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Impact of Virtual Reality on the Visualization of Partial Derivatives in a Multivariable Calculus Class

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ABSTRACT Multivariable calculus is one of the core subjects which engineering students study at university. Students may end up memorizing formulae in multivariable calculus due to the lack of ability in visualizing 3D surfaces, and hence not gain much intuition for the subject. We developed an in-house virtual reality (VR) application with the intention for students to visualize concepts in Multivariable Calculus. In order to evaluate the effectiveness of the VR application (which we term as the treatment), we performed a blinded randomized controlled trial with $n = 312$ students, where we divided them into a control group of $n_{\mathrm{CO}} = 187$ students and treatment group of $n_{\mathrm{TR}} = 125$ students. We gave both groups of students a test immediately after the treatment, as well as asking them to fill in anonymous survey questions using a Likert scale. Our findings show that students perform worse on some questions after using the VR application, and for some other questions students have similar performance to the treatment group. We hypothesize some reasons why this is so, opening the door for future research. We also give recommendations for future developers of VR applications.

INDEX TERMS Education, interdisciplinary, multivariable calculus, smart classroom, virtual reality.

I. INTRODUCTION

Multivariable calculus is one of the first math courses taught in most engineering programmes in higher education. For many students, visualizing objects in three dimensions can prove challenging, affecting their conceptual understanding of the subject [1], [2]. Moreover, some students enter engineering programmes having a weak grasp of the concept of functions [3]–[5], hence moving on from one-variable functions to two-variable functions requires more scaffolding [6], [7]. The difficulty is especially evident when explanations are illustrated with still images, like those on slides or textbooks [8]–[11].

Animated or interactive plots on computers, where a three-dimensional object can be rotated and seen from different angles, as well as physical models, may constitute better tools to present the geometric ideas behind calculus. Recent techniques such as virtual and augmented reality have also been tested for the same purpose [12]–[16] as computing power becomes more ubiquitous.

In this work, we aim to test the effectiveness of VR as a medium to visualize the partial derivatives of two-variable functions, and to see if we could replace part of the standard classroom environment with a flipped classroom [17] with the aid of VR. For this we designed a virtual environment where students can interact with 3D graphs to explore the geometrical meaning of partial derivatives, and compared their learning results against traditional lecture and classroom learning without VR.

This is an ongoing project, as we plan to continuously refine our VR environment over the years based on feedback as well as the results of ongoing experimental studies, hence built it in-house, with the main goal to allow students to learn multivariable calculus at their own pace. Hence we do not use other VR applications for our purposes.

In this paper, we term students who have exposure to the VR medium as being in the treatment group, and students who have exposure to ordinary teaching as being in the

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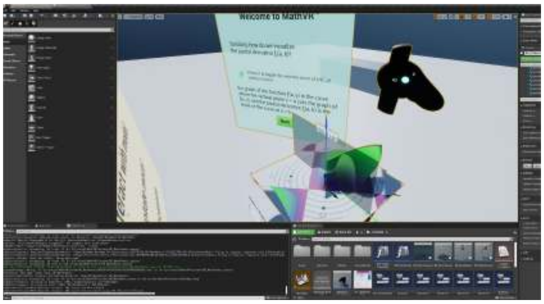

FIGURE 1. **Unreal Engine 4.**

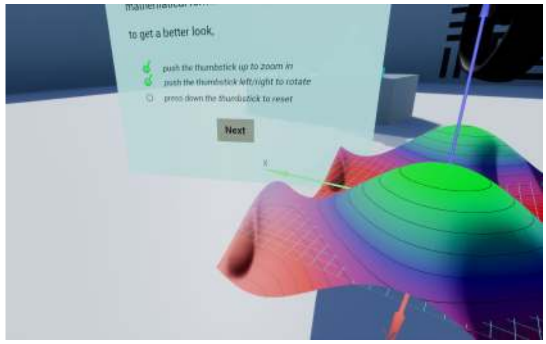

FIGURE 2. **Tutorial with tasks to introduce VR controls.**

control group. To the best of our knowledge, there are no experimental studies which compare the performance of an entire cohort of undergraduate students across the same intake when divided into a treatment group and a control group for multivariable calculus. We believe this is because of two reasons. One - ethical issues in giving a treatment which could be beneficial to students in both groups. Two - manpower issues which may prevent students in the control group from receiving a treatment which is ''provably just as good'' as the students in the treatment group. However, the way which the university the authors are from (Singapore University of Technology and Design) has a unique structure of classes which allows them to do such research.

