Imperfect competition in differentiated credit contract markets

Naoki Kojima*

University of Freiburg

Department of Economics

Platz der Alten Synagoge, KG II,

79085, Freiburg, Germany

Phone: +49 (0)761 203 6888

Fax: +49 (0)761 203 2375

E-mail: naoki.kojima@vwl.uni-freiburg.de

Abstract

This paper studies a duopolistic credit market in which borrowers differ in risk. In our competition game, one lender is in an advantaged position with respect to the other due to past relations with the borrowers. We investigate the features of the equilibrium contract and show that the best borrower is indifferent between the dominant and the opponent lenders' contract while the other borrowers prefer that of the dominant lender. Also, repayment and collateral do not depend upon the borrowers' respective project risk.

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1 Introduction

A great amount of the literature on credit markets has focused on a perfectly competitive banking environment. In practice, however, a given firm has only one or a few banks with which it has closer and more regular business relations. The archetype is the Japanese main bank system. The main bank has a long history of transaction with the firm, through which the bank has invested not a small amount of money to gather information on the firm's management. The main bank is in an advantaged position with respect to other potential lenders in competition for a credit contract because it has better information on the firm and thus cost advantages in screening and monitoring the firm's project.

On the subject of competitive adverse selection, Rochet and Stole (1997) have analyzed two competing sellers in Hotelling's environment and Villas-Boas and Schmidt-Mohr (1999) have investigated the credit contract in the same context. Finally, Champsaur and Rochet (1989) have studied two firms' quality price competition. The present paper analyzes the competition described above via a variant of asymmetric Bertrand competition. Two banks, simultaneously, offer heterogeneous firms incentive contracts composed of repayment and collateral. One lender has closer relations with the firms[1] and thus incurs less costs in screening the firms' respective investment project. The cost-advantaged bank, unsurprisingly, holds a dominant position in this competition. To win a contract, however, it must assure the borrower, at least, of the utility level offered by his competitor's contract. Thus, competition necessarily involves the issue of type-dependent reservation utility. We know—see Lewis and Sappington (1989), Maggi and

[1] As stated above, due at least in part to past investment in information gathering.

Rodriguez-Clare (1995)—that when the reservation utility is type-dependent, there are two countervailing incentives on the part of the agent, namely: understating and overstating. In contrast to standard adverse selection theory, we are uncertain in such a case for which type the participation constraint binds. We shall find that the participation constraint binds at the best type. Bad firms have good cause to resort to their regular bank because the contract terms proposed by the other bank are so disadvantageous. Better firms prefer trading with the alternative bank.

One feature of the literature on type-dependent reservation utility is that the principal is often assumed to provide utility greater than reservation utility, i.e. $U \geq R$ on the whole interval of types.[2] In our context, this assumption is unrealistic. The dominant bank might not necessarily serve all firms. If the competitor offers some customers high utility, the dominant bank may prefer to exclude those customers to extract more rent from the others. Here we obtain, however, that the dominant bank provides all types with a utility at least as high as that provided by the opponent.

The existing literature on credit contracts consists, in general, of two strands: one assuming a perfectly competitive environment (Stiglitz and Weiss (1981), Bester (1985), Bester (1987), Besanko and Thakor (1987)) and the other a monopolistic environment (Besanko and Thakor (1987) and Schmidt-Mohr (1997)). This article's merit is that it assumes the middle ground of the two situations: it deals with a screening contract in imperfect competition.

The paper is novel with respect to the literature on relationship banking as well. Sharpe (1990) has shown that even perfectly competitive banking provides an initial lender with monopolistic power over time since the lender can learn about the borrower's characteristics. Rajan (1992) examined the pros and cons of relationship lending as opposed to capital market debt for a borrower. Boot and Thakor (2000) investigated

[2]Works on competition with incentive contracts naturally consider the possibility of type exclusion (e.g. Champsaur and Rochet (1989)). However, the literature on type-dependent reservation utility such as Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995) assumes the overall type inclusion. The only exception is Jullien (2000).

the effects of competitive pressure from capital markets upon relationship lending and demonstrated that banks might enhance relationship lending activity. Closest in the spirit to the present paper is Dell'Ariccia, Friedman, and Marquez (1999), who studied the competition between two lenders in relationship banking.

