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IMPLEMENTATION GUIDE FOR MINPACK-1

by

Burton S. Garbow, Kenneth E. Hillstrom, Jorge J. More'

Applied Mathematics Division

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ABSTRACT

MINPACK-1 is a package of Fortran subprograms for the numerical solution of systems of nonlinear equations and nonlinear least squares problems. This report describes how to implement the package from the tape on which it is transmitted.

1. Introduction

This report describes the tape containing MINPACK-1, a package of Fortran subprograms for the numerical solution of systems of nonlinear equations and nonlinear least squares problems. Future editions of the MINPACK package will address unconstrained minimization and constrained optimization problems. For each problem area MINPACK-1 contains algorithms that proceed either from an analytic specification of the Jacobian matrix of the problem functions or directly from the problem functions themselves. Since the specification of the Jacobian matrix can be an error-prone task, MINPACK-1 also contains an algorithm to check that the Jacobian matrix is consistent with the functions. Also included on the tape are machine-readable documentation and a complete set of testing aids.

MINPACK-1 may be obtained from either:

National Energy Software Center
Argonne National Laboratory
9700 S. Cass Ave.
Argonne, IL 60439
Phone: (312) 972-7250

or

IMSL
Sixth Floor-NBC Building
7500 Bellaire Blvd.
Houston, TX 77036
Phone: (713) 772-1927

2. The Tape Organization

The tape contains 23 files. The card counts for the respective files are given in Table 1.

TABLE 1

File	Count
1	139
2	4,771
3	3,526
4	186
5	4,778
6	3,528
7	283
8	551
9	879
10	1,022
11	1,033
12	673
13	858
14	284
15	552
16	881
17	1,025
18	1,036
19	675
20	860
21	23
22	29
23	15

3. Overview of Tape Contents

The tape contains both single and double precision versions of the routines in MINPACK-1. For each precision there is a function that provides machine-dependent constants (when an appropriate set of DATA statements is activated); all other MINPACK-1 subprograms are written in a portable subset of ANSI standard Fortran. For those subprograms normally called by the user, machine-readable documentation is provided for both the single and the double precision forms.

The first six files on the tape are:

1. Function SPMPAR, single precision machine-dependent constants.

2. MINPACK-1, single precision, 22 subprograms.

3. MINPACK-1 documentation, single precision, 11 documents.

4. Function DPMPAR, double precision machine-dependent constants.

5. MINPACK-1, double precision, 22 subprograms.

6. MINPACK-1 documentation, double precision, 11 documents.

The remaining 17 files contain test material; their contents are described in Section 6.

4. Machine-Dependent Constants

There are three machine-dependent constants that have to be set before the single or double precision version of MINPACK-1 can be used; for most machines the correct values of these constants are encoded into DATA statements in functions SPMPAR (single precision) and DPMPAR (double precision). As transmitted on the tape, the constants for the IBM 360,370 series are "activated"; that is, the DATA statements encoding all other constants are rendered comments by the presence of 'C' in column 1. If the activated constants are appropriate for the target machine (various non-IBM machines share these constants), then no changes are required to SPMPAR or DPMPAR. Otherwise, a different set of constants will have to be "activated" and the IBM constants "deactivated". The listings of SPMPAR and DPMPAR are included as Appendix A of this report.

We are also including (files 7 and 14) single and double precision versions (named SMCHAR and DMCHAR, respectively) of the environmental inquiry program of W. J. Cody, with which we confirmed many of the constants in SPMPAR and DPMPAR (most of which were obtained from the corresponding Bell Laboratories PORT Library function). These programs should be useful in determining the appropriate constants for machines not already covered in SPMPAR and DPMPAR.

5. Files 1-6: The MINPACK-1 Package and Its Documentation

As described in Section 3, files 1-3 of the tape contain the single precision version of MINPACK-1 and its associated documentation and files 4-6 contain the double precision equivalents. File 2 contains 22 Fortran subprograms: CHKDER, DOGLEG, ENORM, FDJAC1, FDJAC2, HYBRD, HYBRD1, HYBRJ, HYBRJ1, LMDER, LMDER1, LMDIF, LMDIF1, LMPAR, LMSTR, LMSTR1, QFORM, QRFAC, QRSOLV, RWUPDT, R1MPYQ, and R1UPDT. ENORM is of FUNCTION type; the others are all SUBROUTINE's. File 5 contains 22 subprograms with the same names and in the same order. There are no separator cards between members, but the last card of each subprogram has 'END' in columns 7-9.

File 1 contains SPMPAR and file 4 contains DPMPAR. As described in Section 4, these programs may have to be changed, after which the programs in files 1 and 2, or 4 and 5, can be compiled into a library. We might suggest that you choose the precision that corresponds to your normal operating environment.

Machine-readable documentation is provided in files 3 and 6 for the 11 MINPACK-1 members normally called by the user: HYBRD1, HYBRD, HYBRJ1, HYBRJ, LMDER1, LMDER, LMSTR1, LMSTR, LMDIF1, LMDIF, and CHKDER. HYBRD, HYBRJ, LMDER, LMSTR, and LMDIF are the core subroutines in the five general algorithmic paths in MINPACK-1. HYBRD1, HYBRJ1, LMDER1, LMSTR1, and LMDIF1 are "easy-to-use" drivers with simplified calling sequences made possible by assuming default settings for certain parameters and by returning a limited amount of information; many applications do not require full flexibility and in these cases "easy-to-use" drivers can be invoked. Finally, CHKDER should be helpful to users who wish to check the coding of their Jacobian evaluation subroutine.

The documents are recorded with both upper and lower case characters (realizable if the TN print chain is available). They include carriage control characters '1', '0', and ' ' that enable their printing in paged format under program control; one blank page is inserted between successive documents. The printed material can be trimmed to fit onto 8 1/2 x 11 sheets of paper.

6. Files 7-23: The Testing Aids

Files 7-13 on the tape contain testing aids for running the environmental inquiry program (see Section 4 above), the five available algorithmic paths (HYBRD, HYBRJ, LMDER, LMSTR, and LMDIF), and CHKDER, respectively, in single precision. Files 14-20 contain testing aids for the corresponding double precision versions. The testing aids include drivers, initial point specifying subroutines, and subroutines (named FCN) that evaluate the functions and Jacobians. Files 21-23 comprise the input data -- file 21 is used by HYBRD and HYBRJ, file 22 by LMDER, LMSTR, and LMDIF, and file 23 by CHKDER.

In summary, the contents of files 7-23 are as follows:

7. SMCHAR test
8. HYBRD test (single precision)
9. HYBRJ test (single precision)

10. LMDER test (single precision)
11. LMSTR test (single precision)
12. LMDIF test (single precision)
13. CHKDER test (single precision)
14. DMCHAR test
15. HYBRD test (double precision)
16. HYBRJ test (double precision)
17. LMDER test (double precision)
18. LMSTR test (double precision)
19. LMDIF test (double precision)
20. CHKDER test (double precision)
21. HYBRD and HYBRJ data
22. LMDER, LMSTR, and LMDIF data
23. CHKDER data

Small changes may be required to the drivers, for example to renumber the logical input and output units (from 5 and 6) or to include a program card. To facilitate these changes we have placed the drivers at the beginning of the respective test files and, in addition, are including listings of the drivers (double precision versions) as Appendix B of this report.

For each test run, pair one of the program files with the corresponding data file, and make available the library where the MINPACK-1 package has been stored. For example, to run the double precision LMDER test you would point to files 17 and 22.

A special note applies to the drivers for LMSTR in files 11 and 18. Instead of calculating one row of the Jacobian per call, the test FCN subroutine calculates the entire Jacobian at the first call and moves one row at each subsequent call. This strategy depends, therefore, on retention by the system of quantities stored at an earlier call of FCN toward accessing at later calls. Although this feature is nonstandard, we hope that your system enables it.

The test runs for HYBRD, HYBRJ, LMDER, LMSTR, and LMDIF each produce about 1200 lines of output. The test run for CHKDER produces about 300 lines of output, and the SMCHAR (or DMCHAR) test produces only a single page after possibly triggering several underflow messages. The last page of the output for each run contains a summary of the results. We are including listings of the summaries that we produced from our double precision IBM runs (IBM 3033, Fortran H Extended) as Appendix C of this report. Section 7 contains further details toward the evaluation of the test results.

Towards estimating timing, we offer the following two tables: Table 2 records the CPU time in seconds on Argonne's IBM 3033 (including compilation) and Table 3 gives the approximate ratio of the speed of the 3033 to various other machines.

TABLE 2

HYBRD	17
HYBRJ	17
LMDER	21
LMSTR	26
LMDIF	21
CHKDER	4

TABLE 3

CDC 7600	1
CDC 6600	5
Univac 1110	11
Amdahl 470V/6	3
IBM 370/168	3
IBM 360/75	15
Honeywell 6070	10-20
DEC PDP-10	100-150
Burroughs 6700	100

7. Evaluation of the Test Results

The test runs for the machine constants (files 7 and 14) compare the results obtained by the environmental inquiry programs SMCHAR and DMCHAR with the constants specified in SPMPAR and DPMPAR, respectively. The test programs print

```
RERR(1) = (EPSMCH - EPS) / EPSMCH
RERR(2) = (DWARF - XMIN) / DWARF
RERR(3) = (XMAX - GIANT) / GIANT
```

where EPSMCH, DWARF, and GIANT are the constants specified in the MINPACK-1 functions, and EPS, XMIN, and XMAX are the values returned from the environmental inquiry programs.

