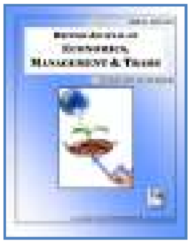

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Impacts of Labor and Capital on Manufacturing Production Function

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Authors' contributions

This work was carried out in collaboration between all authors. They both equally contributed to designing the study, performing the statistical analysis, and writing the protocol and the manuscript including managing the analyses of the study and the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In essence, the power of compounding, together with the effect of discounting, has resulted in calculations of sinking funds and amortization possible. A greater aspect of this concept has allowed the business world to flourish and, at the same time, borrowing, financing, and mortgages have become extensively feasible. In this paper, we will attempt to provide a glimpse of these business concepts using concrete examples and calculations that use mathematical concepts. The Cobb-Douglas production function is given special consideration with a collection of related factors associated with this topic such as elasticity of demand and impacts of labor unions. Furthermore, in terms of the Cobb-Douglas Production Function, the behaviors of $\log\left(f(x,y)/k\right)$ for the different choices of m are significantly linear. An economic scenario in which unskilled and professionals are taken into account, the elasticity of the marginal utility function follows a linear, exponential, and logarithmic relationship for various choices of parameter.

Keywords: Sinking funds; amortization; calculus; future and present values; Cobb-Douglas production function.

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1. INTRODUCTION

The applications of the concepts generally found in calculus textbooks have enormous benefits to those who explore and derive the theory behind business and economic phenomena [1]. It is so effective that every aspect of these ideas such as simple differentiations, the interactive nature of these concepts, and the ability to support and challenge all average citizen out there have an incredible impact on how easily it allows one to comprehend the theory to address the current turbulent market. In addition, substantial experience can be accumulated in modeling and solving of real-world problems through these applications from business to the economy. Equity investment minimizes the debt burden on the business enterprises. In addition, firms increasingly issue shares for the intension of cash savings. This increase is produced by increasing precautionary drives [2]. The tax system promotes this and even accelerates, to a certain degree, the channeling of saving into interest-bearing debt when compounding is possible over time. The formula for the future value of an ordinary annuity has another important application in this regard. For this and others, a few problems are restated in detail to bring this point into consideration [3].A production function exhibits the relationship between inputs of capital, labor, and other factors (if any), as well as the outputs of goods and services in any typical economic environment. The production functions have two main uses in the applications of economics theory. One determines the most efficient combination of factors that a firm can employ and the other is to analyze the relative shares of factors in the output of the production process. Borrowing from this, appropriate analyses are made in the second half of the paper [4].

2. FINANCIAL PRODUCTS

Suppose the parents of a newborn child decide that they will deposit an amount of P in an account on each of the child's birthdays up to the 18th year for college expenses. The account pays an interest rate of 8% compounded annually. We need to find the annual deposit (P) in order for the amount in the account to be \$80,000 after the 18th deposit. Given the future value $(FV), i,$ and n from the formula for the future value, the amount P can be calculated as below.

$$FV = P\frac{(1+i)^n - 1}{i}$$

$$80{,}000 = P\frac{(1.08)^{18} - 1}{0.08}$$

$$P = 80{,}000\ \frac{0.08}{(1.08)^{18} - 1}$$

$$= \$2{,}136.17 \text{ per year}$$

As calculated, an annuity of 18 annual deposits of \$2,136.17 at 8% compounded annually rate will amount to \$80,000 in 18 years.

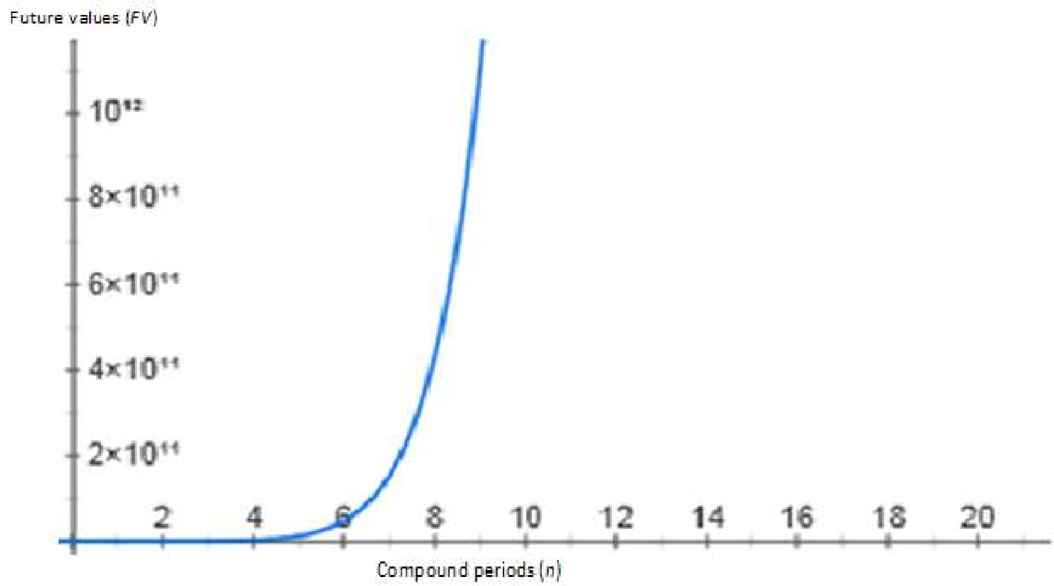

Fig. 1. Future Values as a Function of Compound Periods

From Fig. 1, it is clear that the future value is rapidly increasing over the compound period showing once again that the greater benefit of compounding.