The existing literature on relationship banking rules out the scenario that a bank constructs an incentive mechanism as a loan contract, on the assumption that after the first lending, the bank learns all information about the borrower. We analyze the situation in which a bank, even in an established relationship with a borrower, is subject to asymmetric information and constructs an incentive mechanism to overcome the asymmetry.

Collateral is seldom incorporated in studies of relationship banking. None of the above papers includes collateral in its investigation. Boot and Thakor (1994) is probably the only work to examine collateral, but it does so in the context of moral hazard and not of adverse selection. We find in our model of competition that the lenders' requirement for collateral and reimbursement is independent of the borrower's respective project risk. In practice, banks often grant their borrowers a line of credit, in which the conditions of the loan are prespecified; banks do not examine each and every loan application of the borrowers. The finding of this paper confirms this.

In the next section, we present the model and review the monopolistic case. The third section deals with our main objective, duopolistic competition.

2 The model

All of our agents are risk-neutral. There is a firm with a risky investment project requiring investment cost I.[3] The project succeeds or fails with probabilities p and $1-p$, respectively. p takes its value in the non-empty interval $[\underline{p}, \overline{p}] \subset (0, 1)$. The investment

[3]Alternatively, we can suppose that there are firms different in their project risks, the total mass of which is 1.

cost I is identical for all p. In case of success, the risky project yields the return X and in case of failure, none. The firm is assumed to lack liquidity I and thus resort to a bank in order to finance its project. The distribution of p is public knowledge. $F(p)$ is the absolutely continuous distribution function and $f(p)$ the density function with $f(p) > 0$ on the whole interval $[\underline{p}, \overline{p}]$. Every economic agent knows the return X given success. However, only the firm knows the value of p.

We consider competition for borrowers between two lenders. Usually, a firm with an investment project refers to a bank with which it has regular relations. The bank better knows the practice or character of the firm's management and is more capable of apprising the project's risk than other banks. The bank is also advantaged in terms of the costs necessary to evaluate the project.

Suppose that there are two lenders: Lenders 1 and 2. Lender 1 has established business relations with the firm. Lender i (i=1 or 2) offers a contract to the firm, in which the lender claims repayment R_i if the project is successful and collateral C_i in case of failure. We further assume that the lender is able to gather the amount of money I with zero interest rate; this does not affect the results. The lender incurs costs in examining the borrower's project. We write the cost as $L_i \geq 0$ and assume that

$$L_1 < L_2. \tag{1}$$

This implies that Lender 1 has closer relations with the borrower and is able to screen the borrower at less cost.

The borrower is also subject to incurring costs when soliciting the lender's credit. The borrower must, for instance, prepare documents or presentations of his financial situation in order to convince the lender of his credit-worthiness. Let us denote this cost by $K_i \geq 0$, where we assume

$$0 < K_1 < K_2. \tag{2}$$

The inequality reflects the borrower's closer relationship to Lender 1, which enables a lower cost in applying for this lender's credit.

We assume that the firm possesses collateralizable resources that cannot be liquidated at the start of the investment and that the value of the resources is equal to W regardless of the probability of success p. We assume

$$0 < W \leq L_2 + I. \tag{3}$$

Due to limitations on collateralizable resources, the lenders may not be able to completely secure their loan. Even by requiring the largest possible collateral, the lenders cannot totally dissipate the risk of the project's failure.

Unknowledgeable of the success probability p (referred to from now on as *type*), the lender constructs a contracting mechanism that induces the borrower to reveal his type. We focus on a direct mechanism (c.f. Myerson (1979))[4]. Formally, the mechanism is defined as a map from $[\underline{p}, \overline{p}]$ to E^2 where E is a large enough compact interval of $\mathbb{R}$,

$$p \mapsto (R_i(p), C_i(p)).$$

We restrict ourselves to the study of pure strategies. Facing the mechanism, the borrower is a player of the game and reports his type. By reporting his type to be p', the borrower p obtains the profit: $u_i(p, p') := p(X - R_i(p')) - (1-p)C_i(p') - K_i$.