Every student in our undergraduate cohort goes through one hour of lecture, and two sections consisting of two hours each per week. Each section has two experienced faculty members and an undergraduate teaching assistant assigned to it. The faculty members have PhDs in mathematics or a related field, and have at least one year worth of experience teaching mathematics. This is unlike most research universities where the sections are assigned undergraduate (or graduate) teaching assistants instead.

This allowed us to carry out this experimental study following strong ethical rules. The students in the control group had instruction from experienced instructors without placing any burden on manpower (more on this in Section III-A), and hence had equally effective treatments. Moreover, we gave students in the control group an opportunity to experience the VR environment after the experimental study.

To heighten the comparison with traditional lectures, an important feature of our experimental study was to let students learn by themselves in the virtual set up, without the guidance of an instructor.

II. VR APPLICATION DESIGN

The VR application was developed in Unreal Engine 4 (UE4), an open-sourced 3D game engine. Our target platform was the Oculus Rift, a consumer VR headset with motion controllers that has 6 degrees of freedom tracking.

In the initial stage of technical feasibility check, a method to generate a 3D surface based on any mathematical formula was developed within UE4. Followed by an iterative design cycle, where the user interactions and user interface was explored and optimized. The actual content, specific lesson examples and quiz questions from *Calculus: Single and Multivariable* [18] were then added into the application.

Early on in our prototyping and initial user play-testing, it was apparent that the users did not have sufficient experience with VR devices and applications. Each time, significant effort was required to introduce and guide the user through the VR controls and user interactions. This led to the addition of a tutorial sequence at the start of the VR experience with step by step tasks that guides the user through the different ways they can interact with the 3D surface. Further iterations combined this tutorial with the introduction of basic concepts of partial derivatives, thus streamlining the experience.

With the target platform being full desktop quality headsets, our focus was to design a UI/UX scheme that maximises the potential in interactivity and graphics. This is a shift compared to the other similar software in the market now [19], which are targeted for the mobile smartphone, and accessible to the masses.

The Oculus touch controllers offers 4 buttons and 1 thumbstick of controls per hand (the capacitive touch sensors where intentionally not used to reduced complexity and confusion). Our final design only utilized a single controller as it was sufficient for the basic controls of manipulating the 3D graph, and interact with the menu. Some of the more advanced interactions were left out as we found that they add too much more complexity, and is unnecessary for this initial module. By using only the left controller also avoided the issue of users accidentally exiting the application with the *Oculus Home* button on the right controller.

To facilitate the data collection from each instance of the VR application, a Django server was created to serve as the REST endpoint. Answers to the quiz questions and the time spent on each section were collected for easy consolidation and analysis.

III. EXPERIMENTAL STUDY SETUP

We mention that this research has been cleared by our IRB committee. The multivariable calculus cohort in the semester was split into eight sections of classes with about 50 students each. We used a random number generator to select

20 students from each section to turn up to the VR laboratory, and informed them a week before our experimental study . No students were informed of the details of the experimental study beforehand, in order to prevent students from reading ahead. We only informed students that the class would be split into two. We ran the experimental study over two days, where the first day had students from five sections, and the second day had students from three sections.

Due to some students choosing not to participate in the experimental study , as well as an unforeseen circumstance which caused most of the VR sets to malfunction for one section, we eventually had a control group of $n_{\text{CO}} = 187$ students and treatment group of $n_{\text{TR}} = 125$ students.

The purpose of this experimental study was to assess if a) students perceived VR to be more beneficial in learning, and b) VR was actually more beneficial in learning, where our VR treatment would allow students to visualize surfaces in 3D, as well personalize their own learning by being able to go through examples on their own.