3. SINKING FUND

An annuity is an account created by making a sequence of scheduled payments into or out of itat a given period of time. The most common types of annuities are ordinary annuities and sinking funds. In general, any account that is established for accumulating funds to meet future obligations or debts is called a sinking fund. Other types of annuities are long-term investment accounts, retirement saving accounts, college saving accounts, and lottery payout annuity payments. A sinking fund is a type of fund or an account into which an individual or establishment deposits money on a regular basis in order to repay a debt commitment or other liability that will be due in the future. For example, if one has a loan with a maturity period, one may put aside money into a sinking fund during this period to be prepaid or to pay off the principal when it is due.

If the payments are to be made in the form of an ordinary annuity, then we have only to solve the problem for the sinking fund payment P. In the case of sinking funds, it is a type of annuity in which a specific amount of money in a stated period is invested with a goal amount to be accumulated. If A payments are made n times per year into a sinking fund with interest rate r, compounded n times per year, the payment needed to attain this accumulated fund is described by $P = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^n - 1}$, where A is the accumulated amount. As such, $P = FV\frac{i}{(1+i)^n - 1}$is simply a variation of this formula.We can always find the sinking fund payment by first substituting the appropriate values into the formula and then solving for P.

4. AMORTIZATION

Amortization is a process that divides the payment into the amount of installments that applies to interest and the principal amount of the purchase agreement. Payments close to the beginning of a loan contribute more interest and less principal, than payments towards the end of a loan, mortgage, or purchase [5].

The present value formula for an ordinary annuity has another important use. Suppose that an amount of $7,000 is borrowed from a bank to buy a car and agreed to repay the loan in 36 equal monthly payments, including all interest due. If the bank charges 1.5% per month on the unpaid balance (12% per year compounded monthly), how much should each payment be to eliminate the total debt, including interest in 36 months? Actually, the bank has bought an annuity from the borrower. If the bank pays the borrower $7,000 (present value) for an annuity paying them P per month for 36 months at 12% interest compounded monthly, what are the monthly payments (P)? (Note that the value of the annuity at the end of the 36 months is zero). To find P, we only have to use formula with $PV = \$7{,}000,\ i = 0.015$ and $n = 36$:

$$PV = P\frac{1-(1+i)^{-n}}{i}$$

$$7{,}000 = P\frac{1-(1.015)^{-36}}{0.015}$$

$$P = \$253.07 \text{ per month.}$$

At a payment of $\$253.07$ per month, the car will be yours after 36 months; that is, when you have amortized the debt in 36 equal monthly payments. In general, amortizing a debt means that the debt is retired in a given length of time by equal periodic payments that include compound interest. We are interested in computing the equal periodic payments. Solving the present value formula for P in terms of other variables, we obtain the amortization formula as $P = PV\frac{i}{1-(1+i)^{-n}}$. This formula is simply a variation of the formula above, and either formula can be used to find the periodic payment P.

Computing the payment for a sinking fund is now illustrated. A company estimates that it will have to replace a piece of equipment at a cost of $600,000 in 7 years. To have this money available in 7 years, a sinking fund is established by making equal monthly payments into an account paying 8.5% compounded monthly. The following steps below will explain this calculation.

(A) How much should each payment be? To find P, we choose formula with $FV = \$600{,}000, i = \frac{0.085}{12} = 0.0071$, and $n = 12 \times 7 = 84$:

$$P = FV\frac{i}{(1+i)^n - 1}$$

$$= 600{,}000\frac{0.0071}{(1.0071)^{84} - 1}$$

$$= \$5{,}247.93 \text{ per month}$$

(B) How much interest was earned during the last year? To find the interest earned during the seventh year, we first use the future value formula with $P = \$5{,}247.93$, $I = 0.0071$, and $n = 12 \times 6 = 72$ to find the amount in the account balance after 6 years.

$$FV = P\frac{(1+i)^n - 1}{i}$$

$$= 5{,}247.93\ \frac{(1.0071)^{72} - 1}{0.0071}$$

$$= \$491{,}000.75.$$

If the sinking fund interval is small, then the total future value of all payments can be approximated by $A(n) = \int_0^n Ae^{it}dt = \int_0^{72} 5{,}247.93 \times e^{0.0071\,t}dt = \$493{,}224.66$. This shows that the values obtained from either of these formulas are nearly identical [6].

5. MODELING AVERAGE COST

An example to address this concept adequately is presented next. Given the cost function$C(x) = K + 0.5x^2$, where x is the number of items produced, the graphing strategy to analyze the graph of the average cost function is used. The marginal cost function is plotted on the same set of coordinate axes. The average cost function is described by$\bar{C}(x) = \frac{K+0.5x^2}{x} = \frac{K}{x} + 0.5x$.