The contract $(R_i(p), C_i(p))$ is *implementable* if and only if, for any $p, p' \in [\underline{p}, \overline{p}]$

$$u_i(p) \geq u_i(p, p').$$

where $u_i(p) := p(X - R_i(p)) - (1-p)C_i(p) - K_i$.

[4]In the case of a single principal, it is well known that no generality is lost by focusing on the direct mechanism. This is not generally true of the multi-principal case, but in our context of competition with exclusivity, restriction to the direct mechanism does not raise the problem of generality. See Martimort (1996) and Martimort and Stole (2002) for details.

With an implementable contract, the lender's expected profit from the borrower is

$$\Pi_i(p) := pR_i(p) + (1-p)\alpha C_i(p) - L_i - I,$$
$$= pX - u_i(p) + (\alpha - 1)(1-p)C_i(p) - K_i - L_i - I.$$

where $0 \leq \alpha \leq 1$. α indicates that the valuation of collateral between the lender and the borrower might be different, for instance, on account of transaction costs in liquidating collateral.

Notice that in our setting, the perfect information first best contract is $R(p) = X - \frac{K}{p}$, $C(p) = 0$, and $u(p) = 0$.

As is usual with the mechanism design problem, we transform the implementability condition into the following condition.

Lemma 1 (incentive compatibility condition)**.** *If the contract $(R_i(p), C_i(p))$ is implementable, then the following conditions hold,*

$$u_i(p) \textit{ is convex and absolutely continuous,} \tag{4}$$
$$\dot{u}_i(p) = \frac{u_i(p) + C_i(p) + K_i}{p} \quad a.e.^5 \tag{5}$$

Conversely, if (4) and (5) hold, the implementable contract $(R_i(p), C_i(p))$ can be obtained by setting,

$$R_i(p) = \frac{pX + (p-1)C_i(p) - K_i - u_i(p)}{p}. \tag{6}$$

Proof. See Rochet (1987). □

Observe that, contrary to standard literature (Guesnerie and Laffont (1984) for in-

[5]a.e. stands for almost everywhere.

stance), we make no assumption on $F(p)$ such as the monotone hazard rate condition.

Before analyzing competition, let us examine briefly the case of the monopolist lender, where we suppress the subscript i. We suppose that the project is worth carrying out for at least $\overline{p}$,

$$\overline{p}X - L - K - I > 0.$$

The monopolist maximizes his expected profit $E\Pi(p)$ under the incentive compatibility constraint (4) and (5), $0 \leq C(p) \leq W$ and the participation constraint $u(p) \geq 0$ over types accepting the contract. Here we normalize reservation utility to zero. Since u is non-decreasing from (5), the participation constraint is replaced by $u(\grave{p}) = 0$. where $\grave{p}$ is the lowest type served.

Proposition 1. *The interval of types $[\grave{p}, \overline{p}]$ accepts the lender's contract (i.e. $u(p) \geq 0$ on this interval) and the monopolist's optimal contract exhibits full bunching and is expressed as*

$$R(p) = X - \frac{K}{\grave{p}}, \quad C(p) = 0, \quad u(p) = K\big(\frac{p}{\grave{p}} - 1\big).$$

The lowest type served $\grave{p}$ is determined by the transversality condition:

$$(\grave{p}X - K - L - I)f(\grave{p}) - \frac{K}{\grave{p}^2}\int_{\grave{p}}^{\overline{p}} f(t)tdt = 0.$$

Proof. See the appendix. □

One notable feature of our result is that the lender demands no collateral, irrespective of the presence of liquidation costs. Likewise, the repayment R is independent of type and the optimal contract exhibits full bunching. Section 4.6.1 in Freixas and Rochet (1997) treats a case similar to ours in the two-type setting with type-dependent reservation utility. They show that the monopolist lender requires collateral of the better type. Our result, distinct from theirs, derives from zero reservation utility.

