Impact of Routing Metrics on Path Capacity in Multirate and Multihop Wireless Ad Hoc Networks

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***Abstract*— Finding a path with enough throughput in multihop wireless ad hoc networks is a critical task of QoS Routing. Previous studies on routing algorithms focused on networks with a single channel rate. The capability of supporting multiple channel rates, which is common in wireless systems, has not been carefully studied in routing algorithms. In this paper, we first carry out a comprehensive study on the impacts of multiple rates, interference and packet loss rate on the maximum end-to-end throughput or path capacity. A linear programming problem is formulated to determine the path capacity of any given path. This problem is also extended to a joint routing and link scheduling optimization problem to find a path with the largest path capacity. We show that *interference clique transmission time* is inversely proportional to the upper bound of the path capacity, and hence we propose to use it as a new routing metric. Moreover, we evaluate the capability of various routing metrics such as hop count, expected transmission times, end-to-end transmission delay or medium time, link rate, bandwidth distance product, and interference clique transmission time to discover a high throughput path. The results show that different routing metrics lead to paths with significantly different path capacity, and the interference clique transmission time tends to discover paths with higher throughput than other metrics.**

I. Introduction

Wireless ad hoc networks have attracted a lot of attention in recent years, because they can be easily deployed at low cost and can support wireless communication via multiple wireless hops without relying on existing infrastructures, such as wireless base stations and Internet. They are often referred to as different names in different scenarios, such as wireless sensor networks, mobile ad hoc networks and wireless mesh networks, where there exists multihop wireless communication.

To support end-to-end communication in these networks, routing algorithms play a significant role in finding good paths and forwarding nodes between sources and their destinations. However, finding a good path is not an easy task in a wireless ad hoc network compared with wired networks because wireless links are significantly different from wired ones. First, wireless links are not reliable due to channel errors. Second, achievable channel rates may be different at different links because link quality depends on distance and path loss between two neighbors. Third, links may not exist any more when neighbors move out of the communication range. Finally, wireless transmission is broadcast in nature and a transmission over one link will interfere with transmissions over other links in the neighborhood ([1]).

To address these challenges, considering the features of physical layer and MAC layer is a must for a good routing algorithm. However, existing wireless ad hoc routing protocols typically find routes with the minimum hop-count, the shortcomings of which have been recognized in multihop wireless networks in many prior research works. De Couto etc. ([2]) showed that many of the shortest paths have poor throughput due to large loss rates over the radio links selected in these paths. They ([3]) accordingly proposed a new routing metric called the *expected transmission count (ETX)* to consider the packet loss over wireless links in order to obtain higher throughput. Jain etc. ([4]) studied the impact of interference on performance of multihop wireless network based on an NP-complete optimization problem. They showed that by taking the interference into consideration, routes derived from the optimization problem often yield noticeably better throughput than the shortest path route. In [5] and [6], the authors further proposed heuristic algorithms to address the interference by solving an optimization problem and find paths satisfying a certain bandwidth requirement.

Besides packet loss rate and interference, multirate capability is another common feature of wireless links. A higher data rate can be used to improve throughput if a better signal quality is observed over one link. However, a higher data rate often means a shorter transmission distance and hence more hops in the selected path. The data rate of one link is also subject to change because of a time-varying channel and changing interference in the neighborhood. Notice that packet loss ratio may not be as significant as discussed in [4] if an auto-rate MAC protocol is adopted as in the IEEE 802.11 protocol. A low rate is automatically used when a high packet loss rate is observed and hence leads to a low packet loss rate because of a less strict requirement of SNR (signal-to-noise ratio).

Not surprisingly, multirate capability has a great impact on routing algorithms and hence deserves a careful study in multihop wireless ad hoc networks. It seems intuitive that the end-to-end throughput will be improved if we allow multiple rates to coexist in the network, where a higher channel rate is used over each link if it can deliver more packets in the same period with the consideration of packet loss rates.

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However, in [7], Kawadia and Kumar showed that a single-rate wireless ad hoc network may have better performance than the network where multiple rates coexist if the shortest-hop routing algorithm is used. The reasons behind their findings are that a shortest-hop routing algorithm often choose links with the lowest channel rate while a fixed higher channel rate may be still able to generate a feasible path between the source and its destination and leads to a higher end-to-end throughput.

Several papers in the literature have already started to design good routing metrics in a multirate wireless ad hoc network. In [8], Draves, Padhye and Zill proposed to use the weighted cumulative expected transmission time (WCETT) as a routing metric. In [9], Awerbuch, Holmer and Rubens adopted the medium time metric (MTM). In [10], Zhai and Fang studied the impact of multirate on carrier sensing range and spatial reuse ratio and demonstrated that the bandwidth distance product and the end-to-end transmission delay (the same as the medium time) are better routing metrics than the hop count.

Unfortunately, there is still no comprehensive study on the evaluation of the capability of these routing metrics in maximizing the end-to-end throughput with consideration of coexisting multiple rates and their close relationship with packet loss rate and interference. These factors make it difficult to design a good routing metric to find the path with the widest bandwidth. We use a simple example in Fig. 1 to illustrate why some routing metrics fail to do so.

In Fig. 1, all users are assumed to transmit over the same channel with a fixed transmission power and conform to the IEEE 802.11 protocols. Suppose the highest achievable channel rate over links along path 1 from S_1 and D_1 is 2Mbps, and the highest achievable channel rate over links along path 2 is 54Mbps. Apparently, if the SNR requirement for 1Mbps is larger than 0dB, transmissions over any two hops along path 1 cannot be successful at the same time. Then the maximum end-to-end throughput of path 1 is proportional to $\frac{2}{3}$Mbps. Suppose for the same reason, there is also only one successful transmission allowed at a time along path 2. The maximum end-to-end throughput along path 2 is $\frac{54}{12} = 4.5$Mbps. It is similar for path 3 and 4 except that path 4 passes a large number of short hops resulting in a very long end-to-end transmission delay. Suppose that transmissions along path 4 can be simultaneously successful every other 11 hops and so the maximum end-to-end throughput of path 4 is similar to that of path 2, i.e., 4.5Mbps. It is straightforward that path 1 will be selected from S_1 to D_1 if a routing algorithm minimizes the hop count. Minimizing the transmission times still leads to path 1. Minimizing the end-to-end transmission delay/medium time or maximizing the minimum bandwidth distance product over all links along the path will lead to path 2. For path 3 and 4 from S_2 to D_2, hop count, ETT and the end-to-end transmission delay all lead to path 3 while bandwidth distance product leads to path 4 with a much higher throughput than path 3. It seems that bandwidth distance product works better than all others to find paths with high throughput. However, does it work well in a more general topology? Does there exist an even better routing metric?