A. TREATMENT FOR CONTROL GROUP

Students in the control group remained in their class sections. The students were given a paper survey printed on both sides. The students were asked to fill in the first side, which consisted of two questions assessing their knowledge in partial derivatives. The questions asked are listed in Appendix A. An experienced faculty member (with at least one year worth of experience in teaching multivariable calculus) taught the class in a fifteen minute segment on how to visualize partial derivatives, identify the sign of partial derivatives given a surface, and how to identify the sign of partial derivatives given a contour map.

In order to ensure the same content of instruction, each instructor in the control group referred to the same set of slides, with pictures taken from the textbook *Calculus: Single and Multivariable* [18] from pages 760-761. Instructors went through Figures 14.3, Figures 14.4, Example 3, and Example 4 from this textbook.

The students then had ten minutes to do an ungraded paper quiz, with ten multiple choice questions. The students did not have the opportunity to discuss the questions with each other. The quiz is reproduced in Appendix B. After taking the quiz, the students filled in the other side of the survey. The questions asked are listed in Appendix C.

The instructors then collected the paper copies of the quizzes and the surveys.

B. TREATMENT FOR TREATMENT GROUP

Students in the treatment group went to the VR laboratory. Similar to the control group, they had to fill out the survey before using the VR application (Appendix A).

The VR application allowed students to set their own pace and to go through the same concepts which the experienced instructors went through in the control group, and further allowed students to visualize and manipulate the surfaces and contour maps in 3D to take control of their own learning.

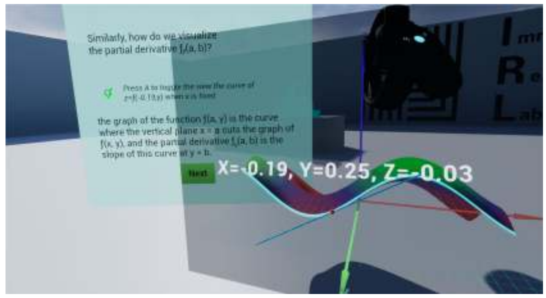

FIGURE 3. A screenshot of the VR application environment, showing the instruction and surface, which students can control dynamically via zooming, rotating and cutting with a vertical plane.

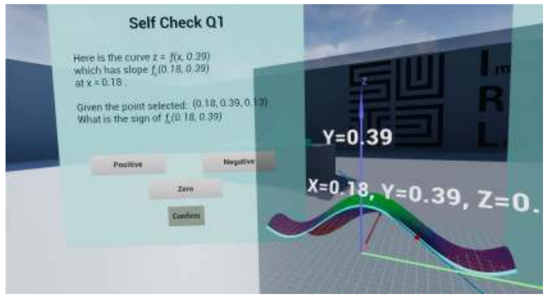

FIGURE 4. An example of a self-test question.

TABLE 1. Summary of t-tests done on survey data.

Null Hypothesis	p-value	95% CI	Conclusion
$\mu_{\text{Q1-CO}} = 0$	2.2e-16	$(0.722, \infty)$	Reject null
$\mu_{\text{Q1-TR}} = 0$	4.0e-13	$(0.577, \infty)$	Reject null
$\mu_{\text{Q2-CO}} = 0$	2.2e-16	$(0.821, \infty)$	Reject null
$\mu_{\text{Q2-TR}} = 0$	2.2e-13	$(0.824, \infty)$	Reject null
$\mu_{\text{Q1-TR}} - \mu_{\text{Q1-CO}} = 0$	0.307	(-0.113,0.358)	Fail to reject null
$\mu_{\text{Q2-TR}} - \mu_{\text{Q2-CO}} = 0$	0.797	(-0.278,0.214)	Fail to reject null
$\mu_{\text{Q2-TR}} - \mu_{\text{Q2-CO}} = 0$	4.6e-08	(0.390,0.812)	Reject null

TABLE 2. Summary statistics of time taken by the treatment group before starting the quiz, and time taken for the quiz.

Time Taken	Min	Q1	Median	Mean	Q3	Max
Before quiz	142	320	415	442	530	1047
For quiz	130	233	302	306	371	642

Students could gesture to any point on the 3D surfaces, and the corresponding slope of the partial derivative f_x (or equivalently f_y) would be displayed. Students could "zoom in", "zoom out", rotate the 3D surfaces, view the partial derivative slope at any point selected and view the projection of the slope onto a contour graph. Figure 3 shows the functionality of the application. Explanations were also provided in the VR application for the student.

