Imperfect Information, Consumers' Expectations and Business Cycles

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Abstract

This paper presents a model of business cycles driven by shocks to consumers' expectations regarding aggregate productivity. Agents are hit by heterogeneous productivity shocks, they observe their own productivity and a noisy public signal regarding aggregate productivity. The shock to this public signal, or "news shock", has the features of an aggregate demand shock: it increases output, employment and inflation in the short run and has no effects in the long run. The dynamics of the economy following an aggregate productivity shock are also affected by the presence of imperfect information: after a productivity shock output adjusts gradually to its higher long run level, and there is a temporary negative effect on inflation and employment.

The fraction of short run fluctuations explained by the news shock is increasing in the level of idiosyncratic noise and is non-monotone in the precision of the public signal. For relatively high levels of idiosyncratic uncertainty news shocks can generate realistic levels of short-run volatility.

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1 Introduction

This paper analyzes business cycle fluctuations in an economy where consumers have imperfect information regarding the level of aggregate productivity. The model formalizes the old idea that business cycles are driven by changes in consumers' expectations. In particular, it formalizes the idea that changes in consumers' expectations can generate fluctuations in expenditure that drive output temporarily away from a natural equilibrium path entirely determined by tastes and technology. In this view cyclical fluctuations in employment and inflation are associated to these temporary deviations of output from the natural path.

In dynamic stochastic general equilibrium models cyclical fluctuations can be driven by a variety of shocks: technology shocks, preference shocks, shocks to public expenditure, and so on. In these models consumers' expectations move together with aggregate variables but do not provide an independent source of fluctuations. In recent work, Danthine and Donaldson (1998) and Beaudry and Portier (2001, 2004) have studied equilibrium models where changes in expectations regarding *future* productivity have real effects on output. However, equilibrium models of this type tend to generate a negative correlation between consumption and investment and between consumption and labor supply following "news shocks". In a standard model of stochastic growth news regarding future productivity induce workers to postpone labor effort and to dissave to finance a current increase in consumption. To overcome this problem one can construct models with embodied technical change or with limited transformation between consumption and investment. In models of this type a future increase in productivity can indeed generate an increase in investment and labor supply. However, in this case consumption now *drops* following a positive expectations shock[1].

An alternative role for shocks to expectations arises in models with increasing returns and multiple equilibria[2]. In models with multiple equilibria changes in expectations can clearly generate positive commovement between consumption, investment and employment. However, in an important dimension models with increasing returns are similar to models with exogenous productivity shocks: a boom is always associated to a contemporaneous increase in total factor productivity and the incentive to work more during booms is fully captured by the corresponding shift in labor productivity[3]. This contrasts with the recent observation by Hall (1997) that a large fraction of business cycle fluctuations seems to be accounted for by shocks that resemble shifts in the marginal rate of substitution between consumption and leisure rather than by shocks to labor

[1] See the thorough discussion in Section 6 of Beaudry and Portier (2004). It is possible to write down models with some type of non-separability of consumption across time such that an increase in future profitability raises both consumption and investment. Whether models of this type can lead to a realistic account of expectation-driven cycles is an open question.

[2] Early contributions to this vast literature are e.g. Kiyotaki (1988) and Benhabib and Farmer (1994).

[3] See Murphy et al. (1989).

productivity.[4] Therefore, we still lack a theoretical account of cycles due to shifts in consumers' expectations where the output and employment response is not driven by a contemporaneous increase in labor productivity.

This paper shows that introducing imperfect information about aggregate productivity in a dynamic equilibrium model it is possible to study consumers' expectations as an independent source of fluctuations. In this context, a positive shock to consumers' expectations tends to increase consumption, employment and inflation. The central point of the paper is that aggregate uncertainty about persistent productivity shocks can generate short-run dynamics that are very different from the long-run dynamics of the model. First, the economy will adjust more gradually to actual productivity changes, second, news about productivity will generate short-run volatility of inflation and employment that would not arise under full information.

The idea that aggregate shifts in productivity are unobservable, and that agents in the economy take time to acknowledge their realization seems consistent with experience, if one just thinks about the productivity slowdown of the 70s or the productivity acceleration of the late 90s. In both cases it took several years for policy makers and economists to figure out the realized path of productivity. The question addressed in the present paper is: what are the aggregate consequences of living in an economy where similar productivity shifts can occur and where no single agent in the economy can observe them?

The mechanism we propose requires some type of friction on the supply side, so that realized output can deviate temporarily from average productivity without triggering an immediate price adjustment. It turns out that the same informational imperfection that generates expectations' shock on the demand side introduces sluggish price adjustment on the supply side. The mechanism on the supply side is very closely related to the mechanism proposed by Phelps (1969) and Lucas (1972) to analyze money non-neutrality.

The paper considers an island economy with local productivity shocks and imperfect information. Each agent observes the productivity shock in his own island and a noisy public signal of the aggregate productivity shock. The presence of a noisy public signal plays a crucial role because it leads agents to make coordinated mistakes regarding the level of aggregate productivity. The shock in the public signal, or "news shock", moves agents' expectations independently of movements in real productivity. Following a news shock average productivity remains constant. However, the average producer believes that productivity has increased in other sectors, moving up the demand for his own good. He responds by increasing his spending, increasing his target level of employment and trying to increase the relative price of his good. This generates an increase in output, in hours worked and in the price level.

The presence of imperfect information also changes the response of the economy to an aggregate productivity shock. In particular a permanent productivity shock has a less than proportional effect on output in the short run and a tem-

[4] Recent estimated DSGE models, such as Smets and Wouters (2003), also tend to attribute a sizeable fraction of output volatility to preference shocks that induce changes on the consumption-leisure margin.

porary negative effect on employment and inflation. Following a productivity shock agents' expectations regarding aggregate productivity increases less than one-for-one with productivity itself. The average producer believes that productivity has increased less in the other sectors than in his own. He responds by increasing his spending less than his productivity, decreasing his target level of employment and trying to decrease the relative price of his good. This reduces aggregate employment and nominal prices. The negative effect of permanent productivity shocks on employment is consistent with the evidence in Gali (1999) and in Francis and Ramey (2002).[5] The latter result, i.e. the fact that with imperfect information a positive productivity shock can have temporary negative effects on employment, has been independently derived in a paper by Kawamoto (2004).

The current paper is related to various strands of literature. First, it is related to the early rational expectations models of the 70s and 80s that focused on monetary shocks, in particular Lucas (1975) and Grossman and Weiss (1982). We have already mentioned the relation with Lucas' work and we will return to it later. Grossman and Weiss (1982) first emphasized the role of imperfect information regarding aggregate productivity shocks, although in a different framework that focuses on investment and the credit market. In their model agents have imperfect information regarding the productivity of capital, and the nominal interest rate provides an imperfect signal about it. Their paper shows that when agents look at the nominal interest rate to draw inference about the marginal product of capital, investment will be responsive to shocks on the money market. In this framework shifts in inflation expectations can affect investment and activity through their effect on the nominal interest rate.

The role of imperfect information and higher order expectations in games with strategic complementarity has been recently analyzed in a number of theoretical and applied papers. In particular Morris and Shin (2002) and Allen, Morris and Shin (2004) have shown that the presence of strategic complementarities amplifies the effect of public information and slows down the adjustment of aggregate variables towards their "fundamental" value. Woodford (2002) shows the link between this literature and the Phelps-Lucas approach to the study of price adjustment. The link has been further studied by Hellwig (2002) who emphasizes that the presence of a public signal regarding monetary policy shocks can have ambiguous effects since it can help agents to coordinate their mistakes. The main difference between our setup and the setup of Woodford (2002) and Hellwig (2002) is that we consider imperfect information regarding productivity shocks rather than money supply shocks. This is an important distinction because the pricing decisions of individual agents depend directly on their realized productivity. Therefore, in this context the price level is not an additional noisy signal of the level of the aggregate shock. Rather, it reflects the average *expectation error* in the economy regarding the aggregate shock, given that prices depends positively on expected productivity and negatively on realized productivity. This means two things. First, that we obtain opposite

[5] See Chrisitano, Eichenbaum and Vigfusson (2003) for a critical view on this evidence.

price responses to the news shock and to the productivity shock. Second, that prices are very informative signals, since they reflect the average error in agents expectations. This means, in particular, that we have to add considerable idiosyncratic noise to the model in order to reduce the speed of learning of the agents in the economy.

The fact that the information contained in prices is used to evaluate the "gap" between realized output and underlying productivity shows the connection between the current paper and a different strand of literature. Namely, the literature on monetary policy with imperfect information. In recent work Aoki (2005), Svensson and Woodford (2003, 2005) and Reis (2003) have considered sticky price models where the monetary authority has imperfect information regarding macroeconomic fundamentals. In those papers the monetary authority uses its observation of current inflation to make inferences on the "natural" level of output (or employment). In our paper the same inference problem is faced by each agent in the economy. As agents observe past output and inflation they update their beliefs regarding productivity and they converge towards the full information equilibrium.

2 The model

2.1 Setup

Preferences and technology. The model is a version of a Dixit-Stiglitz model of monopolistic competition with heterogeneous productivity shocks. Consider an economy with a continuum of islands indexed by $i \in [0,1]$. On each island there is a continuum of infinitely lived households indexed by $k \in [0,1]$. Each household is composed of two agents, a consumer and a producer.

There are two type of commodities. A durable good, H, and a continuum of perishable specialized goods indexed by island and by household. The durable good can be transported across islands and will take the role of commodity money, as it will be used both as the medium of exchange and as the numeraire.

Each period t the consumers from island i travel to a finite set of m islands randomly drawn. They can carry good H with them and exchange it for the goods produced in the island they visit. The assignment of consumers to islands is a uniform random draw from the set of assignments such that each island is visited by a unit mass of consumers coming from m islands. Therefore, the marginal probability of traveling to any other island is uniform for each consumer, and the marginal probability of selling to consumers from any other island is uniform for each producer. The set of islands visited by the consumers from island i is denoted by B_{it}.

Household preferences are represented by the utility function

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_{it}+\alpha\ln X_{it}+\frac{1}{1+\eta}N_{it}^{1+\eta}\right)\right],$$

where C_{it} is a composite good defined below —which aggregates the specialized goods produced on the islands in B_{it}—, X_{it} is consumption of services from the durable good H and N_{it} is hours worked in the production of good i. All households on island i are identical. Therefore, whenever possible we save on notation and use only the island subscript i.

The composite consumption good is a standard CES aggregate including all the varieties produced in the islands visited in period t

$$C_{it} = \left(\sum_{j \in B_{it}} \int C_{ijkt}^{\frac{\sigma-1}{\sigma}} dk \right)^{\frac{\sigma}{\sigma-1}},$$

with $\sigma > 1$, where C_{ijkt} is the consumption of the good produced on island j by producer k.

The production function on island i is linear and is given by

$$Y_{it} = A_{it} N_{it}.$$

where A_{it} is productivity on island i. Different islands receive different productivity shocks. The productivity parameters A_{it} are the crucial source of uncertainty in this economy.

We can describe good H as "hens". Each period the household has an initial endowment of H_{it} hens. Consumers travel with the entire stock H_{it} to the islands in B_{it}. They transfer $H_{it} - X_{it}$ hens to the local producers. The rest of the hens, X_{it}, travels back with the consumers and on the return trip produces a flow of services that is immediately consumed ("eggs"). The $H_{it} - X_{it}$ hens transferred to the producers do not produce a flow of services at date t[6]. At the end of the period household i has a stock $\hat{H}_{it}$ of hens. Hens on island i reproduce at the rate R_{it+1} between period t and $t+1$, so the stock of hens next period is

$$H_{it+1} = R_{it+1} \hat{H}_{it}.$$

The rate of return R_{it} is random and island specific. We will discuss its role below.

Uncertainty and information. At the beginning of each period t all agents in island i observe the productivity level A_{it} and the rate of return R_{it}. Let a_{it} denotes the log of A_{it}. Island i productivity a_{it} has two components: an aggregate component a_t and an island-specific component ϵ_{it}:

$$a_{it} = -\frac{1}{2}\sigma_\epsilon^2 + a_t + \epsilon_{it}$$

The aggregate component a_t follows the random walk[7]

$$a_t = a_{t-1} + u_t$$

[6] That is, when hens change owner they do not make eggs for one period. The role of this assumption is to make sure that the cash-in-advance constraint in the trading stage is never binding.

[7] It is straightforward to add a deterministic component to the growth process. To save on notation we normalize it to zero.

where u_t is an aggregate productivity shock. The log rate of return $r_{it} = \log R_{it}$ also has two components, one aggregate and one island-specific

$$r_{it} = -\frac{1}{2}\sigma_v^2 + r_t + v_{it}.$$

The aggregate component r_t is given by

$$r_t = u_t.$$

This means that the aggregate rate of return r_t is perfectly correlated with the innovation in the aggregate productivity of the specialized goods sector[8]. The role of this assumption is to ensure that the stock of H grows, exogenously, at the same rate as average productivity. This ensures that the price level P_t is stationary. The advantage of this assumption is that it allows the stock of the medium of exchange to adjust to output automatically in a simple real model. This happens without the intervention of a money authority with superior knowledge and without breaking the informational heterogeneity in the model. As we will see, in the full information case this assumption will imply a constant price level. In the case of imperfect information, on the other hand, all fluctuations in the price level will be associated to endogenous changes in the velocity of money.

The cross sectional distribution of the ϵ_{it} and v_{it} satisfies:

$$\int_0^1 \epsilon_{it} di = \int_0^1 v_{it} di = 0.$$

At the beginning of each period agents observe a *public signal* regarding current aggregate productivity. The signal s_t is given by

$$s_t = a_t + e_t.$$

The noise in the public signal e_t will be the source of autonomous shifts in consumers' expectations.

