Impact of Mobility on the Performance of Relaying in Ad Hoc Networks

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Abstract— **We consider a mobile ad hoc network consisting of three types of nodes: source, destination, and relay nodes. All the nodes are moving over a bounded region with possibly different mobility patterns. We introduce and study the notion of** *relay throughput* **, i.e. the maximum rate at which a node can relay data from the source to the destination. Our findings include the results that the relay throughput depends on the node mobility pattern only via its (stationary) node position distribution and that a node mobility pattern that results in a uniform steady-state distribution for all nodes achieves the lowest relay throughput. Random Waypoint and Random Direction mobility models in both one and in two dimensions are studied and approximate simple expressions for the relay throughput are provided. Finally, the behavior of the relay buffer occupancy is examined for the one-dimensional Random Walk, and an explicit form of its mean value is provided in the heavy-traffic case.**

Keywords— **Perfromance evaluation, Packet relaying, Mobility models, MANET, Ad Hoc networks.**

I. Introduction

Grossglauser and Tse [6] observed that mobility in mobile ad hoc networks (MANET) can be used to increase the average network throughput. Their idea was to look at the diversity gain achieved by using the other mobile nodes as relays. Their relay mechanism is simple: if there is no route between the source node (s) and the destination node (d), the source node transmits its packets to one of its neighboring nodes (say, r) for delivery to the node d. It was then shown in [3] that a bounded delay can be guaranteed under this relaying mechanism. The aim of these studies (see also [7]) is the scaling property of the throughput or delay as the number of nodes in the network becomes large. Our interest in the present work is in the performance of the above mentioned relaying mechanism in a network consisting of a fixed finite number of nodes.

It is important to mention that most of the studies of scaling laws of delay and throughput in wireless ad hoc networks assume a uniform spatial distribution of nodes, which is the case, for example, when the nodes perform a symmetric Random Walk over the region of interest [3], [6]. In the present paper, we study the effect of the node mobility pattern on the throughput and delay performance of the relaying scheme of [6]. We are interested in the *maximum relay throughput* of a mobile node, i.e., the maximum that a node can contribute as a relay to the communication between two other nodes. The relaying of data for other nodes requires a relay node to allocate its own resources. In particular, a relay node has to keep the data to be relayed in its buffer. Hence, the study of the buffer behavior of a relay node forms an important topic of research. The present work addresses the above two issues, i.e., the maximum relay throughput and the relay node buffer behavior.

Our point of departure is a simple observation which relates the evolution of a relay node buffer to the evolution of the workload process in a G/G/1 queueing system. The service requirements and inter-arrival times in this queueing system are determined by the characteristics of the mobility pattern of the nodes.

Our main findings are the following:

1) The relay throughput depends only on the stationary distribution of the nodes' position. Hence, any two mobility patterns that have the same stationary distribution will achieve the same relay throughput.
2) It is assumed in [6] that the stationary distribution of a node position is uniform over the region of interest. This has lead to many research efforts which base their work on this particular assumption [3], [7]. We prove that the relay throughput achieved is the lowest when nodes are uniformly distributed.
3) Knowledge of the stationary node location distribution alone is not enough to understand the behavior of relay node buffer. A detailed analysis involving second-order moments of contact times between mobile nodes is necessary to obtain a full picture. We perform such an analysis for the random walk mobility model over a circle where a node can move, by a constant step size, to the right or to the left with equal probability.

An important point that needs to be emphasized is that, unlike [3], [6], [7] which study the system performance when the number of nodes is large, we are interested in a relay node performance while it is involved in relaying data between two particular nodes. Developing models for performance analysis of a relay node buffer and the relay throughput can help in dimensioning a relay node buffer size and on achieving an optimal performance using relaying mechanisms. We note that the model studied in this paper is not restricted to three nodes, nor that the model requires the same mobility pattern for all of the nodes.

The rest of the paper is organized as follows: Section II describes the relaying system considered. In Section III we develop a queueing model for the *relay buffer* (RB). Section IV studies the effect of mobility models on the relaying throughput, and in Section V we find expressions for the relay throughput for the Random Waypoint and the Random Direction models in both one and in two dimensions. Section VI studies the RB behavior for the random walk mobility model. In Section VII, we report numerical results on the stability, relay throughput, contact time distribution, probability of a 2-hop route, and the RB behavior. Section VIII concludes the paper and gives research directions.

II. The System Model

To study the maximum rate at which a node can relay data, we start by considering the scenario where three nodes move in a two-dimensional bounded region. One of these nodes is the source of packets, one is the destination, and the third one is the relaying node. The mobility patterns of the three nodes are independent and may be different from each other; this is in contrast with [3], [6] where the authors assume that the mobility pattern of the nodes is such that the steady-state distribution of the location of all the nodes is uniform over the region of interest. In fact, [3] assumes that nodes perform random walks (there are other mobility models which also result in a uniform stationary distribution, e.g., the Random Direction model [11]). As mentioned earlier, we are interested in the *maximum relay throughput* of a relay node. As a starting point we will restrict ourselves to the case where there is only one relay node. At a later stage we will relax this assumption. Also, we want to study the dependence of the relay node buffer behavior on the mobility model. We assume that a node detects its one-hop neighbor(s) by sending periodically Hello messages. However, to detects two-hop neighbors nodes exchange the addresses of tier neighbors.

The model is the following:

1) The three nodes move independently of each other according to a (possibly node-dependent) mobility model inside a bounded 2-dimensional region.
2) The source node has always data to send to the destination node. This is a standard assumption, also made in [3], [6], [7], because we are interested in the maximum relay throughput of the relay node.
3) When the relay node comes within the transmission range of the source node (we will also say that nodes are *in contact* in this case), and if the destination node is outside the transmission range of the source and of the relay node, then the relay node accrues packets to be relayed to the destination node at a constant rate r_s. [We could allow for a stochastic nature of traffic generated by the source by assuming that r_s is an independent stochastic process. However, such a study is out of the scope of this work.]
4) When the destination node comes within the transmission range of the relay node, and if the destination and the relay node are outside transmission range of the source node, then the relay node sends the relay packets (if any) to the destination node at a constant rate r_d.
5) If the relay node is within transmission range of *both* the source node and the destination node, then the relay node does not contribute to *relaying*. In this case there is either a direct communication between the source and destination or there is a two-hop route via the relay node so that the relay node acts as a forwarding node and not as a relay.

Our objective is to study the properties of the relay buffer (stability, stationary occupancy distribution, throughput). To this end, we first develop a queueing model that will give many insights into the system behavior.

III. A Queueing Model For the Relay Buffer

After addressing the case where there are only three mobile nodes in Section III-A, we investigate the situation of an arbitrary number of source/destination/relay nodes, under the additional assumption that all source and destination nodes are fixed (cf. Section III-B).

A. Single Source, Destination, and Relay Nodes

The state of the relay node at time t is represented by the random variable (r.v.) $S_t \in \{-1, 0, 1\}$ where:

- $S_t = 1$ if at time t the relay node is a neighbor (i.e., within transmission range) of the source, and if the destination is neither a neighbor of the source nor of the relay node. In other words, when $S_t = 1$, the source node sends relay packets to the relay node at time t;
- $S_t = -1$ if at time t the relay node is a neighbor of the destination, and if the source is neither a neighbor of the destination nor of the relay node. When $S_t = -1$ the relay node delivers relay packets (if any) to the destination;
- $S_t = 0$ otherwise.

Mobiles have finite speeds. We will assume that the relay node may only enter state 1 (resp. -1) from state 0: if $S_{t-} \neq S_t$ then necessarily $S_t = 0$ if $S_{t-} = 1$ or $S_{t-} = -1$.

Denote by B_t the RB occupancy at time t. The r.v. B_t evolves as follows:

- it increases at rate r_s if $S_t = 1$. This is because when $S_t = 1$, the relay node receives data to be relayed from the source node at rate r_s;
- it decreases at rate r_d if $S_t = -1$ and if the RB is non-empty. This is because if $S_t = -1$, and if there is any data to be relayed, then the relay node sends data to the destination node at rate r_d.
- it remains unchanged in all other cases.