The students had the opportunity to be self-tested with basic questions asking them the sign of the partial derivatives on the surface of their choice (with an example in Figure 4), with explanations given if they selected the wrong answer. The students also had the opportunity to start the ungraded

TABLE 3. Data from paper surveys for the control group. See Appendices A and C for the questions.

	1	2	3	4	5
Q1 (before)	21	22	73	61	10
Q1 (after)	4	8	34	90	51
Q2 (before)	29	42	62	44	10
Q2 (after)	6	13	40	89	39
Q3	11	15	49	78	34

TABLE 4. Data from paper surveys for the treatment group. See Appendices A and C for the questions.

	1	2	3	4	5
Q1 (before)	4	19	50	42	10
Q1 (after)	0	5	22	65	33
Q2 (before)	10	27	54	26	8
Q2 (after)	0	6	20	74	25
Q3	0	5	18	51	51

quiz in the VR environment whenever they were familiar with visualizing partial derivatives.

In the ungraded quiz, the students had to do the same ten questions as the students in the control group. The quiz format was mostly the same, although surfaces were in the VR environment (Figures 8 and 10) were displayed in 3D. The option to view the partial derivative slope was disabled during the quiz. The time taken for the quiz was also capped at ten minutes. Students could go back and redo questions during the quiz, similar to a paper test.

After taking the quiz, the students filled in the other side of the paper survey. The instructors in the VR laboratory then collected the paper copies of the surveys.

IV. EVALUATION OF DATA

We display the raw survey data for both groups in Tables 3 and 4 in Appendix D for reference.

We looked at every student and computed the difference for the "*before*" and "*after*" questions in Q1 and Q2. More specifically, if a student had a score of x_1 before undergoing the treatment, and then a score of x_2 after undergoing the treatment, we compute the difference $x_2 - x_1$, which we term the students' perceived effect of the treatment.

We then constructed a box plot in Figure 5 which showed the average students' perceived effect of the treatment in the responses for Q1 and Q2 in the paper survey, as well as the scores in Q3 for both the control group and the treatment group.

From the boxplot, it looks like the students from both groups had on average perceived a positive effect of the respective treatments in helping them understand the geometric meaning and visualizing of partial derivatives. Moreover, the average perceived positive effect was about the same in both groups. The only difference was that students in the treatment group perceived VR as being more helpful than the slides in the control group.

As we selected the students randomly for the treatment group, and with the assumptions that the students did not

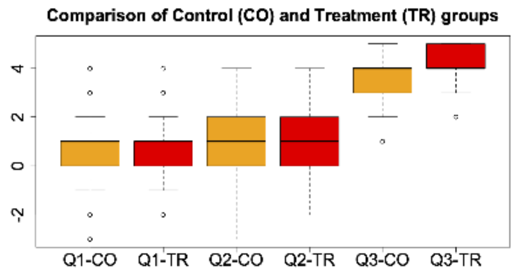

FIGURE 5. Boxplot which compares the students' perceived effect of the treatment in the control group and treatment group.

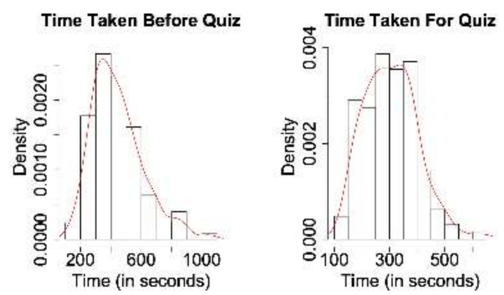

FIGURE 6. Density plots of the time taken by the treatment group before starting the quiz, and time taken for the quiz.

affect each others in writing their survey scores as well as that difference in the recorded scores followed a normal distribution, we set $\mu_{\text{Q1-CO}}$, $\mu_{\text{Q1-TR}}$, $\mu_{\text{Q2-CO}}$, $\mu_{\text{Q2-TR}}$, $\mu_{\text{Q3-CO}}$, $\mu_{\text{Q3-TR}}$ to be the respective recorded means, and further performed

1) four paired one-sided t-tests to test if the means of the perceived differences for each student in both groups were the same, and rejected the null hypothesis that the means of the perceived differences were zero.
2) three two-sample t-tests to test if the means of the perceived differences for each question in the control group and treatment group were equal, and failed to reject the null hypothesis for the first two questions, and rejected the null hypothesis for the last question.