Another feature is that the lender excludes the lower types. We are given the fixed cost and the expected investment return pX approaches 0 when p becomes smaller. The lower the lender sets the cut-off type, the larger the range of types from which the lender makes earnings. Due to the participation constraint, however, the lender thereby makes less profit from each individual type. The cut-off type is determined by this trade-off.

We can see that if K is zero, we have $R(p) = X$, $C(p) = 0$, and $u(p) = 0$: whole bunching, full rent extraction. This is in stark contrast to the standard literature (e.g. Guesnerie and Laffont (1984)), which shows the agents as discriminated by a regularity assumption on F and as enjoying informational rent.

3 Competition

In order to focus on the competition between lenders, we make the following assumption. Every borrower's project is worth realization from a social point of view:

$$pX - K_2 - L_2 - I \geq 0 \quad \text{for } p \in [\underline{p}, \overline{p}].$$

We consider the game in which the borrower can contract with only one principal and in which the two principals maximize $E\Pi_i(p)$ in competition for the borrower, by simultaneously offering an *implementable* contract such that

$$0 \leq C_i(p) \leq W. \tag{7}$$

For Lender 2's profit, we clearly have $\Pi_2(p)$ if $u_2(p) \geq u_1(p)$ and 0 otherwise on the assumption that in the case of a tie $u_2(p) = u_1(p)$, Lender 1 wins the contract as it is the borrower's regular bank.

Stole (1995) analyzed symmetric competition of this type between two identical firms.

Unlike in his study, however, we have the constraint on the range of the control variable (7), which potentially compromises the marginal cost argument in Bertrand competition. However, for the equilibrium pattern of type capturing, we obtain the following lemma, which considerably simplifies our analysis.

Lemma 2. *In equilibrium, Lender 1 offers borrowers utility at least as great as does Lender 2 ($u_1 \geq u_2$) over the whole interval. In other words, Lender 1 wins contracts for all types.*

Proof. Suppose that in equilibrium, Lender i offers u_i. Due to the absolute continuity of u_i, we know that Lender 1 wins a combination of intervals of type. Lender 1 can then imitate Lender 2's contract on the intervals on which the latter lender gives higher utility as doing so does not violate the incentive compatibility constraint (4) and (5). We can verify this by recognizing that the superior envelope of the convex functions $sup\{u_1(p),\, u_2(p)\}$ is a convex function.

Because of the cost advantage, Lender 1 can make a greater profit by offering Lender 2's contact on the intervals initially won by Lender 2. As a result, Lender 1 will win over the whole interval.

□

This lemma allows us to express the participation constraint in our problem in a simple way. Given Lender 2's implementable contract, Lender 1 proposes a contract solving the following problem $\mathcal{O}$:

$$\max_{u_1(p), C_1(p)} \int_{\underline{p}}^{\overline{p}} \Pi_1(p) f(p) dp \tag{8}$$

s. t.

(4), (5) and (7) for $i = 1$,

$$u_2(p) \leq u_1(p) \qquad \text{for } p \in [\underline{p}, \overline{p}]. \tag{9}$$

Here we face an instance of a problem of type-dependent reservation utility. Note that our scenario can be extended to the case of more than two lenders. As long as the costs K_i and L_i are ordered as in (2) and (1), the situation can be reduced to competition between the two most cost-advantaged lenders.

Let us examine more closely Lender 2's strategy in equilibrium. Contrary to standard Bertrand competition, we cannot use the marginal cost argument and deduce Lender 2's nil profit. Due to the restriction on C, there exists no implementable contract that entails zero profit.

First it is obvious that Lender 2 offers a contract such that $\Pi_2(p) \geq 0$. At the very moment of contracting, the lender sees which contract the borrower has chosen and thus learns his type. If the lender thereby incurs a loss, he will not sign the contract. The non-negative profit condition is thus:

$$\Pi_2(p) \geq 0. \tag{10}$$

Then the largest utility which the borrower can obtain with Lender 2 is expressed by the following lemma.