For Analysis of$\bar{C}(x)$:

(A) Domain: Since negative values of x do not make sense and $\bar{C}(0)$ is not defined, the domain is the set of positive real numbers.

(B) Asymptotes: $\frac{a_m x^m}{b_n x^n} = \frac{0.5x^2}{x} = 0.5x$. Thus, there is no horizontal asymptote.

Vertical asymptote: The line $x = 0$ is a vertical asymptote since the denominator is 0 and the numerator is not 0 when$x = 0$.

Oblique asymptotes: If a graph approaches a line that is neither horizontal nor vertical as x approaches ∞or $-\infty$, then that line is called an oblique asymptote. If x is a large positive number, then $K/\,x$ is very small and

$\bar{C}(x) = \frac{K}{x} + 0.5x \approx 0.5x$.That is,

$$\lim_{x\to\infty}[\bar{C}(x) - 0.5x] = \lim_{x\to\infty}\frac{K}{x} = 0.$$

This implies that the graph of $y = \bar{C}(x)$ approaches the line $y = 0.5x$ as $x$approaches to ∞. The line is an oblique asymptote for the graph of $y = \bar{C}(x)$.

For Analysis of $\bar{C}'(x)$:

$$\bar{C}'(x) = -\frac{K}{x^2} + 0.5$$
$$= \frac{0.5x^2 - K}{x^2}$$
$$= \frac{0.5(x - \sqrt{2K})(x + \sqrt{2K})}{x^2}$$

The critical value for $\bar{C}(x)$ is $x = \sqrt{2K}$. So $\bar{C}(x)$ is decreasing during the interval $(0, \sqrt{2K})$, is increasing on $(\sqrt{2K}, \infty)$, and has a local minimum at $x = \sqrt{2K}$.

For Analysis of $\bar{C}''(x)$: $\bar{C}''(x) = \frac{\sqrt{2K}}{x^3}$. $\bar{C}''(x)$ is positive for all positive values of x, so that the graph of $y = \bar{C}(x)$ is concave upward on $(0, \infty)$. A sketch of the graph of $\bar{C}$ is shown in Fig. 2.

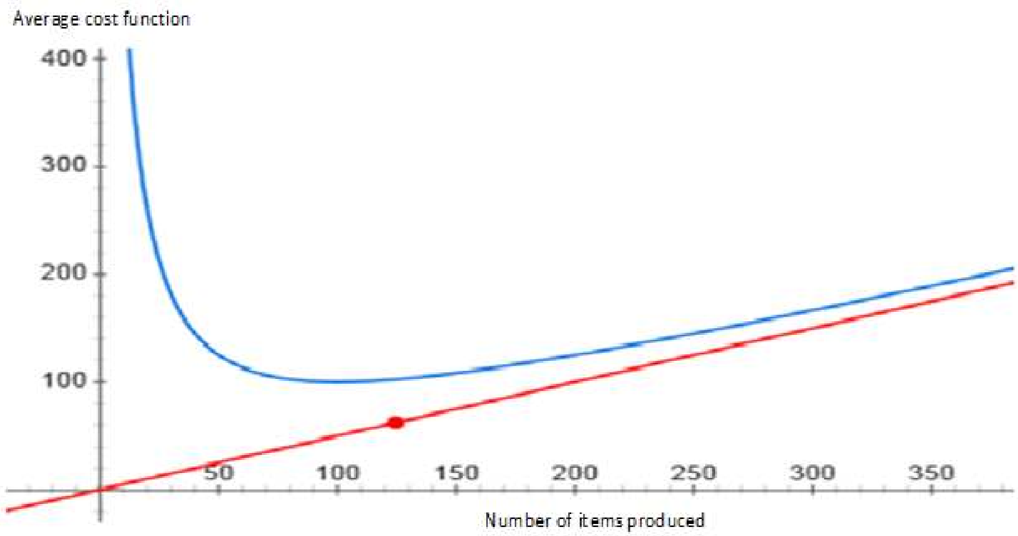

Fig. 2. The Graph of $\bar{C}$ as a function of the number of items produced

$\bar{C}$ is in most part rapidly decreasing before it is asymptotically increasing following the linear equation, $y = 0.5\,x$.

The marginal cost function is $C'(x) = x$ whichexhibits an important principle in economics. In fact, the minimum average cost occurs when the average cost is equal to the marginal cost. It is possible to recover the production function from the cost function [7]. For example, if the cost function is of the form, $C(x, w) = x^2(w_1{}^{1/2} + w_2{}^{1/2})^2$, then the production function corresponding to this cost function is $f(y, z) = \left(\frac{yz}{y+z}\right)^{1/2}$, where y and z are the conditional input demands [8]. In macroeconomics perspective, the output of interest, of course, is Gross Domestic Product (GDP). The technology sector will allow inputs to be substituted for each other, but not at a constant rate. A class of production function that models these situations in which inputs can be substituted for each other at a constant rate is known as a Cobb-Douglas production function.