We summarize the t-tests in Table 1, and can only conclude that the students in the treatment group had a higher perceived benefit of VR than the students in the control group had for slides.

We next looked at the performance of the students on the ungraded quiz. The breakdown of the results of the ungraded quiz are in Appendix E.

Somewhat surprisingly, the mean score of the control group was $\mu_{\text{CO}} = 8.90$, and mean score of the treatment group was $\mu_{\text{TR}} = 7.81$.

Under the same assumptions that the students did not collaborate with each other, and that the scores followed a normal distribution, a two-sided, two-sample t-test with the null hypothesis that $\mu_{\text{TR}} - \mu_{\text{CO}} = 0$ was performed. We ended up rejecting the null hypothesis that the means

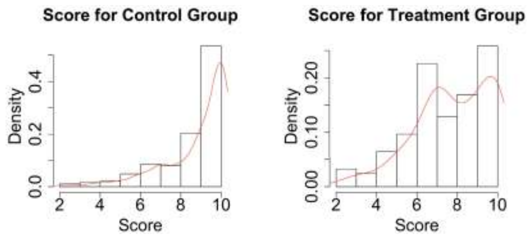

FIGURE 7. Density plots of the scores for the control group and treatment group.

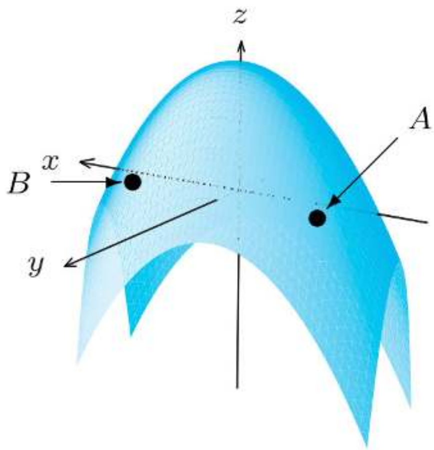

FIGURE 8. Diagram for Question 1 in the quiz for the control group. Source: Figure 14.12, Page 764 in the textbook *Calculus: Single and Multivariable*.

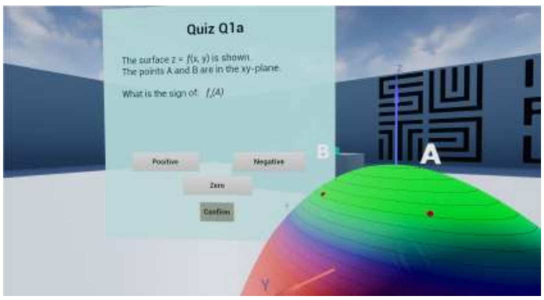

FIGURE 9. Example of diagram for Question 1 in the quiz for the treatment group.

were the same, as we had a p-value of 2.87e-07, with 95% confidence interval $(-1.50, -0.687)$.

V. DISCUSSION

We had hypothesized that students in the treatment group would do better than students in the control group for the test. However, we found out that the students that experienced the VR environment actually did worse.

We thought about this, and came up with the following two hypotheses:

1) In the treatment group, the students had the ability to take the quiz the moment they felt they were ready to do so in the VR environment, as we wanted them to take control of their learning. Conversely, in the control group, the students had to sit through the instructors going through the topic, and take the quiz regardless of whether they were ready.

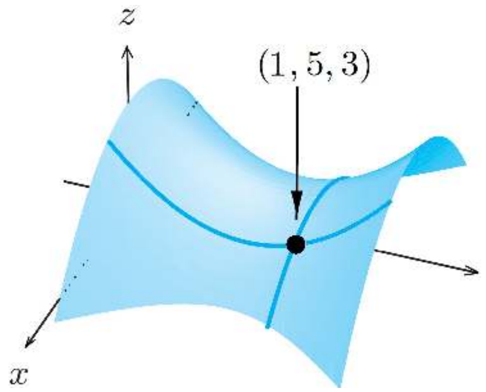

FIGURE 10. Diagram for Question 2 in the quiz for the control group. Source: Figure 14.13, Page 764 in the textbook *Calculus: Single and Multivariable*.