Let $\mathbf{P}_{it}$ denote the vector of the prices of the goods purchased by consumer i at date t,

$$\mathbf{P}_{it} = \{P_{jt}\}_{j \in B_{it}}.$$

Agents in island i can observe the following variables: the local productivity a_{it}, the local rate of return r_{it}, the public signal s_t, the price vector $\mathbf{P}_{it}$ and the quantity of the good sold Y_{it}. In the next paragraphs we will be more specific about the timing of the information flows.

The aggregate shocks in this economy are represented by the productivity shock u_t and the news shock e_t. These shocks are independent and serially

[8] The average expected return on the durable good is $e^{\frac{1}{2}\sigma_u^2}$. Adding a constant one would get an average return $e^{r_0+\frac{1}{2}\sigma_u^2}$. The effect of the constant r_0 is to introduce a nonstochastic trend in the money to output ratio and thus a drift in the price level. For simplicity, we let $r_0 = 0$.

uncorrelated Gaussian random variables with zero mean and variances (σ_u^2, σ_e^2). The island-specific shocks are also Gaussian, with zero mean and variances $(\sigma_\epsilon^2, \sigma_v^2)$. The island-specific shocks are independent across islands for each finite sample of islands, they are also serially uncorrelated and independent of the aggregate shocks.

Trading. The only type of trades allowed in this economy are spot trades and the only good that can be transported across islands is good H. Let us describe in detail the trading sequence and the information flows in this economy. At the beginning of period t, all agents in the young household i observe the productivity a_{it}, the public signal s_t and the rate of return r_{it}. Then, the producer sets the price P_{it} and stands ready to deliver any quantity of the specialized good at that price[9]. This is called the "pricing stage".

After prices are set the consumer travels to the islands in B_{it} and observes the prices of the specialized goods produced in those islands. Let these prices be denoted by the vector $\mathbf{P}_{it} = \{P_{jt}\}_{j \in B_{it}}$. Then, he exchanges good H for specialized goods. Then, he consumes the vector $\{C_{ijt}\}_{j \in B_{it}}$ and the services X_{it} of the durable not used for transactions, where

$$X_{it} = H_{it} - \sum_{j \in B_{it}} P_{jt} C_{ijt}.$$

This is called the "trading stage." Consumers do not communicate with producers during the trading stage, so consumers do not know the quantity sold in the home island when they are making their spending decisions.

At the end of the day, the consumer returns and observes the quantity sold Y_{it} and the total revenues $P_{it}Y_{it}$. The end of period stock of H is $X_{it} + P_{it}Y_{it}$. Thus we can write the law of motion for the stock of H of household i as

$$H_{it+1} = R_{it+1} \left(H_{it} + P_{it}Y_{it} - \sum_{l \in B_{it}} P_{lt} C_{ilt} \right).$$

Aggregating across agents, using market clearing on the goods markets and using the fact that the individual rate of return shocks are i.i.d. one obtains the law of motion for the aggregate stock of H

$$H_{t+1} = R_{t+1} H_t.$$

The pattern of trade across islands is represented in Figure 1.

The crucial ingredients of this setup are three: the presence of unobservable aggregate productivity, the presence of partially revealing prices and quantities, the presence of variable velocity in the circulation of the medium of exchange.

[9] To be more precise let producers set a maximum supply $\overline{Y}_{it}$, satisfy any demand in $[0, \overline{Y}_{it}]$, and adopt some rationing rule if demand exceeds $\overline{Y}_{it}$. As usual in models of price setting we assume that the size of the shocks is small enough that the demand for good i is always smaller than $\overline{Y}_{it}$ in equilibrium.

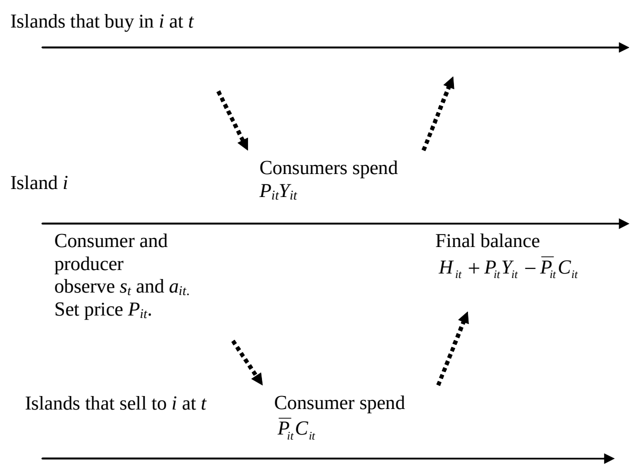

Figure 1: Timeline.

The island structure of the model is constructed to obtain the first two features. The latter feature is necessary if we want to study demand driven fluctuations that are not caused by monetary policy shocks. The presence of a durable good that can be used either for trading or for direct consumption allows us to have variable velocity in a simple framework with commodity money.

Before defining a competitive equilibrium it is useful to describe more formally the information available to market participants. Producer i makes his pricing decision based on the information in $\mathcal{I}_{it}^{I}=\{A_i^t, R_i^t, \mathbf{P}_i^{t-1}, Y_i^{t-1}, s^t\}$[10]. Consumer i observes the prices of the goods in B_{it} and makes his consumption decision based on $\mathcal{I}_{it}^{II}= \{A_i^t, R_i^t, \mathbf{P}_i^t, Y_i^{t-1}, s^t\}$. We will use the following notation for agents expectations at the pricing stage and at the trading stage:

$$\begin{aligned} E_{it}^{I}[.] &= E[.|\mathcal{I}_{it}^{*}] \\ E_{it}^{II}[.] &= E[.|\mathcal{I}_{it}] \end{aligned}$$

Also, we will use the following notation for the cross sectional averages of the

[10] We use the notation $X^t = \{X_t, X_{t-1}, ...\}$.

first order expectations of any variable x_{t+s}

$$\begin{aligned} x^I_{t+s|t} &= \int E^I_{it}\left[x_{t+s}\right] di \\ x^{II}_{t+s|t} &= \int E^{II}_{it}\left[x_{t+s}\right] di. \end{aligned}$$

2.2 Rational Expectations Equilibrium

Consider a consumer from island j shopping on island i. Given his level of consumption C_{jt} his demand for each good k will be

$$C_{jikt} = \left(\frac{P_{ikt}}{\overline{P}_{jt}}\right)^{-\sigma} C_{jt}$$

where the price index for island j, $\overline{P}_{jt}$, is defined[11] as

$$\overline{P}_{jt} = m^{-\frac{1}{1-\sigma}} \left(\sum_{l\in B_{jt}} P_{lt}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{1}$$

Therefore, the demand faced by the producer of good i at date t is given by

$$Y_{ikt} = P_{ikt}^{-\sigma}\frac{1}{m}\sum_{j:i\in B_{jt}} C_{jt}\overline{P}^{\sigma}_{jt}$$

which aggregates the demand of all consumers shopping on island i at date t.

A *symmetric rational expectations equilibrium* is defined by two stochastic processes P_{it} and C_{it} that satisfy

$$P_{it} = P_t\left(\mathcal{I}^I_{it}\right) \tag{2}$$

$$C_{it} = C_t\left(\mathcal{I}^{II}_{it}\right) \tag{3}$$

such that P_{it} and C_{it} maximize household i expected utility subject to the constraints

$$\begin{aligned} Y_{it} &= P_{it}^{-\sigma}\frac{1}{m}\sum_{j:i\in B_{jt}} C_{jt}\overline{P}^{\sigma}_{jt} \\ Y_{it} &= A_{it}N_{it} \\ X_{it} &= H_{it} - \overline{P}_{it}C_{it} \\ H_{it+1} &= R_{it+1}\left(H_{it} - \overline{P}_{it}C_{it} + P_{it}Y_{it}\right) \end{aligned}$$

under the measurability constraints implicit in (2)and (3) and where the prices in $\overline{P}_{it}$ and $\overline{P}_{jt}$ are taken as given.

[11] To simplify notation we use the fact that all prices in island i are identical in equilibrium.

The price P_{it} enters the index $\overline{P}_{jt}$ for each j visiting island i at date t. Nonetheless, each producer in island i takes $\overline{P}_{jt}$ as given, as P_{ikt} has a negligible effect on P_{it}. The assumption of a continuum of monopolists on island i, together with the assumption of an isoelastic demand function, implies that the pricing decision of agent (k,i) has no effect on the information revealed by the sales Y_{ikt}. For any P_{ikt} chosen by agent (k,i) the variable $Y_{ikt}P_{ikt}^{\sigma}$ is a sufficient statistic for all the information revealed by Y_{ikt} and its informational content is independent of the P_{ikt} chosen. This eliminates any experimentation motive from the price-setting problem of the monopolist.[12]

The economy-wide price and quantity indices are defined as

$$\begin{aligned} P_t &= \left(\int P_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}, \\ Y_t &= \frac{\int P_{it} Y_{it} di}{P_t}. \end{aligned}$$

In a symmetric equilibrium the state $Z_t = (a^t, s^t) = \{a_t, s_t, a_{t-1}, s_{t-1}, ...a_0, s_0\}$ is sufficient to characterize the aggregate behavior of the economy. In particular for a given state Z_t it is possible to derive the conditional distribution of past prices P_{jt-s} and quantities C_{jt-s} for $s \geq 0$. Using that distribution and the observed prices and quantities a Bayesian agent can derive the posterior distribution of the state Z_t. Then, using the equilibrium functions $P_t(.)$ and $C_t(.)$ he can derive the distribution of prices and quantities at all future nodes. Since the assignment of consumers to islands is uniformly drawn every period, this is all the information needed by agent i. Therefore, in the pricing stage the decision problem of agent i depends only on his current productivity A_{it} his wealth H_{it} and his beliefs regarding the aggregate state Z_t, described by the CDF F_{it}^{I}. In the trading stage, his decision problem depends on A_{it}, H_{it}, the own price P_{it}, the price index $\overline{P}_{it}$ and his beliefs on Z_t, described by the CDF F_{it}^{II}. Therefore, the own price is a function of the state vector $\left(A_{it}, H_{it}, F_{it}^{I}\right)$ and optimal consumption is a function of $\left(A_{it}, H_{it}, \overline{P}_{it}, F_{it}^{I}, F_{it}^{II}\right)$.

A symmetric equilibrium can be described in a recursive fashion by two functions $\mathcal{P}$ and $\mathcal{C}$ such that

$$\begin{aligned} P_{it} &= \mathcal{P}_t\left(A_{it}, H_{it}, F_{it}^{I}\right) \\ C_{it} &= \mathcal{C}_t\left(A_{it}, H_{it}, \overline{P}_{it}, F_{it}^{I}, F_{it}^{II}\right) \end{aligned}$$

The updating of the beliefs from F_{it}^{I} to F_{it}^{II} is based on the observation of the price index $\overline{P}_{it}$ in the trading stage. The updating from F_{it}^{II} to F_{it+1}^{I} is based on the observation of the quantity sold Y_{it} and of the exogenous signals a_{it}, r_{it} and s_t.

In the following we will study a log-linear approximation of the agents decision rules. In a log-linear approximation only the first moments of F_{it}^{I} and F_{it}^{II}

[12] Indeed, this is the main reason to introduce a continuum of monopolists on each island. Thanks to Giuseppe Moscarini for helping me clarify this point.

will enter the agents decision rule. This enormously simplifies the analysis of the model.

The equilibrium construction above can be extended to define a *stochastic steady state*. To do that one replaces the state Z_t with the infinite dimensional vector $\{a_t, s_t, a_{t-1}, s_{t-1}, ...\}$ and eliminates the time subscript in $\mathcal{P}$ and $\mathcal{C}$. In this way we have a representation of the equilibrium that is recursive and time invariant. In this paper we take a constructive approach, assume the existence of a stochastic steady state and proceed to analyze a linear approximation to steady state behavior. Therefore, from the next section onward Z_t will always be understood as an infinite dimensional vector.

3 Linear Equilibrium

In this section we give a general characterization of a linear equilibrium that approximates the stochastic steady state defined in the previous section and we describe a method to compute the linear equilibrium. This section is mostly methodological and sets the stage for the analysis in next section. We characterize optimal household behavior in pricing and consumption. The objective is to derive an equilibrium relation between current aggregate variables (price level and consumption) and the average first order expectations of current and future aggregate variables. Given a conjecture for equilibrium behavior one can derive the informational content of prices and quantities, construct the average expectations of aggregate variables and verify that they satisfy that equilibrium relation.

The choice of a linear approximation is dictated by the presence of imperfect information. When agents use linear decision rules individual consumption and prices only depend on the first moments of agents' beliefs. Also, when agents use linear decision rules the information revealed by other agents' action can be incorporated into agents beliefs using simple filtering techniques. Finally, when agents use linear decision rules all idiosyncratic shocks wash out when aggregating across individuals. In sum, linearity is useful for three reasons: to simplify the state space for individual decision rules (only first order beliefs matter), to simplify the inference problem faced by each individual, and to simplify the aggregation of individual decision rules.