The constants specified in the MINPACK-1 functions are conservative and do not take into account special features of the system. Therefore, components of RERR are not necessarily zero. For example, rounded arithmetic is reflected by EPS being half as large as EPSMCH, thereby producing a value of 0.5 in RERR(1). Values with magnitude as large as unity should usually be considered suspicious, but we have encountered the following exceptions:

a) In CDC systems, the treatment of small double precision numbers requires a value of DWARF that is large relative to XMIN, and so the value of RERR(2) is close to unity.

b) In Honeywell and Prime systems, the use of extra length registers results in an overly small value of EPS, and so the value of RERR(1) is close to unity; this only occurs in single precision on the Prime but in both precisions on the Honeywell. In addition, these systems do not allow a proper determination of XMAX in double precision and produce a value of RERR(3) close to minus unity.

The interpretation of the test results for the other MINPACK-1 programs depends on the machine precision and on the sensitivity of the problems to the precision of the computations. For those problems sensitive to the precision of the computations we have underlined the (IBM) results in Appendix C. The performances of the MINPACK-1 programs on these problems vary from machine to machine, but in general we expect about the same number of successes and failures as in the IBM runs if the working precision provides at least 10 decimal digits. If this is not the case, there is a deterioration in performance commensurate to the lack of digits. For problems not sensitive to the precision of the computations we expect about the same number of successes and failures as in the IBM runs regardless of the machine precision.

The criteria for the designation "successful" that we have adopted are as follows. For the nonlinear equation solvers success is obtained if the final L2-norm is less than the square root of the machine precision. For the IBM double precision runs (machine precision approximately 10**(-16)) there are 51 successes and four failures. For the nonlinear least squares solvers success is obtained if either the absolute or the relative error in the final L2-norm of the residuals is less than the square root of the machine precision. For the IBM double precision runs there are 53 successes and one failure.

8. Referral for Questions and Comments

We hope that no difficulty will be encountered in installing MINPACK-1, but if a problem occurs we will try to help. We are especially interested in comments about the performance of the codes. Questions and comments may be directed to:

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9700 S. Cass Ave.
Argonne, IL 60439
Phone: (312) 972-7184

