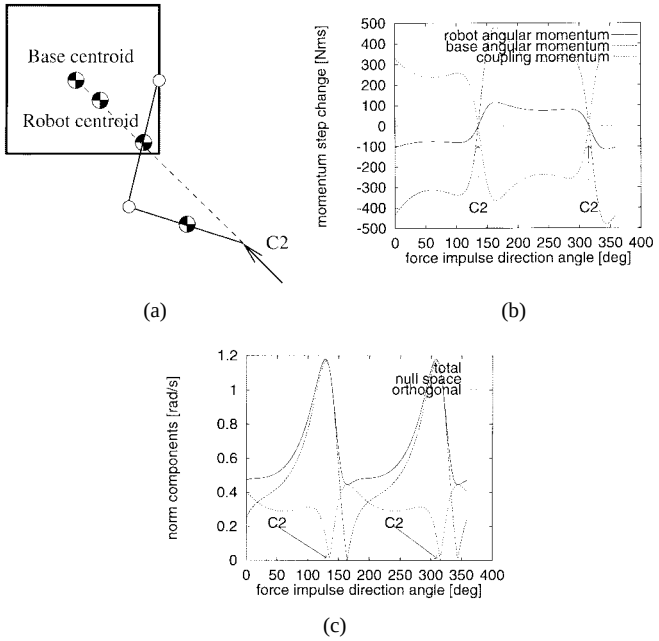


Fig. 5. Impact analysis data demonstrating impact subcase $C2$: (a) pre-impact configuration, (b) step changes of the momenta, and (c) joint reaction norm and its components.

A. The Manipulator Joint Damping Control Strategy

Under the post-impact phase condition $\mathcal{F}_h = \mathcal{F}_b = \mathbf{0}$, from the equation of motion (4), we obtain

$$\hat{\boldsymbol{H}}_\phi \ddot{\boldsymbol{\phi}} + \hat{\boldsymbol{c}} = \boldsymbol{\tau}. \tag{18}$$

Equation (18) shows that there is a possibility for manipulator control, which would not directly account for the base motion. The simplest (linearizing) feedback control in this sense is joint damping control

$$\boldsymbol{\tau}_{jd} = -\boldsymbol{K}_\phi \dot{\boldsymbol{\phi}} + \hat{\boldsymbol{c}} \tag{19}$$

where $\boldsymbol{K}_\phi$ denotes a gain matrix and $\dot{\boldsymbol{\phi}}$ is joint velocity derived from sensor data. The closed-loop system becomes $\ddot{\boldsymbol{\phi}} + \hat{\boldsymbol{H}}_\phi^{-1}\boldsymbol{K}_\phi\dot{\boldsymbol{\phi}} = \boldsymbol{0}$ and with a proper choice of the gain, stable damping can be achieved.

Two main results of joint damping control can be pointed out. First, the manipulator motion stops. Second, the partial base angular momentum becomes equal to the angular momentum of the space robot. The second result is due to the conservation of the angular momentum of the space robot during the post-impact phase. In fact, the effect of joint damping control can be achieved solely by the existing friction in the joints, leaving the joints unactuated after the impact.

B. Reaction Null Space Control

Joint damping control is an effective and simple strategy to transfer the whole impact momentum toward the base. This might be, however, not always desirable. Here we propose an alternative control approach that transfers the whole impact momentum toward the manipulator. Note that this will not necessarily imply increasing the joint velocity norm.