In a linear equilibrium aggregate output and prices, y_t and p_t,[13] are linear functions of the aggregate state Z_t given by

$$p_t = \phi Z_t \tag{4}$$

$$y_t = \psi Z_t \tag{5}$$

where ϕ and ψ are two vectors of coefficients (the constant terms are omitted to save notation). The solution of the model requires solving a fixed point problem

[13] We adopt the convention that lowercase variables represent the logarithm of the corresponding uppercase variables.

to find coefficients ϕ and ψ that are consistent with optimal behavior and with rational equilibrium beliefs.[14]

3.1 Optimality

The first order condition for price setting on island i is

$$E_{it}^{I}\left[\frac{1}{\overline{P}_{it}C_{it}}Y_{it}-\frac{\sigma}{\sigma-1}\frac{1}{A_{it}}\left(\frac{Y_{it}}{A_{it}}\right)^{\eta}\frac{Y_{it}}{P_{it}}\right]=0 \tag{6}$$

where

$$\begin{aligned} Y_{it} &= P_{it}^{-\sigma}\frac{1}{m}\sum_{j:i\in B_{jt}}C_{jt}\overline{P}_{jt}^{\sigma} \\ \overline{P}_{jt} &= m^{-\frac{1}{1-\sigma}}\left(\sum_{l\in B_{jt}}P_{lt}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \end{aligned}$$

When setting the price P_{it} the producer has to form expectations regarding the prices set on the other islands for two reasons: because they determine the prices of the goods bought by the home consumer i (which enter $\overline{P}_{it}$) and because they determine the prices of the goods competing with good i (i.e. the prices which enter $\overline{P}_{jt}$).

The Euler equation for consumption is

$$\frac{1}{\overline{P}_{it}C_{it}}=\alpha\frac{1}{H_{it}-\overline{P}_{it}C_{it}}+\beta E_{it}^{II}\left[\frac{R_{it+1}}{\overline{P}_{it+1}C_{it+1}}\right] \tag{7}$$

which is a familiar expression for the optimal accumulation of a durable good. Since the durable good is the numeraire its price in terms of consumption is equal to $1/\overline{P}_{it}$.

3.2 Benchmark

A simple benchmark case which can be solved analytically is the case of no heterogeneity $\left(\sigma_{\epsilon}^{2}=\sigma_{v}^{2}=0\right)$. In the case of no heterogeneity the economy boils down to an economy with a representative consumer who has full information

[14] Notice that Y_t, and all quantity variables are non-stationary. Therefore, when we take a log-linear approximation we need to normalize all quantity variables by A_t . E.g. we set $\hat{c}_{it}=\ln\left(C_{it}/A_t\right)$ and then we recover $c_{it}=a_t+\hat{c}_{it}$. However, with imperfect information agents do not observe A_t. This means that, for example, in the consumer first order conditions we will have $E_{it}\left[\hat{c}_{it}\right]$ and

$$E_{it}\left[\hat{c}_{it}\right]\neq\hat{c}_{it}.$$

Therefore, to recover agents' decision rules we go back to non-normalized variables, e.g. using the substitution

$$E_{it}\left[\hat{c}_{it}\right]=c_{it}-E_{it}\left[a_{it}\right].$$

For ease of exposition, we report directly the expressions involving non-normalized variables.

about current productivity. In this case employment and prices are constant and given by

$$
\begin{aligned}
N &= \left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{1+\eta}} \\
P &= \frac{1-\beta}{1-\beta+\alpha}\frac{H_0}{A_0}
\end{aligned}
$$

where H_0 is the initial stock of good H and A_0 is the initial level of productivity. Consumption is stochastic and is proportional to the productivity level A_t

$$C_t = A_t N.$$

In this benchmark economy the signal s_t clearly has no role. Moreover, given our specification of the utility function and the fact that there is no capital productivity shocks have no effect on employment, not even in the short run. The intertemporal channel by which productivity shocks can have short run effects on employment are well understood from the RBC literature. In this model that channel is muted. This allows us to study in insulation the effect of imperfect information on the cyclical behavior of employment.

3.3 Individual behavior

As a first step to characterize the equilibrium we characterize optimal individual behavior. Consider a household that takes as given the aggregate relations (4), (5) and the exogenous law of motion for the aggregate state:

$$Z_t = AZ_{t-1} + B\begin{pmatrix} u_t \\ e_t \end{pmatrix} \tag{8}$$

where the matrices A and B are given by the exogenous stochastic process for a_t and s_t.

In a linear equilibrium the functions $\mathcal{P}$ and $\mathcal{C}$ describing individual behavior are replaced by the linear functions

$$p_{it} = q_h h_{it} + q_a a_{it} + q_z E^I_{it}[Z_t], \tag{9}$$

$$c_{it} = b_h h_{it} + b_a a_{it} + b_p \overline{p}_{it} + b^I_z E^I_{it}[Z_t] + b^{II}_z E^{II}_{it}[Z_t]. \tag{10}$$

that only depend on the first moments of agents' beliefs.

Log-linearizing the monopolist first order condition one gets

$$p_{it} = E^I_{it}[\overline{p}_{it}] + E^I_{it}[c_{it} - a_{it}] + \eta E^I_{it}[y_{it} - a_{it}], \tag{11}$$

where the demand curve of the monopolist and the price index are

$$
\begin{aligned}
y_{it} &= -\sigma p_{it} + \frac{1}{m}\sum_{j:i\in B_{jt}} \left(c_{jt} + \sigma \overline{p}_{jt}\right), \\
\overline{p}_{jt} &= \frac{1}{m}\sum_{l\in B_{jt}} p_{lt}.
\end{aligned}
$$

The pricing equation (11) says that agents set their expected relative price $p_{it} - E_{it}^{I}[\overline{p}_{it}]$ based on their productivity a_{it} and on their expected marginal rate of substitution between consumption and leisure, which depends on their consumption c_{it} and on their labor supply $(y_{it} - a_{it})$.

Similarly, one can take a log-linear approximation of the consumer first order condition and the law of motion for the durable to get

$$\overline{p}_{it} + c_{it} = (1-\delta) h_{it} + \delta E_{it}^{II}\left[\overline{p}_{it+1} + c_{it+1}\right], \tag{12}$$

$$h_{it+1} = h_{it} + \theta (p_{it} + y_{it} - \overline{p}_{it} - c_{it}) + r_{it+1}, \tag{13}$$

where δ and θ are two constants given by

$$\delta = \frac{\alpha\beta}{\alpha + (1-\beta)^2},$$

$$\theta = \frac{1-\beta}{\alpha + 1 - \beta}.$$

To characterize individual behavior we need to find the vector of coefficients q and b in (9) and (10) that satisfy the optimality conditions (11), (12) and (13). This vector can be found applying the method of undetermined coefficients described in the Appendix.

For the economic interpretation of the model it will be useful to have the following result:

Lemma 1 *Optimal expenditure $\overline{p}_{it} + c_{it}$ can be written as a linear function*

$$\overline{p}_{it} + c_{it} = (1-\lambda) h_{it} + \lambda W_{it}.$$

where W_{it} is expected permanent income defined as:

$$W_{it} = (1-\mu)\sum_{j=0}^{\infty} \mu^j E_{it}^{II}\left[p_{it+1} + y_{it+j}\right]$$

where λ is the elasticity of expenditure with respect to permanent income and μ is an adjusted discount factor, and $\lambda, \mu \in (0,1)$.

The derivation of this lemma and the expressions for λ and μ are in Appendix A. Notice that we have defined total expenditure including only the consumption of the specialized goods, therefore the elasticity of expenditure with respect to permanent income, λ, is smaller than one, since a fraction of permanent income is allocated to the services of the durable good.

3.4 Learning and aggregation

In this way we have derived optimal individual behavior as a function solely of individual shocks and of expectations regarding aggregate variables. Two additional steps are needed in order to completely characterize an equilibrium.

First, we need to solve the inference problem of the individual agent in order to determine the private expectations. Second, we need to aggregate the individual rules to derive the aggregate behavior of prices and quantities and define a fixed point in terms of ψ and ϕ.

The learning problem of the individual agent can be solved recursively using the Kalman filter. It is convenient to divide the filtering problem in three stages. The first two stages correspond to the pricing and spending stages I and II described above. The last stage correspond to the moment when consumers return home and observe the quantity sold at the end of period t. We denote end of period expectations as $E_{it}[.]$.

First, agents observe the exogenous signals

$$S_{it} = \begin{pmatrix} a_{it} \\ v_{it} \\ s_t \end{pmatrix}.$$

Next, they observe the prices in the islands they visit. The information contained in these prices can be summarized by the price index $\overline{p}_{it}$. Finally, they observe the quantity sold. Given that they know the selling price p_{it} on their own island (given that all producers on island i are identical) the quantity sold is a signal for the intercept of the demand curve for the composite good produced on island i. This intercept is denoted by q_{it}. The cross-sectional variance of q_{it} is due to two factors. First, the demand is random due to the dispersion of the characteristics of the consumers in visiting island i: their expectations, their productivity shocks a_{jt}, their real balances h_{jt}. Second, the demand is random due to the dispersion of the prices of the goods competing with island i producers.

The relation between the signals $S_{it}, \overline{p}_{it}$ and q_{it} and the aggregate state is summarized by the relations:

$$\begin{aligned} S_{it} &= GZ_t + F(\epsilon_{it}, v_{it})' \\ \overline{p}_{it} &= QZ_t + \zeta_{it} \\ q_{it} &= RZ_t + \eta_{it} \end{aligned}$$

The expressions for the matrices G, F, Q and R and the full derivation of the quantity index q_{it} are in the Appendix B. Notice that the variances of the shocks ζ_{it} and η_{it} are not exogenous, as they depend on the cross-sectional dispersion of prices and consumption levels. Therefore, determining values for the variances σ_ζ^2 and σ_η^2 is part of the fixed point problem. The derivation of the cross-sectional dispersion of prices and quantities is lengthy and not central to the argument of the paper, thus it is relegated to Appendix B.

The second step is to aggregate individual behavior and to obtain a fixed point condition involving ϕ and ψ. Aggregating (9) and (10) one obtains an expression for p_t and y_t that includes the average expectations for the state variable. This average expectations can be expressed as

$$Z_{t|t} = \int E_{it}[Z_t]\, di$$

with similar expressions for expectations at stages I and II. Using the recursive expressions for individual expectations derived from the Kalman filter one can write a recursive condition for the average expectations of the form

$$Z_{t|t} = D_1 Z_{t-1|t-1} + D_2 Z_t.$$

One can solve this recursive condition backward and express $Z_{t|t}$ in terms of $Z_t, Z_{t-1}, ...$ Given that Z_t is an infinite dimensional vector one can define a matrix Ξ such that

$$Z_{t|t} = \Xi Z_t. \tag{14}$$

Analogous expressions can be found for $Z^I_{t|t}$ and $Z^{II}_{t|t}$. Clearly, in the numerical implementation we need to truncate the vector Z_t and use an approximation.

Aggregating the individual pricing and consumption rule and substituting for the average expectations of Z_t one obtains the conditions

$$\begin{aligned} p_t &= \left[(q_h + q_a)\, e_1 + q_z \Xi^I\right] Z_t \\ y_t &= \left[(b_h + b_a)\, e_1 + b_p \phi + b^I_z \Xi^I + b^{II}_z \Xi^I\right] Z_t \end{aligned}$$

where e_1 is the vector $(1, 0, 0, ...)$. Let

$$\begin{aligned} \phi' &= (q_h + q_a)\, e_1 + q_z \Xi^I \\ \psi' &= (b_h + b_a)\, e_1 + b_p \phi + b^I_z \Xi^I + b^{II}_z \Xi^I \end{aligned}$$

and recall that the expressions for the coefficients b and q and the matrices Ξ are function of the coefficients ψ and ϕ conjectured by the agents. Therefore we have defined a map T:

$$\begin{pmatrix} \phi' \\ \psi' \end{pmatrix} = T\left(\begin{pmatrix} \phi \\ \psi \end{pmatrix}\right).$$

A linear approximation to a stochastic steady state is found by finding a fixed point of the map T. The computational method we use for the numerical analysis is simply an iteration method based on the map T. The details are in Appendix B.

4 News shocks, prices and output

4.1 Full information

In this section we consider two special cases to illustrate the basic mechanisms of the model. In these cases the linear equilibrium can be solved analytically. First, consider the case of full information. Suppose that agents can observe all the shocks in the economy, private and aggregate. In this case equation (11) boils down to

$$p_{it} = \overline{p}_{it} + c_{it} - (1 + \eta)\, a_{it} + \eta y_{it},$$

that can be aggregated to

$$0 = (1 + \eta)\,(y_t - a_t)\,,$$

and gives

$$y_t = a_t.$$

Notice that this result comes only from the pricing equation. When prices are fully observed output and employment in this economy are fully determined by the supply side of the model and are completely independent of demand and price dynamics.

Given that the expectations are identical across consumers the Euler equation can also be aggregated to obtain

$$p_t + a_t = (1 - \delta)\, a_t + \delta E_t \left[p_{t+1} + a_{t+1}\right],$$

using $E_t [a_{t+1}] = a_t$ and solving forward one finds the unique non-explosive path for p_t

$$p_t = 0,$$

the price level is constant and equal to its steady state level.

Therefore, under full information the economy displays —to a first approximation— identical behavior to the benchmark case with no heterogeneity. In particular: output at date t depends only on current productivity a_t, employment is constant and independent of productivity, prices are constant. Define the log of money velocity as

$$v_t = p_t + y_t - h_t.$$

Since the stock of the medium of exchange adjusts automatically to productivity increases, velocity is constant at a constant price level.

Notice that we have assumed that agents observe all the shocks in the economy. However, in a linear equilibrium the same results can be established in the case where agents observe only the aggregate shock a_t. In that case producers still have no information about the prices faced by consumers and consumers have no information on the quantity sold by producers. However agents' expectations regarding aggregate variables are still identical across agents and this allows us to establish that the aggregate price level is constant and aggregate output is equal to a_t. The cross-sectional dispersion of prices and output will be different from the case of complete information but the aggregate behavior will be –to a first order approximation– unchanged.