Lemma 3. *In the Nash equilibrium of the two-lender game, Lender 2's equilibrium implementable contract* $(u_2(p), C_2(p))$ *is such that it satisfies* (10) *(in inequality) and also that*

$$u_2 \geq \tilde{u}_2 \qquad on\ [\underline{p}, \overline{p}],$$

where $\tilde{u}_2$ *is of any implementable contract* $(\tilde{R}_2(p), \tilde{C}_2(p))$ *which satisfies* (10) *(in inequality).*

Proof. Suppose that Lender 2 offers a contract which gives $\tilde{u}_2(p)$. Lender 1 can then make a counter-offer that gives $u_2(p)$ and brings in positive profit. All borrowers weakly prefer the contract by Lender 1 and Lender 1 therefore wins them over. Lender 2, as a result, is bound to propose $u_2(p)$. Faced with this offer by the opponent, Lender 1 will make an offer giving slightly more utility to the borrowers to win them over. Lender 2 sticks to the contract $u_2(p)$ since it cannot offer a more interesting contract on account of the non-negative profit condition.

$\square$

Some remarks are to be made. First, this is essentially a pointwise maximization problem amongst admissible pairs, i.e, the implementable (u_2, C_2) satisfying (10). Uncertain, however, is that there exists a u_2 that is weakly larger than any other admissible $\tilde{u}_2$ at every point. One can imagine a case in which there are two admissible $\hat{u}_2$ and $\bar{u}_2$ that are weakly larger at every point than any other admissible u_2 but in which $\hat{u}_2$ is larger than $\bar{u}_2$ at some points and contrariwise at others. In this case, there does not exist an equilibrium pair as is stated in the lemma. Second, even if an equilibrium is shown to exist, characterization is not an easy task. Accordingly, we deal with our competitive game by means of the optimal control technique. If an equilibrium exists as in the lemma, it must be a solution of the following problem $\mathcal{R}$:

$$\max_{u_2(p), C_2(p)} \int_{\underline{p}}^{\overline{p}} u_2(p) f(p) dp$$

$$\text{s. t.}$$

$$(4),\ (5),\ (7),\ (10) \text{ for } i = 2.$$

Note that $\mathcal{R}$ might have a solution where it is not the case with the pointwise maximization in the lemma.

The last constraint comprises the state variable u_2 and the control variable C_2. It is difficult to clearly characterize the problem with a mixed state-control constraint. However, when there is no cost to liquidation, C_2 disappears and the constraint is reduced to a pure state constraint. On account of manageability, we content ourselves with this case.

Lemma 4. *Assume that $\alpha = 1$. In the equilibrium of our competition game, then,* *Lender 2's contract $(R_2(p), C_2(p))$ is such that for every $p \in [\underline{p}, \overline{p}]$*

$$R_2(p) = \frac{1}{\underline{p}}\big(L_2 + I\big) + \big(1 - \frac{1}{\underline{p}}\big)W,$$

$$C_2(p) = W,$$

$$u_2(p) = pX - \frac{p}{\underline{p}}\big(L_2 + I\big) + \big(\frac{p}{\underline{p}} - 1\big)W - K_2.$$

Proof. See the appendix. □

When collateral resources are sufficient ($W = L_2 + I$), we obtain the standard result of zero profit in Bertrand competition. By contrast, when the availability of collateral is quite limited, the lender makes profit except from the lowest type. The reason is that in that case, despite the force of competition to drive down the profit, the lender cannot offer the implementable contract leading to zero profit given limitations to collateral resources.

As in the monopolistic case, the collateral and repayment are identical among all types of borrowers; hence bunching. In contrast with monopoly, however, competition induces Lender 2 to require the maximum collateral of every borrower.

We reformulate Problem $\mathcal{O}$ in the light of our lemma. In particular, we replace (9) by

$$u_2(\overline{p}) \leq u_1(\overline{p}) \tag{11}$$

and verify (9) at the end. Note that thanks to the lemma, our problem is formulated in a form close to the discrete one in Section 4.6.1 in Freixas and Rochet (1997). Our case can be viewed as a continuous generalization of theirs.