A P P E N D I X   A

SPMPAR and DPMPAR Listings

```
      REAL FUNCTION SPMPAR(I)                                           SPPR0010
      INTEGER I                                                         SPPR0020
C     **********                                                        SPPR0030
C                                                                       SPPR0040
C     FUNCTION SPMPAR                                                   SPPR0050
C                                                                       SPPR0060
C     THIS FUNCTION PROVIDES SINGLE PRECISION MACHINE PARAMETERS        SPPR0070
C     WHEN THE APPROPRIATE SET OF DATA STATEMENTS IS ACTIVATED (BY      SPPR0080
C     REMOVING THE C FROM COLUMN 1) AND ALL OTHER DATA STATEMENTS ARE   SPPR0090
C     RENDERED INACTIVE. MOST OF THE PARAMETER VALUES WERE OBTAINED     SPPR0100
C     FROM THE CORRESPONDING BELL LABORATORIES PORT LIBRARY FUNCTION.   SPPR0110
C                                                                       SPPR0120
C     THE FUNCTION STATEMENT IS                                         SPPR0130
C                                                                       SPPR0140
C       REAL FUNCTION SPMPAR(I)                                         SPPR0150
C                                                                       SPPR0160
C     WHERE                                                             SPPR0170
C                                                                       SPPR0180
C       I IS AN INTEGER INPUT VARIABLE SET TO 1, 2, OR 3 WHICH          SPPR0190
C         SELECTS THE DESIRED MACHINE PARAMETER. IF THE MACHINE HAS     SPPR0200
C         T BASE B DIGITS AND ITS SMALLEST AND LARGEST EXPONENTS ARE    SPPR0210
C         EMIN AND EMAX, RESPECTIVELY, THEN THESE PARAMETERS ARE        SPPR0220
C                                                                       SPPR0230
C         SPMPAR(1) = B**(1 - T), THE MACHINE PRECISION,                SPPR0240
C                                                                       SPPR0250
C         SPMPAR(2) = B**(EMIN - 1), THE SMALLEST MAGNITUDE,            SPPR0260
C                                                                       SPPR0270
C         SPMPAR(3) = B**EMAX*(1 - B**(-T)), THE LARGEST MAGNITUDE.     SPPR0280
C                                                                       SPPR0290
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.         SPPR0300
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE             SPPR0310
C                                                                       SPPR0320
C     **********                                                        SPPR0330
      INTEGER MCHEPS(2)                                                 SPPR0340
      INTEGER MINMAG(2)                                                 SPPR0350
      INTEGER MAXMAG(2)                                                 SPPR0360
      REAL RMACH(3)                                                     SPPR0370
      EQUIVALENCE (RMACH(1),MCHEPS(1))                                  SPPR0380
      EQUIVALENCE (RMACH(2),MINMAG(1))                                  SPPR0390
      EQUIVALENCE (RMACH(3),MAXMAG(1))                                  SPPR0400
C                                                                       SPPR0410
C     MACHINE CONSTANTS FOR THE IBM 360/370 SERIES,                     SPPR0420
C     THE AMDAHL 470/V6, THE ICL 2900, THE ITEL AS/6,                   SPPR0430
C     THE XEROX SIGMA 5/7/9 AND THE SEL SYSTEMS 85/86.                  SPPR0440
C                                                                       SPPR0450
      DATA RMACH(1) / Z3C100000 /                                       SPPR0460
      DATA RMACH(2) / Z00100000 /                                       SPPR0470
      DATA RMACH(3) / Z7FFFFFFF /                                       SPPR0480
C                                                                       SPPR0490
C     MACHINE CONSTANTS FOR THE HONEYWELL 600/6000 SERIES.              SPPR0500
C                                                                       SPPR0510
C     DATA RMACH(1) / O716400000000 /                                   SPPR0520
C     DATA RMACH(2) / O402400000000 /                                   SPPR0530
C     DATA RMACH(3) / O376777777777 /                                   SPPR0540
C                                                                       SPPR0550
C     MACHINE CONSTANTS FOR THE CDC 6000/7000 SERIES.                   SPPR0560
C                                                                       SPPR0570
C     DATA RMACH(1) / 16414000000000000000B /                           SPPR0580
C     DATA RMACH(2) / 00014000000000000000B /                           SPPR0590
C     DATA RMACH(3) / 37767777777777777777B /                           SPPR0600
```

```
C                                                                       SPPR0610
C     MACHINE CONSTANTS FOR THE PDP-10 (KA OR KI PROCESSOR).            SPPR0620
C                                                                       SPPR0630
C     DATA RMACH(1) / "147400000000 /                                   SPPR0640
C     DATA RMACH(2) / "000400000000 /                                   SPPR0650
C     DATA RMACH(3) / "377777777777 /                                   SPPR0660
C                                                                       SPPR0670
C     MACHINE CONSTANTS FOR THE PDP-11 FORTRAN SUPPORTING               SPPR0680
C     32-BIT INTEGERS (EXPRESSED IN INTEGER AND OCTAL).                 SPPR0690
C                                                                       SPPR0700
C     DATA MCHEPS(1) /  889192448 /                                     SPPR0710
C     DATA MINMAG(1) /    8388608 /                                     SPPR0720
C     DATA MAXMAG(1) / 2147483647 /                                     SPPR0730
C                                                                       SPPR0740
C     DATA RMACH(1) / 006500000000 /                                    SPPR0750
C     DATA RMACH(2) / 000040000000 /                                    SPPR0760
C     DATA RMACH(3) / 017777777777 /                                    SPPR0770
C                                                                       SPPR0780
C     MACHINE CONSTANTS FOR THE PDP-11 FORTRAN SUPPORTING               SPPR0790
C     16-BIT INTEGERS (EXPRESSED IN INTEGER AND OCTAL).                 SPPR0800
C                                                                       SPPR0810
C     DATA MCHEPS(1),MCHEPS(2) / 13568,     0 /                         SPPR0820
C     DATA MINMAG(1),MINMAG(2) /   128,     0 /                         SPPR0830
C     DATA MAXMAG(1),MAXMAG(2) / 32767,    -1 /                         SPPR0840
C                                                                       SPPR0850
C     DATA MCHEPS(1),MCHEPS(2) / 0032400, 0000000 /                     SPPR0860
C     DATA MINMAG(1),MINMAG(2) / 0000200, 0000000 /                     SPPR0870
C     DATA MAXMAG(1),MAXMAG(2) / 0077777, 0177777 /                     SPPR0880
C                                                                       SPPR0890
C     MACHINE CONSTANTS FOR THE BURROUGHS 5700/6700/7700 SYSTEMS.       SPPR0900
C                                                                       SPPR0910
C     DATA RMACH(1) / O1301000000000000 /                               SPPR0920
C     DATA RMACH(2) / O1771000000000000 /                               SPPR0930
C     DATA RMACH(3) / O0777777777777777 /                               SPPR0940
C                                                                       SPPR0950
C     MACHINE CONSTANTS FOR THE BURROUGHS 1700 SYSTEM.                  SPPR0960
C                                                                       SPPR0970
C     DATA RMACH(1) / Z4EA800000 /                                      SPPR0980
C     DATA RMACH(2) / Z400800000 /                                      SPPR0990
C     DATA RMACH(3) / Z5FFFFFFFF /                                      SPPR1000
C                                                                       SPPR1010
C     MACHINE CONSTANTS FOR THE UNIVAC 1100 SERIES.                     SPPR1020
C                                                                       SPPR1030
C     DATA RMACH(1) / O147400000000 /                                   SPPR1040
C     DATA RMACH(2) / O000400000000 /                                   SPPR1050
C     DATA RMACH(3) / O377777777777 /                                   SPPR1060
C                                                                       SPPR1070
C     MACHINE CONSTANTS FOR THE DATA GENERAL ECLIPSE S/200.             SPPR1080
C                                                                       SPPR1090
C     NOTE - IT MAY BE APPROPRIATE TO INCLUDE THE FOLLOWING CARD -      SPPR1100
C     STATIC RMACH(3)                                                   SPPR1110
C                                                                       SPPR1120
C     DATA MINMAG/20K,0/,MAXMAG/77777K,177777K/                         SPPR1130
C     DATA MCHEPS/36020K,0/                                             SPPR1140
C                                                                       SPPR1150
C     MACHINE CONSTANTS FOR THE HARRIS 220.                             SPPR1160
C                                                                       SPPR1170
C     DATA MCHEPS(1) / '20000000, '00000353 /                           SPPR1180
C     DATA MINMAG(1) / '20000000, '00000201 /                           SPPR1190
C     DATA MAXMAG(1) / '37777777, '00000177 /                           SPPR1200
```

```
C
C     MACHINE CONSTANTS FOR THE CRAY-1.
C
C     DATA RMACH(1) / 0377224000000000000000B /
C     DATA RMACH(2) / 0200034000000000000000B /
C     DATA RMACH(3) / 0577777777777777777776B /
C
C     MACHINE CONSTANTS FOR THE PRIME 400.
C
C     DATA MCHEPS(1) / :10000000153 /
C     DATA MINMAG(1) / :10000000000 /
C     DATA MAXMAG(1) / :17777777777 /
C
      SPMPAR = RMACH(I)
      RETURN
C
C     LAST CARD OF FUNCTION SPMPAR.
C
      END
```

```
      DOUBLE PRECISION FUNCTION DPMPAR(I)
      INTEGER I
C     **********
C
C     FUNCTION DPMPAR
C
C     THIS FUNCTION PROVIDES DOUBLE PRECISION MACHINE PARAMETERS
C     WHEN THE APPROPRIATE SET OF DATA STATEMENTS IS ACTIVATED (BY
C     REMOVING THE C FROM COLUMN 1) AND ALL OTHER DATA STATEMENTS ARE
C     RENDERED INACTIVE. MOST OF THE PARAMETER VALUES WERE OBTAINED
C     FROM THE CORRESPONDING BELL LABORATORIES PORT LIBRARY FUNCTION.
C
C     THE FUNCTION STATEMENT IS
C
C       DOUBLE PRECISION FUNCTION DPMPAR(I)
C
C     WHERE
C
C       I IS AN INTEGER INPUT VARIABLE SET TO 1, 2, OR 3 WHICH
C         SELECTS THE DESIRED MACHINE PARAMETER. IF THE MACHINE HAS
C         T BASE B DIGITS AND ITS SMALLEST AND LARGEST EXPONENTS ARE
C         EMIN AND EMAX, RESPECTIVELY, THEN THESE PARAMETERS ARE
C
C         DPMPAR(1) = B**(1 - T), THE MACHINE PRECISION,
C
C         DPMPAR(2) = B**(EMIN - 1), THE SMALLEST MAGNITUDE,
C
C         DPMPAR(3) = B**EMAX*(1 - B**(-T)), THE LARGEST MAGNITUDE.
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER MCHEPS(4)
      INTEGER MINMAG(4)
      INTEGER MAXMAG(4)
      DOUBLE PRECISION DMACH(3)
      EQUIVALENCE (DMACH(1),MCHEPS(1))
      EQUIVALENCE (DMACH(2),MINMAG(1))
      EQUIVALENCE (DMACH(3),MAXMAG(1))
C
C     MACHINE CONSTANTS FOR THE IBM 360/370 SERIES,
C     THE AMDAHL 470/V6, THE ICL 2900, THE ITEL AS/6,
C     THE XEROX SIGMA 5/7/9 AND THE SEL SYSTEMS 85/86.
C
      DATA MCHEPS(1),MCHEPS(2) / Z34100000, Z00000000 /
      DATA MINMAG(1),MINMAG(2) / Z00100000, Z00000000 /
      DATA MAXMAG(1),MAXMAG(2) / Z7FFFFFFF, ZFFFFFFFF /
C
C     MACHINE CONSTANTS FOR THE HONEYWELL 600/6000 SERIES.
C
C     DATA MCHEPS(1),MCHEPS(2) / O606400000000, O000000000000 /
C     DATA MINMAG(1),MINMAG(2) / O402400000000, O000000000000 /
C     DATA MAXMAG(1),MAXMAG(2) / O376777777777, O777777777777 /
C
C     MACHINE CONSTANTS FOR THE CDC 6000/7000 SERIES.
C
C     DATA MCHEPS(1) / 15614000000000000000B /
C     DATA MCHEPS(2) / 15010000000000000000B /
C
```

```
C     DATA MINMAG(1) / 00604000000000000000B /                          DPPR0610
C     DATA MINMAG(2) / 00000000000000000000B /                          DPPR0620
C                                                                       DPPR0630
C     DATA MAXMAG(1) / 37767777777777777777B /                          DPPR0640
C     DATA MAXMAG(2) / 37167777777777777777B /                          DPPR0650
C                                                                       DPPR0660
C     MACHINE CONSTANTS FOR THE PDP-10 (KA PROCESSOR).                  DPPR0670
C                                                                       DPPR0680
C     DATA MCHEPS(1),MCHEPS(2) / "114400000000, "000000000000 /         DPPR0690
C     DATA MINMAG(1),MINMAG(2) / "033400000000, "000000000000 /         DPPR0700
C     DATA MAXMAG(1),MAXMAG(2) / "377777777777, "344777777777 /         DPPR0710
C                                                                       DPPR0720
C     MACHINE CONSTANTS FOR THE PDP-10 (KI PROCESSOR).                  DPPR0730
C                                                                       DPPR0740
C     DATA MCHEPS(1),MCHEPS(2) / "104400000000, "000000000000 /         DPPR0750
C     DATA MINMAG(1),MINMAG(2) / "000400000000, "000000000000 /         DPPR0760
C     DATA MAXMAG(1),MAXMAG(2) / "377777777777, "377777777777 /         DPPR0770
C                                                                       DPPR0780
C     MACHINE CONSTANTS FOR THE PDP-11 FORTRAN SUPPORTING               DPPR0790
C     32-BIT INTEGERS (EXPRESSED IN INTEGER AND OCTAL).                 DPPR0800
C                                                                       DPPR0810
C     DATA MCHEPS(1),MCHEPS(2) /  620756992,           0 /              DPPR0820
C     DATA MINMAG(1),MINMAG(2) /    8388608,           0 /              DPPR0830
C     DATA MAXMAG(1),MAXMAG(2) / 2147483647,          -1 /              DPPR0840
C                                                                       DPPR0850
C     DATA MCHEPS(1),MCHEPS(2) / 004500000000, 000000000000 /           DPPR0860
C     DATA MINMAG(1),MINMAG(2) / 000040000000, 000000000000 /           DPPR0870
C     DATA MAXMAG(1),MAXMAG(2) / 017777777777, 037777777777 /           DPPR0880
C                                                                       DPPR0890
C     MACHINE CONSTANTS FOR THE PDP-11 FORTRAN SUPPORTING               DPPR0900
C     16-BIT INTEGERS (EXPRESSED IN INTEGER AND OCTAL).                 DPPR0910
C                                                                       DPPR0920
C     DATA MCHEPS(1),MCHEPS(2) /   9472,      0 /                       DPPR0930
C     DATA MCHEPS(3),MCHEPS(4) /      0,      0 /                       DPPR0940
C                                                                       DPPR0950
C     DATA MINMAG(1),MINMAG(2) /    128,      0 /                       DPPR0960
C     DATA MINMAG(3),MINMAG(4) /      0,      0 /                       DPPR0970
C                                                                       DPPR0980
C     DATA MAXMAG(1),MAXMAG(2) /  32767,     -1 /                       DPPR0990
C     DATA MAXMAG(3),MAXMAG(4) /     -1,     -1 /                       DPPR1000
C                                                                       DPPR1010
C     DATA MCHEPS(1),MCHEPS(2) / 0022400, 0000000 /                     DPPR1020
C     DATA MCHEPS(3),MCHEPS(4) / 0000000, 0000000 /                     DPPR1030
C                                                                       DPPR1040
C     DATA MINMAG(1),MINMAG(2) / 0000200, 0000000 /                     DPPR1050
C     DATA MINMAG(3),MINMAG(4) / 0000000, 0000000 /                     DPPR1060
C                                                                       DPPR1070
C     DATA MAXMAG(1),MAXMAG(2) / 0077777, 0177777 /                     DPPR1080
C     DATA MAXMAG(3),MAXMAG(4) / 0177777, 0177777 /                     DPPR1090
C                                                                       DPPR1100
C     MACHINE CONSTANTS FOR THE BURROUGHS 6700/7700 SYSTEMS.            DPPR1110
C                                                                       DPPR1120
C     DATA MCHEPS(1) / 01451000000000000 /                              DPPR1130
C     DATA MCHEPS(2) / 00000000000000000 /                              DPPR1140
C                                                                       DPPR1150
C     DATA MINMAG(1) / 01771000000000000 /                              DPPR1160
C     DATA MINMAG(2) / 07770000000000000 /                              DPPR1170
C                                                                       DPPR1180
C     DATA MAXMAG(1) / 00777777777777777 /                              DPPR1190
C     DATA MAXMAG(2) / 07777777777777777 /                              DPPR1200
```

```
C                                                                       DPPR1210
C     MACHINE CONSTANTS FOR THE BURROUGHS 5700 SYSTEM.                  DPPR1220
C                                                                       DPPR1230
C     DATA MCHEPS(1) / O1451000000000000 /                              DPPR1240
C     DATA MCHEPS(2) / O0000000000000000 /                              DPPR1250
C                                                                       DPPR1260
C     DATA MINMAG(1) / O1771000000000000 /                              DPPR1270
C     DATA MINMAG(2) / O0000000000000000 /                              DPPR1280
C                                                                       DPPR1290
C     DATA MAXMAG(1) / O0777777777777777 /                              DPPR1300
C     DATA MAXMAG(2) / O0007777777777777 /                              DPPR1310
C                                                                       DPPR1320
C     MACHINE CONSTANTS FOR THE BURROUGHS 1700 SYSTEM.                  DPPR1330
C                                                                       DPPR1340
C     DATA MCHEPS(1) / ZCC6800000 /                                     DPPR1350
C     DATA MCHEPS(2) / Z000000000 /                                     DPPR1360
C                                                                       DPPR1370
C     DATA MINMAG(1) / ZC00800000 /                                     DPPR1380
C     DATA MINMAG(2) / Z000000000 /                                     DPPR1390
C                                                                       DPPR1400
C     DATA MAXMAG(1) / ZDFFFFFFFF /                                     DPPR1410
C     DATA MAXMAG(2) / ZFFFFFFFFF /                                     DPPR1420
C                                                                       DPPR1430
C     MACHINE CONSTANTS FOR THE UNIVAC 1100 SERIES.                     DPPR1440
C                                                                       DPPR1450
C     DATA MCHEPS(1),MCHEPS(2) / O170640000000, O000000000000 /         DPPR1460
C     DATA MINMAG(1),MINMAG(2) / O000040000000, O000000000000 /         DPPR1470
C     DATA MAXMAG(1),MAXMAG(2) / O377777777777, O777777777777 /         DPPR1480
C                                                                       DPPR1490
C     MACHINE CONSTANTS FOR THE DATA GENERAL ECLIPSE S/200.             DPPR1500
C                                                                       DPPR1510
C     NOTE - IT MAY BE APPROPRIATE TO INCLUDE THE FOLLOWING CARD -      DPPR1520
C     STATIC DMACH(3)                                                   DPPR1530
C                                                                       DPPR1540
C     DATA MINMAG/20K,3*0/,MAXMAG/77777K,3*177777K/                     DPPR1550
C     DATA MCHEPS/32020K,3*0/                                           DPPR1560
C                                                                       DPPR1570
C     MACHINE CONSTANTS FOR THE HARRIS 220.                             DPPR1580
C                                                                       DPPR1590
C     DATA MCHEPS(1),MCHEPS(2) / '20000000, '00000334 /                 DPPR1600
C     DATA MINMAG(1),MINMAG(2) / '20000000, '00000201 /                 DPPR1610
C     DATA MAXMAG(1),MAXMAG(2) / '37777777, '37777577 /                 DPPR1620
C                                                                       DPPR1630
C     MACHINE CONSTANTS FOR THE CRAY-1.                                 DPPR1640
C                                                                       DPPR1650
C     DATA MCHEPS(1) / 0376424000000000000000B /                        DPPR1660
C     DATA MCHEPS(2) / 0000000000000000000000B /                        DPPR1670
C                                                                       DPPR1680
C     DATA MINMAG(1) / 0200034000000000000000B /                        DPPR1690
C     DATA MINMAG(2) / 0000000000000000000000B /                        DPPR1700
C                                                                       DPPR1710
C     DATA MAXMAG(1) / 0577777777777777777777B /                        DPPR1720
C     DATA MAXMAG(2) / 0000007777777777777776B /                        DPPR1730
C                                                                       DPPR1740
C     MACHINE CONSTANTS FOR THE PRIME 400.                              DPPR1750
C                                                                       DPPR1760
C     DATA MCHEPS(1),MCHEPS(2) / :10000000000, :00000000123 /           DPPR1770
C     DATA MINMAG(1),MINMAG(2) / :10000000000, :00000100000 /           DPPR1780
C     DATA MAXMAG(1),MAXMAG(2) / :17777777777, :37777677776 /           DPPR1790
C                                                                       DPPR1800
```

```
      DPMPAR = DMACH(I)
      RETURN
C
C     LAST CARD OF FUNCTION DPMPAR.
C
      END
```