Let $\{Z_n\}_n$ ($Z_1 < Z_2 < \cdots$) denote the consecutive jump times of the process $\{S_t, t \geq 0\}$. An instance of the evolution of S_t and B_t as a function of t is displayed in Figure 1

The evolution of the discrete indexed process $\{S_{Z_k}, k \geq 1\}$ consists of sequences of 1, 0 and -1. This naturally motivates us to look at the times when the relay node returns to the source node after being neighbor of the destination node at least once. This is done in the following.

We define a *cycle* as the interval of time that starts at $t = Z_k$, for some k with $S_t = 1$, and (necessarily) $S_{t-} = 0$ and $S_{Z_{k-2}} = -1$, and ends at the smallest time $t + \tau$ such that

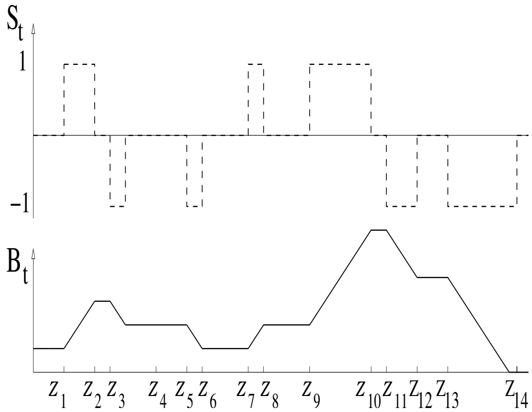

Fig. 1. Evolution of $\{S_t\}_t$ and relay node buffer occupancy.

$S_{t+\tau} = 1$ and $S_{t+s} = -1$ for some $s < \tau$. In Figure 1, the time-interval $[Z_7, Z_{17})$ constitutes a cycle. Note that there is no restriction on the number of times the relay node becomes neighbor of the source node or of the destination node during a cycle. Hence, during a cycle the relay node will transmit packets to the destination and will receive packets from the source.

Let W_n be the time at which the n^{th} cycle begins. Let

$$\sigma_n \triangleq \int_{t=W_n}^{W_{n+1}} \mathbf{1}_{\{S_t=1\}} dt \tag{1}$$

be the amount of time spent by the relay node in state 1 during the n^{th} cycle. Similarly, let

$$\alpha_n \triangleq \int_{t=W_n}^{W_{n+1}} \mathbf{1}_{\{S_t=-1\}} dt \tag{2}$$

be the amount of time spent by the relay node in state -1 during the n^{th} cycle. Observe that during the amount of time σ_n, the RB increases at rate r_s, and it is decreases at rate r_d during the amount of time α_n. Let $\tilde{B}_n$ be the RB occupancy at the beginning of the n^{th} cycle. Clearly,

$$\tilde{B}_{n+1} = [\tilde{B}_n + r_s\sigma_n - r_d\alpha_n]^+ \tag{3}$$

where $[x]^+ = \max(x, 0)$. In other words, $\tilde{B}_{n+1}$ can be interpreted as the workload seen by the $(n+1)^{st}$ arrival in a G/G/1 queue, where $r_s\sigma_n$ is the service requirement of the n^{th} customer, and $r_d\alpha_n$ is the inter-arrival time between the n^{th} and the $(n+1)^{st}$ customer. This interpretation will be used next.

Assumption A: Throughout Section III-A we assume that the sequence $\{C_n, \sigma_n, \alpha_n\}_n$ is stationary and ergodic, with $0 < E[C_n] < \infty$, $0 < E[\sigma_n] < \infty$ and $0 < E[\alpha_n] < \infty$.

Clearly, the statistical properties of the random variables C_n, σ_n, and α_n will depend on the node mobility patterns. Hence, our study will be restricted to the class of mobility models under which the stationarity and ergodicity assumptions hold for the sequence $\{C_n, \sigma_n, \alpha_n\}_n$.

Definition: The long-term fraction of time the RB receives data is

$$\pi_s \triangleq \lim_{t\to\infty} \frac{1}{t} \int_{u=0}^{t} \mathbf{1}_{\{S_u=1\}} du, \tag{4}$$

and the long-term fraction of time that the destination node is the neighbor of only the relay node (i.e., the fraction of time that the RB is draining off) is

$$\pi_d \triangleq \lim_{t\to\infty} \frac{1}{t} \int_{u=0}^{t} \mathbf{1}_{\{S_u=-1\}} du. \tag{5}$$

It can be shown that these limits exist under Assumption A. The proof is beyond the scope of this paper. Moreover, Assumption A implies that [1]

$$\pi_s = \lim_{t\to\infty} P(S_t = 1) = \frac{E[\sigma_n]}{E[C_n]} \tag{6}$$

and

$$\pi_d = \lim_{t\to\infty} P(S_t = -1) = \frac{E[\alpha_n]}{E[C_n]}. \tag{7}$$

Theorem 1: If $r_s E[\sigma_n] < r_d E[\alpha_n]$ then $\tilde{B}_n$ converges in probability to a proper and finite r.v. $\tilde{B}$ (i.e., $\lim_n P(\tilde{B}_n < x)$ $= P(\tilde{B} < x)$). If $r_s E[\sigma_n] > r_d E[\alpha_n]$, then $\tilde{B}_n$ converges to $+\infty$ $P-a.s.$ □

Proof. Follows from the relation to the G/G/1 queue made above and [10]. ■

Remark 1: In terms of π_s and π_d the stability condition of Theorem 1 writes

$$r_s\pi_s < r_d\pi_d.$$

Remark 2: If all nodes have the same mobility model, then clearly $\pi_s = \pi_d$, since the relay node is equally likely to be within the transmission range of the source and of the destination. Therefore, by Remark 1, the stability condition is

$$r_s < r_d.$$

Theorem 2: If $r_s\pi_s < r_d\pi_d$, then the *relay throughput* T_r, defined as the stationary output rate of the relay node, is given by

$$T_r = r_s\pi_s.$$ □

Proof. In steady-state the RB can be thought of as a standard G/G/1 queue so that the output rate is the same as the input rate and is given by $r_s\pi_s$. ■

Remark 3: The relay throughput T_r only depends on r_s and the stationary distribution of the node mobility pattern. In particular, two different mobility patterns with the same stationary distribution (for the location of the nodes) will yield the same relay throughput.
It is clear from Theorem 2 and Remark 1 that π_s and π_d play an important role in determining the stability and the throughput of the RB. Much of the rest of this paper will be devoted to the study of these quantities.

B. Multiple Source, Destination, and Relay Nodes

We now assume that there are K source nodes, M destination nodes and N relay nodes, all with the same transmission range R (the latter will be assumed throughout). The source and destination nodes are stationary. The relay nodes move *independently of each other* inside a connected area A according to the *same mobility pattern*. The distance between any two source nodes, and between any two destination nodes, is assumed to be greater than $2R$. This implies that a relay node can not receive (resp. transmit) data from (resp. to) two or more source (resp. destination) nodes at the same time.

Furthermore, assume that the routing protocol generates routes of length no more than h-hops, i.e., the lifetime of a packet in number of hops is not greater than h. The distance between any source and any destination node is set to be greater than hR. Therefore, there does not exist a direct route from any source to any destination node, which implies that packets have to use mobile relay nodes to transfer data.

The RB of a relay node is composed of M queues; one for each of the M destinations. The system behaves as follows:

1) When there are i relay nodes inside the transmission range of source node k, where $i \in \{1 \cdots N\}$ and $k \in \{1 \cdots K\}$, then the source transmits to the i relay nodes the packets addressed to destination node $m \in \{1 \cdots M\}$ with probability P_k^m in a round-robin scenario, where $\sum_{m=1}^{M} P_k^m = 1$. So, queue m of the relay node accrues packets at a fixed rate $r_{S_k} P_k^m / i$, where r_{S_k} is the transmission rate of source node k.
2) When the relay node receives a packet from a source that is destined to destination node m, it buffers this packet in its queue of index m.
3) When there are j relay nodes with non-empty queue m inside the transmission range of destination m, these relay nodes share the channel bandwidth fairly. More precisely, queue m of these j relay nodes drains off at a fixed rate r_{D_m}/j, where r_{D_m} is the transmission rate of a relay node to the destination node m. The service discipline in queue m of the relay node is FIFO.