To begin with the derivation, first we note that from the equation of motion (8) it is possible to derive an alternative expression to (18) for the joint torque. The upper part of (8) (with $\mathcal{F}_h = \mathcal{F}_b = \boldsymbol{0}$) is

$$\tilde{\boldsymbol{H}}_\omega\dot{\boldsymbol{\omega}} + \tilde{\boldsymbol{H}}_{\omega\phi}\ddot{\boldsymbol{\phi}} + \tilde{\boldsymbol{c}}_\omega = \boldsymbol{0}. \tag{20}$$

Solve for $\ddot{\boldsymbol{\phi}}$ and substitute the solution into the lower part of (8), to obtain:

$$\overline{\boldsymbol{H}}\dot{\boldsymbol{\omega}} + \bar{\boldsymbol{c}} + \tilde{\boldsymbol{H}}_\phi\ddot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega} = \boldsymbol{\tau} \tag{21}$$

where $\ddot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}$ denotes joint acceleration from the angular reaction null space, $\overline{\boldsymbol{H}} = (\tilde{\boldsymbol{H}}_{\omega\phi}^T - \tilde{\boldsymbol{H}}_\phi\tilde{\boldsymbol{H}}_{\omega\phi}^+\tilde{\boldsymbol{H}}_\omega)$ and $\bar{\boldsymbol{c}} = \tilde{\boldsymbol{c}}_\phi - \tilde{\boldsymbol{H}}_\phi\tilde{\boldsymbol{H}}_{\omega\phi}^+\tilde{\boldsymbol{c}}_\omega$. Equation (21) shows that via joint torque control input two control tasks can be performed *simultaneously*: a satellite base control task (through the joint torque component $\overline{\boldsymbol{H}}\dot{\boldsymbol{\omega}}$) and a manipulator control task within the angular reaction null space (through the component $\tilde{\boldsymbol{H}}_\phi\ddot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega}$). The main idea is to use the former component for transferring the base angular momentum toward the manipulator arm and the latter component for reducing the joint velocity. These two tasks may seem contradictory at a first glance. Recall, however, the impact analysis which has shown that, in most cases, the reaction null space component of the joint velocity has a much larger norm than the orthogonal component. Since the reaction null space component does not contribute to momentum, it would be desirable to reduce it to zero. Of course, this will not influence the momentum distribution whatsoever, or in other words, the base attitude control subtask will be not disturbed. In fact, the above strategy shows that *total decoupling* of the manipulator dynamics from the base attitude dynamics can be achieved. We refer to this strategy as *reaction null space control*.

To reduce the joint velocity without affecting the transfer of momentum, we propose a manipulator joint damping control technique within the angular reaction null space. Define the reaction null space component as

$$\ddot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega} = -\boldsymbol{P}_{\mathrm{RNS}\omega}\boldsymbol{K}_\phi\dot{\boldsymbol{\phi}} \tag{22}$$

and the base attitude component as

$$\dot{\boldsymbol{\omega}} = -\boldsymbol{K}_\omega\boldsymbol{\omega} \tag{23}$$

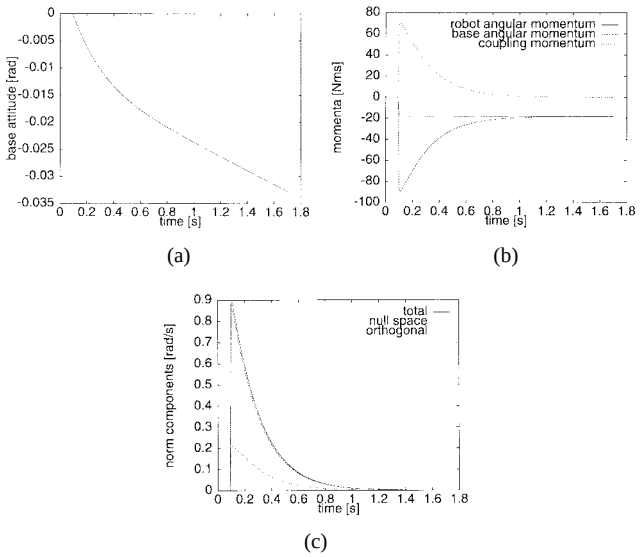
where $\boldsymbol{K}_\omega$ is a gain matrix for base attitude motion damping, and $\boldsymbol{\omega}$ is measured base attitude velocity. The control torque can be now written as

$$\boldsymbol{\tau}_{ns} = -\overline{\boldsymbol{H}}\boldsymbol{K}_\omega\boldsymbol{\omega} + \bar{\boldsymbol{c}} - \tilde{\boldsymbol{H}}_\phi\boldsymbol{P}_{\mathrm{RNS}\omega}\boldsymbol{K}_\phi\dot{\boldsymbol{\phi}}. \tag{24}$$