6. THE COBB-DOUGLAS PRODUCTION FUNCTION

Every business enterprise produces a good or a service. Of course, technologies enable them to flourish in terms of profitability. Roughly speaking, all input for the production included labor, land, and capital. If it is defined specifically, engineering, mechanics, electricity, insurance, water, safety equipment, and services such as postal, telephone, internet, and software products can also be included. The production function is a relationship between those of input and output. The Cobb-Douglas production function is an economic function resulted from the utilization of two variables, x and y, such as labor and capital, and is written as $f(x,y) = kx^m y^n$, where $k, m,$ and$n$ are positive constants with $m + n = 1$. It can be shown that the cost function will have the form, $C(x,w,r) = k^{-\frac{1}{m+n}}\left[\left(\frac{m}{n}\right)^{\frac{n}{m+n}} + \left(\frac{m}{n}\right)^{-\frac{m}{m+n}}\right] w^{\frac{m}{m+n}} \cdot r^{\frac{n}{m+n}} \cdot x^{\frac{1}{m+n}}$. This cost function, derived from the Cobb-Douglas production function, is nonlinear in x unless $m + n = 1$. Under this condition, the cost function will take the form, $C(x,w,r) = \frac{1}{k} w^{\alpha} \cdot r^{1-\alpha} \cdot x$.

In economic theory, the production function for the good and services, together with the physical constraints that the enterprise regularly deals with, will assist to achieve the maximum output possible by employing a combination of inputs. Table 1 is obtained to accommodate $\log(f(x,y)/k)$ considering labor productivity for the$x$ variable and capital productivity as the y variable for China from 1978 to 2001. Note that labor and capital productivities in this analysis are calculated as being the ratios of GDP to labor and capital so that the Cobb-Douglas relation will not hold exactly. In fact, there were large regional variations in labor productivity that seem to have widened overtime [9].

Interest and depreciation are two separate concepts even though one typically depends on the other [10].The depreciation is essentially a decline in the value over a period of time largely due to physical deterioration and obsolescence. Analysis of depreciation by various methods has widespread use. Let $B_t$be the book value of the asset at the end of the t^{th}period. Clearly, $B_0 = A$,is the value of the asset at the beginning of the n period and $B_n = S$,is the salvage value of the asset at the end of the n period. If $D_t$is the depreciation charge for the n^{th} period, then $D_t = B_{t-1} - B_t$.The calculation of depreciation charges is based on the sinking fund method or the compound interest method using the book value $B_t = \left[\frac{A-S}{s_{\overline{n}|j}}\right] \bar{s}|j$. The depreciation charge is $D_t = B_{t-1} - B_t = \left[\frac{A-S}{s_{\overline{n}|j}}\right](1+j)^{t-1}$,where $\bar{s}|j$ denotes the sum of the accumulated values of each payment calculated at the interest rate, j[11]. An exponential decay model can be used to model depreciation as given by$y = Ce^{-kt}, k > 0$, featuring the horizontal asymptote to $y = 0$ to right, passing through $(0, C), C$ is the initial value, and the function is decreasing, but bounded below by $y = 0$. Proportional capital depreciation with depreciation rate δ from $t \to t+1$ can be expressed using the iterative equation,$K_{t+1} = (1-\delta)K_t + I_t$.An equilibrium with constant capital occurs when $K_{t+1} = K_t$ showing that$I_t = \delta K_t$.That is, I_t is a fraction of K_t.As such, the cost of capital with depreciation and interest is,$C(K_t) = (r+\delta)K_t$, where r is the prevailing rate of interest.

Production functions include certain input costs that can be assigned monetary values, such as the stock of labor employed, land used, and stock of real capital that went into the production process. The capital costs mostly comprise physical units are often subject to loosely called "depreciation." A distinction has to be made between "time depreciation" and "output deterioration" of capital during a period. The economists are interested in knowing how much of this loss was due to time depreciation and how much resulted from the use deterioration. If capital and labor employed remain fixed, a production function yielded gross value added to show the distribution of gross income among factor claimants. Furthermore, change resulted from time depreciation rates has an immediate effect on the net incomes. However, the workers' effectiveness is not initially reflected in the production factor calculations [4].

As we know the second-derivative tests in calculus give us sufficient conditions for a critical point to produce a local extremum or a saddle point. For this, calculation of first and second-derivatives of the Cobb-Douglas production function are needed and derived below.

$$f_x = kmx^{m-1}y^{1-m} = km\left(\frac{x}{y}\right)^{m-1} > 0$$

$$f_y = kx^m(1-m)y^{-m} = k(1-m)\left(\frac{x}{y}\right)^{m} > 0$$

$$f_{xx} = km(m-1)x^{m-2}y^{1-m} = km\frac{(m-1)\left(\frac{x}{y}\right)^{m-2}}{y} < 0$$

$$f_{xy} = kmx^{m-1}(1-m)y^{-m} = km\frac{(1-m)\left(\frac{x}{y}\right)^{m}}{x} > 0$$

$$f_{yy} = kx^m(1-m)(-m)y^{-m-1} = -km\frac{(1-m)\left(\frac{x}{y}\right)^{m}}{y} < 0.$$