4.2 Uninformative private signals

A second case that can be solved analytically is the case where idiosyncratic uncertainty is very large with respect to the innovations in aggregate productivity. As the ratios $\frac{\sigma_\epsilon^2}{\sigma_u^2}$ and $\frac{\sigma_v^2}{\sigma_u^2}$ approach infinity the cross sectional dispersion of the private signals $a_{it}, r_{it}, \overline{p}_{it}$ and q_{it} increases relative to the size of aggregate shocks. Thus, agents put less and less weight on their private information to forecast aggregate variables. In the limit agents' forecasts of aggregate variables are identical across agents and they are solely based on the public signal s_t.

Furthermore, given the high dispersion of price signals agents' expectations are the same in stages I and II, that is we have

$$E_{it}^{I}[Z_t] = E_{it}^{II}[Z_t] = E[Z_t | s_t, s_{t-1}, ...]$$

for all i. Notice that private shocks still matter for pricing and consumption decisions and agents do not have common information regarding them, for example $E_{it}^{I}[a_{it}] = a_{it} \neq E_{jt}^{I}[a_{it}]$ for $j \neq i$.

To simplify matters assume that at the end of each period t the aggregate productivity shock is publicly revealed. In this way the learning dynamics are muted, a_{t-1} is common knowledge and agents only have to form expectations about the current shock u_t. Denote the expectations based only on public information as $E_t[.]$. Then the following proposition characterizes the equilibrium of this economy.

Proposition 2 *In the case of* $\sigma_\epsilon^2/\sigma_u^2 = \sigma_v^2/\sigma_u^2 = \infty$ *and full information on* a_{t-1} *equilibrium output and prices are given by*

$$p_t = \frac{\tilde{\lambda} + \eta}{1 + \eta(\hat{\sigma} - 1)} \left(E_t[a_t] - a_t \right) \quad (15)$$

$$y_t = \left(1 - \tilde{\lambda}\right) a_t + \tilde{\lambda} E_t[a_t] - p_t \quad (16)$$

where $\tilde{\lambda}$ *is a coefficient in* $(0, 1)$ *and where* $\hat{\sigma}$ *is*

$$\hat{\sigma} = \sigma \left(1 - \frac{1}{m}\right) + \frac{1}{m}.$$

The expectation of aggregate output and prices in the current and future periods are equal across agents and given by

$$E_t[y_{t+j}] = E_t[a_t] \text{ for } j = 0, 1, ...$$
$$E_t[p_{t+j}] = 0 \text{ for } j = 0, 1, ...$$

Let us provide an interpretation of equations (15) and (16), starting from the latter.

The coefficient $\tilde{\lambda}$ represents the elasticity of individual expenditure with respect to the expected net present value of current and future *aggregate* nominal output, $y_t + p_t$. The coefficient $\tilde{\lambda}$ differs from the coefficient λ derived in Lemma (1) because λ represents the elasticity of individual expenditure with respect to the expected net present value of *individual* nominal income, $p_{it} + y_{it}$. The calculation of $\tilde{\lambda}$ takes into account that current and future prices p_{it} are endogenously determined and depend on the optimal consumption path. The formal derivation of $\tilde{\lambda}$ is in Appendix A. The main difference between λ and $\tilde{\lambda}$ is that while λ only depends on the parameters α and β (i.e. the preference for the durable good and the discount factor), $\tilde{\lambda}$ depends on all the parameters of the model since it captures the result of the optimal adjustment of both consumption and prices by consumers.

The price level p_t appears in equation (16) for the following reason. Notice that $E_t[p_{t+1}] = 0$ so an increase in the price level today corresponds to a proportional reduction in the real rate of return on the durable good, which is equal to $-p_t$. Since the intertemporal elasticity of substitution is 1 this results in a proportional reduction in consumption.

Thus, (16) bears only a casual resemblance to an aggregate demand equation of the type $y_t = m_t - p_t$ that would arise in a model with a fixed money supply and fixed money velocity. Consumers here are free to adjust velocity by varying their consumption of the durable good, prices appear in (16) only because of an intertemporal effect and the consumption response incorporates optimal pricing behavior in the current and future period.

On the other hand, (15) is very closely related to the "aggregate supply" relation derived in Lucas (1972) where confusion between relative and absolute price changes allow temporary output deviations from its natural level. When expected aggregate productivity is large relative to individual productivity, agents try to increase the relative price of their good. They do so for two reasons: because high expected productivity shifts the demand curve faced by each monopolist and because high expected productivity leads to higher consumption by the same monopolist. This leads to a higher price level.

Notice that in this model agents cannot disentangle two shocks that have opposite effects on inflation. In the simple case we are considering here the effects of the two shocks exactly balance so that expected inflation $E_t[p_t]$ is zero. Therefore aggregate inflation is completely unexpected and corresponds to the expectation error in the estimate of current productivity. In the general case analyzed in the next section shocks will have an effect on both expected and unexpected inflation.

Now consider the effects of the two underlying shocks e_t and u_t on prices and output. The news shock e_t increases $E_t[a_t]$ without affecting a_t. This leads to an increase in prices. On the quantity side an increase in permanent income $E_t[a_t]$ shifts equation (16) to the right. However as prices have increased the net effect on quantities can be positive or negative. The total effect of a news shock on output is positive if the following inequality holds

$$\tilde{\lambda} - \frac{\tilde{\lambda} + \eta}{1 + \eta(\hat{\sigma} - 1)} > 0. \tag{17}$$

We will examine conditions under which this inequality holds in the following. When output increases also employment must increase given that productivity has not changed.

Let us turn to the effect of a productivity shock u_t, which increases both a_t and $E_t[a_t]$. Due to imperfect information the effect of u_t on $E_t[a_t]$ is smaller than the effect on a_t itself. Therefore $E_t[a_t] - a_t$ declines and so do prices. This happens because the expected upward shift in the demand curve faced by each producer is less than proportional than the productivity increase itself.

The effect of a productivity shock on total output is unambiguously positive. However, the effect on employment depends on wether the effect on y_t is larger or smaller than one. Inspecting the equilibrium conditions one can see that if inequality (17) is satisfied the response of output will be smaller than one and employment will decrease on impact after a positive productivity shock.

Notice that $\tilde{\lambda}$ is a function of $\hat{\sigma}$ so condition (17) cannot be immediately interpreted in terms of fundamental parameters. However, it is easy to show that $\tilde{\lambda}$ is bounded above and bounded away from zero so for $\hat{\sigma}$ large enough (17) is satisfied. This fact and the previous discussion are summarized in the next proposition.

Proposition 3 *There is a finite cutoff $\overline{\sigma} > 1$ such that if $\hat{\sigma} \geq \overline{\sigma}$ then:*
a positive productivity shock, $u_t > 0$, increases output, and decreases prices, employment and velocity;
a positive news shock, $e_t > 0$, increases output, prices, employment and velocity.

The model has a further implication regarding the response of consumers' expectations to the two shocks.

Remark 4 *If $\hat{\sigma} \geq \overline{\sigma}$ then:*
after a positive productivity shock, $u_t > 0$, consumers' expectations satisfy $E_t[y_t] < y_t$;
after a positive news shock, $e_t > 0$, consumers' expectations satisfy $E_t[y_t] > y_t$.

Therefore, after a news shock consumers' confidence reacts more compared to actual output growth, while after a productivity shock consumers' confidence reacts less. We will return to this prediction when we analyze the dynamic case.

We conclude this section with a comparative static result, regarding the effects of the news shock e_t.

Proposition 5 *An increase in $\hat{\sigma}$, keeping $\tilde{\lambda}$ constant, increases the response of consumption to the news shock e_t and reduces the response of the price level to the same shock. An increase in $\tilde{\lambda}$, everything else equal, increases the response of both consumption and the price level to the news shock e_t.*

Even though this result is not stated in terms of fundamental parameters, it is of practical relevance because, for the range of parameter values we use in the numerical section, $\tilde{\lambda}$ is very responsive to variations in the corrected discount factor δ and not very responsive to changes in $\hat{\sigma}$. So by changing δ and $\hat{\sigma}$ we will obtain similar implications: an increase in δ determines an increase in the responses of both consumption and prices, while an increase in $\hat{\sigma}$ will increase the response of consumption and decrease the response of prices.

The coefficient $\hat{\sigma}$ is associated to the degree of strategic complementarity in producers' pricing. This connection has been studied extensively in the literature on sticky prices[15]. Its role in economies with imperfect information has

[15] See the discussion in Chapter 3 (§1.3) of Woodford (2003),

been emphasized by Woodford (2002) who presents a model where strategic complementarity leads to slower adjustment of prices after a monetary shock. In the present model the degree of strategic complementarity in pricing is given by

$$\hat{\sigma} = \sigma \left(1 - \frac{1}{m}\right) + \frac{1}{m}$$

where m is the number of islands visited by each consumer. A producer can immediately adjust to a price increase by other local producers, since he has full information about these prices. On the other hand, a producer has to form expectations on the prices set by his competitors located on other islands. The fraction $1/m$ is the fraction of prices, competing with good (i, k), set on the same island, while $1 - 1/m$ is the fraction of prices set on other islands. A higher m implies a higher degree of uncertainty in price setting and a higher degree of complementarity. The fraction $1/m$ here plays a role analogous to the fraction of firms that can adjust their price in models with sticky prices. A larger value for m increases the uncertainty regarding other producers' prices and, given that $\sigma > 1$, it reduces the response of the aggregate price level to both shocks.

It is interesting to ask what determines the level of δ and thus of $\tilde{\lambda}$. A high value for δ corresponds to an economy where the value of the durable good is large relative to total transactions even though the value of the services provided by the durable good is small compared to total output. This happens when α is small and β is close to one, so that the present value of the services of the durable good is high, even though the flow value of the services of the durable good may be small. If we consider a sequence of economies with $\beta_n = 1 - \frac{1}{n}$ and $\alpha_n = \frac{1}{n}$ then δ_n and $\tilde{\lambda}_n$ both converge to 1. Moreover the parameters λ and μ derived in Lemma 1 will also converge to 1. In this way we can approximate a "pure credit" economy where consumption at date t responds one-for-one to expected permanent changes in income.

5 Equilibrium dynamics

In this section we use numerical simulations of the model in order to further study its qualitative and quantitative implications. On the qualitative side, we look at the response of the economy to a news shock, e_t, and a productivity shock, u_t. The main predictions derived in the simple case of uninformative private signals in section 4.2 extend to the general model. Namely, prices and employment increase following a positive news shock and decrease following a positive productivity shock. Consumers' expectations over-react after a news shock and under-react after a productivity shock. The response of prices and quantities depends on the elasticity of substitution σ and on the propensity to spend out of permanent income λ. Moreover, in the dynamic case we can characterize the adjustment dynamics and show that they depend in a non-monotone way on the informativeness of the public signal.

Next, we turn to the quantitative implications and we ask what fraction of output volatility can be explained by the news shock. In particular, we look at the ability of the model to replicate the variance decompositions at various horizons obtained in existing VAR studies.

To compute the equilibrium we use the method of undetermined coefficients described in section 3. The state space is approximated using the truncated state vector $Z_t^{(T)} = \{a_t, s_t, ..., a_{t-T}, s_{t-T}\}$. For T sufficiently large the choice of T does not affect the results.

The model is very stylized in at least three respects: there is no capital, labor is immobile across sectors and the durable good is the only asset available. Still, we try to choose reasonable values for the benchmark model parameters. The objective of the quantitative analysis at this stage is mostly to understand the role of the various parameters in determining the relative response of prices and quantities and the speed of adjustment of the economy to the full information path.

The parameter β is set at 0.99 so one can interpret the time period as a quarter. The parameter η is set at 0.33 corresponding to a Frisch labor elasticity of 3. The parameter σ is set equal to 15 which implies a mark-up of around 7%. This value for σ is relatively high compared to the range of values commonly used in business cycle models with price rigidities. As we will see later, a high value of σ is needed in order to replicate the relative responses of prices and quantities.

The parameter α is set equal to 0.01 in our benchmark parametrization. This preference parameter, together with $\beta = 0.99$ implies that the ratio of the value of the durable good to output in steady state is equal to 2. Considering that the durable good is the only type of financial wealth in the economy this is not an unreasonable parametrization. On the other hand, this clearly implies a small value for money velocity (which is 1/2) so we also experiment with larger values for α. In terms of the permanent income equation in Lemma 1 $\alpha = 0.01$ implies that $(\lambda, \mu) = (0.82, 0.90)$ which imply a propensity to spend out of permanent income equal to 0.82, and that consumers use a discount factor of 0.9 when evaluating permanent income. Both values are relatively low, and generate a downward bias in the response of spending to expected permanent changes in income. This will tend to give us conservative estimates of the effects of news shocks[16].

As discussed in section 4 the parameters σ and λ are crucial because they determine the responses of prices and quantities to the news shock. The effect of changes in these two parameters on equilibrium dynamics will be analyzed in

[16]The relation between α and λ (for a given β) is non-monotone and this value for λ is the maximum attainable given that $\beta = .99$. As noticed above, as we let α and $1 - \beta$ go to zero, both λ and μ converge to 1, however the convergence is slow, which explains the relatively low values we get for λ and μ.

By modifying the utility function to include a term $\alpha \ln \left(X_{it} - \bar{X} \right)$ it is possible to overcome this problem and disconnect the elasticity of spending from the money-to-output ratio. In this way one can obtain values of λ close to 1 for any choice of β. However, at this stage this seems an unnecessary complication, given the stylized nature of the model.

detail in 5.2.