In this paper, we endeavor to address all the factors together using an extended link conflict graph model. A linear

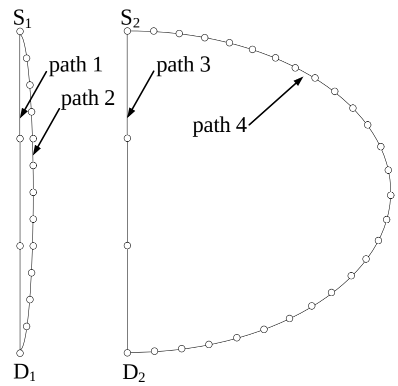

Fig. 1. Paths between the source S and the destination D

programming optimization problem is formulated to solve the path capacity or the maximum end-to-end throughput of a given path. The solution of the path capacity in some scenarios implies that the interference clique transmission time is a good routing metric to find paths with high throughput. The solution of the optimization problem establishes a foundation for the evaluation of the relative performance of different routing metrics. Moreover, the model is extended to a joint optimization problem of link scheduling and routing algorithm to find the optimum path between the source and the destination that have the largest end-to-end throughput. Though the joint optimization problem requires a centralized implementation and is NP-complete, it provides a measure how good the routing metrics really are comparing to the best possible one. The results show that the end-to-end transmission delay and the interference clique transmission time are the best two among all the metrics mentioned above, and the interference clique transmission time consistently leads to paths with throughput close to the optimum one and higher than those obtained from other routing metrics. In addition, the interference clique transmission time can find paths with up to 10% more throughput than the end-to-end transmission delay especially when the distance between the source and its intended destination is long, say, about more than 4 hops in the shortest hop routing algorithm. Furthermore, we illustrate that good routing metrics can generate paths with higher throughput in a multirate wireless ad hoc network than any routing metrics in a single-rate wireless ad hoc network with any single possible channel rate.

The rest of this paper is organized as follows. Section II studies the impact of multirate capability on the network performance. In Section III, we extend the link conflict graph to characterize multirate, interference and packet loss rate together in order to find the path capacity of any given path in the network. In Section IV, we generalize the Bellman-Ford routing algorithm for several different routing metrics. The relative performance of different routing metrics is evaluated in Section V. Finally, Section VI concludes this paper.

II. Impact of Multirate Capability on Path Selection In Wireless Ad Hoc Networks

In wireless ad hoc networks, a channel rate over each link can be adaptively selected according to the link signal

TABLE I

SIGNAL-TO-NOISE RATIO AND RECEIVER SENSITIVITY

Rates (Mbps)	SNR (dB)	Receiver sensitivity (dBm)
54	24.56	-65
48	24.05	-66
36	18.80	-70
24	17.04	-74
18	10.79	-77
12	9.03	-79
9	7.78	-81
6	6.02	-82

quality. When the signal quality is good, a high channel rate is used. Otherwise, a low channel rate is used. This auto rate selection has been widely adopted by the 802.11 products. In this section, we study the impact of multiple channel rates on the path selection in wireless ad hoc networks and attempt to identify the important factors we should consider in the path selection.

A. Receiver Sensitivity and SNR for Multiple Rates

Wireless devices have to satisfy two conditions to correctly decode one received packet. First, the received signal strength of the intended packet must be larger than a threshold, which is called receiver sensitivity. Second, the signal to noise-plus-interference ratio (SNR) has to be larger than a certain threshold. Receiver sensitivity defines a transmission range only in which a transmission can be successful. SNR indicates how much interference can be tolerated and determines the spatial reuse ratio, i.e., the maximum number of concurrent successful transmissions in a certain area.

Wireless systems normally support multiple channel rates as in UWB and 802.11 systems. For example, all the IEEE 802.11 a/b/g standards support multiple channel rates. Specifically, 1, 2, 5.5, and 11Mbps are supported by the 802.11b. 6, 9, 18, 24, 36, and 54Mbps are supported by the 802.11a/g. Different channel rates have different requirements of the receiver sensitivity and SNR. Table I shows the requirement of one 802.11a product [11]. Therefore, transmission radius and spatial reuse ratio may be significantly different for different channel rates.

B. Tradeoff between the rate and the transmission distance

A higher channel rate can achieve higher throughput than a lower channel rate over one link. However, it often has a shorter maximum transmission distance [12] because of its higher requirement of the receiver sensitivity and SNR. Therefore, using higher channel rates at the forwarding nodes often results in more hops between a source and its intended destination. On the other hand, a path with the smallest number of hops often travel through links with low channel rates, and hence may suffer from throughput loss.

C. Carrier Sensing Range, Interference and Spatial Reuse

In the CSMA/CA (carrier sense multiple access with collision avoidance) MAC protocol, like the IEEE 802.11 MAC protocols, each node should sense an idle channel before any transmission. The area around one node, in which it can sense transmissions from other nodes, is called its carrier sense range. Therefore, in each carrier sense range, there is at most one successful transmitter or transmission.

Because a higher channel rate may have a shorter transmission distance, it may require more hops to travel through one carrier sense range than a lower channel rate. Therefore, the spatial reuse ratio may be low for high channel rates. Here the *spatial reuse ratio* is measured by the reciprocal of the number of hops between any two concurrent successful transmissions. For example, using 54Mbps, the maximum spatial reuse ratio can be achieved by scheduling concurrent transmissions at links that are at least 8 or more hops away from each other [10]. On the other hand, this hop number, when the maximum spatial reuse ratio is achieved, can be 3 for 1Mbps.

The other reason that a high channel rate has a low spatial reuse ratio is its high requirement of SNR. Assuming that the transmission power is the same for the intended signal and the interference signal, the SNR is proportional to

$$SNR \propto (\frac{d_i}{d_h})^{\gamma} \tag{1}$$

where d_h is hop distance or the distance between the transmitter and the receiver, d_i is the distance between the receiver and the interfering node, and γ is the path loss exponent. Thus a higher SNR requires a large value of $(\frac{d_i}{d_h})^{\gamma}$, leading to a lower spatial reuse ratio.

D. Effective Data Rate and Protocol Overhead

Although the channel rates have nominal values, the effective data rates seen by an application may be much smaller than these values. They are closely related to the packet size and protocol overhead. In wireless systems, a preamble is often used for synchronization between the sender and the receiver. It has a fixed value per standard and can be regarded as the physical layer overhead. Besides the physical layer overhead, MAC layer head, IP head and TCP head of each packet also have fixed length, and does not change with the channel rate.