Let $f(x)$, $x \in A$, be the stationary node location probability density. Denote by x_{S_k} and x_{D_m} the fixed location in A of source $k \in \{1, \cdots, K\}$ and destination $m \in \{1, \cdots, M\}$, respectively.

Hence, the probability that a relay node is the neighbor of a node located in $x \in A$ is

$$\pi(x) = \int_{\{y \in A : d(x,y) \le R\}} f(y) dy, \tag{8}$$

where $d(u, v)$ is the Euclidean distance between vectors u and v.

By conditioning on the number of nodes within range of source node k, we find that the input rate at queue m of each relay node is

$$\begin{aligned} \tau_{S_k}^m =& P_k^m r_{S_k} \sum_{i=1}^{N} \frac{1}{i} \begin{pmatrix} N-1 \\ i-1 \end{pmatrix} \pi(x_{S_k})^i \, \overline{\pi(x_{S_k})}^{N-i} \\ =& P_k^m r_{S_k} \frac{1 - \left(1 - \pi(x_{S_k})\right)^N}{N}, \end{aligned} \tag{9}$$

where $\overline{a} := 1 - a$.

The overall long-term arrival rate to queue m of a relay node from all of the sources is

$$\tau_S^m := \sum_{k=1}^{K} \tau_{S_k}^m = \frac{1}{N} \sum_{k=1}^{K} P_k^m r_{S_k} \Big[1 - \big(1 - \pi(x_{S_k})\big)^N\Big]. \tag{10}$$

The exact derivation of τ_D^m, the long-term service rate of queue m at a relay node, is intractable since it depends on the (stationary distribution of) location of the other relay nodes with respect to the destination D_m, and on whether or not queue m at each relay node is empty or not and located within transmission range of D_m. More precisely, if i relay nodes are within the transmission range of destination D_m, and if queue m in each of these relay nodes is non-empty, then the service rate in queue m at each of the i relay nodes is r_{D_m}/i. The above reasoning indicates that r_{D_m}/N is the minimum instantaneous service rate at each queue m. This yields the following lower bound—called $\hat{\tau}_D^m$—on the long-term service rate of queue m:

$$\hat{\tau}_D^m = r_{D_m} \frac{1 - \left(1 - \pi(x_{D_m})\right)^N}{N}. \tag{11}$$

As a result, a sufficient condition for the stability of queue m at each relay node is

$$\tau_S^m < \hat{\tau}_D^m. \tag{12}$$

If queue m at a relay node is stable, then the relay throughput T_r^m at this queue is equal to its long-term arrival rate, i.e,

$$T_r^m = \tau_S^m = \sum_{k=1}^{K} \tau_{Sk}^m = \frac{1}{N} \sum_{k=1}^{K} P_k^m r_{Sk} \left[1 - \overline{\pi(x_{Sk})}^N\right]. \tag{13}$$

The network throughput, T, is the sum of the relay throughputs at all the M queues of all the N relay nodes, namely

$$T = \sum_{n=1}^{N} \sum_{m=1}^{M} T_r^m = \sum_{k=1}^{K} r_{Sk} \Big[1 - \big(1 - \pi(x_{Sk})\big)^N\Big]. \tag{14}$$

Observe that $1 - \big(1 - \pi(x_{Sk})\big)^N$ is the probability that there is at least one relay node inside the transmission range of the source node k.

We conclude this section by briefly addressing the situation where all of the nodes are moving. Since an exact calculation of the throughput of queue m at a relay node is very difficult, we will derive an approximation for this quantity. This approximation is based on the assumption that routes cannot exceed two hops. We assume that all nodes move independently of each other with the same mobility pattern, and that they have the same transmission range. Let p_1 be the probability that two nodes are within transmission range of one another. Let p_2 be the probability that three nodes constitute a two-hop route. Then, under the above simplifying assumption

$$\begin{aligned} & \sum_{i=1}^{N} \frac{P_k^m r_{S_k}}{i} \binom{N-1}{i-1} (p_1 - p_2)^i \, (1 - p_1)^{N+1-i} \\ =& P_k^m r_{S_k} \frac{(1 - p_1)\left((1 - p_2)^N - (1 - p_1)^N\right)}{N} \end{aligned} \tag{15}$$

is the contribution of source node k to the long-term arrival rate in queue m at any relay node. Therefore, the overall long-term input rate at queue m at any relay node can be approximated by summing up the r.h.s. of the above identity over all the values of k. This gives

$$\tau_S^m \approx \frac{(1-p_1)\big((1-p_2)^N-(1-p_1)^N\big)}{N}\sum_{k=1}^{K} P_k^m r_{S_k}. \quad (16)$$

When $P_k^m = 1/M$ (that is, there is a uniform probability that source node k sends to destination node m) and when the transmission rates of all sources are equal to r_S, then (16) becomes

$$\tau_S^m \approx r_S\frac{(1-p_1)\big((1-p_2)^N-(1-p_1)^N\big)}{MN}. \quad (17)$$

In the next section, we will investigate the impact of the mobility pattern on the relay throughput. We will show that the throughput is minimized when in steady-state the nodes are uniformly distributed over the area.

IV. Comparison of Mobility Models

We consider the scenario where nodes move independently of each other according to some mobility pattern. Assume that the nodes location distribution is stationary. The nodes position can take values in a discrete set X with cardinality $\#X = G$. Let $G(x), x \in X$ denote the set of all points in the transmission range of a node located at x. We assume that there is complete symmetry, so that $\#G(x) = \#G(y)$ for all $x,\ y \in X$ and that if $x \in G(y)$ then $y \in G(x)$. This can be assumed when there is no *boundary effect*, for example, as is the case of motion over a torus or over a circle (representing, respectively, motion over a plane or line with wrap around).

Let P be the probability measure over X that represents the stationary node location distribution. As the cardinality of X is equal to G, P can be represented as an G-dimensional (column) vector. The uniform stationary node location over X, called U, is a G-dimensional vector whose entries are all equal to $\frac{1}{G}$. Let $e_x, x \in X$, denote a probability measure over X which gives all mass to position x, i.e., e_x is an G-dimensional vector whose entries are all equal to 0 except for the x^{th} components which is equal to 1.

For any stationary node location distribution P over X, let $g(P)$ denote the probability that two nodes are neighbor of each other. Let H denote the neighborhood matrix, i.e., $H_{x,y} = 1$ if $y \in G(x)$ and $H_{x,y} = 0$ otherwise. Note H is a symmetric matrix. In terms of P_x (resp. P_y), the probability that a node is at location x (resp. y) in the stationary regime $g(P)$ writes

$$g(P) = \sum_{x\in X} P_x \sum_{y\in G(x)} P_y = \sum_{x\in X} P_x \sum_{y\in X} H_{x,y}P_y = P^T HP$$

where P^T is the transpose of P and we use the fact that the locations of the nodes are independent.

Theorem 3: A uniform distribution of nodes over the region of interest achieves the minimum probability of contact between any two nodes. □

Proof: Consider any P of the form

$$P = U + \delta e_x - \delta e_y, \quad (18)$$

for some $0 < \delta < 1$ and $x, y \in X$, $x \neq y$. Then

$$\begin{aligned} g(P) =& P^T HP \\ =& g(U) + \delta^2(e_x^T He_x + e_y^T He_y - e_x^T He_y - e_y^T He_x) \\ &+ 2\delta(e_x - e_y)^T HU, \end{aligned} \quad (19)$$

where we have used the fact that for all $x \in X$ $e_x^T He_x = 1$ as $x \in G(x)$. Since H is a symmetric matrix, we have $P^T HQ = Q^T HP$ for all P, Q probability measures on X. Also, it is easy to see that $e_y^T He_x = 1$ if $x \in G(y)$ and is 0 otherwise. Hence we get

$$\begin{aligned} g(P) =& g(U) + 2\delta^2(1 - \mathbf{1}_{\{x\in G(y)\}}) + 2\delta(e_x - e_y)^T HU \\ =& g(U) + 2\delta^2\mathbf{1}_{\{x\notin G(y)\}} + \frac{2\delta(\#G(x) - \#G(y))}{G}, \end{aligned}$$

where in the last expression we have used the, easy to observe, fact that $e_x^T HU = \frac{\#G(x)}{G}$. Hence, since $\#G(x) = \#G(y)$ for all $x,\ y \in X$, it is seen that $g(P) - g(U) = 2\delta^2\mathbf{1}_{\{\mathbf{x}\notin\mathbf{G(y)}\}}) \geq \mathbf{0}$. Which implies that

$$U \in \underset{P=U+\delta e_x-\delta e_y}{\operatorname{argmin}} g(P). \quad (20)$$