A P P E N D I X B

Test Driver Listings

```
C     **********
C
C     THIS PROGRAM CHECKS THE CONSTANTS OF MACHINE PRECISION AND
C     SMALLEST AND LARGEST MACHINE REPRESENTABLE NUMBERS SPECIFIED IN
C     FUNCTION DPMPAR, AGAINST THE CORRESPONDING HARDWARE-DETERMINED
C     MACHINE CONSTANTS OBTAINED BY DMCHAR, A SUBROUTINE DUE TO
C     W. J. CODY.
C
C     DATA STATEMENTS IN DPMPAR CORRESPONDING TO THE MACHINE USED MUST
C     BE ACTIVATED BY REMOVING C IN COLUMN 1.
C
C     THE PRINTED OUTPUT CONSISTS OF THE MACHINE CONSTANTS OBTAINED BY
C     DMCHAR AND COMPARISONS OF THE DPMPAR CONSTANTS WITH THEIR
C     DMCHAR COUNTERPARTS. DESCRIPTIONS OF THE MACHINE CONSTANTS ARE
C     GIVEN IN THE PROLOGUE COMMENTS OF DMCHAR.
C
C     SUBPROGRAMS CALLED
C
C       MINPACK-SUPPLIED ... DMCHAR,DPMPAR
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER IBETA,IEXP,IRND,IT,MACHEP,MAXEXP,MINEXP,NEGEP,NGRD,
     *        NWRITE
      DOUBLE PRECISION DWARF,EPS,EPSMCH,EPSNEG,GIANT,XMAX,XMIN
      DOUBLE PRECISION RERR(3)
      DOUBLE PRECISION DPMPAR
C
C     LOGICAL OUTPUT UNIT IS ASSUMED TO BE NUMBER 6.
C
      DATA NWRITE /6/
C
C     DETERMINE THE MACHINE CONSTANTS DYNAMICALLY FROM DMCHAR.
C
      CALL DMCHAR(IBETA,IT,IRND,NGRD,MACHEP,NEGEP,IEXP,MINEXP,MAXEXP,
     *            EPS,EPSNEG,XMIN,XMAX)
C
C     COMPARE THE DPMPAR CONSTANTS WITH THEIR DMCHAR COUNTERPARTS AND
C     STORE THE RELATIVE DIFFERENCES IN RERR.
C
      EPSMCH = DPMPAR(1)
      DWARF = DPMPAR(2)
      GIANT = DPMPAR(3)
      RERR(1) = (EPSMCH - EPS)/EPSMCH
      RERR(2) = (DWARF - XMIN)/DWARF
      RERR(3) = (XMAX - GIANT)/GIANT
C
C     WRITE THE DMCHAR CONSTANTS.
C
      WRITE (NWRITE,10)
     *      IBETA,IT,IRND,NGRD,MACHEP,NEGEP,IEXP,MINEXP,MAXEXP,EPS,
     *      EPSNEG,XMIN,XMAX
C
C     WRITE THE DPMPAR CONSTANTS AND THE RELATIVE DIFFERENCES.
C
      WRITE (NWRITE,20) EPSMCH,RERR(1),DWARF,RERR(2),GIANT,RERR(3)
      STOP
   10 FORMAT (17H1DMCHAR CONSTANTS /// 8H IBETA =, I6 // 8H IT    =,
     *        I6 // 8H IRND  =, I6 // 8H NGRD  =, I6 // 9H MACHEP =,
```

```
     *         I6 // 8H NEGEP =, I6 // 7H IEXP =, I6 // 9H MINEXP =,       DPPD0620
     *         I6 // 9H MAXEXP =, I6 // 6H EPS =, D15.7 // 9H EPSNEG =,    DPPD0630
     *         D15.7 // 7H XMIN =, D15.7 // 7H XMAX =, D15.7)              DPPD0640
   20 FORMAT ( /// 42H DPMPAR CONSTANTS AND RELATIVE DIFFERENCES ///       DPPD0650
     *          9H EPSMCH =, D15.7 / 10H RERR(1) =, D15.7 //               DPPD0660
     *          8H DWARF =, D15.7 / 10H RERR(2) =, D15.7 // 8H GIANT =,    DPPD0670
     *          D15.7 / 10H RERR(3) =, D15.7)                              DPPD0680
C                                                                          DPPD0690
C     LAST CARD OF DRIVER.                                                 DPPD0700
C                                                                          DPPD0710
      END                                                                  DPPD0720
```

```
C     **********
C
C     THIS PROGRAM TESTS CODES FOR THE SOLUTION OF N NONLINEAR
C     EQUATIONS IN N VARIABLES. IT CONSISTS OF A DRIVER AND AN
C     INTERFACE SUBROUTINE FCN. THE DRIVER READS IN DATA, CALLS THE
C     NONLINEAR EQUATION SOLVER, AND FINALLY PRINTS OUT INFORMATION
C     ON THE PERFORMANCE OF THE SOLVER. THIS IS ONLY A SAMPLE DRIVER,
C     MANY OTHER DRIVERS ARE POSSIBLE. THE INTERFACE SUBROUTINE FCN
C     IS NECESSARY TO TAKE INTO ACCOUNT THE FORMS OF CALLING
C     SEQUENCES USED BY THE FUNCTION SUBROUTINES IN THE VARIOUS
C     NONLINEAR EQUATION SOLVERS.
C
C     SUBPROGRAMS CALLED
C
C       USER-SUPPLIED ...... FCN
C
C       MINPACK-SUPPLIED ... DPMPAR,ENORM,HYBRD1,INITPT,VECFCN
C
C       FORTRAN-SUPPLIED ... DSQRT
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER I,IC,INFO,K,LWA,N,NFEV,NPROB,NREAD,NTRIES,NWRITE
      INTEGER NA(60),NF(60),NP(60),NX(60)
      DOUBLE PRECISION FACTOR,FNORM1,FNORM2,ONE,TEN,TOL
      DOUBLE PRECISION FNM(60),FVEC(40),WA(2660),X(40)
      DOUBLE PRECISION DPMPAR,ENORM
      EXTERNAL FCN
      COMMON /REFNUM/ NPROB,NFEV
C
C     LOGICAL INPUT UNIT IS ASSUMED TO BE NUMBER 5.
C     LOGICAL OUTPUT UNIT IS ASSUMED TO BE NUMBER 6.
C
      DATA NREAD,NWRITE /5,6/
C
      DATA ONE,TEN /1.0D0,1.0D1/
      TOL = DSQRT(DPMPAR(1))
      LWA = 2660
      IC = 0
   10 CONTINUE
         READ (NREAD,50) NPROB,N,NTRIES
         IF (NPROB .LE. 0) GO TO 30
         FACTOR = ONE
         DO 20 K = 1, NTRIES
            IC = IC + 1
            CALL INITPT(N,X,NPROB,FACTOR)
            CALL VECFCN(N,X,FVEC,NPROB)
            FNORM1 = ENORM(N,FVEC)
            WRITE (NWRITE,60) NPROB,N
            NFEV = 0
            CALL HYBRD1(FCN,N,X,FVEC,TOL,INFO,WA,LWA)
            FNORM2 = ENORM(N,FVEC)
            NP(IC) = NPROB
            NA(IC) = N
            NF(IC) = NFEV
            NX(IC) = INFO
            FNM(IC) = FNORM2
            WRITE (NWRITE,70) FNORM1,FNORM2,NFEV,INFO,(X(I), I = 1, N)
            FACTOR = TEN*FACTOR
```

```
   20       CONTINUE
         GO TO 10
   30 CONTINUE
      WRITE (NWRITE,80) IC
      WRITE (NWRITE,90)
      DO 40 I = 1, IC
         WRITE (NWRITE,100) NP(I),NA(I),NF(I),NX(I),FNM(I)
   40    CONTINUE
      STOP
   50 FORMAT (3I5)
   60 FORMAT ( //// 5X, 8H PROBLEM, I5, 5X, 10H DIMENSION, I5, 5X //)
   70 FORMAT (5X, 33H INITIAL L2 NORM OF THE RESIDUALS, D15.7 // 5X,
     *        33H FINAL L2 NORM OF THE RESIDUALS  , D15.7 // 5X,
     *        33H NUMBER OF FUNCTION EVALUATIONS  , I10 // 5X,
     *        15H EXIT PARAMETER, 18X, I10 // 5X,
     *        27H FINAL APPROXIMATE SOLUTION // (5X, 5D15.7))
   80 FORMAT (12H1SUMMARY OF , I3, 16H CALLS TO HYBRD1 /)
   90 FORMAT (39H NPROB   N    NFEV  INFO  FINAL L2 NORM /)
  100 FORMAT (I4, I6, I7, I6, 1X, D15.7)
C
C     LAST CARD OF DRIVER.
C
      END
      SUBROUTINE FCN(N,X,FVEC,IFLAG)
      INTEGER N,IFLAG
      DOUBLE PRECISION X(N),FVEC(N)
C     **********
C
C     THE CALLING SEQUENCE OF FCN SHOULD BE IDENTICAL TO THE
C     CALLING SEQUENCE OF THE FUNCTION SUBROUTINE IN THE NONLINEAR
C     EQUATION SOLVER. FCN SHOULD ONLY CALL THE TESTING FUNCTION
C     SUBROUTINE VECFCN WITH THE APPROPRIATE VALUE OF PROBLEM
C     NUMBER (NPROB).
C
C     SUBPROGRAMS CALLED
C
C       MINPACK-SUPPLIED ... VECFCN
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER NPROB,NFEV
      COMMON /REFNUM/ NPROB,NFEV
      CALL VECFCN(N,X,FVEC,NPROB)
      NFEV = NFEV + 1
      RETURN
C
C     LAST CARD OF INTERFACE SUBROUTINE FCN.
C
      END
```

```
C     **********
C
C     THIS PROGRAM TESTS CODES FOR THE SOLUTION OF N NONLINEAR
C     EQUATIONS IN N VARIABLES. IT CONSISTS OF A DRIVER AND AN
C     INTERFACE SUBROUTINE FCN. THE DRIVER READS IN DATA, CALLS THE
C     NONLINEAR EQUATION SOLVER, AND FINALLY PRINTS OUT INFORMATION
C     ON THE PERFORMANCE OF THE SOLVER. THIS IS ONLY A SAMPLE DRIVER,
C     MANY OTHER DRIVERS ARE POSSIBLE. THE INTERFACE SUBROUTINE FCN
C     IS NECESSARY TO TAKE INTO ACCOUNT THE FORMS OF CALLING
C     SEQUENCES USED BY THE FUNCTION AND JACOBIAN SUBROUTINES IN
C     THE VARIOUS NONLINEAR EQUATION SOLVERS.
C