The effective data rate r_d can be computed as

$$r_d = \frac{L_{pl}}{T_{preamble} + \frac{L_H + L_{pl}}{r_c}} \tag{2}$$

where $T_{preamble}$ is the time not related to the channel rate r_c, L_{pl} is the length of payload we intend to transmit, and L_H is the length of protocol overhead transmitted with the channel rate r_c. $T_{preamble}$ includes the physical layer preamble and may also includes some MAC layer overhead, e.g., the interframe spacing. L_H includes the header of MAC, IP and TCP layers. For an example, in 802.11, if RTS/CTS/ACK are transmitted with the basic rate and DATA is transmitted with the selected channel rate r_c, then

$$\begin{aligned} T_{preamble} &= (T_{RTS} + T_{CTS} + 2T_{SIFS})\varphi + \\ &\quad T_{SIFS} + T_{DIFS} + T_{phy} + T_{ACK} \\ \varphi &= \begin{cases} 1, \text{(if RTS/CTS are used)} \\ 0, \text{(if RTS/CTS are not used)} \end{cases} \end{aligned} \tag{3}$$

where T_{RTS}, T_{CTS}, and T_{ACK} are the time for the transmission of RTS, CTS, and ACK frames, respectively. T_{phy} is the time for the transmission of the physical preamble of the MAC DATA frame. T_{SIFS} and T_{DIFS} are the interframe spacing time of SIFS and DIFS, respectively. If L_{pl} approaches infinity, r_d approaches r_c.

Given the length of a packet payload L_{pl}, the higher the channel rate, the larger ratio the preamble occupies in the transmission time of a packet, which means a higher protocol overhead. A high channel rate is normally preferred, but the corresponding high protocol overhead must be take into consideration ([10], [13]).

III. Path Capacity in Wireless Ad Hoc Networks

It is a fundamental issue to know the maximum end-to-end throughput, referred to as *path capacity* thereafter, of a given path or multiple paths in the wireless ad hoc networks. Any traffic load higher than the path capacity is not supported and even deteriorates the performance as a result of excessive medium contention [1], [14], [15]. The knowledge of path capacity can be used to reject any excessive traffic in the admission control for real-time services. It can also be used in routing algorithms to find a path with the largest capacity or to evaluate the performance of different routing algorithms. Furthermore, the derivation of path capacity may also suggest novel and efficient routing metrics.

However, it is not easy to derive path capacity for paths in the wireless ad hoc networks, considering all the factors discussed previously. In this section, we first extend the link conflict graph model to describe necessary conditions required by those factors. Then we formulate the problem into a link scheduling problem with the help of the flow conflict graph.

In this paper, we assume that there is no power control scheme and the transmission power at each node is known before link scheduling.

A. Link Conflict Graph

According to the interference relationships between links, we can construct the link conflict graph, where each node represents one link and each edge represents that there is a conflict between the two corresponding links. For example, a five-link chain topology and its link conflict graph are shown in Fig. 2. Link 1 and 2 conflict with each other because node B cannot transmit and receive at the same time. Link 1 and 3 conflict with each other because node C's transmission will introduce enough interference for the reception at node B. Link 1 and 4 do not conflict with each other if node D's transmission does not interfere with the receiving at node B.

The link conflict graph can be constructed on different physical layer models. In the protocol model, any other transmitter has to be at least a certain distance away from an ongoing receiver. In the carrier sensing model, any other transmitter has to be at least a certain distance away from an ongoing transmitter. In the physical model, the aggregate power from all other ongoing transmissions plus the noise power must be less than a certain threshold so that the SNR requirement at an ongoing receiver is satisfied. In the bi-directional transmission model, such as the 802.11, where the two-way handshake DATA/ACK or four-way handshake

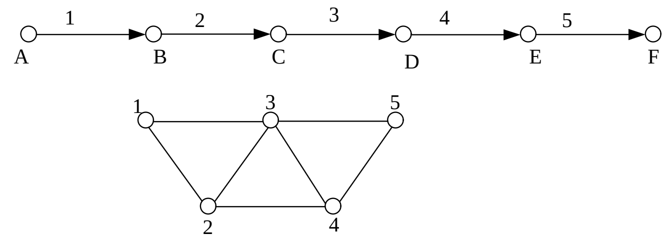

Fig. 2. A five-link chain topology and its link Conflict graph

RTS/CTS/DATA/ACK are used for each transmission, both the transmitter and the receiver of one link has to satisfy the requirements from one or more of the above models. Some mixed models can also be adopted, such as a model considering the requirements from both the carrier sensing model and the physical model.

In this paper, we call a model as a *distance model* if it involves the distance between the considered link and one other link at a time as in the carrier sensing model. A model is called an *interference model* if it considers the impact of interference power level from other links as in the physical model. A *mixed model* considers the requirements of both models. All these models can be characterized by a weighted conflict graph. A wight w_{ij} describes the impact of link i on link j, and

$$w_{ij} = \begin{cases} \frac{Pr_j(i)}{\frac{Pr_j(j)}{SNR_j} - P_N}, & \text{(interference model)} \\ b \ (0 \text{ or } 1), & \text{(distance model)} \\ \max\{\frac{Pr_j(i)}{\frac{Pr_j(j)}{SNR_j} - P_N}, b\}, & \text{(mixed model)} \end{cases} \quad (4)$$

where $Pr_j(i)$ and $Pr_j(j)$ are the received power at link j from the transmissions over link i and j, respectively, P_N is the noise power, SNR_j is the required SNR for a successful transmission at link j, and $\frac{Pr_j(j)}{SNR_j} - P_N$ is the maximum allowable interference at link j.

Given a link set S and a link $j \in S$ satisfying $\sum_{i \in S, i \neq j} w_{ij} < 1$, the transmission at link j will be successful even if all links belonging to the set S are simultaneously transmitting. If this condition is true for all $j \in S$, the transmissions at all the links in S can be scheduled successfully at the same time. Such a set is called an *independent set*. If adding any one more link into an independent set S results in a non-independent set, S is called a *maximum independent set*. For a set of links, if any two links in the set cannot be scheduled to transmit successfully at the same time, we refer to the set as an *interference clique*. If the set is not an interference clique any more after adding any link, it is also referred to as a *maximum interference clique*.

B. Upper Bound of Path Capacity in the Single Interference Model

In the single interference model, any two links L_i and L_j conflict with each other if the weight w_{ij} defined in Equation (4) is larger than or equal to 1 and do not conflict otherwise, and the conflict relationship is independent of any other links.