Now, any other probability distribution over the set X is a point in $G-$dimensional canonical simplex. The uniform distribution is at the centroid of this simplex and any other distribution P, when viewed as an G dimensional vector (a point in the simplex), can be written as

$$P = U + \epsilon, \quad (21)$$

where U is the uniform distribution and ϵ is an G-dimensional vector whose entries are in the interval $[-\frac{1}{G}, \frac{G-1}{G}]$ and the entries sum to zero. Clearly, any such ϵ can be written as a (possibly non-unique) finite sum

$$\epsilon = \sum_{x\in I(P)} (e_x - e_{y(x)})\delta_x, \quad (22)$$

where $I(P) \subset X$ is some index set, $y(x) \in X$, and $\delta_x > 0,\ x \in I(P)$. This is because e_x forms a basis for the $G-$dimensional space and because P is a probability vector with $\sum_{x\in X} \epsilon_x = 0$.

Recall that if the stationary node distribution is P, we can write $g(P)$ as $g(P) = P^T HP$, where H is an $G \times G$ symmetric matrix indicating the neighborhood relation. We have already shown that when $P = U$, the uniform distribution, the directional derivative of $P^T HP$ is positive along any direction of the form $(e_x - e_y)$ where e_x is $G-$dimensional vector with all except the x^{th} entry equal to zero. We now use continuity of the derivative of $g(U)$ to conclude that its directional derivative along any direction is positive. Hence $U \in \underset{P}{\operatorname{argmin}}\, g(P)$. ∎

The above result does not imply that the *relay throughput* achieves its minimum under the uniform stationary node distribution. This is because the relay throughput under distribution P, denoted $T_r(P) = r_s\pi_s(P)$ and with $\pi_s(P) = \lim_{t\to\infty} P(S_t = 1)$ under the probability measure P, is

$$T_r(P) = r_s\Big(g(P) - \sum_{x\in X} P_x \sum_{y\in G(x)} P_y \sum_{z\in G(x)\cup G(y)} P_z\Big), \quad (23)$$

and it can be easily seen that for any $x \in X$, $\pi_s(e_x) = 0$. Since $\pi_s(\cdot)$ is a probability, this implies that $P = e_x$ achieves minimum of $\pi_s(\cdot)$. However, it is reasonable to assume that the uniform distribution is a local minimum for $\pi_s(\cdot)$ because the second term in expression for $\pi_s(\cdot)$ above is of smaller order as compared to the first term.

Observe that if the source node and the destination node are fixed, and if they are far apart (so that a two-hop communication between them via a relay node is not possible), then a uniform distribution of relay node achieves the minimum relay throughput.

In the next section, we will find expressions for the relay throughput in the case nodes move according to the Random Waypoint and the Random Direction models.

V. Throughput in Random Waypoint and Random Direction Models

In this section, we compute the relay throughput in the case where (i) the relay node moves along a finite interval according either to the Random Waypoint model or to the Random Direction model, the source and destination nodes being stationary (Section V-A.1), (ii) all nodes move independently of each other, with the same mobility model (Random Direction or Random Waypoint), either along a finite interval (Section V-A.2) or inside a square (Section V-B).

We have shown in Theorem 2 that the relay throughput T_r is given by $T_r = r_s\pi_s$, where r_s is the transmission rate of the source to the relay node (r_s is a given parameter), and π_s is the stationary probability that the source is sending packets to the relay node (see Section III). In the following, we will compute π_s for each case mentioned above. This will be carried out under the assumption that all nodes have the same transmission range R.

A. One Dimension

For the Random Waypoint mobility model over the interval $[0, L]$, the stationary probability density function of a node location is [2]

$$f(x) = \frac{6(L-x)x}{L^3}, \quad x \in [0, L]. \tag{24}$$

The stationary probability density function under the Random Direction mobility model is uniform [11], i.e.,

$$f(x) = \frac{1}{L}, \quad x \in [0, L]. \tag{25}$$

1) Only Relay Node is Mobile: We assume that the source and the destination nodes are fixed in $[0, L]$, and that the relay node moves along this interval according to either the Random Direction or the Random Waypoint mobility model.

We first focus on the stability condition. From Remark 1 the stability condition is given by $r_s\pi_s < r_d\pi_d$, where these quantities are defined in Section III. Let us compute π_s and π_d for either mobility model (recall that r_s and r_d are given parameters). We have

$$\pi_s = \int_{x=(s-R)^+}^{(s+R)\wedge L} f(x)dx, \quad \pi_d = \int_{x=(d-R)^+}^{(d+R)\wedge L} f(x)dx,$$

where $f(\cdot)$ is the stationary node location distribution, and $a \wedge b = \min(a, b)$. Thus, the stability condition reads

$$r_s \int_{x=(s-R)^+}^{(s+R)\wedge L} f(x)dx < r_d \int_{x=(d-R)^+}^{(d+R)\wedge L} f(x)dx. \tag{26}$$

Consider now the relay throughput. In the stable case it is given by (see Theorem 2)

$$r_s \int_{x=(s-R)^+}^{(s+R)\wedge L} f(x)dx. \tag{27}$$

In the particular case where the relay node moves according to the Random Direction mobility model, the stability condition is (use (26)) with $f(x)$ given in (25))

$$r_s((s+R)\wedge L)-(s-R)^+ < r_d((s+R)\wedge L)-(s-R)^+,$$

and the relay throughput, T_{RD}^f, achieved is (cf. (25) and (27))

$$T_{RD}^f = r_s \frac{((s+R)\wedge L)-(s-R)^+}{L}. \tag{28}$$

For $R < s < L - R$ and $R < d < L - R$, $T_{RD}^f = r_s \frac{2R}{L}$ and $\pi_s = \pi_d$, regardless of the position of the source node and of the destination node. In this case, the stability condition reduces to $r_s < r_d$.

When the relay moves according to the Random Waypoint mobility model, then the stability condition is (use (24) and (26))

$$r_s[2(A-B)(3-(A^2+AB+B^2))] < r_d[2(C-D)(3-(C^2+CD+D^2))],$$

with

$$A := (s+R)\wedge L, \qquad C := (d+R)\wedge L,$$
$$B := (s-R)^+, \qquad D := (d-R)^+,$$

and the relay throughput, T_{RW}^f, is given by

$$T_{RW}^f = r_s \frac{2(A-B)(3-(A^2+AB+B^2))}{L^3}. \tag{29}$$

2) All Nodes are Mobile: We now assume that the source, destination and relay nodes are all mobile, and move along $[0, L]$ according to the same mobility model: the Random Waypoint or the Random Direction model. First, observe from Remark 2, that in this case the stability condition is given by $r_s < r_d$.

Let us now compute the throughput $T_r = r_s\pi_s$ for each mobility model. We have

$$\pi_s = \int_0^L f(x) \int_{(x-R)^+}^{(x+R)\wedge L} f(y)dydx - \int_0^L f(x) \left[\int_{(x-R)^+}^{(x+R)\wedge L} f(y)dy\right]^2 dx$$
$$- \int_0^L f(x) \int_{(x-R)^+}^{(x+R)\wedge L} f(y) \int_{(y-R)^+}^{(y+R)\wedge L} f(z)dzdydx, \tag{30}$$

where $f(x)$ is given either by (24) or by (25), depending on the mobility model in use. It is easy to compute π_s in explicit form for both functions $f(x)$. We will instead provide compact approximation formulas, since the exact ones are lengthy. We approximate π_s by the first term in the r.h.s. of (30). Note that this term is the probability that two nodes are neighbors. This approximation is justified by the fact that, for each function $f(x)$ in (24) and (25), the second and the third term in the r.h.s. of (30) are much smaller than the first term, when the ratio $\rho := R/L$ is small with respect to 1. This yields the following approximate throughputs:

$$T_{RW} \approx r_s \frac{\rho(12 - 2\rho^5 - 20\rho^2 + 15\rho^3)}{5} \quad (31)$$

for the Random Waypoint mobility model, and

$$T_{RD} \approx r_s \rho(2 - \rho), \quad (32)$$

for the Random Direction mobility model. Observe that these formulas depend on R and L only through their ratio.