C     SUBPROGRAMS CALLED
C
C       USER-SUPPLIED ...... FCN
C
C       MINPACK-SUPPLIED ... DPMPAR,ENORM,HYBRJ1,INITPT,VECFCN
C
C       FORTRAN-SUPPLIED ... DSQRT
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER I,IC,INFO,K,LDFJAC,LWA,N,NFEV,NJEV,NPROB,NREAD,NTRIES,
     1        NWRITE
      INTEGER NA(60),NF(60),NJ(60),NP(60),NX(60)
      DOUBLE PRECISION FACTOR,FNORM1,FNORM2,ONE,TEN,TOL
      DOUBLE PRECISION FNM(60),FJAC(40,40),FVEC(40),WA(1060),X(40)
      DOUBLE PRECISION DPMPAR,ENORM
      EXTERNAL FCN
      COMMON /REFNUM/ NPROB,NFEV,NJEV
C
C     LOGICAL INPUT UNIT IS ASSUMED TO BE NUMBER 5.
C     LOGICAL OUTPUT UNIT IS ASSUMED TO BE NUMBER 6.
C
      DATA NREAD,NWRITE /5,6/
C
      DATA ONE,TEN /1.0D0,1.0D1/
      TOL = DSQRT(DPMPAR(1))
      LDFJAC = 40
      LWA = 1060
      IC = 0
   10 CONTINUE
         READ (NREAD,50) NPROB,N,NTRIES
         IF (NPROB .LE. 0) GO TO 30
         FACTOR = ONE
         DO 20 K = 1, NTRIES
            IC = IC + 1
            CALL INITPT(N,X,NPROB,FACTOR)
            CALL VECFCN(N,X,FVEC,NPROB)
            FNORM1 = ENORM(N,FVEC)
            WRITE (NWRITE,60) NPROB,N
            NFEV = 0
            NJEV = 0
            CALL HYBRJ1(FCN,N,X,FVEC,FJAC,LDFJAC,TOL,INFO,WA,LWA)
            FNORM2 = ENORM(N,FVEC)
            NP(IC) = NPROB
            NA(IC) = N
            NF(IC) = NFEV
            NJ(IC) = NJEV
```

```
            NX(IC) = INFO
            FNM(IC) = FNORM2
            WRITE (NWRITE,70)
     1            FNORM1,FNORM2,NFEV,NJEV,INFO,(X(I), I = 1, N)
            FACTOR = TEN*FACTOR
   20       CONTINUE
         GO TO 10
   30 CONTINUE
      WRITE (NWRITE,80) IC
      WRITE (NWRITE,90)
      DO 40 I = 1, IC
         WRITE (NWRITE,100) NP(I),NA(I),NF(I),NJ(I),NX(I),FNM(I)
   40    CONTINUE
      STOP
   50 FORMAT (3I5)
   60 FORMAT ( //// 5X, 8H PROBLEM, I5, 5X, 10H DIMENSION, I5, 5X //)
   70 FORMAT (5X, 33H INITIAL L2 NORM OF THE RESIDUALS, D15.7 // 5X,
     1        33H FINAL L2 NORM OF THE RESIDUALS  , D15.7 // 5X,
     2        33H NUMBER OF FUNCTION EVALUATIONS  , I10 // 5X,
     3        33H NUMBER OF JACOBIAN EVALUATIONS  , I10 // 5X,
     4        15H EXIT PARAMETER, 18X, I10 // 5X,
     5        27H FINAL APPROXIMATE SOLUTION // (5X, 5D15.7))
   80 FORMAT (12H1SUMMARY OF , I3, 16H CALLS TO HYBRJ1 /)
   90 FORMAT (46H NPROB   N    NFEV   NJEV  INFO  FINAL L2 NORM /)
  100 FORMAT (I4, I6, 2I7, I6, 1X, D15.7)
C
C     LAST CARD OF DRIVER.
C
      END
      SUBROUTINE FCN(N,X,FVEC,FJAC,LDFJAC,IFLAG)
      INTEGER N,LDFJAC,IFLAG
      DOUBLE PRECISION X(N),FVEC(N),FJAC(LDFJAC,N)
C     **********
C
C     THE CALLING SEQUENCE OF FCN SHOULD BE IDENTICAL TO THE
C     CALLING SEQUENCE OF THE FUNCTION SUBROUTINE IN THE NONLINEAR
C     EQUATION SOLVER. FCN SHOULD ONLY CALL THE TESTING FUNCTION
C     AND JACOBIAN SUBROUTINES VECFCN AND VECJAC WITH THE
C     APPROPRIATE VALUE OF PROBLEM NUMBER (NPROB).
C
C     SUBPROGRAMS CALLED
C
C       MINPACK-SUPPLIED ... VECFCN,VECJAC
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER NPROB,NFEV,NJEV
      COMMON /REFNUM/ NPROB,NFEV,NJEV
      IF (IFLAG .EQ. 1) CALL VECFCN(N,X,FVEC,NPROB)
      IF (IFLAG .EQ. 2) CALL VECJAC(N,X,FJAC,LDFJAC,NPROB)
      IF (IFLAG .EQ. 1) NFEV = NFEV + 1
      IF (IFLAG .EQ. 2) NJEV = NJEV + 1
      RETURN
C
C     LAST CARD OF INTERFACE SUBROUTINE FCN.
C
      END
```

```
C     **********
C
C     THIS PROGRAM TESTS CODES FOR THE LEAST-SQUARES SOLUTION OF
C     M NONLINEAR EQUATIONS IN N VARIABLES. IT CONSISTS OF A DRIVER
C     AND AN INTERFACE SUBROUTINE FCN. THE DRIVER READS IN DATA,
C     CALLS THE NONLINEAR LEAST-SQUARES SOLVER, AND FINALLY PRINTS
C     OUT INFORMATION ON THE PERFORMANCE OF THE SOLVER. THIS IS
C     ONLY A SAMPLE DRIVER, MANY OTHER DRIVERS ARE POSSIBLE. THE
C     INTERFACE SUBROUTINE FCN IS NECESSARY TO TAKE INTO ACCOUNT THE
C     FORMS OF CALLING SEQUENCES USED BY THE FUNCTION AND JACOBIAN
C     SUBROUTINES IN THE VARIOUS NONLINEAR LEAST-SQUARES SOLVERS.
C
C     SUBPROGRAMS CALLED
C
C       USER-SUPPLIED ...... FCN
C
C       MINPACK-SUPPLIED ... DPMPAR,ENORM,INITPT,LMDER1,SSQFCN
C
C       FORTRAN-SUPPLIED ... DSQRT
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER I,IC,INFO,K,LDFJAC,LWA,M,N,NFEV,NJEV,NPROB,NREAD,NTRIES,
     *        NWRITE
      INTEGER IWA(40),MA(60),NA(60),NF(60),NJ(60),NP(60),NX(60)
      DOUBLE PRECISION FACTOR,FNORM1,FNORM2,ONE,TEN,TOL
      DOUBLE PRECISION FJAC(65,40),FNM(60),FVEC(65),WA(265),X(40)
      DOUBLE PRECISION DPMPAR,ENORM
      EXTERNAL FCN
      COMMON /REFNUM/ NPROB,NFEV,NJEV
C
C     LOGICAL INPUT UNIT IS ASSUMED TO BE NUMBER 5.
C     LOGICAL OUTPUT UNIT IS ASSUMED TO BE NUMBER 6.
C
      DATA NREAD,NWRITE /5,6/
C
      DATA ONE,TEN /1.0D0,1.0D1/
      TOL = DSQRT(DPMPAR(1))
      LDFJAC = 65
      LWA = 265
      IC = 0
   10 CONTINUE
         READ (NREAD,50) NPROB,N,M,NTRIES
         IF (NPROB .LE. 0) GO TO 30
         FACTOR = ONE
         DO 20 K = 1, NTRIES
            IC = IC + 1
            CALL INITPT(N,X,NPROB,FACTOR)
            CALL SSQFCN(M,N,X,FVEC,NPROB)
            FNORM1 = ENORM(M,FVEC)
            WRITE (NWRITE,60) NPROB,N,M
            NFEV = 0
            NJEV = 0
            CALL LMDER1(FCN,M,N,X,FVEC,FJAC,LDFJAC,TOL,INFO,IWA,WA,
     *                  LWA)
            CALL SSQFCN(M,N,X,FVEC,NPROB)
            FNORM2 = ENORM(M,FVEC)
            NP(IC) = NPROB
            NA(IC) = N
```

```
            MA(IC) = M
            NF(IC) = NFEV
            NJ(IC) = NJEV
            NX(IC) = INFO
            FNM(IC) = FNORM2
            WRITE (NWRITE,70)
     *            FNORM1,FNORM2,NFEV,NJEV,INFO,(X(I), I = 1, N)
            FACTOR = TEN*FACTOR
   20       CONTINUE
         GO TO 10
   30 CONTINUE
      WRITE (NWRITE,80) IC
      WRITE (NWRITE,90)
      DO 40 I = 1, IC
         WRITE (NWRITE,100) NP(I),NA(I),MA(I),NF(I),NJ(I),NX(I),FNM(I)
   40    CONTINUE
      STOP
   50 FORMAT (4I5)
   60 FORMAT ( //// 5X, 8H PROBLEM, I5, 5X, 11H DIMENSIONS, 2I5, 5X //
     *         )
   70 FORMAT (5X, 33H INITIAL L2 NORM OF THE RESIDUALS, D15.7 // 5X,
     *        33H FINAL L2 NORM OF THE RESIDUALS  , D15.7 // 5X,
     *        33H NUMBER OF FUNCTION EVALUATIONS  , I10 // 5X,
     *        33H NUMBER OF JACOBIAN EVALUATIONS  , I10 // 5X,
     *        15H EXIT PARAMETER, 18X, I10 // 5X,
     *        27H FINAL APPROXIMATE SOLUTION // (5X, 5D15.7))
   80 FORMAT (12H1SUMMARY OF , I3, 16H CALLS TO LMDER1 /)
   90 FORMAT (49H NPROB   N    M   NFEV  NJEV  INFO  FINAL L2 NORM /)
  100 FORMAT (3I5, 3I6, 1X, D15.7)
C
C     LAST CARD OF DRIVER.
C
      END
      SUBROUTINE FCN(M,N,X,FVEC,FJAC,LDFJAC,IFLAG)
      INTEGER M,N,LDFJAC,IFLAG
      DOUBLE PRECISION X(N),FVEC(M),FJAC(LDFJAC,N)
C     **********
C
C     THE CALLING SEQUENCE OF FCN SHOULD BE IDENTICAL TO THE
C     CALLING SEQUENCE OF THE FUNCTION SUBROUTINE IN THE NONLINEAR
C     LEAST-SQUARES SOLVER. FCN SHOULD ONLY CALL THE TESTING
C     FUNCTION AND JACOBIAN SUBROUTINES SSQFCN AND SSQJAC WITH
C     THE APPROPRIATE VALUE OF PROBLEM NUMBER (NPROB).
C
C     SUBPROGRAMS CALLED
C
C       MINPACK-SUPPLIED ... SSQFCN,SSQJAC