In this subsection, we assume that the link rate is determined by the received power and is equal to the maximum available rate satisfying the requirement of receiver sensitivity. We will discuss in Section III-D a more general case where the link rate is determined by both the receiver sensitivity and the surrounding interference.

Let i be the index of available channel rates and $P_{se}(i)$ be the receiver sensitivity for the ith channel rate r_i. Index i increases when the channel rate increases, and if $j > i$, $r_j > r_i$ and $P_{se}(j) > P_{se}(i)$. Then the link rate r_c is determined by the receiving power Pr at the receiver of the link.

$$r_c = r_i \text{ if } P_{se}(i+1) > Pr \geq P_{se}(i) \tag{5}$$

Given r_c for each link, w_{ij} can be calculated for any two links, and the link conflict graph can be constructed accordingly for a given topology.

Now let us define a new metric called the *interference clique transmission time* T_C for one clique C in the link conflict graph, and

$$T_C = \sum_{l \in C} T_l \tag{6}$$

where T_l is the transmission time for a packet over link l. For a given path P, find the set S of all the maximum interference clique C for the links belonging to P. Let T_P^* be the maximum value of T_C for all cliques of P and

$$T_P^* = \max_{C \in S} T_C \tag{7}$$

Notice that finding all the maximum cliques for a graph is a NP hard problem. However, the number of links of a path in wireless networks is normally limited to a very small number. The brute-force algorithm can find them in a reasonable time if the number of links is small.

Given T_P^*, the path capacity C_P is upper bounded by

$$C_P \leq \frac{L_p}{T_P^*} \tag{8}$$

where L_p is the packet length. This can be easily shown by the following observation. T_P^* is the interference clique transmission time of one clique C of P. Considering one link l in C and any one packet successfully delivered from the source to the destination, the packet takes time T_P^* to travel through all the links in C, and link l cannot schedule any other transmission during the period T_P^*. That means the packet takes at least time T_P^* at link l, and the throughput at link l is less than or equal to $\frac{L_p}{T_P^*}$. Because the end-to-end throughput cannot be larger than the throughput of any one link of the path, path capacity $C_P \leq \frac{L_p}{T_P^*}$.

It can be shown that if there is an odd cycle [16] in the link conflict graph, e.g. in Fig. 3, the equal sign in Equation (8) does not hold. Suppose the transmission time of a packet over all links are the same and is equal to T. It can be easily shown that $C_P = \frac{2L_p}{7T} < \frac{L_p}{3T}$, where L_p is the packet length and $3T$ is the T_P^* or the maximum value of interference clique transmission time of all cliques of the path.

However, a large number of paths found by routing algorithms have no odd cycles when minimizing or maximizing some metrics, as in the shortest hop routing algorithm. Most

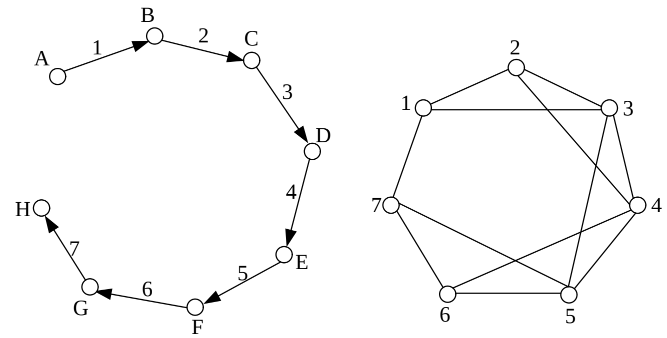

Fig. 3. A path with an odd cycle in the link conflict graph

of these paths may have a unique feature: if two links of a path conflict with each other, all the links between them along the path conflict with both of them. We call these paths as the **direct routes**, and other paths as the **detour routes**. For direct routes, the problem to find all maximum clique can be simplified. To find all the maximum cliques including one link, we only need to consider other links close to this one along the path. We refer to these cliques as the *local interference clique* of a path. For direct routes, the maximum value of the interference clique transmission time of all local cliques, or $\hat{T}_P^*$, is equal to that for all cliques, or T_P^*. Some polynomial algorithms can be designed to find all local cliques, which is omitted in this paper due to the limited space.

For direct routes, $C_P = \frac{L_p}{T_P^*} = \frac{L_p}{\hat{T}_P^*}$ and the following simple scheduling can achieve the path capacity:

- The first link or the source node schedules a transmission every other T_P^*.
- Each link starts the transmission at the same time the upstream link finishes a transmission.

It can be easily shown that there will be no conflicting links being scheduled to transmit at the same time so that it is a feasible scheduling.

In this subsection, we define a new metric called the interference clique transmission time and show it can more or less represent the path capacity. We will show later both metrics T_P^* and $\hat{T}_P^*$, i.e., the maximum value of the interference clique transmission time of all cliques and that of all local cliques, can be used as a routing metrics to find paths with high throughput, and $\hat{T}_P^*$ can be more easily computed than T_P^*. Apparently,

$$C_P \leq \frac{L_p}{T_P^*} \leq \frac{L_p}{\hat{T}_P^*} \tag{9}$$

C. Exact Path Capacity in Single Interference Model

Let the link conflict graph be constructed in the same way as in the above subsection. Then we can find all the independent sets $\{E_1, E_2, E_3, ..., E_\alpha, ..., E_M\}$, and $E_\alpha \in P$ for all $1 \leq \alpha \leq M$, where P is the set of all links in the considered path, and M is the maximum number of independent sets for the set P. Although it is a NP hard problem to find all independent sets, some brute-force algorithm can finish in a reasonable time because the number of links of a path in wireless networks is not large.

At any time, at most one independent set will be chosen to be scheduled to transmit packets for all links in that set. Let $\lambda_\alpha \geq 0$ denote the time share scheduled to the independent set E_α, and

$$\sum_{1\leq\alpha\leq M} \lambda_\alpha \leq 1, \lambda_\alpha \geq 0 (1 \leq \alpha \leq M) \tag{10}$$

Let $R_\alpha = \{r_e : e \in P\}$ be a row vector of size $|P|$, where $r_e = 0$ if $e \notin E_\alpha$; r_e is the effective data rate over link e, defined in Equation(2), otherwise.