The closed-loop equation becomes

$$\overline{\boldsymbol{H}}(\dot{\boldsymbol{\omega}} + \boldsymbol{K}_\omega\boldsymbol{\omega}) + \tilde{\boldsymbol{H}}_\phi(\ddot{\boldsymbol{\phi}}_{\mathrm{RNS}\omega} + \boldsymbol{P}_{\mathrm{RNS}\omega}\boldsymbol{K}_\phi\dot{\boldsymbol{\phi}}) = \boldsymbol{0}. \tag{25}$$

This represents a superposition of two decoupled dynamical subsystems. Since $\overline{\boldsymbol{H}}$ and $\tilde{\boldsymbol{H}}_\phi$ are positive-definite (recall that we assumed matrices $\boldsymbol{H}_{b\phi}, \boldsymbol{H}_{\omega\phi}$ to be full rank), with proper choice of the two gains $\boldsymbol{K}_\omega$ and $\boldsymbol{K}_\phi$, we can achieve stable base attitude and (reaction null space) joint velocity damping.



(a) (b) (c)

Fig. 6. Post-impact damping control: (a) base attitude, (b) transfer of momentum, and (c) joint velocity norm and its components.

C. Illustrative Computer Simulation Study

We use the same model as above, with the pre-impact configuration ($-80°$, $160°$). We choose a relatively unfavorable impact direction, that of $170°$ [cf., Fig. 4(b)]. The impact occurs at $t = 0.1$ s. In all simulations a 4th-order Runge–Kutta integration method with 0.01 s time interval is used. The gains are set to $K_\omega = 7\ \mathrm{s}^{-1}$ and $\boldsymbol{K}_\phi = \mathrm{diag}[5\,5]\ \mathrm{s}^{-1}$.

Three cases are studied. First, Fig. 6 shows the joint damping control case. It is clearly seen that the momentum is transferred toward the base [Fig. 6(b)] and the joint motion stops [Fig. 6(c)]. Second, Fig. 7 shows the results from reaction null space control without using the null space component, however. The momentum is transferred from the base successfully [Fig. 7(b)], the manipulator moves however relatively fast [Fig. 7(c)]. Finally, Fig. 8 displays the results of reaction null space control, but this time joint damping in the reaction

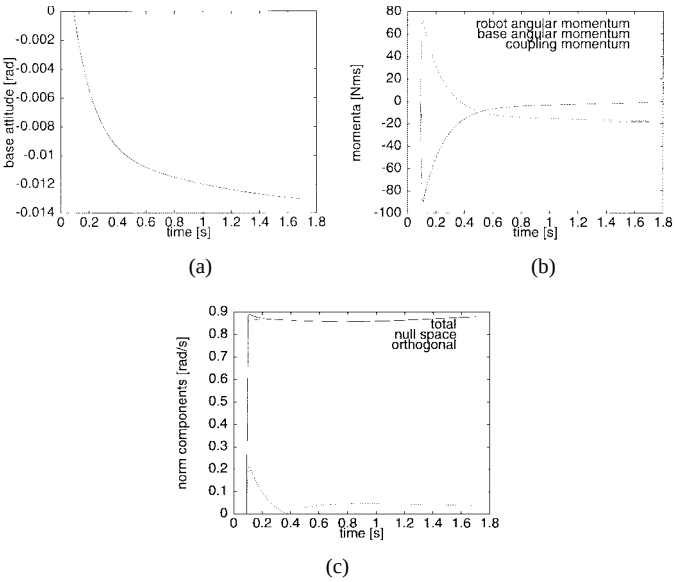


Fig. 7. Post-impact reaction null space control without joint damping: (a) base attitude, (b) transfer of momentum, and (c) joint velocity norm and its components.