Proposition 2. *Suppose that $\alpha = 1$. Then the solution of Problem $\mathcal{Q}$ is as follows:*

$$
\begin{aligned}
C_1(p) &= W,\\
R_1(p) &= (1 - \frac{1}{\underline{p}})W + \frac{1}{\underline{p}}\big(L_2 + I\big) + \frac{1}{\bar{p}}\big(K_2 - K_1\big),\\
u_1(p) &= pX + \big(\frac{p}{\underline{p}} - 1\big)W - \frac{p}{\underline{p}}\big(L_2 + I\big) - \frac{p}{\bar{p}}K_2 + \big(\frac{p}{\bar{p}} - 1\big)K_1.
\end{aligned}
$$

Proof. See the appendix. □

As in the monopolistic case, we obtain full bunching: the repayment and collateral are identical across all types. A noticeable difference brought by competition is that both lenders require collateral of every borrower. (5) shows that the larger the collateral is, the larger utility the borrower obtains. As seen in Lemma 4, competition drives Lender 2's profit lower because the competitive pressure forces him to offer the borrower higher utility. Lender 2 uses collateral to this end. To counter the competition, Lender 1 also has to propose high utility to the borrower and therefore both lenders use collateral.

The theoretical literature on competition over credit contracts with exclusivity (for instance Besanko and Thakor (1987) and Schmidt-Mohr (1997)) asserts that a good borrower pledges more collateral than a bad one because the former is likely to succeed in the investment project and unlikely to lose the collateral. Our main difference is that we introduce heterogeneity between banks, whereas the rest of the existing literature generally deals with perfect competition. To the best of my knowledge, our result is new in the universal risk-neutral environment.

The empirical works of Berger and Udell (1990) and Berger and Udell (1995) report that riskier borrowers more often pledge collateral than safer ones. However, it is unclear in those studies how much collateral banks initially wanted or whether there were enough collateralizable resources at the time of contracting. Further empirical work must test the coherence of our result with the actual loan case.

To interpret our result, let us distinguish between borrower risk and loan risk. Borrower risk is learned over time by the lender through a series of business relationships and borrowers are classified according to the risk. Loan risk varies, in general, with each investment project of the same borrower. In the present model, K_1 and L_1 can be interpreted as the lender's classification of borrowers due to borrower risk whereas loan risk is represented by p. The proposition states that the loan contract depends upon borrower risk (K_1 and L_1) but not loan risk (p). This contract is identical to the line of credit widely used in practice in which the loan terms are fixed in advance and the borrower can draw a loan when needed.

In contrast to standard principal-agent problems, the participation constraint (9) is binding at the best type. With $u_1(p) - u_2(p)$ decreasing, the worst type obtains positive rent whereas the best type has all rent extracted. Lender 1 wishes to extract a borrower's rent and must at the same time leave the borrower reservation utility. Lender 1 therefore faces a dilemma between rent extraction and rent assurance.

Good borrowers are offered a rather nice contract by Lender 2; at the limits, the best borrower is indifferent between the two banks. This result is well borne out by observation of the actual economy in which a bad firm often has no alternative to its regular bank since other banks offer either a contract of very poor terms or even flatly deny the borrower; a good firm, by contrast, can resort to other banks with much greater ease to obtain an alternative offer.

In general, customer capturing pits the lender against a trade-off. Serving a wider range of types brings in more sources of profit but there is a negative effect. Serving more types implies that the lender needs to satisfy the incentive compatibility and participation constraints for a wider range of types. This is more demanding than serving fewer types. The lender may therefore wish to forego some borrowers when the negative effect outweighs the positive on profit. In doing so, the lender can extract enough rent from the rest of all types to offset losses from giving up some types. In our case, Lemma 2 tells us that the positive effect is larger and that Lender 1 chooses to capture all customers.

By examining $u_1 - u_2$, we can assert that the borrower sees the smaller difference between the two lenders' contracts the smaller $K_2 - K_1$ is. Harsher competition drives contract terms to be favorable to the borrower.