Therefore, $\lambda_\alpha R_\alpha$ is a flow vector that the network can support in the time share λ_α for the independent set E_α. We define a schedule S as a frequency vector $S = \{\lambda_\alpha : 1 \leq \alpha \leq M\}$. For a given demand vector $\vec{f} = \{f_e : e \in P\} \in R^{|P|}$, $\vec{f}$ is feasible if there exists a schedule S satisfying

$$\vec{f} = \sum_{1\leq\alpha\leq M} \lambda_\alpha R_\alpha \tag{11}$$

Path capacity is the maximum end-to-end throughput, which only counts the traffic traveling through all links from the source to the destination, so

$$C_P = \max \min_{e\in P} f_e \tag{12}$$

Now, we can formulate the path capacity problem as follows:

$$\begin{array}{l} \text{Maximize } \min_{e\in P} f_e \\ \text{Subject to:} \\ \quad \sum_{1\leq\alpha\leq M} \lambda_\alpha \leq 1 \\ \quad \sum_{1\leq\alpha\leq M} \lambda_\alpha R_\alpha - \vec{f} = 0 \\ \quad \lambda_\alpha \geq 0, 1 \leq \alpha \leq M \end{array} \tag{13}$$

It can be easily shown that the set of all feasible demand vectors is a convex set, and given a feasible demand vector $\vec{f} = \{f_e : e \in P\}$, the new vector $\vec{f^*} = \min_{e\in P} f_e(1, 1, ..., 1) = \min_{e\in P} f_e I$ is also feasible, where I is the all-one vector in $R^{|P|}$. Thus the Problem (13) can be converted to a linear programming problem:

$$\begin{array}{l} \text{Maximize } f_e \\ \text{Subject to:} \\ \quad \sum_{1\leq\alpha\leq M} \lambda_\alpha \leq 1 \\ \quad \sum_{1\leq\alpha\leq M} \lambda_\alpha R_\alpha - f_e I = 0 \\ \quad \lambda_\alpha \geq 0, 1 \leq \alpha \leq M, f_e \geq 0 \end{array} \tag{14}$$

Now we can interpret the schedule S as the following link scheduling for a given path. The time is divided into slots of duration τ. Each time slot is partitioned into a set of subslots indexed by α $(1 \leq \alpha \leq M)$, such that the αth subslot has a length of $\lambda_\alpha \tau$ seconds. In the αth subslot, all links in the set E_α will be scheduled to transmit. Thus, during each time slot of length τ, the throughput f_e over link e is

$$f_e = \frac{1}{\tau} \sum_\alpha \lambda_\alpha \tau R_\alpha(e) = \sum_\alpha \lambda_\alpha R_\alpha(e) \tag{15}$$

Since in the solution of Problem (14) f_e is the same for all links, the path capacity is equal to $\min_{e\in P} f_e = f_e$.

D. Path Capacity in Multi-Interference Model with Variable Link Rate

In above two subsections, we only consider interference one by one, and link rate is determined by the receiver sensitivity. In this subsection, we will study the aggregate effect of all existing interferences on transmissions, and the link rate is determined not only by the receiver sensitivity but also by the interference level contributed by all surrounding transmissions.

In the multi-interference model, link conflict graph is a weighted graph and the weight w_{ij} between link i and j is defined in Equation (4). Independent sets will be significantly different from those obtained in the single-interference model, and the highest achievable link rate of each link may be also different when the link is in different independent sets due to different interference level.

Given a set of links E_α, the interference level at each link is determined easily if we assume each user uses a predefined transmission power. When all links in E_α are scheduled to transmit at the same time, SNR at link L_i in E_α is given by

$$SINR_{i\alpha} = \begin{cases} \frac{\Pr_{ii}}{P_N + \sum_{\{j: L_j \in E_\alpha \setminus \{L_i\}\}} \Pr_{ji}} & \text{(multi-interference)} \\ \min_j \frac{\Pr_{ii}}{P_N + \Pr_{ji}} & \text{(single-interference)} \end{cases} \tag{16}$$

where Pr_{ii} is the received power level of the intended signal at link L_i, and Pr_{ij} for all $L_j \in E_\alpha \setminus \{L_i\}$ is the received interference power at link L_i from the transmission at link L_j. If two different links L_i and L_j have a common node, we set $Pr_{ji} = Pr_{ij} = \infty$ because one node cannot transmit and receive at the same time. Notice that if bidirectional transmission is allowed, Pr_{ij} can be interference level of either DATA transmission or ACK transmission, and we also need to check if the SNR requirement for receiving both DATA and ACK frames is satisfied at Link i.

If there is a link whose SNR is less than the requirement of the lowest link rate, then the transmission over that link cannot be scheduled at the same time with other links in E_α, and E_α is not an independent set. Otherwise, E_α is an independent set. For an independent set E_α, the link rate of each link in E_α will be selected as the highest possible channel rate satisfying both requirements of receiver sensitivity and SNR.

According to the above description of independent sets, we can use some brute-fore algorithms to find all independent sets and determine the link rates for all links in them for one path. Then the same method in the previous section can be used to derive the path capacity of any given path.

E. Extension to Multiple Paths between a Source and Its Destination or between Multiple Pairs of Source and Destination

Given k paths P_1, P_2, ..., P_K between the source node S and the destination node D, let f_k denote the path throughput of the kth path.

Let $P = \bigcup_i P_i$, find all independent sets E_α $(1 \leq \alpha \leq M)$ and calculate R_α for each E_α of P. Let $I(P_k)$ is an row indicator vector in $R^{|P|}$, and

$$I_e(P_k) = \begin{cases} 1, & \text{if } e \in P_k \\ 0, & otherwise \end{cases} \tag{17}$$

Then the problem to find the maximum aggregate throughput over all the paths can be formulated as

$$\begin{aligned}
&\text{Maximize } \sum_{1\le k\le K} f_k \\
&\text{Subject to:} \\
&\quad \textstyle\sum_{1\le\alpha\le M} \lambda_\alpha \le 1 \\
&\quad \textstyle\sum_{1\le\alpha\le M} \lambda_\alpha R_\alpha - \sum_k f_k I(P_k) = 0 \\
&\quad \lambda_\alpha \ge 0 \ (1 \le \alpha \le M),\ f_k \ge 0 \ (1 \le k \le K)
\end{aligned} \tag{18}$$

If k paths P_1, P_2, ..., P_K belong to k pairs of source and destination, the problem formulation is the same if we want to maximize the aggregate throughput of all source-destination pairs. If the fairness is considered, some other objective functions (or utility functions) can be used [17].