We conclude this section by considering the case where $r_s = r_d := r$. In this case, the relay buffer is not stable. To handle this situation, it is proposed in [6] to use a probability of relaying, p_r, which is close to 1, so that when the relay node enters the neighborhood of the source node, the source node transmits data to be relayed with probability $p_r < 1$, and does not transmit with the complementary probability. Note that this scheme ensures stability and gives near maximum throughput as well.

B. Two Dimensions

In this section we consider nodes moving in a square. We start by computing the relay throughput; then we find the probability of these nodes form a two-hop route.

1) Three Nodes Moving: Nodes move independently of each other inside a square of side length L. They move according to the same mobility model, the Random Direction or the Random Waypoint model.

Similarly to Section V-A.2 the stability condition is $r_s < r_d$. The probability that two nodes are neighbors is

$$\pi(f) = \int_{x \in [0,L]^2} f(x) \int_{x_2 : |x - x_2| \le R} f(x_2) dx_2 dx \approx \pi R^2 \int_{x \in [0,L]^2} f^2(x) dx,$$

where we have used the continuity of $f(\cdot)$, and the assumption that R is negligible with respect to L. This approximation is in agreement with Theorem 3, which states that the minimum probability of contact is achieved by the uniform distribution, since the latter integral is the L_2-norm of $f(\cdot)$, which is minimized when $f(\cdot)$ is the uniform distribution.

When the RB is stable, the relay throughput is approximated by $r_s \pi(f)$, where r_s is the source transmission range. It can be shown, numerically, that for the Random Waypoint mobility model over a square $\pi(f) \approx 1.36\pi(R/L)^2$ [5], this implies that the relay throughput, T_{RW}^{2d}, is approximated by

$$T_{RW}^{2d} \approx 1.36 \pi r_s \rho^2, \quad (33)$$

where $\rho := R/L$. Under the Random Direction mobility we find that $\pi(f) \approx \pi\rho^2$. Hence, the relay throughput, T_{RD}^{2d}, is approximated by

$$T_{RD}^{2d} \approx \pi r_s \rho^2. \quad (34)$$

Note that $T_{RD}^{2d} < T_{RW}^{2d}$.

2) Probability of a Two-Hop Route: The throughput of data between a pair of nodes when there exists a route between them, called *forwarding throughput*, is a function of the route length in hops. A first step to derive the forwarding throughput is to compute the distribution of the route length in hops. In this section, we compute the probability of a two-hop route between two nodes s and d, assuming there are N other nodes. All the nodes are mobile inside a square $A = [0, L]^2$ and moving (independently) according to either the Random Waypoint or Random Direction models.

Let x_s (resp. x_d) represents the position of node s (resp. d). There is a two-hops route between node s and d, if the node d is inside the annulus of center x_s and of interior and exterior radius equal to R and $2R$, and there is at least an intermediate node inside, D_I, the intersection of the two disks of radius R centered around x_s and x_d. The probability of the two-hop routes between nodes s and d when $R << L$ writes

$$P^N(2) \approx \int_A f(x_s) \int_C f(x_d) \Big[1 - \overline{\Big(\int_{D_I} f(x)dx\Big)^N}\Big] dx_d dx_s$$
$$\approx 2\pi \sum_{i=1}^{N} (-1)^{i+1} \binom{N}{i} u(i) v_f(i) \Big(\frac{R}{L}\Big)^{2(i+1)},$$

where $u(i) = \int_1^2 r A(r)^i dr$, and $v_f(i) = \int_{[0,1]^2} f(x)^{i+2} dx$. Here $A(r)$ is the area of D_I when $R = 1$ and the distance between nodes s and d is equal to r. This gives

$$A(r) = 2 \arccos\Big(\frac{r}{2}\Big) - \frac{r}{2}\sqrt{4 - r^2}. \quad (35)$$

We observe that $P^N(2)$ is a function of $(R/L)^2$. The values of $u(i)$ and $v_f(i)$ are easy to obtain numerically.

Until now we have looked at the effect of mobility patterns on the "average" throughput and showed that throughput depends only on the stationary node distribution. We then showed that minimum throughput is achieved when the stationary distribution of node position is uniform. In the next section, we will study the relay buffer behavior in the case where the relay node performs a Random Walk over a circle, and the end of the section we derive the mean relay buffer size in heavy-traffic.

VI. Relay Buffer Behavior

In this section, the effect of mobility model on the relay buffer occupancy is studied. We assume that the mobility models under consideration have stationary node location distribution. The plan is to view this system as a G/G/1 queue in heavy-traffic and then to look at the effect of mobility pattern on the relay buffer occupancy. We know from heavy-traffic analysis of a G/G/1 queue [9] that the tail behavior (the large deviation exponent) of the buffer occupancy is determined by the variance of the service and inter-arrival times.

Moreover, it is also to be understood that the effective arrival process in the queueing model (introduced in Section III-A) is

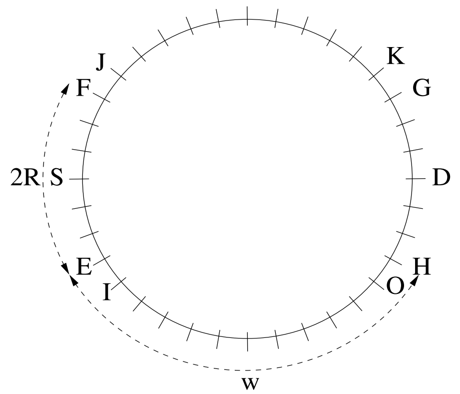

Fig. 2. Random walk on circle.

not the contact time between the relay and source nodes, i.e.,

$$\int_{u=Z_n}^{Z_{n+1}} \mathbf{1}_{\{S(u)=1\}} du, \tag{36}$$

but is composed of many (random number of) such contact times since

$$\sigma_n = \int_{u=W_n}^{W_{n+1}} \mathbf{1}_{\{S(u)=1\}} du. \tag{37}$$

That this is the case can be easily seen by studying the evolution of the $\{S_t\}$ process. Since it is possible that in a cycle, the $\{S_t\}$ process alternates between values 0 and 1 for many times before taking the value of -1.

Clearly, a larger relay buffer occupancy would imply that the amount of time required to deliver all the packets would be composed of many contact periods between the relay node and the destination, hence there can be several inter-visit between the relay node and the destination required to deliver the packets. This implies that we can not study the delay incurred by the nodes by considering only one inter-visit time (or the meeting time) or only one contact time. This shows that the buffer behavior (hence the delays) will depend on *both* contact times and the inter-visit times. This section is devoted to such a study for the particular case where the relay node is performing Random walk and the source and destination are fixed. This section is meant for illustration of the above phenomenon.

We consider the following scenario. The relay node is moving according to a symmetric random walk (RW) on a circle of circumference[1] $4R + 2w$ steps – see Figure 2. The RW step size is fixed and is equal to μ meters. The speed of the relay node is assumed to be constant and equal to v, so the time required to jump from one step to the next one, is equal to μ/v seconds. The source and the destination are held fixed, and they are located as shown in Figure 2. The quantities w and R are assumed to be integers. Also, the data transmission between source and destination only takes place through the relay node.

When the relay node becomes a neighbor of the source (when passing points E or F), it starts to accumulate data to be relayed to the destination at rate r_s. When the relay node enters the neighborhood of the destination, via points G or H, it delivers the data to destination at rate r_d. Once in the interval $[E, F]$, the relay node remains there for a random amount of time before exiting via points E or F. Symmetry implies that this time has the same distribution whether the relay node enters $[E, F]$ through the point E or F. Similar is the case for the segment $[G, H]$. We call this (random) time the *contact time* between the relay node and the source (or the destination). Once the relay node exits $[E, F]$, it either enters $[J, K]$ or $[I, O]$. Now, the relay node stays in this region for a random amount of time (during which it neither receives nor transmits), and then either reenters $[E, F]$ or enters $[G, H]$.