So that the calculation of $f_{xx}f_{yy} - \left(f_{xy}\right)^2 = 0$. The second-derivative test for local extrema fails requiring further investigation of $f(x,y)$. Further, the marginal products are positive everywhere. From the second derivatives, it can be concluded that the total products are rising at a diminishing rate to show that the marginal products are declining. The marginal productivity of capital and labor if the price is p and capital has cost r per unit capital, and w is the hourly wage, we have $r = pf_x$ and $w = pf_y$. Consequently, the total cost = total capital cost + total labor cost. We now consider a logarithmic production function instead. Thus, we obtain from the Cobb-Douglas production function, $\log f(x,y) = \log k + m\log x + (1-m)\log y$and so that $\log(f(x,y)/k) = m\log x + (1-m)\log y$. Since, for a given value of m, $\log(f(x,y)/k)$ is linear with respect to time variable t, we have $\log(f(x,y)/k) = \alpha + \beta(t-t_0)$, $\beta > 0$. And, also $f(x,y) = ke^{\alpha+\beta(t-t_0)} = \gamma e^{\beta t}$with $\gamma = ke^{\alpha-\beta t_0}$.This allows us, to obtain either x or y from each other, if the values of t are already known. For example, $x = \sqrt[m]{Ay^{1-m}e^{\beta t}} = By^{(1-m)/m}e^{\beta t/m}$,where A, B, and β are known constants. One observation made from Fig. 3 is that for a fixed year, the values of $\log(f(x,y)/k)$ for small increment of m have the same differencesthat can be easily explained. For a small increment ofα, $0 < \alpha < 1$, we have, the difference between two values of $\log(f(x,y)/k)$ at $m = m + \alpha$ and $m = m$is$(m+\alpha)\log x + (1-m-\alpha)\log y - m\log x - (1-m)\log y = \alpha(\log x - \log y) = \alpha\log\left(\frac{x}{y}\right)$, showing that the difference remains constant and in the same time, it is independent of m.Using the data found in [8], Table 1 has been constructed for the values of m as specified to verify this observation. For simple national economic models, the production function includes household consumption, government spending, and investment. In consideration of macroeconomic models, waste is generally not included. Thus, the national income is $Y = C + G + I$, where C is household consumption, G is government spending, and $I$is investment. The total output of an economy is an aggregate production function, with output as a function of the factors of production. However, economists usually consider the function with two factors, labor and capital for production, but it is more accurate as to include land as a separate factor, $Y = f(N, K, L)$, where Nstands for the number of workers so that the logarithmic equivalent of production function is $\log(f(x, y, z)/k) = m\log x + n\log y + (1 - m - n)\log z$,where $n, m > 0$ and $n + m < 1$.

Table 1. Values of $log(f(x, y)/k)$over choices of m

$log(f(x,y)/k)$								
m = 0.10	**m = 0.20**	**m = 0.30**	**m = 0.40**	**m = 0.50**	**m = 0.60**	**m = 0.70**	**m = 0.80**	**m = 0.90**
1.69452	1.83275	1.97097	2.10919	2.24741	2.38563	2.52385	2.66208	2.80030
1.70781	1.84741	1.98702	2.12662	2.26623	2.40584	2.54544	2.68505	2.82465
1.72066	1.86154	2.00242	2.14329	2.28417	2.42505	2.56593	2.70680	2.84768
1.72132	1.86287	2.00441	2.14595	2.28749	2.42903	2.57057	2.71211	2.85366
1.72376	1.86773	2.01171	2.15568	2.29966	2.44363	2.58761	2.73158	2.87556
1.74689	1.89172	2.03656	2.18139	2.32622	2.47105	2.61588	2.76072	2.90555
1.77096	1.91867	2.06639	2.21410	2.36181	2.50952	2.65723	2.80495	2.95266
1.78486	1.93625	2.08764	2.23903	2.39042	2.54181	2.69320	2.84459	2.99598
1.77772	1.93218	2.08665	2.24111	2.39558	2.55004	2.70451	2.85898	3.01344
1.78134	1.93942	2.09751	2.25560	2.41368	2.57177	2.72986	2.88795	3.04603
1.79445	1.95543	2.11641	2.27740	2.43838	2.59936	2.76034	2.92132	3.08230
1.78631	1.94936	2.11242	2.27548	2.43854	2.60159	2.76465	2.92771	3.09077
1.77801	1.94324	2.10846	2.27369	2.43892	2.60415	2.76937	2.93460	3.09983
1.79028	1.95732	2.12435	2.29138	2.45842	2.62545	2.79248	2.95952	3.12655
1.82334	1.99347	2.16360	2.33372	2.50385	2.67398	2.84411	3.01424	3.18436
1.86335	2.03651	2.20967	2.38283	2.55598	2.72914	2.90230	3.07545	3.24861
1.89201	2.06802	2.24403	2.42004	2.59606	2.77207	2.94808	3.12409	3.30010
1.90443	2.08459	2.26474	2.44489	2.62505	2.80520	2.98536	3.16551	3.34567
1.91608	2.09977	2.28345	2.46714	2.65083	2.83451	3.01820	3.20188	3.38557
1.91992	2.10745	2.29498	2.48250	2.67003	2.85756	3.04509	3.23261	3.42014
1.92508	2.11777	2.31045	2.50314	2.69582	2.88851	3.08119	3.27388	3.46656
1.92831	2.12422	2.32013	2.51605	2.71196	2.90787	3.10378	3.29970	3.49561
1.91592	2.11584	2.31575	2.51567	2.71559	2.91550	3.11542	3.31533	3.51525
1.92185	2.12769	2.33354	2.53938	2.74522	2.95107	3.15691	3.36276	3.56860

Fig. 3 provides the evaluations of the Cobb-Douglas function over the different choices of m from 1978 to 2000.