F. Consideration of the packet error rate

If we know the packet error rate pe_i over each link L_i, to find the path capacity, we only need to modify the link rate vector R_α in the above problem formulation, and

$$R'_\alpha = R_\alpha Diag\{(1-pe_1),(1-pe_2),...,(1-pe_{|P|})\} \tag{19}$$

where $Diag\{(1-pe_1),(1-pe_2),...,(1-pe_{|P|})\}$ is a diagonal matrix with $(1-pe_i)$ $(1 \le i \le |P|)$ on the diagonal.

The interference clique transmission time T_C becomes the expected interference clique transmission time T'_C, given by

$$T'_C = \sum_{l\in C} \frac{T_l}{1-pe_l} \tag{20}$$

T^*_P and $\hat{T}^*_P$ defined in Section III-B should also be recalculated accordingly.

IV. Path Selection in Wireless Ad Hoc Networks

In this section, we study how to select a good path with high bandwidth by using various routing metrics. First, we formulate an linear/integer programming optimization problem to find the best possible path to achieve the maximum end-to-end throughput or path capacity. Though it is a centralized algorithm, this provides the maximum of capacities of all paths found by any distributed routing algorithms and makes it possible to evaluate how close the path capacity found by different routing metrics is to the maximum. Then we propose several heuristic routing algorithms to utilize various routing metrics, including the expected interference clique transmission time, to find a good path. The new routing metric accounts for the multirate capability and interference, which the previously proposed routing metrics may not take advantage of, and hence may obtain significant performance gain.

A. Optimal Path Selection

The maximization of the end-to-end throughput between a source and a destination is a max-flow problem, which can be formulated as a linear programming problem as follows:

$$\begin{aligned}
&\text{Maximize } v \\
&\text{Subject to:} \\
&\sum_{\{j:(i,j)\in E\}} x_{ij} - \sum_{\{j:(j,i)\in E\}} x_{ji} = \begin{cases} v & i=s, \\ 0 & i\in N\backslash\{s,t\} \\ -v & i=t \end{cases} \\
&x_{ij} \ge 0,\ (i,j)\in E \\
&\textstyle\sum_{1\le\alpha\le M} \lambda_\alpha R_\alpha - \vec{f} = 0 \\
&\textstyle\sum_{1\le\alpha\le M} \lambda_\alpha \le 1, \lambda_\alpha \ge 0
\end{aligned} \tag{21}$$

where x_{ij} is the flow from node i to node j over link L_{ij}, $\vec{f}$ is the flow demand vector and $\vec{f} = \{x_{ij} + x_{ji},\ (i,j) \in E,\ i<j\}$. The first two rows are the standard formulation of a max-flow problem. The last two rows are feasibility conditions of the flow vector that considers the wireless interference as well as the multirate capability, and they replace the original condition that flow over each link is less than or equal to the link capacity. Normally, the solution of this problem will lead to multiple paths between the source and the destination.

In this paper, we focus on the unicast and single-path routing algorithm. Therefore we need to modify the above problem into a single-path problem as follows:

$$\begin{aligned}
&\text{Maximize } v \\
&\text{Subject to:} \\
&\sum_{\{j:(i,j)\in E\}} x_{ij} - \sum_{\{j:(j,i)\in E\}} x_{ji} = \begin{cases} v & i=s, \\ 0 & i\in N\backslash\{s,t\} \\ -v & i=t \end{cases} \\
&0 \le x_{ij} \le Cap_{ij}\cdot z_{ij},\ (i,j)\in E \\
&\textstyle\sum_{\{j:(i,j)\in E\}} z_{ij} \le 1, z_{ij} \in \{0,1\} \\
&\textstyle\sum_{1\le\alpha\le M} \lambda_\alpha R_\alpha - \vec{f} = 0 \\
&\textstyle\sum_{1\le\alpha\le M} \lambda_\alpha \le 1, \lambda_\alpha \ge 0
\end{aligned} \tag{22}$$

where Cap_{ij} is the maximum achievable link rate over link L_{ij}. $z_{ij} = 1$ means that L_{ij} may have a nonzero flow. The third row means that there is at most one outgoing link from each node with a nonzero flow. The first three rows specify that there is only one path between the source and the destination. The links along that path have the same flow and all other links have zero flow. This problem is an mixed integer-linear programming.

B. Using Routing Metrics in Path Selection

There are already many different routing metrics for ad hoc networks as discussed in Section I, including hop count, end-to-end transmission delay (or medium time), link rate, and bandwidth-distance product (BDiP). We also propose a new routing metric, i.e., interference clique transmission time (CTT) in Section III-B. To reduce the computation time, local interference clique transmission time (LCTT) can be used.

If the packet loss rate is considered, all aforementioned routing metrics become expected transmission count (ETX), expected end-to-end transmission delay, expected link rate, expected BDiP, expected CTT, and expected LCTT. To use these routing metrics, we should find paths to minimize ETX, expected end-to-end transmission delay, expected CTT, or expected LCTT; or to maximize expected link rate or expected

BDiP. Thereafter, we refer to the routing algorithms using them as min-hop, min-delay, max-rate, max-BDiP, min-CTT and min-LCTT, respectively. Specifically, a min-hop routing algorithm finds the path with the smallest hop count or ETX; a min-delay routing algorithm finds the path with the shortest (expected) end-to-end transmission delay. A max-rate or max-BDiP routing algorithm finds a path which has the widest bottleneck link, where the bottleneck link of a path is defined as the link with the lowest (expected) link rate or the smallest value of (expected) BDiP among all the links of that path. A min-CTT or min-LCTT routing algorithm finds a path which has the smallest value of bottleneck clique, where the bottleneck clique of a path is defined as the clique with the largest value of (expected) CTT or LCTT among all cliques or local cliques of that path.

Among these routing metrics, the hop count and the end-to-end transmission delay are end-to-end additive routing metrics, so the Bellman-Ford algorithm can be used to minimize them. Other routing metrics can be used with some widest path routing algorithm. Bellman-Ford algorithm can be also used for this purpose because it is well suited to computation of a matrix with the maximum bandwidth or the largest/smallest value of other metrics for a given number of hops [18].

These routing metrics can also be used in some distributed routing algorithms, such as AODV and DSR. When a node overhears a repeated routing request message, it only forwards or rebroadcasts the request message when the recalculated routing metric of the path that the received request message travels through has a better value than that for the previously received request message, such as a smaller hop count.

V. Performance Evaluation

In this section, we use Matlab to evaluate the performance of various routing metrics in finding good paths in terms of path capacity, and investigate which metric finds paths with larger path capacity and how close path capacity of the found paths is to the optimal value.