The number of times that the relay node enters $[E, F]$ without entering $[G, H]$ is denoted by the r.v. L, and is geometrically distributed with parameter p, independent of whether the relay node exited $[E, F]$ via E or F, that is,

$$P(L = k) = (1 - p)p^{k-1}.$$

The parameter p is the probability that a symmetric random walker starting at point J hits point F before reaching G.

Let $A_j, j \geq 1$, be independent and identically distributed random variables representing the first time that a random walker, starting at point F, exits $[E, F]$, so that the service requirement in the queueing model of Section III-A is $r_s\sigma$, where

$$\sigma = \sum_{j=1}^{L} A_j. \tag{38}$$

In the following, A denotes a generic r.v. with the same distribution as A_j.

Using results from random walk literature (for example [4]), it can be shown that

$$\begin{aligned} E[A] &= 2R\frac{\mu}{v}, \\ Var[A] &= \left(\frac{\mu}{v}\right)^2 \cdot \frac{4R}{3}(2R+1)(R+1), \\ p &= 1 - \frac{1}{w}, \\ E[L] &= w, \\ Var[L] &= w(w-1). \end{aligned}$$

Since L is independent of A, we get

$$\begin{aligned} E[\sigma] &= E[A]E[L] = 2wR\frac{\mu}{v} \\ Var[\sigma] &= Var[A]E[L] + (E[A])^2 Var[L] \\ &= 4w^2R^2\left(\frac{\mu}{v}\right)^2 + \frac{4}{3}wR(2R^2+1)\left(\frac{\mu}{v}\right)^2. \end{aligned}$$

In our case, the relay buffer occupancy at the instants when the relay node enters $[E, F]$ after having been in $[G, H]$, is

$$\tilde{B}_{n+1} = (\tilde{B}_n + r_s\sigma_n - r_d\alpha_n)^+. \tag{39}$$

In the scenario under study, σ_n and α_n have the same probability distribution. When $r_s \approx r_d$ with $r_s < r_d$, then (39) gives the evolution of the customer waiting times in a G/G/1 queue in heavy-traffic. We will approximate the mean

[1]R is the transmission range of source, destination, and relay node.

customer waiting time in this queue by the corresponding quantity in a $GI/GI/1$ in heavy-traffic. In the latter queue, it is known [9, P. 29] that the stationary waiting time is *exponentially distributed* with mean

$$E[\tilde{B}] = \frac{(r_s^2 + r_d^2)Var(\sigma)}{2E[\sigma](r_d - r_s)} = \frac{\mu(r_s^2 + r_d^2)}{v(r_d - r_s)}(Rw + \frac{1}{3}(2R^2 + 1)), \tag{40}$$

where we have used the fact that α_n and σ_n have the same probability distribution.

VII. Numerical Results

In this section we present simulation results to validate results in Theorem 1 (stability issues), Theorem 2 (throughput depends only on stationary distribution), Section V (throughputs obtained by Random Waypoint and Random Direction Models), the probability of two-hop route, the relay buffer behavior as studied in Section VI, and the probability distribution of the contact time. Throughout this section, we will assume that the transmission range of the nodes is constant and is equal to R.

A. Validation of Theorem 1

We consider the scenario of three nodes: a source, a destination, and a relay node. each moving according to a symmetric random walk over a circle. It follows from Theorem 1 that the relay node buffer occupancy is stable *iff* the ratio $p = \frac{r_s}{r_d} < 1$. Figure 3 plots the evolution of relay node buffer with time for different values of p. It is evident from the figure that when

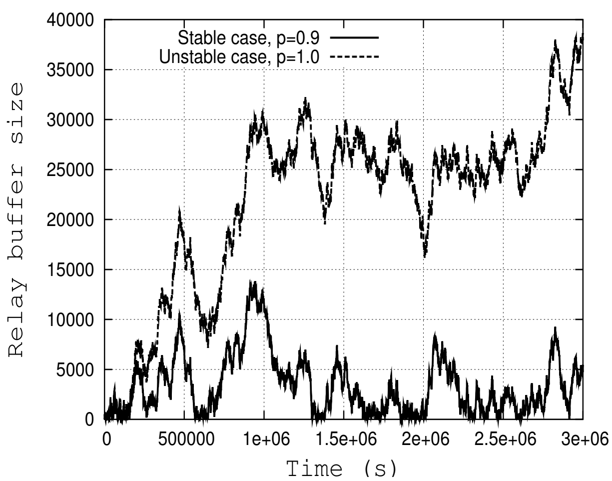

Fig. 3. Time-evolution of relay node buffer for Random Walk model over a circle for different values of ratio, $p = \frac{r_s}{r_d}$.

$p = 1.0$, the buffer occupancy process is unstable. While for the case $p = 0.9 < 1.0$, this process is stable. Similar results were obtained even for $p \approx 1.0$ with $p < 1.0$ but are not shown here.

B. Validation of Theorem 2 and Section V

Theorem 2 states that the relay throughput depends only on the stationary node distribution. Section V provides the value of relay throughput under Random Direction and Random Waypoint mobility models. To validate both of these results, we ran simulations to find the relay throughput for case of the Random Direction and the Random Waypoint mobility models with different parameters.

We illustrate that the throughput depends only on the stationary distribution of the node position by looking at the scenario where three nodes move over a line of length $L = 4$ kilometers according to the Random Direction model. We assume that the time between two consecutive decision instants (travel time) is fixed and equal to 15 seconds and the distribution of speed was chosen to be i) Uniform over some interval, ii) Exponentially distributed, and iii) fixed. Note that the case where speed is fixed corresponds to the Random Walk. Since the stationary node location distribution is same for all the three choice of speed distribution, Theorem 2 implies that the relay throughput will be identical. The numerical results plotted in Figure 4 are in accordance with this result. Also evident is the fact that relay throughput $T_r = \pi_s$, for $r_s = 1$. In Figure 5 we plot values of π_s when the three nodes move

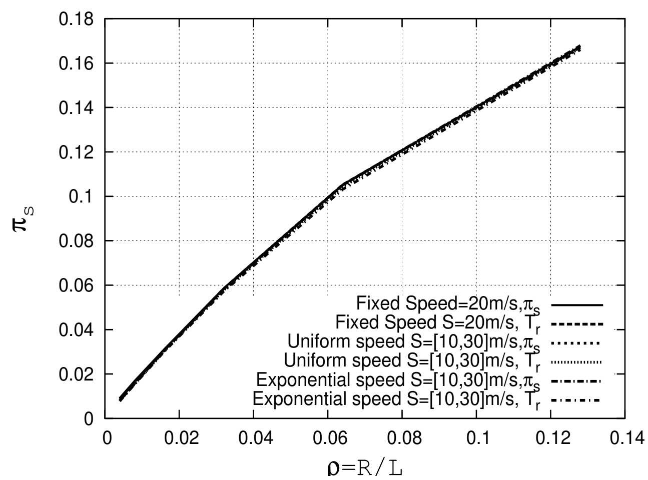

Fig. 4. Values of π_s obtained for the Random Direction model over a segment of length L. Various distributions for the speed were taken.

according to the Random Waypoint model over a square of side-lengths L for different values of transmission range, R. We keep the speed of the mobiles fixed. The plot shows that π_s (and hence T_r) is a function of $\rho = \frac{R}{L}$ alone. The numerical values also support the result of Section V-B where for the Random Waypoint model in square, the throughput is approximately $1.36\pi\rho^2$. Similarly, the values of the throughput from theory and simulations provide a good match for all of the scenario studied in Section V. Because of the space restriction, we did not include these numerical results due to space constraints.