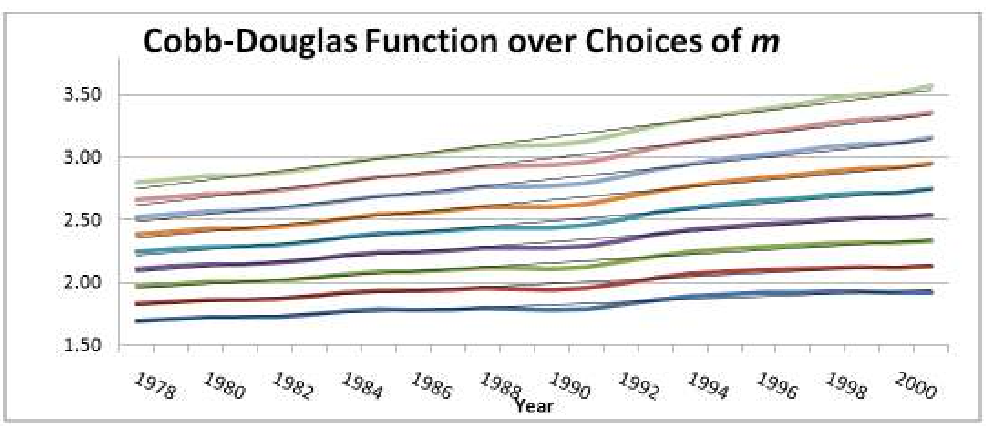

Fig. 3. Values of Cobb-Douglas function over choices of m

The contents of lines in Fig. 3 are nothing but the behaviors of $\log\left(f(x,y)/k\right)$ over the period from 1978 to 2001 by taking into account the different choices of m. These behaviors are almost linear providing a significant linear relationship among the variables under consideration over the parameter.

7. ELASTICITY OF DEMAND

It is natural to believe that a price increase leads to an increase in revenue. Economists use the notion of elasticity of demand to address relationships among price, demand, and revenue. The price elasticity of demand measures the extent of responsiveness or sensitivity of consumers to changes in the price of a good or service. Consumer responsiveness to a price change is measured by the price elasticity of demand (E), defined by the relationship, $E = \frac{\%\text{ of}\Delta Q}{\%\text{ of}\Delta P}$, which is the percentage change in quantity demanded (Q)against percentage change in price(P). The relative rate of change of a function $f(x)$ is defined as $\frac{f(x)'}{f(x)} = \frac{d}{dx}\ln f(x)$. Because of this equivalency, this is also referred to as the logarithmic derivative of $f(x)$. If the price p and demand x are related by $x = f(p)$, then the elasticity of demand as a function of p is given by $E(p) = -\frac{pf'(p)}{f(p)} = -p\frac{d}{dp}\ln f(p)$. Either a price increase will decrease revenue or a price decrease will increase revenue when demand is elastic so that $E(p) > 1$. This easily follows from the revenue as a function of $p, R(p) = x \cdot p = p \cdot f(p)$so that $R'(p) = pf'(p) + f(p) = f(p)[1 - E(p)]$. In the case of inelastic demand $(E(p) < 1)$, a price increase or decrease follows revenue increase or decrease, respectively. Furthermore, the Constant Elasticity of Substitution (CES) production function is a type of production function that exhibits constant elasticity of substitution that inherits a property typical of production functions and utility functions [12]. Recently, much has been focused on the substitution elasticity between labor and capital [13]. Borrowing the commonly used notations, Q being output, F being factory production $(F > 0)$, a being the share parameter $(0 < a < 1)$, $K,$ and$L$ being primary production factors such as capital and labor, and $-1 < r = \frac{(s-1)}{s} \neq 0$, in which $s$representsthe elasticity of substitution, this takes the form, $Q = F \cdot (a \cdot K^r + (1-a) \cdot L^r)^{\frac{1}{r}}$.Evidently, isoquants of the CES production functions are negatively sloped and strictly convex towards the original. One thing to note is that a scalar multiplier of $K$and Lwill produce a same multiple of the production output.

Apair of two market combinations, $A = (p,r)$ and $B = (q,r)$, where $p < q$is shown as points A and Bin Fig. 4. If X is considered a normal good, Marginal Rate of Substitution (MRS) is the slope of the indifference curve, that must be greater at B. If not, the consumer would not purchase more X at higher income. If X is an inferior good, MRSat point B would be less than at point A.