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER NPROB,NFEV,NJEV
      COMMON /REFNUM/ NPROB,NFEV,NJEV
      IF (IFLAG .EQ. 1) CALL SSQFCN(M,N,X,FVEC,NPROB)
      IF (IFLAG .EQ. 2) CALL SSQJAC(M,N,X,FJAC,LDFJAC,NPROB)
      IF (IFLAG .EQ. 1) NFEV = NFEV + 1
      IF (IFLAG .EQ. 2) NJEV = NJEV + 1
      RETURN
C
C     LAST CARD OF INTERFACE SUBROUTINE FCN.
```

```
C                                                             LMDD1230
      END                                                     LMDD1240
```

```
C     **********
C
C     THIS PROGRAM TESTS CODES FOR THE LEAST-SQUARES SOLUTION OF
C     M NONLINEAR EQUATIONS IN N VARIABLES. IT CONSISTS OF A DRIVER
C     AND AN INTERFACE SUBROUTINE FCN. THE DRIVER READS IN DATA,
C     CALLS THE NONLINEAR LEAST-SQUARES SOLVER, AND FINALLY PRINTS
C     OUT INFORMATION ON THE PERFORMANCE OF THE SOLVER. THIS IS
C     ONLY A SAMPLE DRIVER, MANY OTHER DRIVERS ARE POSSIBLE. THE
C     INTERFACE SUBROUTINE FCN IS NECESSARY TO TAKE INTO ACCOUNT THE
C     FORMS OF CALLING SEQUENCES USED BY THE FUNCTION AND JACOBIAN
C     SUBROUTINES IN THE VARIOUS NONLINEAR LEAST-SQUARES SOLVERS.
C
C     SUBPROGRAMS CALLED
C
C       USER-SUPPLIED ...... FCN
C
C       MINPACK-SUPPLIED ... DPMPAR,ENORM,INITPT,LMSTR1,SSQFCN
C
C       FORTRAN-SUPPLIED ... DSQRT
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER I,IC,INFO,K,LDFJAC,LWA,M,N,NFEV,NJEV,NPROB,NREAD,NTRIES,
     *        NWRITE
      INTEGER IWA(40),MA(60),NA(60),NF(60),NJ(60),NP(60),NX(60)
      DOUBLE PRECISION FACTOR,FNORM1,FNORM2,ONE,TEN,TOL
      DOUBLE PRECISION FJAC(40,40),FNM(60),FVEC(65),WA(265),X(40)
      DOUBLE PRECISION DPMPAR,ENORM
      EXTERNAL FCN
      COMMON /REFNUM/ NPROB,NFEV,NJEV
C
C     LOGICAL INPUT UNIT IS ASSUMED TO BE NUMBER 5.
C     LOGICAL OUTPUT UNIT IS ASSUMED TO BE NUMBER 6.
C
      DATA NREAD,NWRITE /5,6/
C
      DATA ONE,TEN /1.0D0,1.0D1/
      TOL = DSQRT(DPMPAR(1))
      LDFJAC = 40
      LWA = 265
      IC = 0
   10 CONTINUE
         READ (NREAD,50) NPROB,N,M,NTRIES
         IF (NPROB .LE. 0) GO TO 30
         FACTOR = ONE
         DO 20 K = 1, NTRIES
            IC = IC + 1
            CALL INITPT(N,X,NPROB,FACTOR)
            CALL SSQFCN(M,N,X,FVEC,NPROB)
            FNORM1 = ENORM(M,FVEC)
            WRITE (NWRITE,60) NPROB,N,M
            NFEV = 0
            NJEV = 0
            CALL LMSTR1(FCN,M,N,X,FVEC,FJAC,LDFJAC,TOL,INFO,IWA,WA,
     *                  LWA)
            CALL SSQFCN(M,N,X,FVEC,NPROB)
            FNORM2 = ENORM(M,FVEC)
            NP(IC) = NPROB
            NA(IC) = N
```

```
         MA(IC) = M
         NF(IC) = NFEV
         NJ(IC) = NJEV
         NX(IC) = INFO
         FNM(IC) = FNORM2
         WRITE (NWRITE,70)
     *         FNORM1,FNORM2,NFEV,NJEV,INFO,(X(I), I = 1, N)
         FACTOR = TEN*FACTOR
   20    CONTINUE
      GO TO 10
   30 CONTINUE
      WRITE (NWRITE,80) IC
      WRITE (NWRITE,90)
      DO 40 I = 1, IC
         WRITE (NWRITE,100) NP(I),NA(I),MA(I),NF(I),NJ(I),NX(I),FNM(I)
   40    CONTINUE
      STOP
   50 FORMAT (4I5)
   60 FORMAT ( //// 5X, 8H PROBLEM, I5, 5X, 11H DIMENSIONS, 2I5, 5X //
     *         )
   70 FORMAT (5X, 33H INITIAL L2 NORM OF THE RESIDUALS, D15.7 // 5X,
     *        33H FINAL L2 NORM OF THE RESIDUALS  , D15.7 // 5X,
     *        33H NUMBER OF FUNCTION EVALUATIONS  , I10 // 5X,
     *        33H NUMBER OF JACOBIAN EVALUATIONS  , I10 // 5X,
     *        15H EXIT PARAMETER, 18X, I10 // 5X,
     *        27H FINAL APPROXIMATE SOLUTION // (5X, 5D15.7))
   80 FORMAT (12H1SUMMARY OF , I3, 16H CALLS TO LMSTR1 /)
   90 FORMAT (49H NPROB   N    M   NFEV  NJEV  INFO  FINAL L2 NORM /)
  100 FORMAT (3I5, 3I6, 1X, D15.7)
C
C     LAST CARD OF DRIVER.
C
      END
      SUBROUTINE FCN(M,N,X,FVEC,FJROW,IFLAG)
      INTEGER M,N,IFLAG
      DOUBLE PRECISION X(N),FVEC(M),FJROW(N)
C     **********
C
C     THE CALLING SEQUENCE OF FCN SHOULD BE IDENTICAL TO THE
C     CALLING SEQUENCE OF THE FUNCTION SUBROUTINE IN THE NONLINEAR
C     LEAST SQUARES SOLVER. IF IFLAG = 1, FCN SHOULD ONLY CALL THE
C     TESTING FUNCTION SUBROUTINE SSQFCN. IF IFLAG = I, I .GE. 2,
C     FCN SHOULD ONLY CALL SUBROUTINE SSQJAC TO CALCULATE THE
C     (I-1)-ST ROW OF THE JACOBIAN. (THE SSQJAC SUBROUTINE PROVIDED
C     HERE FOR TESTING PURPOSES CALCULATES THE ENTIRE JACOBIAN
C     MATRIX AND IS THEREFORE CALLED ONLY WHEN IFLAG = 2.) EACH
C     CALL TO SSQFCN OR SSQJAC SHOULD SPECIFY THE APPROPRIATE
C     VALUE OF PROBLEM NUMBER (NPROB).
C
C     SUBPROGRAMS CALLED
C
C       MINPACK-SUPPLIED ... SSQFCN,SSQJAC
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER NPROB,NFEV,NJEV,J
      DOUBLE PRECISION TEMP(65,40)
      COMMON /REFNUM/ NPROB,NFEV,NJEV
      IF (IFLAG .EQ. 1) CALL SSQFCN(M,N,X,FVEC,NPROB)
```

```
      IF (IFLAG .EQ. 2) CALL SSQJAC(M,N,X,TEMP,65,NPROB)                LMSD1230
      IF (IFLAG .EQ. 1) NFEV = NFEV + 1                                 LMSD1240
      IF (IFLAG .EQ. 2) NJEV = NJEV + 1                                 LMSD1250
      IF (IFLAG .EQ. 1) GO TO 120                                       LMSD1260
      DO 110 J = 1, N                                                   LMSD1270
         FJROW(J) = TEMP(IFLAG-1,J)                                     LMSD1280
  110    CONTINUE                                                       LMSD1290
  120 CONTINUE                                                          LMSD1300
      RETURN                                                            LMSD1310
C                                                                       LMSD1320
C     LAST CARD OF INTERFACE SUBROUTINE FCN.                            LMSD1330
C                                                                       LMSD1340
      END                                                               LMSD1350
```

```
C     **********
C
C     THIS PROGRAM TESTS CODES FOR THE LEAST-SQUARES SOLUTION OF
C     M NONLINEAR EQUATIONS IN N VARIABLES. IT CONSISTS OF A DRIVER
C     AND AN INTERFACE SUBROUTINE FCN. THE DRIVER READS IN DATA,
C     CALLS THE NONLINEAR LEAST-SQUARES SOLVER, AND FINALLY PRINTS
C     OUT INFORMATION ON THE PERFORMANCE OF THE SOLVER. THIS IS
C     ONLY A SAMPLE DRIVER, MANY OTHER DRIVERS ARE POSSIBLE. THE
C     INTERFACE SUBROUTINE FCN IS NECESSARY TO TAKE INTO ACCOUNT THE
C     FORMS OF CALLING SEQUENCES USED BY THE FUNCTION AND JACOBIAN
C     SUBROUTINES IN THE VARIOUS NONLINEAR LEAST-SQUARES SOLVERS.
C
C     SUBPROGRAMS CALLED
C
C       USER-SUPPLIED ...... FCN
C
C       MINPACK-SUPPLIED ... DPMPAR,ENORM,INITPT,LMDIF1,SSQFCN
C
C       FORTRAN-SUPPLIED ... DSQRT
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER I,IC,INFO,K,LWA,M,N,NFEV,NJEV,NPROB,NREAD,NTRIES,NWRITE
      INTEGER IWA(40),MA(60),NA(60),NF(60),NJ(60),NP(60),NX(60)
      DOUBLE PRECISION FACTOR,FNORM1,FNORM2,ONE,TEN,TOL
      DOUBLE PRECISION FNM(60),FVEC(65),WA(2865),X(40)
      DOUBLE PRECISION DPMPAR,ENORM
      EXTERNAL FCN
      COMMON /REFNUM/ NPROB,NFEV,NJEV
C
C     LOGICAL INPUT UNIT IS ASSUMED TO BE NUMBER 5.
C     LOGICAL OUTPUT UNIT IS ASSUMED TO BE NUMBER 6.
C