It remains to choose values for the variances of the shocks. We set $\sigma_u = 0.01$. For the variance decomposition exercise this choice of σ_u is merely a normalization. However, this value of σ_u roughly matches observed medium run volatility of TFP growth. In particular it implies that the standard deviation of TFP growth over 10 year intervals is equal to 0.63%, the corresponding value for US postwar data is 0.67% a year[17]. For the idiosyncratic component of productivity we set $\sigma_\epsilon = 6\sigma_u$. To choose this parameter we look at the dispersion in the innovation of productivity shocks across firms, documented in Franco and Philippon (2004).[18] On the other hand, it is difficult to calibrate σ_v as it does not have any obvious empirical counterpart. For now, we set $\sigma_v = 10\sigma_u$ to limit the amount of information revealed through this channel.

The parameter σ_e has a central role in our analysis since it determines the precision of the public signal. As a benchmark let us set $\sigma_e = 3\sigma_u$..We will experiment with different values for $\sigma_e, \sigma_\epsilon$ and σ_v and look at their implications for short-run output volatility. In particular, we will attempt to replicate the variance decompositions obtained in Shapiro and Watson (1988) and Gali (1992).

Finally, we set the number of islands visited to $m = 2$. We choose a small value for m in order to limit the speed of learning in the economy. When m is large consumers have a large number of independent price observations and quantity observations, thus, given the simple structure of the model, they can learn quickly the underlying value of a_t.

5.1 Dynamic responses

Figure 2 shows the responses of output, employment and the price level to a productivity shock u_t and to a news shock e_t. The last panel of Figure 2 illustrates the dynamics of average expectations regarding aggregate productivity, $a_{t|t}$.

The qualitative responses are analogous to the ones obtained in section 4. It is interesting to see that after a productivity shock u_t output adjusts gradually to the new equilibrium level. Along the transition path output grows less than actual productivity. The average agent realizes that aggregate productivity has

[17] Using BLS data for private nonfarm businesses for the post-war period. In the model this statistic is approximately equal to $\sqrt{\frac{4}{10}}\sigma_u$.

[18] In particular, they report that the average coefficient of correlation between the technology shocks of any pair of firms is equal 2.6% (p. 9), which implies:

$$1 + \frac{\sigma_\epsilon^2}{\sigma_u^2} = \frac{1}{0.026}.$$

Here we use their result in order to make assumptions about the informativeness of local shocks on the aggregate economy.

On the other hand, that their results suggest that the idiosyncratic shocks ϵ_{it} should be modeled as permanent rather than temporary. Unfortunately, with permanent shocks the wealth distribution in our model is no longer stationary. In the conclusions we discuss how to extend the model to allow for persistent idiosyncratic shocks.

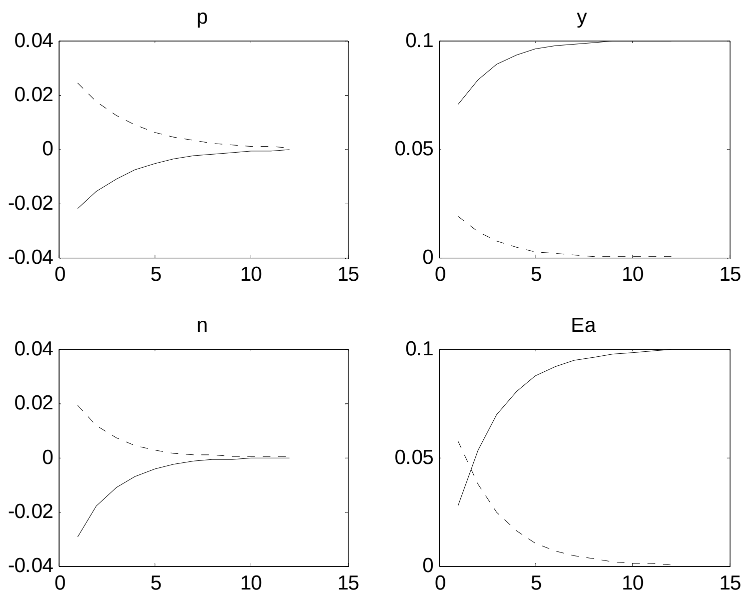

Figure 2: Impulse responses of prices, output, employment and expected productivity.
Solid line: response to $u_t = 0.1$, dashed line: response to $e_t = \frac{\sigma_e}{\sigma_u} 0.1$.

increased, but believes that his individual productivity has increased more. His optimal response is to reduce labor supply and reduce the price of his good. This generates a temporary reduction in aggregate employment and in the price level.

From a qualitative standpoint the conditional covariances of inflation and output are consistent with the evidence from identified VAR exercises if we match e_t with the IS shock and u_t with the supply shock (see e.g. Table III in Gali (1992)). Following a news shock output and inflation have positive correlation, while following a productivity shock they have negative correlation. A similar result holds for the conditional correlation of output and employment. The negative conditional correlation of output and employment after a permanent technology shocks is consistent with the evidence presented in Gali (1999) and Francis and Ramey (2003).

Recently, there has been substantial controversy regarding this empirical finding[19] and more generally regarding the use of VAR evidence with semi-structural identification assumptions[20]. Part of the controversy is due to the lack of theoretical models that can be used to interpret the "demand shocks" or "IS shocks" identified in the VAR literature. The present model has the advantage of having a theoretical structure consistent with the identification assumptions made in the VAR literature and offers a micro-founded theory of the "demand shocks" thus identified[21].

Let us turn to the model predictions regarding the relative reaction of output and output expectations following news shocks and productivity shocks. Once more, the dynamic results confirm the results obtained in the simple static case of section 4. Figure 3 (panel (a)) shows the response of output y_t (solid line) and the average first order expectation of output $y_{t|t}$ (dashed line) after a news shock. After a positive news shock output expectations tend to increase more than actual output. On the other hand after a positive productivity shock output tends to increase less than expected productivity, as we can see from panel (b) of Figure 3.

To give an economic interpretation for this behavior it is useful to obtain a simple approximate expression for equilibrium output. Given that agents are forward looking and use a discount factor relatively close to 1 ($\mu = 0.9$) they tend to focus on long-run income to estimate their permanent income. Since they know that the economy will converge to a level of output equal to productivity a_t their permanent income is roughly approximated by $a_{t|t}$. Moreover, they expect the price level to revert to its steady state relatively fast. Therefore, increases in the current price level are associated to a proportional increase in the real interest rate. Since agents have a propensity to spend out of permanent income equal to $\lambda = 0.8$ and their elasticity of intertemporal substitution is 1

[19]See Christiano, Eichenbaum and Viguffson (2003) and Gali and Rabanal (2004).

[20]Chari, Kehoe and McGrattan (2004).

[21]Some VAR studies follow Blanchard and Quah (1989) and use long-run restrictions to separate "supply shocks" from "demand shocks" others, e.g. Blanchard (1989), use sign restrictions on the responses of prices. Our model is consistent with both approaches.

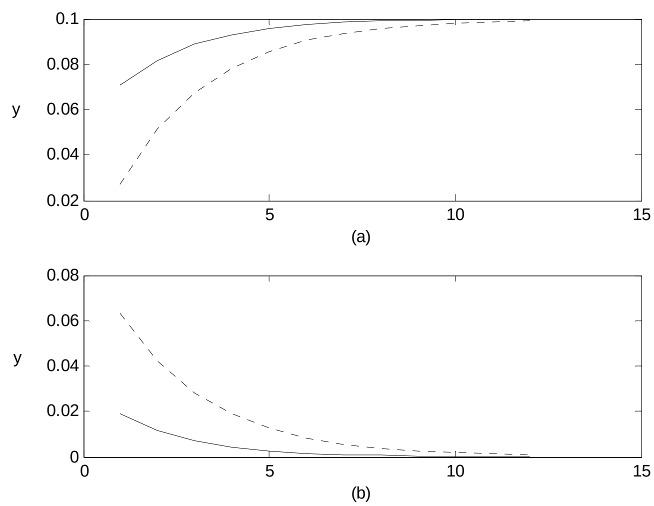

Figure 3: Responses of output and average expected output to a productivity shock (panel (a)) and to a news shock (panel (b)).
Output: solid line; average expected output: dashed line. Shocks as in Figure 3.

we can reasonably approximate output behavior as:

$$y_t \approx 0.8a_{t|t} - p_t$$

Figure 4 illustrates this approximation for the output response to news and productivity shocks.

Using this approximation we can now study why expected output responds more than realized output to a news shock. First, expected output depends on the average expectation of $a_{t|t}$, that is, the second order expectation of productivity which we denote by $a^{(2)}_{t|t}$. As is common in models with higher order expectations higher order expectation tends to be more sensitive to the public signal (s_t) and less sensitive to the private signals (a_{it} in this case)[22]. Therefore, after a (public) noise shocks $a^{(2)}_{t|t}$ responds more than $a_{t|t}$. This is illustrated in panel (a) of Figure 5. Secondly, output depends negatively on prices. However, since prices tend to respond to the average expectation error $a_t - a_{t|t}$ the first order expectation of prices, $p_{t|t}$, tends to be closer to zero than actual prices (see Figure 5, panel (b)). Therefore, conditional on a news shocks agents tend to overestimate the average perceived change in permanent income and to underestimate the associated inflation. Both reasons concur to induce agents to overestimate the output increase .

[22] See Morris and Shin (2002) for a thorough discussion of this phenomenon.

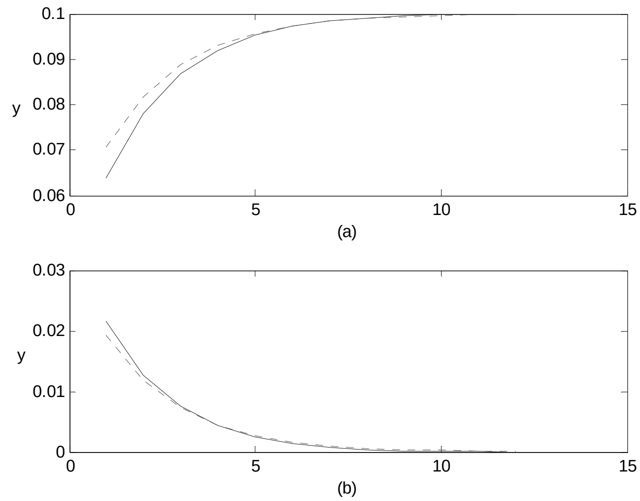

Figure 4: Approximation for output responses to productivity (panel (a)) and news shock (panel (b)).
Using $0.8 * a_{t|t} - p_t$ (dashed line) to approximate output (solid line). Shocks as in Figure 3.

A similar (symmetric) reasoning applies to the case of a productivity shock and can be used to explain the underreaction of expectations following that type of shock.

These predictions regarding the relative reaction of consumers' expectations following the two type of shocks are amenable to empirical testing using available measures of consumers' expectations. This is something we plan to pursue in future work.

5.2 Strategic complementarity in pricing and spending elasticity

In section 4 we noticed that the two crucial parameters that determine the responses of prices and output to the various shocks are σ and λ, the first affects the degree of strategic complementarity in pricing while the second determines the elasticity of spending with respect to permanent income.

Here we illustrate this point using two examples. In Figure 6 we illustrate the responses of prices and output for different levels of σ. In the top two panels we report the results for the benchmark level $\sigma = 15$ and in the bottom two panels we report the results for $\sigma = 35$. A higher level of elasticity of substitution mutes the price response to both shocks, increases the response of output to a

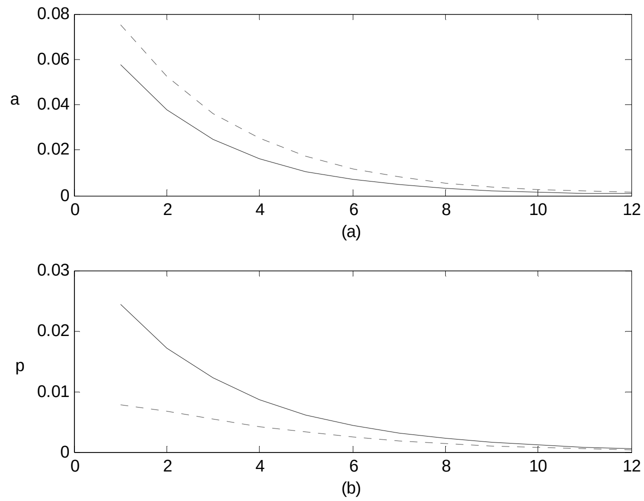

Figure 5: Decomposition of output and expected output response.
Panel (a): Response of $a_{t|t}$ (solid line) and $a^{(2)}_{t|t}$ (dashed line) to a news shock.
Panel (b): Response of p_t (solid line) and $p_{t|t}$ (dashed line) to a news shock.
Shocks as in Figure 3.

news shock and slows down output adjustment following a productivity shock.

In Figure 7 we illustrate the effect of changing β from 0.99 to 0.94. Again, the top two panels report the benchmark impulse responses, for comparison. A decrease in the discount factor keeping the preference for the durable good constant ($\alpha = 0.1$) has the effect of reducing the effective supply of means of payments in this economy. This reduces the elasticity of spending with respect to permanent income from $\lambda = 0.8$ to $\lambda = 0.5$. The endogenous response of money velocity to changing expectations regarding permanent income is much smaller, and news shocks have a smaller effect on this economy, both in terms of prices and in terms of quantities.