C. Validation of Section III-B

Section III-B studies multiple relay nodes with fixed source nodes and destination nodes. It reports stability condition and derives the value of the relay throughput and the network throughput. To validate the stability condition, we take the scenario of one source node, one destination node, and 3 relay nodes move according to the Random Waypoint model inside a square of side-length $L = 4000m$. The source and

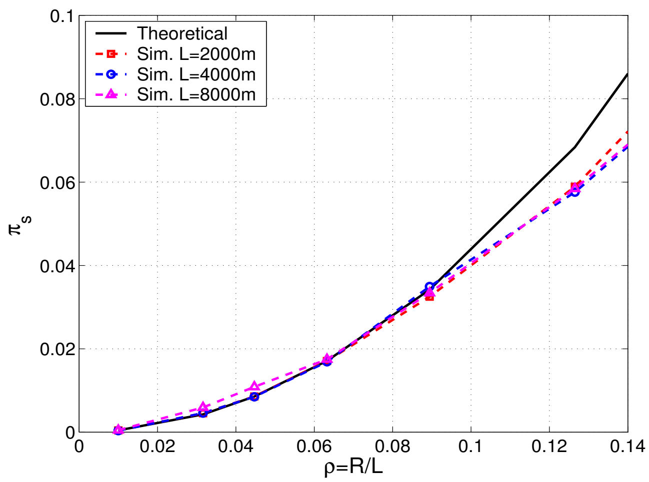

Fig. 5. Simulation results showing π_s for Random Waypoint model over a square of side-lengths L for different values of transmission range, R. Also shown are corresponding values from Section V-B.

destination nodes are fixed and they are symmetric according to the center of the square, and the separated distance between them is of $2000m$. The stable case is shown in Figure 6 where $r_s = 0.9r_d$, and the unstable case is shown in Figure 7 where $r_s = 1.1r_d$. The relay throughput and the network throughput as a function of the number of the relay nodes are shown in, respectively, Figures 8 and 9 for different value of R/L. In this scenario, the probability that the relay node is neighbor of the source of location $(1000, 1000)$ is equal to 0.0485, 0.0275, and 0.0126 for R/L equal to 0.1, 0.075, and 0.05 respectively.

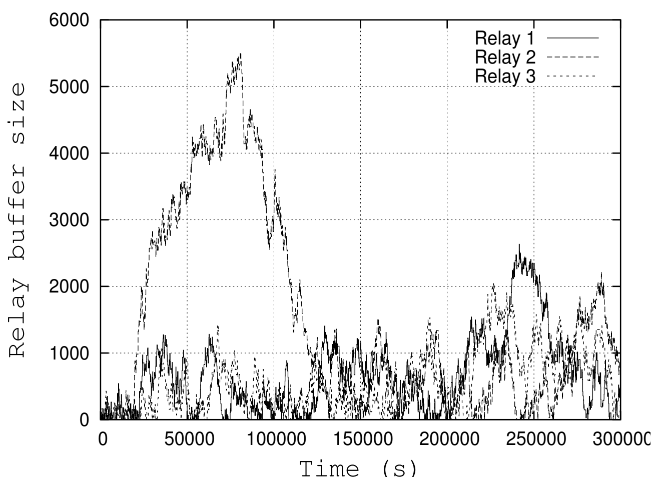

Fig. 6. Time-evolution of relay buffer for three relay nodes moving according to Random Waypoint model inside a square of side-length $4000m$, with fixed and symmetric source and destination node w.r.t. to square center. Here $r_s = 0.9r_d$ (the stable case)

D. Validation of Section III-B

In this section we validate the approximation of Section III-B of the case where all source and destination nodes are moving (c.f, Equation 16). We consider a scenario of N relay nodes and K source nodes and M destination nodes. All the nodes move according to the Random Waypoint model within

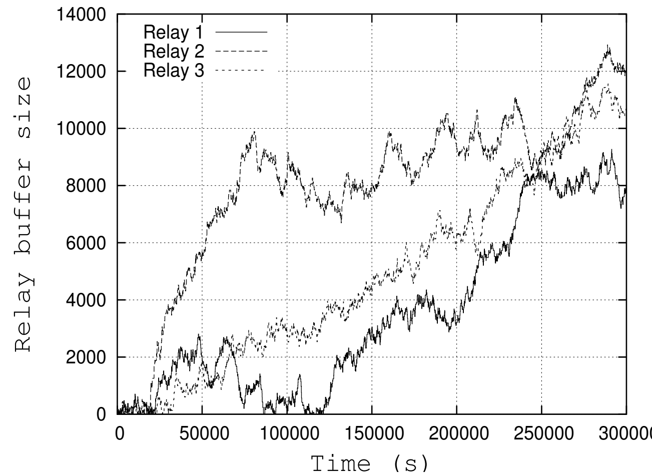

Fig. 7. Time-evolution of relay buffer for relay nodes moving according to Random Waypoint model inside a square of side-length $4000m$, with fixed and symmetric source and destination nodes w.r.t.to square center. Here $r_s = 1.1r_d$ (the unstable case)

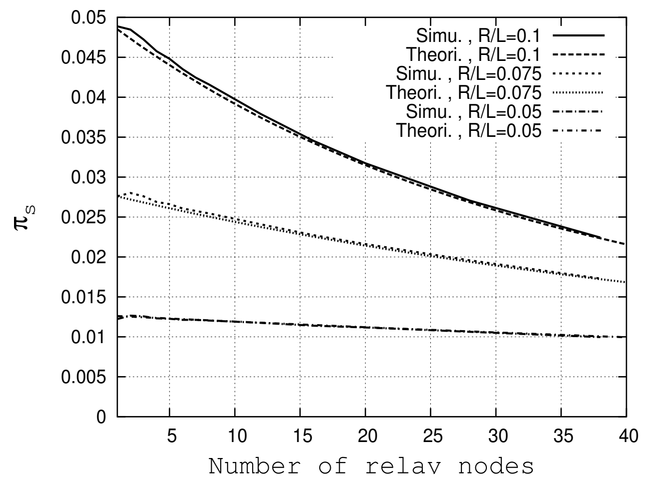

Fig. 8. Relay throughput as function of number of relay nodes in a square region of side-length $4000m$ with fixed source and fixed destination nodes.

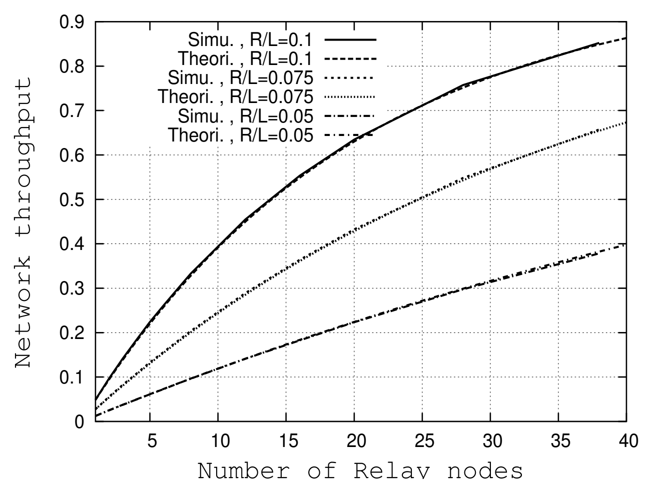

Fig. 9. Network throughput as function of number of relay nodes in square region of side-length $4000m$ with fixed source and fixed destination nodes.

a square region of side-length L equal to 4000. We assume that the lifetime of the packets in hops is equal to 4 hops. In

Figure 10, we show the approximation of Equation 16 as well as the simulation result of, τ_S^m, the long-term arrival rate to the queue m of the relay node n from the source nodes. We observe that the above approximation is accurate for $R/L \leq 0.05$, and the relative error between the approximation and simulation is less than 5%, for $R/L = 0.05$.

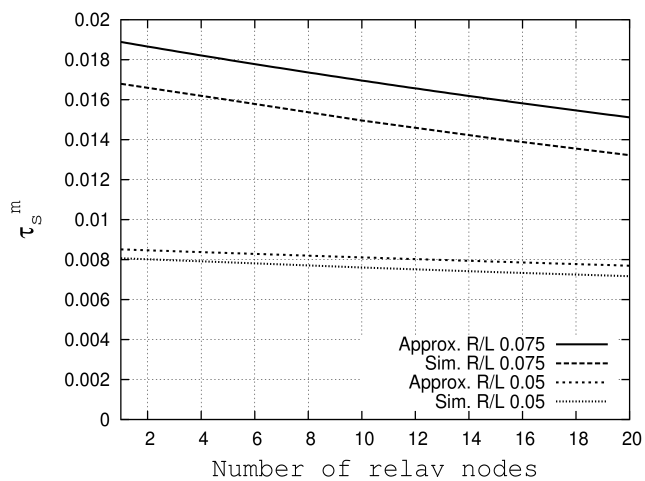

Fig. 10. Long-term arrival rate at queue m of relay node n from source node k as function of number of relay nodes N, where $K = 4$, $M = 5$, $P_k^m = 1/M$, and $r_{S_k} = 1$.