      DATA NREAD,NWRITE /5,6/
C
      DATA ONE,TEN /1.0D0,1.0D1/
      TOL = DSQRT(DPMPAR(1))
      LWA = 2865
      IC = 0
   10 CONTINUE
         READ (NREAD,50) NPROB,N,M,NTRIES
         IF (NPROB .LE. 0) GO TO 30
         FACTOR = ONE
         DO 20 K = 1, NTRIES
            IC = IC + 1
            CALL INITPT(N,X,NPROB,FACTOR)
            CALL SSQFCN(M,N,X,FVEC,NPROB)
            FNORM1 = ENORM(M,FVEC)
            WRITE (NWRITE,60) NPROB,N,M
            NFEV = 0
            NJEV = 0
            CALL LMDIF1(FCN,M,N,X,FVEC,TOL,INFO,IWA,WA,LWA)
            CALL SSQFCN(M,N,X,FVEC,NPROB)
            FNORM2 = ENORM(M,FVEC)
            NP(IC) = NPROB
            NA(IC) = N
            MA(IC) = M
            NF(IC) = NFEV
            NJEV = NJEV/N
```

```
            NJ(IC) = NJEV
            NX(IC) = INFO
            FNM(IC) = FNORM2
            WRITE (NWRITE,70)
     *            FNORM1,FNORM2,NFEV,NJEV,INFO,(X(I), I = 1, N)
            FACTOR = TEN*FACTOR
   20       CONTINUE
         GO TO 10
   30 CONTINUE
      WRITE (NWRITE,80) IC
      WRITE (NWRITE,90)
      DO 40 I = 1, IC
         WRITE (NWRITE,100) NP(I),NA(I),MA(I),NF(I),NJ(I),NX(I),FNM(I)
   40    CONTINUE
      STOP
   50 FORMAT (4I5)
   60 FORMAT ( //// 5X, 8H PROBLEM, I5, 5X, 11H DIMENSIONS, 2I5, 5X //
     *         )
   70 FORMAT (5X, 33H INITIAL L2 NORM OF THE RESIDUALS, D15.7 // 5X,
     *        33H FINAL L2 NORM OF THE RESIDUALS  , D15.7 // 5X,
     *        33H NUMBER OF FUNCTION EVALUATIONS  , I10 // 5X,
     *        33H NUMBER OF JACOBIAN EVALUATIONS  , I10 // 5X,
     *        15H EXIT PARAMETER, 18X, I10 // 5X,
     *        27H FINAL APPROXIMATE SOLUTION // (5X, 5D15.7))
   80 FORMAT (12H1SUMMARY OF , I3, 16H CALLS TO LMDIF1 /)
   90 FORMAT (49H NPROB   N    M   NFEV  NJEV  INFO  FINAL L2 NORM /)
  100 FORMAT (3I5, 3I6, 1X, D15.7)
C
C     LAST CARD OF DRIVER.
C
      END
      SUBROUTINE FCN(M,N,X,FVEC,IFLAG)
      INTEGER M,N,IFLAG
      DOUBLE PRECISION X(N),FVEC(M)
C     **********
C
C     THE CALLING SEQUENCE OF FCN SHOULD BE IDENTICAL TO THE
C     CALLING SEQUENCE OF THE FUNCTION SUBROUTINE IN THE NONLINEAR
C     LEAST-SQUARES SOLVER. FCN SHOULD ONLY CALL THE TESTING
C     FUNCTION SUBROUTINE SSQFCN WITH THE APPROPRIATE VALUE OF
C     PROBLEM NUMBER (NPROB).
C
C     SUBPROGRAMS CALLED
C
C       MINPACK-SUPPLIED ... SSQFCN
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER NPROB,NFEV,NJEV
      COMMON /REFNUM/ NPROB,NFEV,NJEV
      CALL SSQFCN(M,N,X,FVEC,NPROB)
      IF (IFLAG .EQ. 1) NFEV = NFEV + 1
      IF (IFLAG .EQ. 2) NJEV = NJEV + 1
      RETURN
C
C     LAST CARD OF INTERFACE SUBROUTINE FCN.
C
      END
```

```
C     **********
C
C     THIS PROGRAM TESTS THE ABILITY OF CHKDER TO DETECT
C     INCONSISTENCIES BETWEEN FUNCTIONS AND THEIR FIRST DERIVATIVES.
C     FOURTEEN TEST FUNCTION VECTORS AND JACOBIANS ARE USED. ELEVEN OF
C     THE TESTS ARE FALSE(F), I.E. THERE ARE INCONSISTENCIES BETWEEN
C     THE FUNCTION VECTORS AND THE CORRESPONDING JACOBIANS. THREE OF
C     THE TESTS ARE TRUE(T), I.E. THERE ARE NO INCONSISTENCIES. THE
C     DRIVER READS IN DATA, CALLS CHKDER AND PRINTS OUT INFORMATION
C     REQUIRED BY AND RECEIVED FROM CHKDER.
C
C     SUBPROGRAMS CALLED
C
C       MINPACK SUPPLIED ... CHKDER,ERRJAC,INITPT,VECFCN
C
C     ARGONNE NATIONAL LABORATORY. MINPACK PROJECT. MARCH 1980.
C     BURTON S. GARBOW, KENNETH E. HILLSTROM, JORGE J. MORE
C
C     **********
      INTEGER I,LDFJAC,LNP,MODE,N,NPROB,NREAD,NWRITE
      INTEGER NA(14),NP(14)
      LOGICAL A(14)
      DOUBLE PRECISION CP,ONE
      DOUBLE PRECISION DIFF(10),ERR(10),ERRMAX(14),ERRMIN(14),
     *                 FJAC(10,10),FVEC1(10),FVEC2(10),X1(10),X2(10)
C
C     LOGICAL INPUT UNIT IS ASSUMED TO BE NUMBER 5.
C     LOGICAL OUTPUT UNIT IS ASSUMED TO BE NUMBER 6.
C
      DATA NREAD,NWRITE /5,6/
C
      DATA A(1),A(2),A(3),A(4),A(5),A(6),A(7),A(8),A(9),A(10),A(11),
     *     A(12),A(13),A(14)
     *     /.FALSE.,.FALSE.,.FALSE.,.TRUE.,.FALSE.,.FALSE.,.FALSE.,
     *      .TRUE.,.FALSE.,.FALSE.,.FALSE.,.FALSE.,.TRUE.,.FALSE./
      DATA CP,ONE /1.23D-1,1.0D0/
      LDFJAC = 10
   10 CONTINUE
         READ (NREAD,60) NPROB,N
         IF (NPROB .LE. 0) GO TO 40
         CALL INITPT(N,X1,NPROB,ONE)
         DO 20 I = 1, N
            X1(I) = X1(I) + CP
            CP = -CP
   20       CONTINUE
         WRITE (NWRITE,70) NPROB,N,A(NPROB)
         MODE = 1
         CALL CHKDER(N,N,X1,FVEC1,FJAC,LDFJAC,X2,FVEC2,MODE,ERR)
         MODE = 2
         CALL VECFCN(N,X1,FVEC1,NPROB)
         CALL ERRJAC(N,X1,FJAC,LDFJAC,NPROB)
         CALL VECFCN(N,X2,FVEC2,NPROB)
         CALL CHKDER(N,N,X1,FVEC1,FJAC,LDFJAC,X2,FVEC2,MODE,ERR)
         ERRMIN(NPROB) = ERR(1)
         ERRMAX(NPROB) = ERR(1)
         DO 30 I = 1, N
            DIFF(I) = FVEC2(I) - FVEC1(I)
            IF (ERRMIN(NPROB) .GT. ERR(I)) ERRMIN(NPROB) = ERR(I)
            IF (ERRMAX(NPROB) .LT. ERR(I)) ERRMAX(NPROB) = ERR(I)
   30       CONTINUE
         NP(NPROB) = NPROB
```

```
         LNP = NPROB                                                    CHKD0620
         NA(NPROB) = N                                                  CHKD0630
         WRITE (NWRITE,80) (FVEC1(I), I = 1, N)                         CHKD0640
         WRITE (NWRITE,90) (DIFF(I), I = 1, N)                          CHKD0650
         WRITE (NWRITE,100) (ERR(I), I = 1, N)                          CHKD0660
         GO TO 10                                                       CHKD0670
   40 CONTINUE                                                          CHKD0680
      WRITE (NWRITE,110) LNP                                            CHKD0690
      WRITE (NWRITE,120)                                                CHKD0700
      DO 50 I = 1, LNP                                                  CHKD0710
         WRITE (NWRITE,130) NP(I),NA(I),A(I),ERRMIN(I),ERRMAX(I)        CHKD0720
   50    CONTINUE                                                       CHKD0730
      STOP                                                              CHKD0740
   60 FORMAT (2I5)                                                      CHKD0750
   70 FORMAT ( /// 5X, 8H PROBLEM, I5, 5X, 15H WITH DIMENSION, I5, 2X,  CHKD0760
     *         5H IS   , L1)                                            CHKD0770
   80 FORMAT ( // 5X, 25H FIRST FUNCTION VECTOR    // (5X, 5D15.7))     CHKD0780
   90 FORMAT ( // 5X, 27H FUNCTION DIFFERENCE VECTOR // (5X, 5D15.7))   CHKD0790
  100 FORMAT ( // 5X, 13H ERROR VECTOR // (5X, 5D15.7))                 CHKD0800
  110 FORMAT (12H1SUMMARY OF , I3, 16H TESTS OF CHKDER /)               CHKD0810
  120 FORMAT (46H NPROB   N    STATUS     ERRMIN         ERRMAX /)      CHKD0820
  130 FORMAT (I4, I6, 6X, L1, 3X, 2D15.7)                               CHKD0830
C                                                                       CHKD0840
C     LAST CARD OF DERIVATIVE CHECK TEST DRIVER.                        CHKD0850
C                                                                       CHKD0860
      END                                                               CHKD0870
```