Looking at Figure 7, one could be tempted to draw implications regarding the desirability of a tight control over monetary aggregates. The economy with a smaller λ is an economy where money velocity is less responsive to consumers' expectations due to the relative scarcity of the durable good that serves as medium of exchange. When such is the case consumers' spending is more stable and expectations shocks have a smaller cyclical impact. This seems to resonate with the monetarist view that the banking system should be regulated so as to keep under control changes in money velocity. However, it is important to remember that, in order to keep monetary disturbances to the minimum, we have made a strong assumption, namely that the stock of the durable good

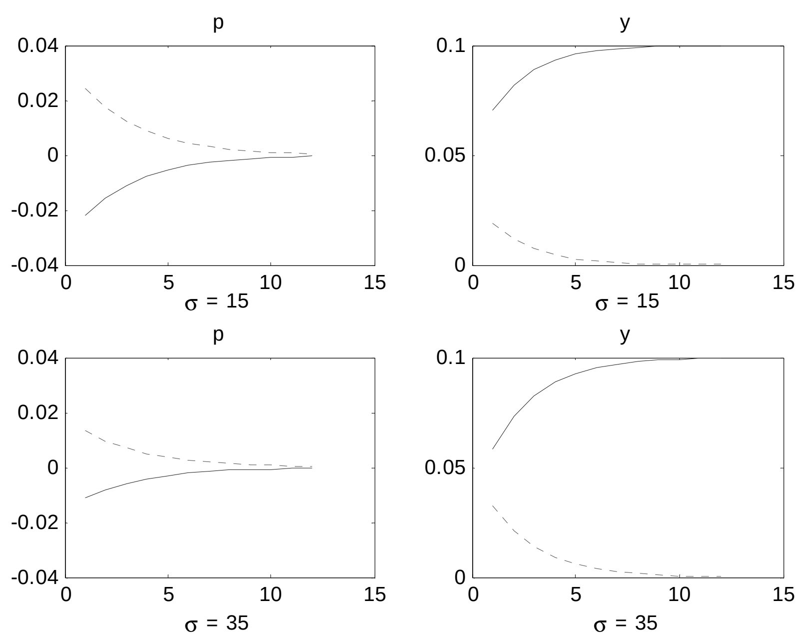

Figure 6: Strategic complementarity in pricing and price and output responses. Solid line: response to productivity shock. Dashed line: response to news shock. Shocks as in Figure 3.

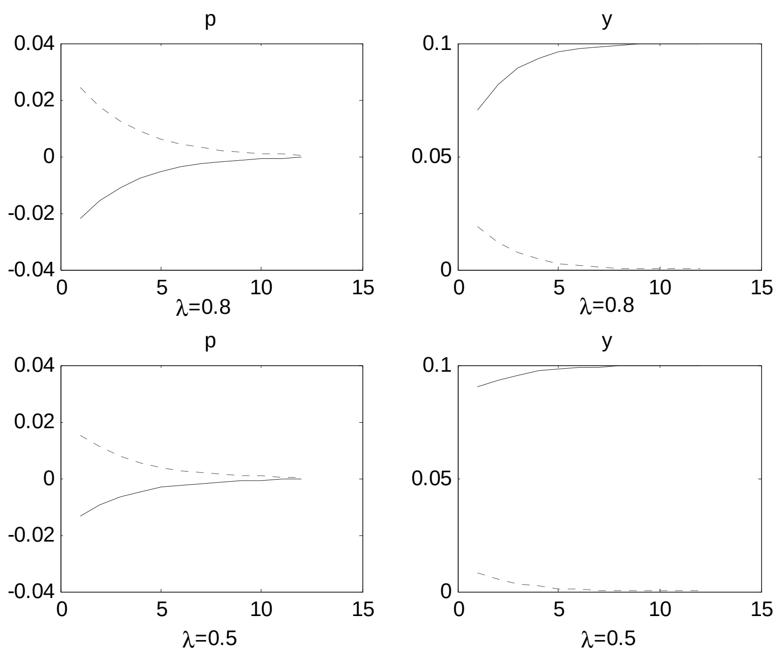

Figure 7: Spending elasticity and price and output responses.
Solid line: response to productivity shock. Dashed line: response to news shock. Shocks as in Figure 3.

adjusts *automatically* to the average productivity level. This assumption is analogous to assuming that there is a monetary authority with perfect knowledge regarding a_t that adjusts money supply one-for-one with productivity growth. Therefore, at this stage one should be wary of drawing any implications for monetary policy. In current related research the author is analyzing a version of the model with an explicit role for monetary policy. That model can be used to evaluate monetary policy in a more realistic setup where the monetary authority has imperfect information regarding aggregate productivity.

5.3 Noise shocks and persistence

By construction, the news shock can only affect output in the short run, while all long-run output volatility is due to the productivity shock. An important quantitative question is: What fraction of short-run output volatility can be explained by the news shock? The structure of the model imposes a bound on the fraction of output volatility that can be explained by the news shock. If the public signal is very noisy agents would disregard it altogether, while if the signal is very precise the economy will converge very fast to the full information equilibrium. In both cases the noise will explain a small fraction of output volatility. Therefore, the question is whether intermediate levels of signal precision can generate realistic values for the fraction of output volatility explained by the news shock.

Figure 8 illustrates the effects of changing the precision of the signal (i.e. changing σ_e) on the dynamic responses of output[23]. In the first panel of Figure 8 the public signal is very precise, after a productivity shock the economy converges very fast to the long-run equilibrium and a news shock has a very small and temporary effect on output. As we move to the second and third panel we see that the effect of a news shock increases *and* becomes more persistent. However, in the fourth panel we see that as the noise is very large agents stop relying on the public signal and the impact effect becomes smaller. On the other hand, as the quality of the signal deteriorates agents take a longer time to learn the long-run equilibrium, so the demand shock becomes very persistent. Also, as information becomes less precise output takes a long time to adjust after a real productivity shock.

Table 1 summarizes the result of experimenting with different values for the precision of the private and public signals. In the table we report the fraction of variance accounted for by the news shock. For comparison, the last column reports the values obtained by Gali (1992) (Table IV), under the assumption that the news shock and productivity shock correspond, respectively, to the "IS shock" and "supply shock" identified in that paper. The last two rows of the table report the cross sectional standard deviation of the price and quantity signals $\overline{p}_{it}$ and q_{it} as a fraction of the volatility of the quarterly innovation in the underlying aggregate variables. This ratios reflect the precision of the *endogenous* private signals.

[23] To make the graphs easier to read we set $\sigma = 35$ for Figure 9.

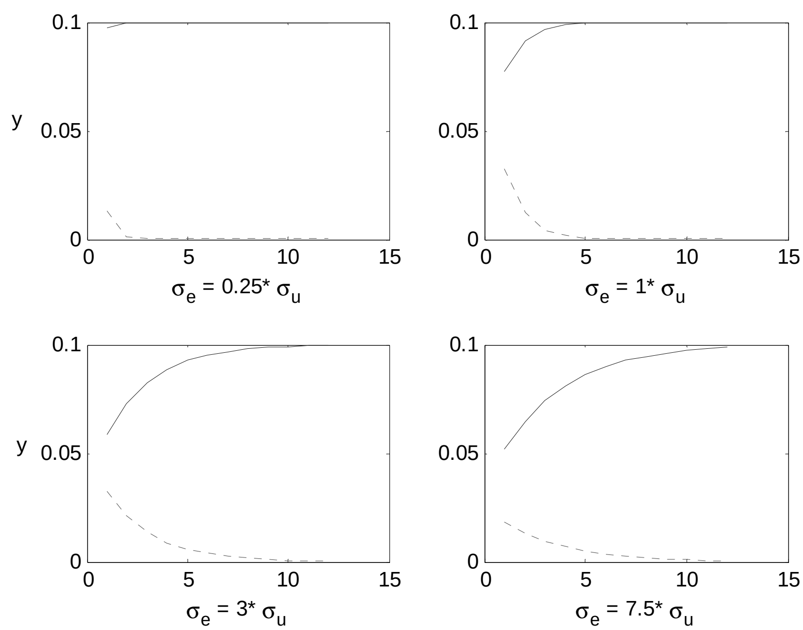

Figure 8: Signal precision and output responses.
Solid line: response to productivity shock. Dashed line: response to news shock. Shocks as in Figure 3.

In the first column we report the variance decomposition for the benchmark parameters. Under that set of parameters the noise shocks only account for a small fraction of output volatility at any horizon. To obtain more realistic values one needs to decrease the precision of both private signals, as in column (ii). Notice, however, that while the volatility of output on impact increases agents are still learning very fast and the output volatility explained by the news shock over a 1 year horizon is very small. This happens because the endogenous price signal is still very precise. This means that at the end of each period agents can make precise inferences about the aggregate price level and use it to update their beliefs about the underlying value of the productivity shock. Notice that we made two simplifying assumptions that make price signals very informative: we completely abstracted from monetary shocks (there are no aggregate shocks to the money supply and no money demand shocks) and we abstracted from preference shocks that change the relative demands for goods produced in different islands (as in Lucas (1972)). With these assumptions the structure of the model is very simple and the price level essentially reflects aggregate deviation between the perceived level of a_t and actual a_t. Furthermore, we have assumed that all the news shocks are i.i.d. and all the price and quantity observations made by agents are independent. This allows every agent to collect a large sample of price and quantity observations in a short amount of time. Allowing for a more realistic autocorrelation structure for the shocks would both slow down learning and introduce an additional source of persistence. Finally, the model has no propagation mechanism aside from information diffusion, in particular there are no temporary technology shocks and no capital. Given these assumptions, it is not surprising that the news shock has a only short-lived effects.

The variance decomposition for the prices, on the other hand, does not depend much on the values we choose for the precision of private and public signals. This is not surprising because the price level does not have a long run component, so the effect of the two shocks tend to vanish over time and their relative weights tend to be similar.

Table 1. Variance decomposition

	i	ii	iii	iv	Gali (1992)
$\frac{\sigma_e}{\sigma_u}$	3	3	3	8	
$\frac{\sigma_\epsilon}{\sigma_u}$	6	12	25	25	
$\frac{\sigma_v}{\sigma_u}$	10	20	25	25	
Output					
1 quarter	0.07	0.19	0.25	0.31	0.31
2 quarters	0.04	0.13	0.18	0.27	—
5 quarters	0.02	0.05	0.08	0.18	0.19
10 quarters	0.01	0.02	0.04	0.10	0.10
Prices					
1 quarter	0.56	0.58	0.58	0.50	0.37
2 quarters	0.56	0.58	0.58	0.50	—
5 quarters	0.56	0.58	0.58	0.50	0.52
10 quarters	0.56	0.58	0.58	0.49	0.51
Endogenous signals					
$\frac{\sigma_{\overline{p}_{it}}}{\sigma_{p_t}}$	0.70	2.75	11.38	9.93	
$\frac{\sigma_{q_{it}}}{\sigma_{q_t}}$	9.03	35.09	142.67	125.81	
Learning time					
	1.8	2.0	2.1	5.0	

Note: Fraction of forecast volatility explained by the e_t shock.
Last column from Table IV in Gali (1992).
See the text for the definition of learning time.

Going back to output volatility, if one wants to replicate the variance decomposition for output obtained in keynesian VAR exercises like Shapiro and Watson (1988) or Gali (1992) one has to use quite large values for the idiosyncratic shocks. This is what we do in columns (iii) and (iv). Once the idiosyncratic shocks are sufficiently dispersed then it is possible to change the volatility of σ_e in order to match the time profile of the variance decomposition. As noticed when discussing Figure 8, increasing σ_e increases the persistence of the effects of the news shocks. In column (iv) we find a combination of variance parameters that replicates well the evidence in Gali (1992).

The calibration in (iii) and (iv) is clearly not realistic in terms of the cross-sectional implications of the model. On the other hand it is realistic in terms of the implied speed of learning of the private sector. The speed of learning can be measured in terms of how many quarters it take for agents in the economy to realize that there has been a permanent productivity shock. In particular consider the time it takes for $a_{t|t}$ to reach $\frac{1}{2}a_t$ after a permanent productivity shock u_t. In the last row of Table 1 we report this statistic. Notice that, with the exception of column (iv), we obtain very fast learning, where agents learn half of the productivity increase in the first 2 quarters. Considering that economists and central bankers have often taken several years to acknowledge the occurrence of major shifts in productivity growth, the only realistic parametrization seem to be the one in column (iv).

Naturally, this means that in order to obtain accurate quantitative predictions on the speed of learning in the economy without incredible assumptions about idiosyncratic shocks we need a richer model that allows for some of the ingredients listed above (monetary shocks, autocorrelated shocks, etc.).

The quantitative conclusion we draw at this stage is that in a model with a realistic speed of learning regarding shifts in aggregate productivity public news shocks can account for a realistic fraction of short run volatility.

6 Concluding remarks

The main objective of this paper was to analyze a simple setup where agents spending adjusts endogenously to changes in expected output and where these shifts in expectations are non-neutral in the short term due to the informational separation between buyers and sellers. In this setup the lack of full information bites twice, on the one hand the lack of full information means that demand shifts can be non-neutral, through a version of the Phelps-Lucas mechanism, on the other hand since consumers have imperfect information about future productivity consumers spending can move away from actual productivity and generate demand shifts, absent any shocks to the money supply. This model has a rich set of implications regarding the conditional covariances of output, prices and employment that are amenable to quantitative analysis. In this paper we have showed that the main qualitative predictions seem consistent with the existing VAR evidence.

The basic message of our approach is that uncertainty over growth in the medium-long run is key to understanding short run fluctuations. The first quantitative question that arises is: given the observed uncertainty about productivity shifts over the medium-long run, can we explain short run fluctuations as "aggregate mistakes" in the process of adjusting to these productivity shifts?