FIGURE 11. Example of diagram for Question 2 in the quiz for the treatment group.

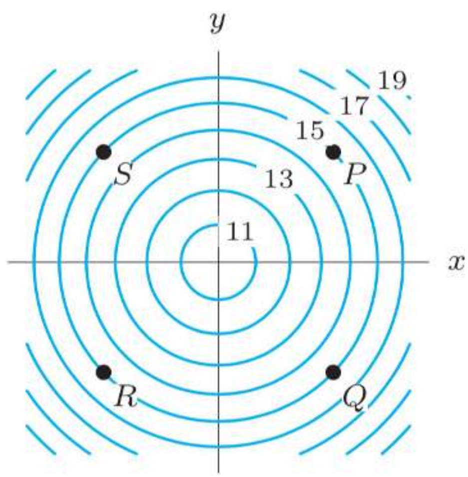

FIGURE 12. Diagram for Question 2 in the quiz for the control group. Source: Figure 14.7, Page 763 in the textbook *Calculus: Single and Multivariable*.

We first hypothesized that the weaker students may have over-estimated their level of understanding, and jumped to do the quiz when they were unprepared.

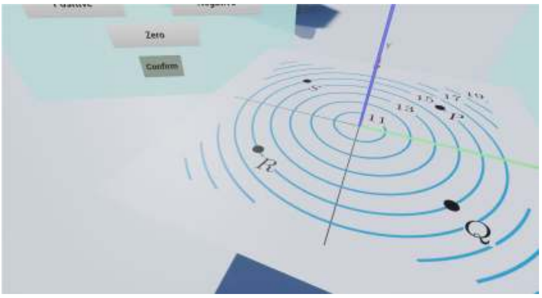

FIGURE 13. Example of diagram for Question 3 in the quiz for the treatment group.

TABLE 5. Recorded answers for the control group. See Appendix B for the questions. Correct answers are bolded.

Questions	-1	0	1
Q1ai)	25	2	**160**
Q1aii)	**164**	3	20
Q1bi)	25	19	**143**
Q1bii)	8	**162**	17
Q2a)	**164**	6	17
Q2b)	6	8	**173**
Q3a)	5	1	**181**
Q3b)	9	2	**176**
Q3c)	12	0	**175**
Q3d)	**167**	1	19

We show density plots in Figure 6 of the time the students spent in the VR application before taking the quiz, and the time taken during the quiz, as well as summary statistics in Table 2.

We see that the students in the treatment group had varying times of learning from the VR application, and varying times taking the quiz. The interquartile range of the time taken for students using the VR application to visualize partial derivatives is from 320 to 530 seconds (about five to nine minutes), and interquartile range of the time taken to take the quiz is from 233 to 371 seconds (about four to six minutes).

On the other hand, students in the control group had about 900 seconds (15 minutes) of teaching from experienced faculty before taking the quiz for 600 seconds (10 minutes).

Hence there could be some weight to our hypothesis that students have over-estimated their level of understanding, and opted to take the quiz earlier.

The implications of this hypothesis would mean that technology in itself cannot be a panacea for understanding advanced mathematics [20], and some form of intervention (whether via better implemented technology, or via in-person diagnosis of the student's ability) is required [21].

2) Looking at Table 6 in Appendix E, it seems that students in the treatment group were more prone to getting the first three questions and the fifth question wrong. In the VR environment, we tried to replicate the textbook questions from *Calculus: Single and Multivariable* as much as possible, which included the different orientation of the axes as in Figures 8 and 10. We next hypothesized that students may have assumed that there would always be one orientation of the axes, and hence may have made more careless mistakes compared to the students in the control group.

We show density plots in Figure 7 of the scores (total number of questions the students answered correctly) in both the control group and treatment group. It seems there are two modes in the treatment group, and this gives some evidence that there are students who consistently get the four questions wrong (due to different orientation of axes).

The implications of this hypothesis would mean that the VR environment by itself may be beneficial by itself (subject to positive results for our tests in future years), and we might be able to use a flipped classroom for this segment, in order to better deploy our instructors for more challenging concepts.