A. Simulation Setup

In the simulations, there are N nodes randomly distributed in the network. The channel rates 54, 18, 11, 6 and 1Mbps are studied, and their transmission radii are 76, 183, 304, 396, 610m [12], respectively. As discussed in [10], 802.11 systems have very close interference ranges and the optimum carrier sensing ranges for different channel rates, so we use a single interference range 900m for all channel rates for simplicity. That is to say, as long as two nodes are at least 900m away from each other, the transmission from a node does not interfere with the reception at the other. The data packet size is 1000bytes. The IEEE 802.11b/g protocol parameters are adopted to calculate the effective data rate at each link. 1 and 11 Mbps are 802.11b rates and 6, 18 and 54 are 802.11g rates. Two-way handshake DATA/ACK is used. Both DATA and ACK rates are transmitted with the same link rate.

We fix the node nearest to the upper left corner as the source, and find the paths from it to all other nodes. Therefore, there are total $N-1$ different source-destination pairs or paths considered in the evaluation. We compare seven routing algorithms consisting of optimal, min-hop, min-delay, max-rate, max-BDiP, min-CTT and min-LCTT routing algorithms. Here, the optimal one is the one obtained from the mixed integer-linear problem (22). The performance metric is the path capacity. Paths are computed using these routing algorithms and the path capacities of these paths are computed by solving the linear programming defined in Equation (14).

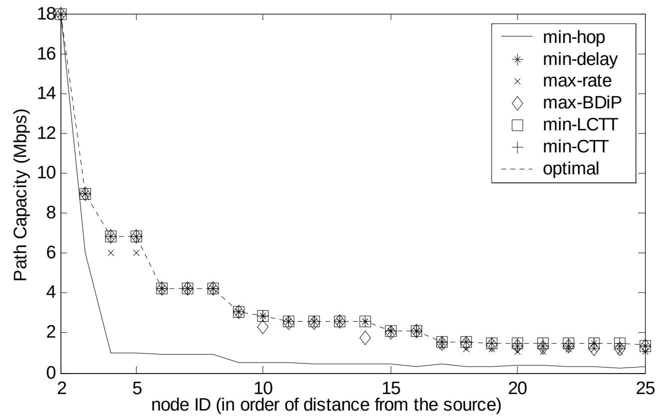

Fig. 4. Path capacity for different routing algorithms

B. Comparison with Optimal Routing

The optimal routing algorithm is formulated as a mixed integer-linear problem as in Equation (22). Normally it is a NP hard problem. Therefore, we can only solve the problem for a small topology in a reasonable time. In this set of simulation, 25 nodes are randomly distributed in a 200m X 2500m topology.

Figure 4 shows the path capacities of paths discovered by different routing algorithms. We can observe that min-CTT and min-LCTT routing algorithms can always find the path with a path capacity equal to the optimal value in this topology. Min-delay routing algorithms can find the path with optimum path capacity when the source-destination distance is not large. However it fails to do so when the source-destination distance is large although it finds a value close to the optimum value. Max-rate and max-BDiP may not be able to find a path with the optimum path capacity whether the source-destination distance is large or small. In addition, min-hop routing algorithm has much worse performance in finding a path with a high throughput than all other routing algorithms because it does not consider the multirate capability of the wireless nodes.

C. Performance Evaluation of Six Routing Metrics in a Larger Topology

In this set of simulation, 400 nodes are randomly distributed in a 1500m X 300m topology. To obtain a better vision effect, we only show the results of 26 random pairs of source-destinations. All other pairs have the similar results.

Fig. 5 shows the path capacities of paths selected by different routing algorithms. First, min-hop routing algorithm has