In the paper we have made a first attempt at answering this question and we have been able to obtain a realistic amount of short run volatility driven purely by news shocks. This comes at the cost of assuming irrealistically large idiosyncratic shocks. At this stage, these large idiosyncratic shocks are a stand-in for all the unmodelled sources of uncertainty that can complicate the learning problem of the individual agents. Clearly, more quantitative work is required to describe more explicitly these additional sources of uncertainty and assess their relevance.

In the present model, we have left aside any role for monetary policy by introducing a commodity money that "by magic" adjusts to the productivity shifts in the economy. We have done so on purpose to try and abstract as much as possible from monetary disturbances as a source of business cycles. However, the question that remains open is whether *systematic* monetary interventions can dampen the response of private agents to expected future productivity and in this way reduce the output fluctuations generated by news shocks. In order for the monetary authority to do so it is not necessary for the monetary authority to have superior information. The monetary authority can intervene *ex post*, when all agents in the economy have realized that output has exceeded its potential in the past, and do so in such a way as to increase the expected real rate of interest *ex ante* when output is above potential. Studying the feasibility and desirability of such interventions is the matter of a separate paper (Lorenzoni (2005)).

Notice also, that the present model already embeds a strong automatic stabilizer. In the model the price level is stationary. This means that a positive inflation shock generates an expected *deflation*, i.e. an increase in the real interest rate. This is why price level increases substantially dampen the effect of news shock on output. In a fully fledged monetary model the real interest rate will also respond to inflation if the monetary authority follows an active monetary policy rule. It is an important task for future research to evaluate whether the degree of automatic stabilization we assume in the present model is higher or lower than the level of stabilization attained by actual (or optimal) policy rules. Again, for all these questions it is unavoidable to turn to models with an explicit role for monetary policy.

Appendix A

Proof of Lemma 1.

We conjecture the following form for optimal consumption

$$\overline{p}_{it} + c_{it} = (1-\lambda)\, h_{it} + \lambda\,(1-\mu) \sum_{j=0}^{\infty} \mu^j E_{it}^{II} \left[p_{it+j} + y_{it+j}\right]$$

substituting in the Euler equation (12) and using (13) one obtains:

$$\begin{aligned} \overline{p}_{it} + c_{it} \quad = \quad & (1-\delta)\, h_{it} + \delta[(1-\lambda)\, h_{it} + (1-\lambda)\,\theta \left(E_t^{II}\left[p_{it} + y_{it}\right] - \overline{p}_{it} - c_{it}\right) + \\ & \lambda\,(1-\mu) \sum_{j=0}^{\infty} \mu^j E_t^{II} \left[p_{it+j+1} + y_{it+j+1}\right]] \end{aligned}$$

This confirms our conjecture, provided that the parameters λ and μ satisfy the equations:

$$\begin{aligned} (1+\delta\,(1-\lambda)\,\theta)\,(1-\lambda) \quad &= \quad (1-\delta) + \delta\,(1-\lambda) \\ (1+\delta\,(1-\lambda)\,\theta)\,\lambda\,(1-\mu) \quad &= \quad \delta\,(1-\lambda)\,\theta \\ (1+\delta\,(1-\lambda)\,\theta)\,\lambda\,(1-\mu)\,\mu^j \quad &= \quad \delta\lambda\,(1-\mu)\,\mu^{j-1} \text{ for } j = 1, 2, ... \end{aligned}$$

There is a unique $\lambda \in (0,1)$ that satisfies the first quadratic equation. All the remaining equations are satisfied by setting

$$\mu = \frac{\delta}{1+\theta\delta(1-\lambda)}.$$

Proof of Proposition 2.
Conjecture the following linear form for prices and output

$$\begin{aligned} p_t &= \phi_u u_t + \phi_e e_t \\ y_t &= a_{t-1} + \psi_u u_t + \psi_e e_t \end{aligned}$$

which implies

$$\begin{aligned} E_t[p_{t+k}] &= 0 \text{ for } k>0 \quad (18) \\ E_t[y_{t+k}] &= E_t[a_t] \text{ for } k>0 \quad (19) \end{aligned}$$

Since the prices in island i do not affect the information of agent i regarding future aggregate variables the expected demand faced by the producer in island i is

$$E_{it}[y_{it}] = -\hat{\sigma} p_{it} + E_{it}[y_t + \hat{\sigma} p_t].$$

Conjecture the following form for the individual decision rule:

$$c_{it} = b_h h_{it} + b_{a_i} a_{it} + b_y E_{it}[y_t + \hat{\sigma} p_t] + b_a E_{it}[a_t] - \overline{p}_{it}$$

Write the wealth accumulation equation as

$$E_{it}[h_{it+1}] = h_{it} - \theta(\hat{\sigma}-1) p_{it} + \theta E_{it}[y_t + \hat{\sigma} p_t] - \theta(\overline{p}_{it} + c_{it})$$

using the optimal pricing condition (11) this becomes

$$\begin{aligned} E_{it}[h_{it+1}] &= h_{it} - \theta\left[1 + \frac{\hat{\sigma}-1}{1+\eta(\hat{\sigma}-1)}\right](\overline{p}_{it} + c_{it}) - \theta\frac{(1+\eta)(1-\hat{\sigma})}{1+\eta(\hat{\sigma}-1)} a_{it} + \\ &\quad + \theta\left[1 - \frac{\eta(1-\hat{\sigma})}{1+\eta(\hat{\sigma}-1)}\right] E_{it}[y_t + \hat{\sigma} p_t]. \end{aligned}$$

The Euler equation can be written as

$$\overline{p}_{it} + c_{it} = (1-\delta) h_{it} + \delta E_{it}[b_h h_{it+1} + (b_{a_i} + b_y + b_a) a_t].$$

Substituting in the wealth accumulation equation one obtains

$$\begin{aligned} \left(1 + \delta\theta b_h \left(1 + \frac{\hat{\sigma}-1}{1+\eta(\hat{\sigma}-1)}\right)\right) b_h &= 1 - \delta + \delta b_h \\ \left(1 + \delta\theta b_h \left(1 + \frac{\hat{\sigma}-1}{1+\eta(\hat{\sigma}-1)}\right)\right) b_{a_i} &= \delta\theta b_h \frac{(1+\eta)(\hat{\sigma}-1)}{1+\eta(\hat{\sigma}-1)} \\ \left(1 + \delta\theta b_h \left(1 + \frac{\hat{\sigma}-1}{1+\eta(\hat{\sigma}-1)}\right)\right) b_y &= \delta\theta b_h \left(1 - \frac{\eta(\hat{\sigma}-1)}{1+\eta(\hat{\sigma}-1)}\right) \\ \left(1 + \delta\theta b_h \left(1 + \frac{\hat{\sigma}-1}{1+\eta(\hat{\sigma}-1)}\right)\right) b_a &= \delta(b_{a_i} + b_y + b_a) \end{aligned}$$

The first equation is a quadratic equation with a unique solution in $[0,1]$. The remaining equations can be used to determine b_{a_i}, b_y and b_a. Summing side by side one can show that the solution satisfies:

$$b_h + b_{a_i} + b_y + b_a = 1.$$

Aggregating the consumption equation across consumers and using (18) and (19) one obtains

$$\begin{aligned} y_t &= (b_h + b_{a_i}) a_t + b_y E_t [y_t + \hat{\sigma} p_t] + b_a E_t [a_t] - p_t \\ &= (b_h + b_{a_i}) a_t + (b_y + b_a) E_t [a_t] - p_t \end{aligned}$$

and defining

$$\tilde{\lambda} = b_y + b_a$$

one obtains (16).

Substituting the individual consumption rule in the optimal pricing condition and aggregating one obtains

$$p_t = E_t [p_t] + \left(1 - \tilde{\lambda}\right) a_t + \tilde{\lambda} E_t [a_t] - (1 + \eta) a_t + \eta E_t [y_t + \hat{\sigma} p_t] + \eta (1 - \hat{\sigma}) p_{it},$$

(16) follows immediately.

Appendix B

The law of motion for Z_t is:

$$Z_{t+1} = A Z_t + B \begin{pmatrix} u_t \\ e_t \end{pmatrix}$$

For computational purposes we will consider the truncated version of Z_t, $Z_t^{[T]} = \{a_t, s_t, ...a_{t-T}, s_{t-T}\}$. Then A and B are:

$$\begin{aligned} A &= \begin{bmatrix} 1 & \mathbf{0}_{1\times 2T-1} \\ 1 & \mathbf{0}_{1\times 2T-1} \\ \mathbf{I}_{2(T-1)} & \mathbf{0}_{2(T-1),2} \end{bmatrix}; \\ B &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \end{aligned}$$

Guess a linear form for prices and quantities

$$\begin{aligned} p_t &= \phi Z_t \\ y_t &= \psi Z_t \end{aligned}$$

Optimality

First we want to find linear rules of the type

$$\begin{aligned} p_{it} &= q_x h_{it} + q_a a_{it} + q_z E^I_{it}[Z_t] \\ c_{it} &= b_x h_{it} + b_a a_{it} + b_p \overline{p}_{it} + b^I_z E^I_{it}[Z_t] + b^{II}_z E^{II}_{it}[Z_t] \end{aligned}$$

that satisfy the individual optimality conditions:

$$\begin{aligned} p_{it} &= E^I_{it}[p_t + c_{it}] - (1+\eta)\, a_{it} + \eta E^I_{it}[y_{it}] \\ \overline{p}_{it} + c_{it} &= (1-\delta)\, h_{it} + \delta E^{II}_{it}\left[\overline{p}_{it+1} + c_{it+1}\right] \\ h_{it+1} &= h_{it} + \theta\left(p_{it} + y_{it} - \overline{p}_{it} - c_{it}\right) \end{aligned}$$

The random assignment rule between consumers and islands implies that

$$\begin{aligned} E_{it}[y_{it}] &= \frac{1}{m} E_{it}\left[\sum c_{jt}\right] - \sigma \frac{m-1}{m}\left(p_{it} - E_{it}[p_t]\right) \\ &= E_{it}[y_t] - \left(\sigma \frac{m-1}{m} - \psi_p \frac{1}{m}\right)\left(p_{it} - E_{it}[p_t]\right) \\ &= E_{it}[y_t] - \hat{\sigma}\left(p_{it} - E_{it}[p_t]\right) \end{aligned}$$

where ψ_p is the parameter that determines the response of demand in island i to the prices in island i and will be determined in the following.

Using this relation we can substitute the individual rules and use the law of iterated expectations to get

$$\begin{aligned} p_{it} = \; & \frac{1}{1+\eta\hat{\sigma}}\left\{\left[(1 + b_p + \eta\hat{\sigma})\,\phi + b^I_z + b^{II}_z + \eta\psi\right] E^I_{it}[Z_t] + \right. \\ & \left. b_x h_{it} + (b_a - 1 - \eta)\, a_{it}\right\} \end{aligned}$$

and

$$\begin{aligned} \overline{p}_{it} + c_{it} = \; & (1-\delta)\, h_{it} + \delta\,(1+b_p)\,\phi A E^{II}_{it}[Z_t] + \delta b_x E^{II}_{it}[h_{it+1}] + \\ & \delta b_a e_1 A E^{II}_{it}[Z_t] + \delta\left(b^I_z + b^{II}_z\right) A E^{II}_{it}[Z_t], \end{aligned}$$

$$E^{II}_{it}[h_{it+1}] = h_{it} + \theta\left(p_{it} + E^{II}_{it}[y_{it}] - \overline{p}_{it} - c_{it}\right),$$

$$E^{II}_{it}[y_{it}] = (\psi + \hat{\sigma}\phi)\, E^{II}_{it}[Z_t] - \hat{\sigma} p_{it},$$

where e_1 is the vector $[1, 0, 0...]$.

Substituting and matching coefficients one obtains

$$\begin{aligned} q_x &= \frac{1}{1+\eta\hat{\sigma}} b_x \\ q_a &= \frac{1}{1+\eta\hat{\sigma}}\left(b_a - 1 - \eta\right) \\ q_z &= \frac{1}{1+\eta\hat{\sigma}}\left[(1 + b_p + \eta\hat{\sigma})\,\phi + b^I_z + b^{II}_z + \eta\psi\right] \end{aligned}$$

and

$$
\begin{aligned}
b_x &= \frac{1}{1+\delta b_x \theta}\left[\left((1-\delta)+\delta b_x\right)+\delta b_x \theta\,(1-\hat{\sigma})\,q_x\right] \\
b_a &= \frac{1}{1+\delta b_x \theta}\delta b_x \theta\,(1-\hat{\sigma})\,q_a \\
b_p &= -1 \\
b_z^I &= \frac{1}{1+\delta b_x \theta}\delta b_x \theta\,(1-\hat{\sigma})\,q_z \\
b_z^{II} &= \frac{1}{1+\delta b_x \theta}\left[\delta\,(1+b_p)\,\phi A+\delta b_x \theta\,(\psi+\hat{\sigma}\phi)+\delta b_a e_1 A+\delta\left(b_z^{II}+b_z^I\right)A\right]
\end{aligned}
$$

which can be solved for q and b. Notice that the parameters q_x, q_a, b_x, b_a, b_p can be solved separately without knowledge of ψ and ϕ.

Kalman filter

For given ϕ and ψ we can derive the expressions for the Kalman filter. Agent i observes first the vector of signals S_{it}, where

$$
S_{it} = \begin{pmatrix} a_{it} \\ r_{it} \\ s_t \end{pmatrix}
$$

and then observes $\overline{p}_{it}$.