TABLE 6. Recorded answers for the treatment group. See Appendix B for the questions. Correct answers are bolded.

Questions	-1	0	1
Q1ai)	32	16	**77**
Q1aii)	**79**	13	33
Q1bi)	23	42	**60**
Q1bii)	7	**110**	8
Q2a)	**87**	5	33
Q2b)	11	8	**106**
Q3a)	3	3	**119**
Q3b)	6	0	**119**
Q3c)	8	3	**114**
Q3d)	**105**	6	14

Both of the above hypotheses are plausible, but we do not have any strong evidence to advance either (or both) of them. In the next iteration of our VR application, we intend to close up the above gap and resolve these hypotheses. We plan to do so by a) ensuring that the students spend a minimum amount of time in the VR environment before doing a similar quiz, and that b) the quizzes are done on paper, rather than within the VR environment.

VI. CONCLUSION

The results of our experimental study indicate that while VR learning may be perceived as beneficial, it does not necessarily translate in better understanding. Subjects in the treatment group felt that their understanding and visualization skills improved thanks to the VR application, but did not perform as well as the control group in the quiz.

Beside the technical modifications discussed in Section V above, this study suggests that VR is not always suitable as a replacement to lectures and classroom learning – at least not in the early stages of advanced math education. Instead, using this technology in addition to lectures may be a more appropriate way to exploit its potential.

VII. COMPETING INTERESTS

The authors declare no competing interests.

APPENDIXES

APPENDIX A
SURVEY QUESTIONS BEFORE TREATMENT

Here are the two questions which students in both groups had to do. For each question, students had the option of circling numbers on a scale from 1 to 5.

1) Rate how well you understand the geometric meaning of partial derivatives.
2) Rate your competency in visualizing the partial derivatives.

APPENDIX B
QUIZ FOR BOTH GROUPS

Here are the ten questions from the ungraded quiz which both groups took. The students in the control group took the paper version, with 2D diagrams taken from the textbook *Calculus: Single and Multivariable* [18]. The students in the treatment group took the quiz in the VR application.

Students were given three choices for every question, and the students had to circle the correct answer.

The three choices for the parts in Q1a), Q2, and Q3) were: **Positive**, **Negative**, **Zero**. The three choices for the parts in Q1b) were: **Positive to Negative**, **Negative to Positive**, and **No change**.

1) We are given the surface $z = f(x, y)$, with the points A and B in the xy-plane.
   a) What is the sign of
      i) $f_x(A)$?
      ii) $f_x(B)$?
   b) The point P in the xy-plane moves along a straight line from A to B.
      i) How does the sign of $f_x(P)$ change?
      ii) How does the sign of $f_y(P)$ change?
2) We are given the saddle-shaped surface $z = f(x, y)$.
   a) What is the sign of $f_x(1, 5)$?
   b) What is the sign of $f_y(1, 5)$?
3) We are given the contour diagram of f.
   a) What is the sign of $f_x(P)$?
   b) What is the sign of $f_y(P)$?
   c) What is the sign of $f_x(Q)$?
   d) What is the sign of $f_y(Q)$?

APPENDIX C
SURVEY QUESTIONS AFTER TREATMENT

Students in both groups had to do three questions, of which the first two questions were the same. For each question, students had the option of circling numbers on a scale from 1 to 5.

1) Rate how well you understand the geometric meaning of partial derivatives.
2) Rate your competency in visualizing the partial derivatives.

For students in the control group, they had the following third question

3) Rate how effective the slides were in helping you visualize partial derivatives.

For students in the treatment group, they had the following third question

3) Rate how effective VR was in helping you visualize partial derivatives.

APPENDIX D
BREAKDOWN OF SURVEY QUESTIONS

Here is the tabulated raw data of the survey questions.

APPENDIX E
BREAKDOWN OF QUIZ RESULTS FOR BOTH GROUPS

Here are the quiz breakdowns for both the control group and the treatment group.

We use the following coding:

- −1: For an answer that is **Negative** or **Negative to Positive**.
- 0: For an answer that is **Zero** or **No change**.
- 1: For an answer that is **Positive** or **Positive to Negative**.

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