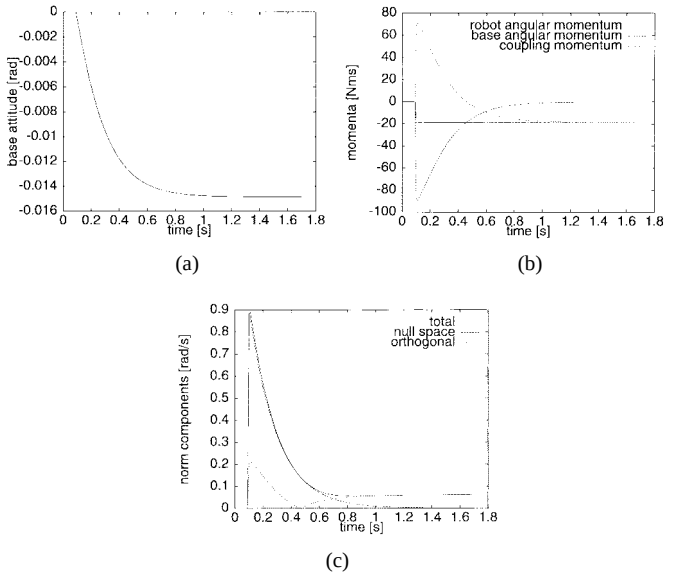


Fig. 8. Post-impact reaction null space control with joint damping: (a) base attitude, (b) transfer of momentum, and (c) joint velocity norm and its components.

null space is applied in addition to momentum transfer control. The momentum transfer data is the same as in the previous case [Fig. 8(b)], but the joint velocity decreased very much [Fig. 8(c)], in comparison to the case without null space damping. It is clearly seen that with this control approach the angular motion of the base can be effectively stopped in a relatively short time after the impact. The impact momentum, conserved now in manipulator motion with the lowest possible velocity (which is due to the reaction null space damping strategy), can be *gradually* re-transfered to the base to be then managed by the additional base actuators. The reason we consider such a strategy useful is that the dynamics of the manipulator allow fast and in the same time high precision control of angular momentum transfer, something which can be difficult to be achieved if only the base actuators are used for momentum management.

VI. Conclusion

In this paper, we presented the results from the analysis of the impact phenomenon for a free-floating serial rigid-body chain. We especially focused on the study of the joint reaction and the base reaction, and the change of the respective partial momenta of the space robot. It has been shown that preferable directions of the impulsive force exist, such that impact momentum transfer toward the base can be minimized. An interesting result we obtained is that for specific force impulse directions, there is a set of pre-impact configurations, such that the impact will not change the angular momentum of the space robot. In this case, the two partial momenta of the system are of equal magnitude and opposite sign. This has an important consequence: in the post-impact phase it suffices to apply just joint damping control, which stops the angular motion of the whole system. Another particular case are impacts that do not change neither the angular momentum of the robot, nor its two partial momenta. As a consequence, the satellite base attitude is not disturbed at all, neither during the impact, nor in the post-impact phase, when manipulator control is applied. This situation has been related to the existence of the angular reaction null space. These cases are mainly of theoretical interest. In practice, it would be difficult to achieve such favorable distribution of the partial momenta.

We introduced a manipulator control law for the post-impact phase, that is based on the concept of reaction null space. The advantage of this control is that it decouples entirely the manipulator dynamics from the satellite attitude dynamics. We have shown that it is possible to transfer angular momentum from the base toward the manipulator, and in the same time, to decrease the joint velocity. This control strategy is important in view of possible time-critical or emergency situations that might occur immediately after the impact, when the attitude control system would not be able to respond in an appropriate way.

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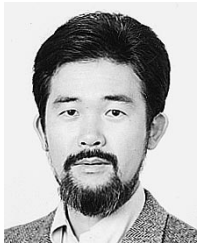

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