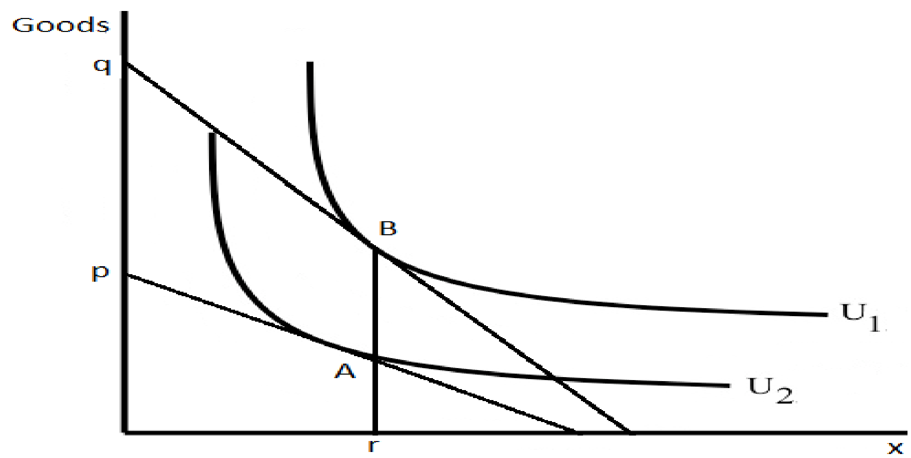

Fig. 4. Marginal Rate of Substitution (MRS) for Two Market Combinations

Measuring the degree of substitutability between any pair of factors is an interest to many economists. The most famous, the elasticity of substitution has been introduced independently by John R. Hicks and Joan V. Robinson [14,15,16,17]. In fact, the elasticity of substitution measures the percentage change in factor proportions due to a change in the marginal rate of the technical substitution. If the productions function is $f(K,L)$, then the elasticity of substitution between capital (K) and labor (L) is given by $\sigma = d\ln(L/K)/d\ln(f_K/f_L) = [d(L/K)/d(f_K/f_L)] \times [(f_K/f_L)(L/K)]$.

8. IMPACTS OF LABOR UNIONS

In some situations, exorbitant benefits and retirement packages for employees are negotiated with employers by labor unions. American adults also believe that, by joining labor unions, the living standards of workers in the US would be greater than that of the workforce in other countries. The impact of wage rates and employment in the union and nonunion sectors is worth considering in addressing this issue. The theory will provide some answers to many questions.

The formation of a labor union increases the wages and rights in the union sector, but only at the cost of employment reduction as some have already argued. Workers who are unable to find a job in the union sector would increase the workforce in the nonunion sector and as such, wages will dramatically go down in the nonunion sector as observed in Fig. 5. This certainly impacts the underlying economic reality. The union negotiated benefit and pension plans appear to consume a large portion of the state budget. The impact of labor unions on wages and employment in the union combined with nonunion sectors is vast and potentially contributes to the severe crisis being unfold.

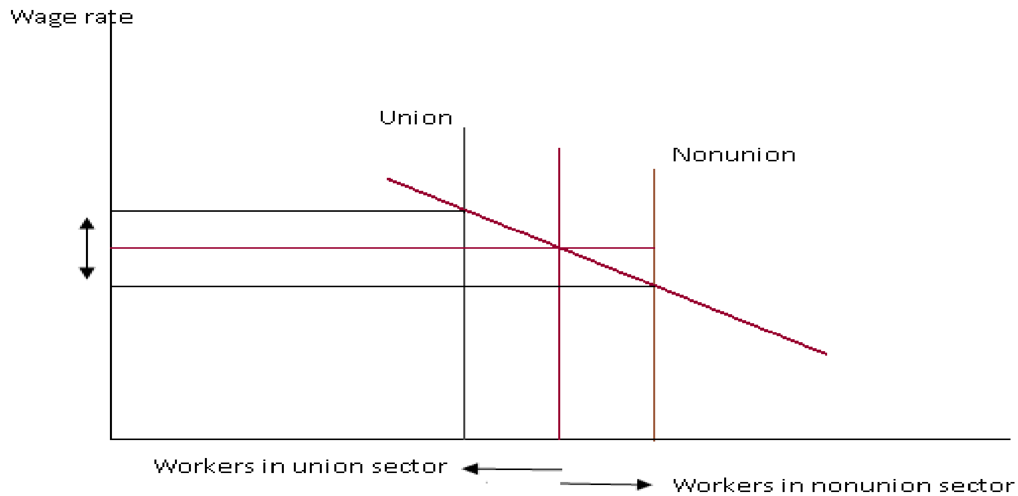

Fig. 5. Union and nonunion sectors vs. wage rates

The effect of labor unions on productivity has been extensively studied. There has been a study using a two-step approach [18]. One of them considered the economic influence of unionization on labor productivity. Other stage is the study on estimation of a production function model of plants using the transcendental logarithmic production function, $\ln\left(\frac{Q}{L}\right) = \ln C + \alpha_1 \ln\left(\frac{K}{L}\right) + \alpha_2 \ln L + \gamma_1 (\ln K)^2 + \gamma_2 \ln K \ln L + \gamma_3 (\ln L)^2 + \sum_{i=1}^{n} \beta_i Z$,where $QIL =$ productivity (quantity output/labor hours); $C =$ constant; $K =$ capital (rentals and depreciation of assets); $L =$ labor (labor hours of direct production workers); and $Z =$ control variables, including: ln of management, ln of the employee turnover rate, ln of absenteeism rate etc. It is found that unionization exerted a positive effect on labor productivity and the unionized plants, absenteeism and negative turnover on productivity rose as compared to that of the nonunionized plants [18].