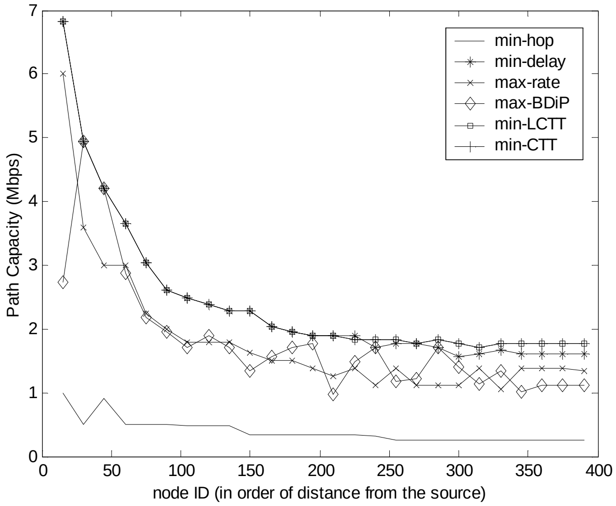

Fig. 5. Path capacity for different routing algorithms

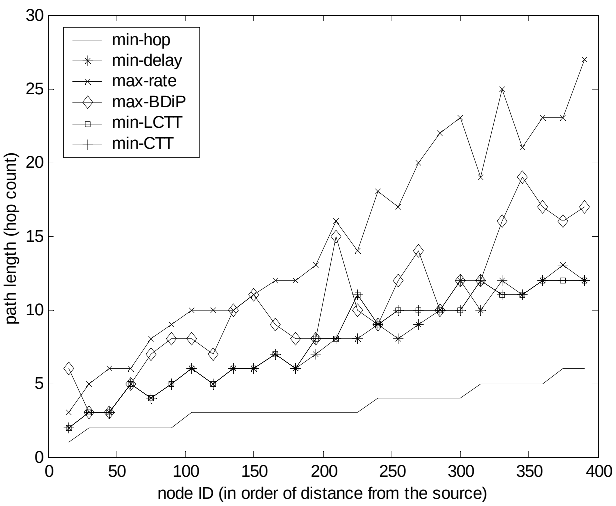

Fig. 6. Path lengths for different routing algorithms

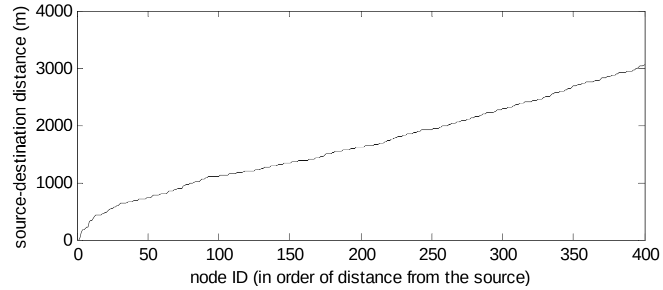

Fig. 7. Source-destination distance

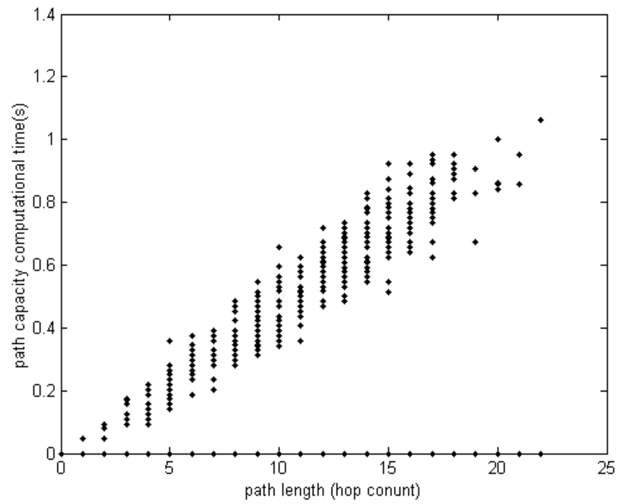

Fig. 8. Path capacity computational time

much worse performance than all other algorithms. Second, min-CTT always finds a path which has the largest path capacity among paths found by all algorithms. Third, min-LCTT almost has the same performance as min-CTT for all pairs of source-destinations. Fourth, min-delay routing algorithm can only find a path with a capacity equal to that found by min-CTT when the source-destination distance is less than 2000 meters, and the path capacity is 10% lower than that found by min-CTT or min-LCTT otherwise. Furthermore, max-rate and max-BDiP routing algorithms can find paths with capacities several times of that found by min-hop algorithms, but up to 60% less than that found by min-CTT and min-LCTT routing algorithms.

Fig. 6 shows the hop count of paths found by these routing algorithms. Apparently, min-hop routing algorithm finds the path with the smallest hop count. Max-rate and max-BDiP routing algorithms often find paths with a very large hop count. Min-delay, min-CTT, and min-LCTT routing algorithms find paths with similar hop counts.

Fig. 7 shows the source-destination distance for all the source-destination pairs. This distance ranges from about 0m to 3000m. It is meaningful when comparing with other figures. For example, when the source-destination distance is larger than 2000m, min-hop routing algorithm finds paths with 4 or more hops, min-delay, min-CTT and min-LCTT routing algorithms finds paths with 7 or more hops, and min-CTT and min-LCTT find paths with capacities significantly higher than those found by other routing algorithms.

Fig. 8 shows the computational time of the path capacities problem defined in Equation (14) for all paths found by these routing algorithms. Since this problem requires the information of all the independent sets, the computational time also includes the time to find all the independent sets for all the links of the considered path. Each point shows a computational time for one path. We can observe that the computational time almost linearly increases with the number of hop count of paths. It illustrates that the path capacity problem can be solved in a short time when the hop count is less than 22.

Table II shows the path finding time and the path capacity computational time for all the routing algorithms. The values in the table are aggregate values for all 399 paths. We can observe that max-CTT has a much larger value of path finding time because there is no polynomial algorithm to calculate CTT. Other routing algorithms have a reasonable path finding time. Path capacity computational time is approximately linear to the hop count which is shown in Fig. 6.

TABLE II

RUN TIME OF DIFFERENT ROUTING ALGORITHMS

Algorithm	Path finding time(s)	Capacity computational time(s)
min-hop	1.9840	85.7190
min-delay	10.8280	140.1250
max-rate	4.2030	275.6880
max-BDiP	12.0160	201.6710
min-maxLCTT	24.8750	155.2660
min-maxCTT	289.3130	164.8590

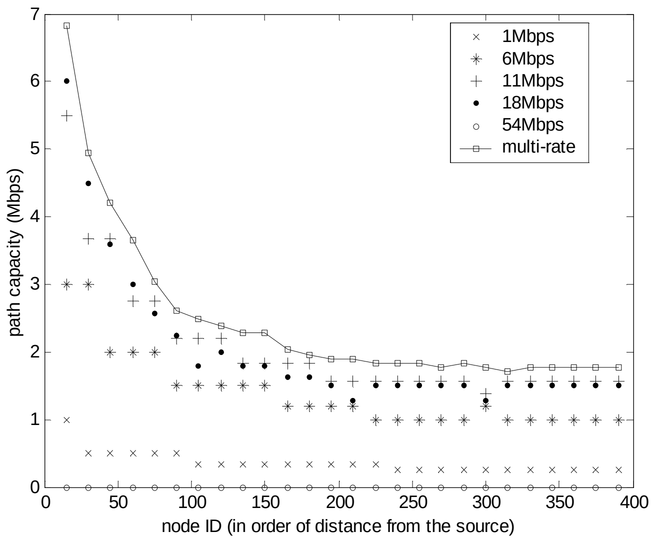

Fig. 9. Path capacity for a single rate network

D. Path Capacity of a Single-Rate Network

In this subsection, we illustrate that if an appropriate routing metric is used, better end-to-end throughput can be achieved by allowing multiple rates at each node, which may not be the case when hop count is used as the routing metric [7]. The topology is the same with that in the above subsection. Min-CTT routing algorithm is used because it can always find a path with higher throughput. Only a single link rate, 1, 6, 11, 18, or 54 Mbps, is allowed in the single-rate scenario. We compare the results from single-rate scenarios with the scenario where all these five link rates are allowed. Notice that in the single-rate scenarios, a scenario using a lower link rate has more links in the network because a lower link rate has a larger transmission range.

Fig. 9 shows the path capacities found for all these scenarios. Apparently, much higher path capacity can be found in the multirate scenario than all the single-rate scenarios. Notice that, if only 54 Mbps is allowed in the network, the network is partitioned into many parts and there is often no feasible path between a source and a destination. Therefore, path capacity is equal to zero for the scenario with 54 Mbps in this topology.

VI. CONCLUSIONS

In this paper, we first investigate the impact of multirate capability and interference on the path capacity, and formulate a linear programming to solve the path capacity of a given path in a wireless multirate and multihop ad hoc network. A new routing metric called the *interference clique transmission time* is proposed to find a path with higher throughput than previously proposed routing metrics. A joint routing and MAC scheduling problem is also formulated to address the impact of multirate and interference in a wireless multirate and multihop ad hoc network, which provides the maximum of path capacities of paths found by all routing algorithms. The routing metrics, interference clique transmission time, hop count, link rate, end-to-end transmission delay, and bandwidth distance product, are evaluated in a random topology. The results demonstrate that interference clique transmission time is the best routing metric to find a path with much higher path capacity than other routing metrics. It also finds paths with path capacity equal to the optimum one found by the joint optimization problem in the simulated topology.

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