In the pricing stage and in the trading stage he forms the expectations

$$
\begin{aligned}
E_{it}^I[Z_t] &= E_{it-1}[Z_t]+C\left(S_{it}-E_{it-1}[S_{it}]\right) \\
E_{it}^{II}[Z_t] &= E_{it}^I[Z_t]+L\left(\overline{p}_{it}-E_{it}^I[\overline{p}_{it}]\right) \\
E_{it}[Z_t] &= E_{it}^{II}[Z_t]+M\left(q_{it}-E_{it}^{II}[q_{it}]\right)
\end{aligned}
\tag{20}
$$

To derive the Kalman gains use the orthogonality conditions

$$
\begin{aligned}
E_{it-1}\left[\left(Z_t-E_{it-1}Z_t-C\left(S_{it}-E_{it-1}S_{it}\right)\right)\left(S_{it}-E_{it-1}[S_{it}]\right)'\right] &= 0 \\
E_{it}^I\left[\left(Z_t-E_{it}^I Z_t-L\left(p_t-E_{it}^I[p_t]\right)\right)\left(\overline{p}_{it}-E_{it}^I[\overline{p}_{it}]\right)'\right] &= 0 \\
E_{it}^{II}\left[\left(Z_t-E_{it}^{II} Z_t-M\left(q_{it}-E_{it}^{II}[q_{it}]\right)\right)\left(q_{it}-E_{it}^{II}[q_{it}]\right)'\right] &= 0
\end{aligned}
$$

the law of motion (8) and the relations

$$
\begin{aligned}
S_{it} &= GZ_t+FV_{it} \\
\overline{p}_{it} &= QZ_t+\zeta_{it} \\
q_{it} &= RZ_t+\eta_{it}
\end{aligned}
$$

where V_{it} is

$$
V_{it} = \begin{bmatrix} \epsilon_{it} \\ v_{it} \end{bmatrix}
$$

and

$$\Sigma_V = \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}.$$

G and F are known matrices given by:

$$\begin{aligned} G &= \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} \\ 1 & 0 & -1 & \mathbf{0} \\ 0 & 1 & 0 & \mathbf{0} \end{bmatrix} \\ F &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \\ Q &= \phi \\ R &= \psi + \hat{\sigma}\phi \end{aligned}$$

Let

$$\begin{aligned} \Omega &= Var_{it-1}[Z_t] \\ \Omega_I &= Var_{it}^{I}[Z_t] \\ \Omega^{II} &= Var_{it}^{II}[Z_t] \end{aligned}$$

then the orthogonality conditions give us the Kalman gains C, L, M

$$\begin{aligned} C' &= \left(G\Omega G' + F\Sigma_V F'\right)^{-1} G\Omega \\ L' &= \left(Q\Omega_I Q' + \sigma_\zeta^2\right)^{-1} Q\Omega_I \\ M' &= \left(R\Omega_{II} R' + \sigma_\eta^2\right)^{-1} R\Omega_{II} \end{aligned}$$

The expressions for the residual variance are

$$\Omega_I = \Omega - \Omega G' \left(G\Omega G' + F\Sigma_V F'\right)^{-1} G\Omega \tag{21a}$$

$$\Omega_{II} = \Omega_I - \Omega_I Q' \left(Q\Omega_I Q' + \sigma_\zeta^2\right)^{-1} Q\Omega_I \tag{21b}$$

$$\hat{\Omega} = \Omega_{II} - \Omega_{II} R' \left(R\Omega_{II} R' + \sigma_\eta^2\right)^{-1} R\Omega_{II} \tag{21c}$$

Using the law of motion of Z_t we obtain the steady state condition:

$$Var_t[Z_{t+1}] = A\hat{\Omega}A' + B\Sigma B' = \Omega$$

where

$$\Sigma = \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_e^2 \end{bmatrix}.$$

Iterations on the conditions (21) gives us the matrices Ω and allows us to derive the steady state Kalman gains matrices.

Prices and output

Now we need to express the average first order expectations in terms of the current state as

$$
\begin{aligned}
Z_{t|t^I} &= \Xi_I Z_t \\
Z_{t|t^{II}} &= \Xi_{II} Z_t \\
Z_{t|t} &= \Xi Z_t
\end{aligned}
$$

Using the updating equations and aggregating across consumers one obtains

$$
Z_{t|t} = (I - MR)(I - LQ)(I - CG)AZ_{t-1|t-1} + ((I - MR)((I - LQ)CG + LQ) + MR)Z_t
$$

For Ξ one obtains the expression

$$
\Xi = (I - MR)(I - LQ)(I - CG)A\Xi + ((I - MR)((I - LQ)CG + LQ) + MR)
$$

and similar expressions for Ξ_I and Ξ_{II}. These matrices are infinite dimensionals, when using the truncated vector $Z_t^{[T]}$ we find finite dimensional matrices $\Xi^{[T]}$ that approximate Ξ (more on this approximation below). This is the only step where the use of the truncated vector $Z_t^{[T]}$ requires an approximation.

Having expressions for $Z_{t|t^I}$ and $Z_{t|t^{II}}$ in terms of the current state variable we can use the equilibrium relations to obtain:

$$
\begin{aligned}
\phi &= (q_x + q_a)e_1 + q_z \Xi_I \\
\psi &= (b_x + b_a)e_1 + b_p \phi + b_z^I \Xi_I + b_z^{II} \Xi_{II}
\end{aligned}
$$

In order to evaluate the accuracy of the approximation due to the truncation of the state space, one can compute $\Xi_I^{[T+k]}, \Xi_{II}^{[T+k]}$ for any k and evaluate wether the vectors $q_z\left(\Xi_I^{[T]} - \Xi_I^{[T+k]}\right)$, $b_z^I\left(\Xi_I^{[T]} - \Xi_I^{[T+k]}\right)$ and $b_z^{II}\left(\Xi_{II}^{[T]} - \Xi_{II}^{[T+k]}\right)$ are close to zero.

Cross-sectional dispersion

The computation above takes σ_ζ^2 and σ_η^2 (and ψ_p) as given. The last step is to derive the cross sectional dispersion of prices and quantities. Given the cross sectional dispersion of prices and quantities one obtains an expression for σ_ζ^2 and σ_η^2. Therefore, to compute the equilibrium of a given economy we need to solve a fixed point problem in terms σ_ζ^2 and σ_η^2.

The first step in deriving the volatility of η_{it} and ζ_{it} is to derive the volatility of the individual expectations $E_{it}^I Z_t$ and $E_{it}^{II} Z_t$. Define the idiosyncratic component of agents' expectations as

$$
J_{it} = E_{it}[Z_t] - Z_{t|t}
$$

and define J_{it}^{I} and J_{it}^{II} in a similar way. We can use the relations (20) and obtain the following recursive expression for the individual forecast errors

$$\begin{aligned} J_{it}^{I} &= (I - CG)\, AJ_{it-1} + CFV_{it} \\ J_{it}^{II} &= (I - LQ)\, J_{it}^{I} + L\zeta_{it} \\ J_{it} &= (I - MR)\, J_{it}^{II} + M\eta_{it} \end{aligned}$$

This gives us the law of motion for the individual forecast errors:

$$\begin{aligned} J_{it} = {} & (I - MR)(I - LQ)(I - CG)\, AJ_{it-1} + \\ & + (I - MR)(I - LQ)\, CFV_{it} + (I - MR)\, L\zeta_{it} + M\eta_{it} \end{aligned}$$

Define

$$x_{it-1} = h_{it} - r_{it}$$

the wealth dynamics are then given by

$$\begin{aligned} x_{it} = {} & \left[\theta\left((1-\hat{\sigma})\, q_z - b_z^I\right)(I - CG)\, A + b_z^{II}(I - LQ)(I - CG)\, A\right] J_{it-1} + \\ & + \left[1 + \theta(1-\hat{\sigma})\, q_x - \theta b_x\right] x_{it-1} + \\ & + \left[\theta\left((1-\hat{\sigma})\, q_z - b_z^I\right) CF + \theta b_z^{II}(I - LQ)\, CF\right] V_{it} + \\ & + \theta\left((1-\hat{\sigma})\, q_a - b_a\right)\epsilon_{it} + \\ & + \left[1 + \theta\left((1-\hat{\sigma})\, q_x - b_x\right)\right] v_{it} + \\ & + \theta b_z^{II} L\zeta_{it} + \theta\eta_{it} \end{aligned}$$

Using the relations just derived one can write the joint dynamics of individual wealth and expectations in matrix form as

$$\begin{pmatrix} J_{it} \\ x_{it} \end{pmatrix} = W_1 \begin{pmatrix} J_{it-1} \\ x_{it-1} \end{pmatrix} + W_2 \begin{pmatrix} \epsilon_{it} \\ v_{it} \\ \eta_{it} \\ \zeta_{it} \end{pmatrix}$$

The steady state distribution of J_{it} and x_{it} is normal with variance-covariance matrix $\Sigma_{J,x}$

$$\Sigma_{J,x} = W_1 \Sigma_{J,x} W_1' + W_2 \Sigma_{id} W_2'.$$

Using the joint cross sectional dispersions of agents' expectations and wealth we can derive the cross sectional dispersions of prices and quantities.

Using the agents decision rules we can write p_{it} and c_{it} in terms of independent components

$$\begin{aligned} p_{it} &= p_t + q_x x_{it-1} + q_x v_{it} + q_a \epsilon_{it} + q_z\left((I - CG)\, AJ_{it-1} + CFV_{it}\right) \\ c_{it} &= y_t + b_x x_{it-1} + b_x v_{it} + b_a \epsilon_{it} + b_p \zeta_{it} + b_z^I\left((I - CG)\, AJ_{it-1} + CFV_{it}\right) + \\ & \quad + b_z^{II}\left((I - LQ)\left((I - CG)\, AJ_{it-1} + CFV_{it}\right) + L\zeta_{it}\right) \end{aligned}$$

Define

$$\begin{aligned}
\hat{p}_{it} &= q_x x_{it-1} + q_x v_{it} + q_a \epsilon_{it} + q_z \left(\left(I - CG \right) AJ_{it-1} + CFV_{it} \right) \\
\hat{c}_{it} &= b_x x_{it-1} + b_x v_{it} + b_a \epsilon_{it} + b_z^I \left(\left(I - CG \right) AJ_{it-1} + CFV_{it} \right) + \\
&\quad + b_z^{II} \left(\left(I - LQ \right) \left(\left(I - CG \right) AJ_{it-1} + CFV_{it} \right) + L\zeta_{it} \right)
\end{aligned}$$

or in matrix form

$$\begin{aligned}
\hat{p}_{it} &= W_{1,p} \begin{pmatrix} J_{it-1} \\ x_{it-1} \end{pmatrix} + W_{2,p} \begin{pmatrix} \epsilon_{it} \\ v_{it} \\ \eta_{it} \\ \zeta_{it} \end{pmatrix} \\
\hat{c}_{it} &= W_{1,c} \begin{pmatrix} J_{it-1} \\ x_{it-1} \end{pmatrix} + W_{2,c} \begin{pmatrix} \epsilon_{it} \\ v_{it} \\ \eta_{it} \\ \zeta_{it} \end{pmatrix}
\end{aligned}$$

so one can write

$$\begin{aligned}
p_{it} &= p_t + \hat{p}_{it} \\
c_{it} &= y_t + \hat{c}_{it} + \psi_p \left(\overline{p}_{it} - p_t \right).
\end{aligned}$$

This gives us the consistency condition for the demand response ψ_p

$$\psi_p = b_p + b_z^{II} L.$$

The variances $\sigma_{\hat{p}}^2$ and $\sigma_{\hat{c}}^2$ are then

$$\begin{aligned}
\sigma_{\hat{p}}^2 &= W_{1,p} \Sigma_{J,x} W_{1,p}' + W_{2,p} \Sigma_{id} W_{2,p}' \\
\sigma_{\hat{c}}^2 &= W_{1,c} \Sigma_{J,x} W_{1,c}' + W_{2,c} \Sigma_{id} W_{2,c}'
\end{aligned}$$

The demand curve for the monopolist can be written in terms of independent components

$$y_{it} = y_t - \sigma \left(p_{it} - \frac{1}{m^2} \sum_{j \in \tilde{H}_{it}} \sum_{k \in H_{jt}} p_{kt} \right) + \frac{1}{m} \sum_{j \in \tilde{H}_{it}} \left(\hat{c}_{jt} + \psi_p \frac{1}{m} \sum_{k \in H_{jt}} \left(p_{kt} - p_t \right) \right)$$

where the $\hat{c}_{jt}$ are independent of the $p_{kt} - p_t$. Define the intercept of the demand curve of monopolist i, q_{it}, as

$$\begin{aligned}
q_{it} &= y_{it} + \left(\frac{m-1}{m} \sigma - \frac{1}{m} \psi_p \right) p_{it} = \\
&= y_t + \hat{\sigma} p_t + \eta_{it}
\end{aligned}$$

where

$$\hat{\sigma} = \left(\frac{m-1}{m} \sigma - \frac{1}{m} \psi_p \right)$$

then

$$\eta_{it} = \left(\sigma + \psi_p\right) \frac{1}{m^2} \sum_{j \in \tilde{H}_{it}} \sum_{k \in H_{jt}, k \neq i} \hat{p}_{kt} + \frac{1}{m} \sum_{j \in \tilde{H}_{it}} \hat{c}_{jt}$$

Therefore the consistency conditions for σ_ζ^2 and σ_η^2 are:

$$\begin{aligned} \sigma_\zeta^2 &= \frac{1}{m}\sigma_{\hat{p}}^2, \\ \sigma_\eta^2 &= \left(\sigma + \psi_p\right)^2 \left(\frac{m-1}{m}\right)^2 \sigma_{\hat{p}}^2 + \frac{1}{m}\sigma_{\hat{c}}^2. \end{aligned}$$

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