E. Validation of Two-Hop Route Probability

We have $N+2$ nodes moving inside a square of side length $L = 4000$ according to the Random Waypoint model. We validate the approximation formula for the probability of a two hops route for different values of N (cf., section V-B.2). In Figure 11, we show the results of the simulation and the approximation for $R/L \in \{0.025, 0.0375, 0.05\}$. We observe that for $R/L \leq 0.05$ the approximation is accurate.

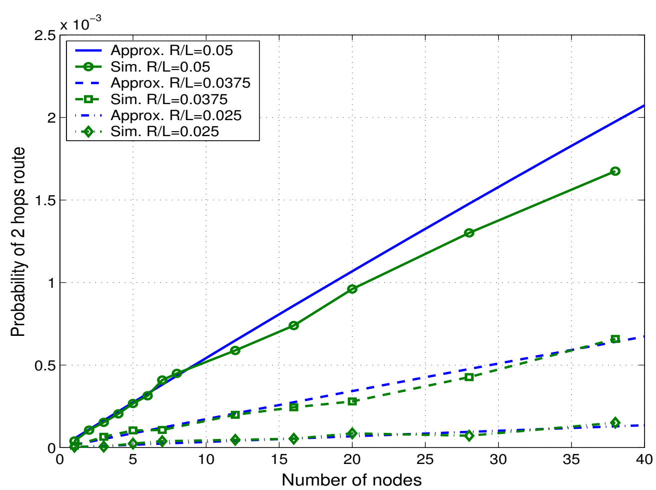

Fig. 11. Probability of two-hop route as function of number of nodes. All nodes move according to Random Waypoint model inside square of side length $4000m$.

F. Validation of Section VI

We consider the relay node buffer occupancy in the scenario in Section VI. The third line of Table I reports the percentage of the relative error of the relay buffer occupancy found in (40) and the corresponding simulated value $E[B_{sim}]$, for different values of the parameters R and w with $\mu = 50$ meters and $r_d\mu/v = 200$ data units. Parameters R and w are chosen so that the circumference of the circle is equal to 3000 meters (i.e. $(4R+2w)\mu = 3000$ meters). In the simulation the relay node buffer is sampled at the beginning of each cycle (as defined in Section III). Throughout these experiments $\frac{r_s}{r_d} = 0.95$, so as to reflect the heavy-traffic scenario under which (40) was established. We observe that the relative error between Equation (40) and the simulation is very small.

R	11	9	7	5
w	8	12	16	20
$\frac{\lvert E[B_{sim}]-E[\tilde{B}]\rvert}{E[\tilde{B}]}$ (%)	2	1	1	2

TABLE I
VALIDATION OF EQUATION (40) (FOR $\mu = 50$ METERS AND $r_d\mu/v = 200$ DATA UNITS).

G. Validation of the Contact Time Distribution

We consider the scenario where the nodes move according to the Random Direction model and Random Waypoint model inside a square of side length $L = 4000m$. The nodes speed is constant and is equal to V. For the Random Direction model, the travel time, T, is greater than R/V. The contact time is defined as the interval of time that a pair of nodes stay inside one another transmission range after being outside one another transmission range. In Figure 12 and 13, we show the normalized contact time distribution defined as the contact time divided by R/V of the Random Waypoint and the Random Direction model. We observe in both cases that for $R/V \leq 20s$ and $R/L \leq 0.05$, the normalized contact time distribution is almost independent of R/V, of R/L, and T. So in the case Random Direction model, we deduce numerically that the mean contact time is equal to $1.26\frac{R}{V}$. Similarly in the case of Random Waypoint model, the mean contact time is equal to $1.24\frac{R}{V}$. Knowing the mean contact time and the probability that a pair of nodes are inside each other transmission range [2], we deduce that the mean intermeeting time the time when the pair of nodes are not in contact is equal to: $1.26\frac{R}{V}\left(\frac{1}{\pi}\left(\frac{L}{R}\right)^2 - 1\right)$, in the case of the Random Direction model, $1.24\frac{R}{V}\left(\frac{0.73}{\pi}\left(\frac{L}{R}\right)^2 - 1\right)$, in the case of the Random Waypoint model.

[2] See section V-B.1

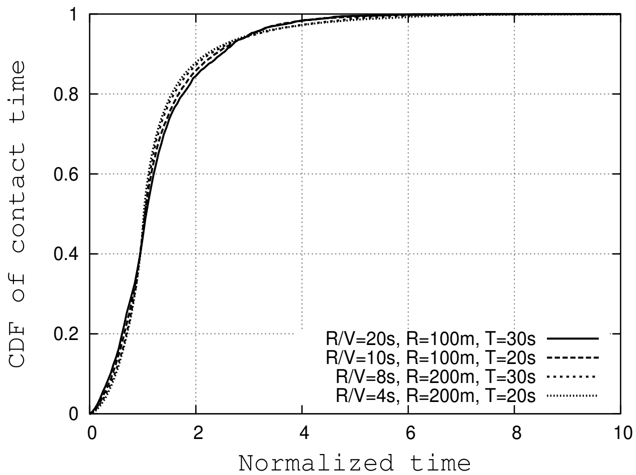

Fig. 12. Normalized contact time distribution for different values of R/V and travel time T. Nodes move inside square with length $L = 4000m$ according to Random Waypoint model.

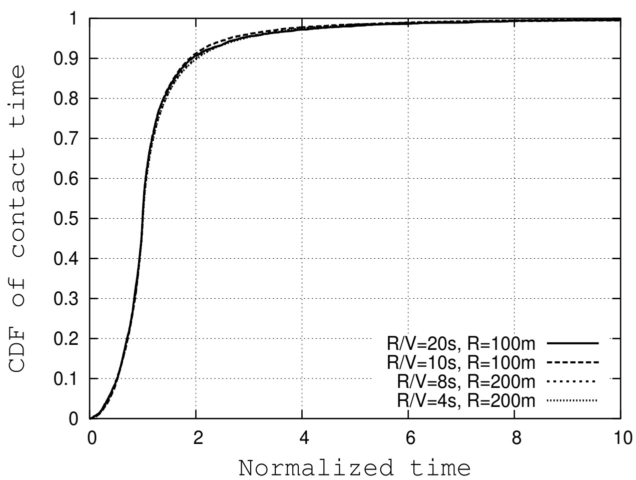

Fig. 13. Normalized contact time distribution. Nodes move inside square with length $L = 4000m$ according to Random Waypoint model.

VIII. Conclusions

We have studied the performance of relaying in mobile ad hoc networks by developing a queueing model. The parameters of the queueing model depend on the node mobility pattern.

Our main findings are that (under the assumptions placed on our model) the relay throughput only depends on the stationary node location distribution, and that uniform stationary distribution of nodes results in the smallest relay throughput. Approximate throughput formulas have been derived for both the Random Waypoint and the Random Direction mobility models; these formulas have been found to be in agreement with simulation results. Approximation formula for the mean buffer occupancy of the relay node has been obtained for the Random Walk mobility model.

We have implicitly assumed that the order of delivery of data does not matter. This is an important simplifying assumption that has allowed us to assume that the relay node does not transmit data from its relay buffer when there is either a direct or two-hop route (via the relay node) between the source and destination. Relaxing this assumption would be important for applications using TCP-like protocols.

Most of our work in this paper has focused on the average behavior of the relay node buffer occupancy process. It would be interesting to study the tail of the relay buffer occupancy process for various mobility models. Also, the process $\{B_n\}_n$ is embedded at regeneration instants; it would interesting to study the relay node buffer at arbitrary times.

This study forms a research effort towards developing performance models for relay protocol, and understanding the impact of mobility on their performance.

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