The literature on type-dependent reservation utility (Lewis and Sappington (1989), Feenstra and Lewis (1991) and Maggi and Rodriguez-Clare (1995)) reports that when reservation utility is linear, a non-degenerate interval of types earns profit equal to reservation utility and on that interval, pooling results. They treat a model similar to a labor contract in which the principal's objective function is strictly concave. In our standard financial contract model, the principal's objective function is linear in the control variable, where only the highest type obtains utility equal to reservation utility. Our findings suggest that the standard results of principal-agents schemes with type-dependent reservation utilities of the above works depend crucially on the specification of principals' preferences. Further, as mentioned in the introduction, these authors assume ad hoc the satisfaction of the participation constraint over the entire type space. This assumption also contributes to their conclusion.

4 Appendix

4.1 Proof of Proposition 1

We first ignore the convexity of u and check finally if it is satisfied. The Hamiltonian is written with an adjoint variable λ:

$$H = \Pi(p)f(p) + \lambda(p)\frac{u(p) + C(p) + K}{p}.$$

Then, we obtain $\lambda = -\frac{1}{p}\int_p^{\overline{p}} f(\tau)\tau d\tau \leq 0$ from $\dot{\lambda} = f - \frac{\lambda}{p}$ a.e. and the transversality condition $\lambda(\overline{p}) = 0$. Maximization leads to $C = 0$. Then u can be found from (5):

$$u(p) = u(\grave{p})\frac{p}{\grave{p}} + K\big(\frac{p}{\grave{p}} - 1\big).$$

$\lambda(\grave{p}) \neq 0$ leads to $u(\grave{p}) = 0$ due to the transversality condition $\lambda(\grave{p})u(\grave{p}) = 0$. The ignored convexity condition is satisfied and R can be retrieved from (6). The cut-off type $\grave{p}$ is determined by the free-time transversality condition (see Neustadt (1976)).

4.2 Proof of Lemma 4

Let us solve Problem $\mathcal{R}$ while replacing (10) by $\Pi_2(\grave{p}) \geq 0$. At the end, we verify that the original condition is satisfied. The Hamiltonian is written as;

$$H = u_2 f + \lambda\frac{u_2 + C_2 + K_2}{p}$$

where λ is an absolutely continuous adjoint variable. We have

$$\dot{\lambda} = -f - \frac{\lambda}{p} \quad \text{a.e.}$$

The transversality conditions are

$$\lambda(\underline{p}) \geq 0, \qquad \lambda(\underline{p})\Pi_2(\underline{p}) = 0, \qquad \lambda(\overline{p}) = 0.$$

We have immediately $\lambda = \frac{1}{p}\int_p^{\acute{p}} f(t)t dt > 0$. Maximizing the Hamiltonian, we obtain $C_2 = W$. u_2 can also be found if we notice that $\lambda(\underline{p}) \neq 0$ and thus $\Pi_2(\underline{p}) = 0$. R_2 can be retrieved as well. It is easily seen that the original constraint (10) ignored is satisfied.

4.3 Proof of Proposition 2

The Hamiltonian is:

$$H = \big((\alpha - 1)(1-p)f + \frac{\lambda}{p}\big)C_1 + (pX - K_1 - L_1 - I - u_1)f$$
$$+ \lambda \frac{u_1 + K_1}{p}.$$

The necessary conditions to optimality are

$$\dot{\lambda} = f - \frac{\lambda}{p} \quad \text{a.e.} \quad \text{on } [\underline{p}, \overline{p}],$$

$$\lambda(\underline{p}) = 0, \quad \lambda(\overline{p}) \geq 0, \quad \lambda(\overline{p})\big(u_1(\overline{p}) - \overline{p}X + K_2 + L_2 + I\big) = 0,$$

Easily established is the fact that $\lambda(p) = \frac{1}{p}\int_{\underline{p}}^{p} f(t)tdt$ and therefore

$$u_1(\overline{p}) = \overline{p}X - K_2 - L_2 - I.$$

We obtain $C_1(p)$ and $u_1(p)$ as in the proposition.

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