If an economy consisted of unskilled workforce (L) and professionals (S), then elasticity of the marginal utility is $\int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt,$ where $C(t)$is consumption at time $t$and $\theta \geq 0$ [19]. Using the integration by parts formula, the latter will be equivalent to

$$\frac{1}{1-\theta}\int_0^\infty \left(C(t)^{1-\theta}-1\right)\, d\left(\frac{e^{-\rho t}}{-\rho}\right) = \frac{1}{-\rho(1-\theta)}\left\{e^{-\rho t}\left(C(t)^{1-\theta}-1\right) - \int_0^\infty e^{-\rho t} d\left(C(t)^{1-\theta}-1\right)\right\}$$

$$= \frac{1}{-\rho(1-\theta)}\left\{e^{-\rho t}\left(C(t)^{1-\theta}-1\right) - \int_0^\infty e^{-\rho t}(1-\theta)C(t)^{-\theta}C'(t)dt\right\}$$

$$= \frac{1}{-\rho(1-\theta)}\left\{e^{-\rho t}\left(C(t)^{1-\theta}-1\right) - (1-\theta)\int_0^\infty C(t)^{1-\theta}e^{-\rho t}\frac{C'(t)}{C(t)}dt\right\}$$

$$= \frac{1}{-\rho(1-\theta)}\left\{e^{-\rho t}\left(C(t)^{1-\theta}-1\right) - (1-\theta)\int_0^\infty C(t)^{1-\theta}e^{-\rho t}d(\log C(t))\right\}$$

$$= \frac{1}{-\rho(1-\theta)}\Bigg\{e^{-\rho t}\big(C(t)^{1-\theta}-1\big) - (1-\theta)\Bigg\{C(t)^{1-\theta}e^{-\rho t}\log C(t) - \int_0^{\infty}\log C(t)\,d\big(C(t)^{1-\theta}e^{-\rho t}\big)\Bigg\}\Bigg\}.$$

If $\theta = 0$, then the utility function is linear, and the representative agent is risk-neutral. The utility function is logarithmic as $\theta \to 1$.From above, if $C(t)^{1-\theta}e^{-\rho t}$ is a constant, the third term vanishes, thus giving us$C(t) = C_0 e^{\left(\frac{\rho}{1-\theta}\right)t}$which is exponentialas $t \to \infty$. Fig. 6 provides all of these scenarios as $t \to \infty$.

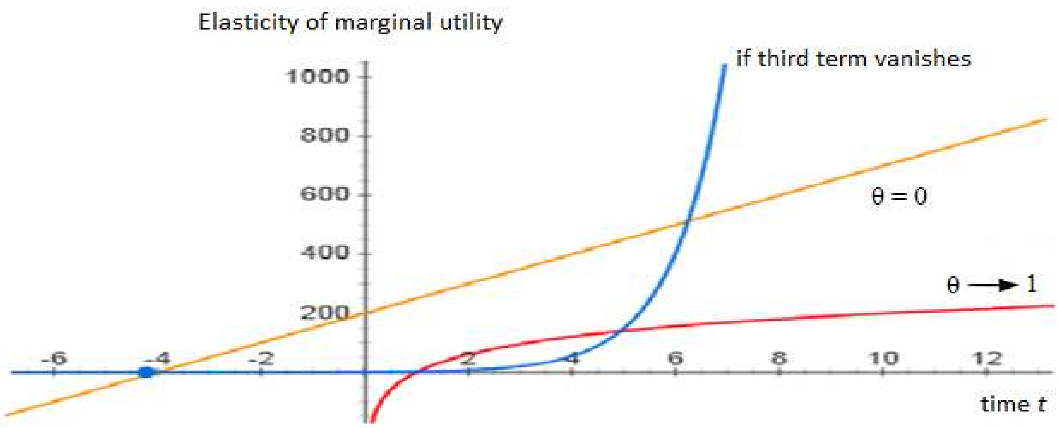

Fig. 6. All possibilities of utility functions for all θ as $t \to \infty$

9. CONCLUSIONS

This paper has provided a glimpse of concepts generally found in economic literature related to labor, capital, and production function. The themes require rigorous explanations and arguments to be understood by the general public unless assisted by the mathematical theories. It is noted throughout the first part of the paper from the calculations of financial products such as sinking fund, amortization, and also, the modeling average cost that the compounding greatly assisted these financial concepts to be viable and to exist in the same time. In terms of the Cobb-Douglas Production Function, the behaviors of $\log\big(f(x,y)/k\big)$ for the different choices of m are significantly linear. An economic scenario in which unskilled and professionals are taken into account, the elasticity of the marginal utility function follows a linear, exponential, and logarithmic relationship as $t \to \infty$ for various values of θ as pointed out.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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