A P P E N D I X   C

Test Output Summaries

```
DMCHAR CONSTANTS

IBETA =     16

IT    =     14

IRND  =      0

NGRD  =      1

MACHEP =   -13

NEGEP =   -14

IEXP =      7

MINEXP =   -65

MAXEXP =    63

EPS =  0.2220446D-15

EPSNEG =  0.1387779D-16

XMIN =  0.5397605D-78

XMAX =  0.7237006D+76


DPMPAR CONSTANTS AND RELATIVE DIFFERENCES

EPSMCH =  0.2220446D-15
RERR(1) =  0.0

DWARF =  0.5397605D-78
RERR(2) =  0.0

GIANT =  0.7237006D+76
RERR(3) =  0.0
```

SUMMARY OF 55 CALLS TO HYBRD1

NPROB	N	NFEV	INFO	FINAL L2 NORM
1	2	24	1	0.0
1	2	9	1	0.3469447D-14
1	2	9	1	0.0
2	4	107	4	0.1680600D-32
2	4	114	4	0.2602497D-33
2	4	125	4	0.1103789D-32
3	2	181	1	0.1712493D-08
3	2	11	1	0.3744485D-07
4	4	94	1	0.3993906D-10
4	4	233	1	0.3341714D-08
4	4	505	1	0.1731553D-09
5	3	27	1	0.2753435D-12
5	3	32	1	0.2250875D-09
5	3	40	1	0.1623431D-12
6	6	96	1	0.3316730D-12
6	6	171	1	0.1615190D-11
6	9	124	1	0.1580959D-12
6	9	476	4	0.3079344D-01
7	5	17	1	0.3252653D-11
7	5	238	1	0.1533203D-09
7	5	482	1	0.4930352D-10
7	6	25	1	0.3757052D-09
7	6	172	1	0.1029402D-09
7	6	253	1	0.1824689D-11
7	7	20	1	0.2742537D-08
7	7	616	1	0.1809471D-09
7	7	139	4	0.1006304D+14
7	8	120	4	0.6440508D-01
7	9	41	1	0.1881985D-08
8	10	31	1	0.6195651D-14
8	10	31	1	0.2954633D-14
8	10	77	1	0.3316379D-13
8	30	112	1	0.7826137D-14
8	40	92	1	0.4549521D-12
9	10	16	1	0.2516594D-14
9	10	19	1	0.1725957D-12
9	10	52	1	0.4174341D-09
10	1	7	1	0.2775558D-16
10	1	9	1	0.5551115D-16
10	1	16	1	0.0
10	10	16	1	0.5013184D-14
10	10	19	1	0.2188251D-12
10	10	39	1	0.3045701D-14
11	10	130	4	0.5296388D-02
11	10	84	1	0.5915050D-10
11	10	83	1	0.3427482D-08
12	10	60	1	0.0
12	10	46	1	0.3767526D-13
12	10	61	1	0.4598020D-10
13	10	21	1	0.1493879D-07
13	10	59	1	0.5104291D-08
13	10	42	1	0.9878854D-10
14	10	30	1	0.2057898D-08
14	10	45	1	0.7953612D-08
14	10	58	1	0.4526428D-09

SUMMARY OF 55 CALLS TO HYBRJ1

NPROB	N	NFEV	NJEV	INFO	FINAL L2 NORM
1	2	17	3	1	0.0
1	2	7	1	1	0.0
1	2	7	1	1	0.0
2	4	163	10	4	0.4634665D-33
2	4	122	8	4	0.3678147D-35
2	4	128	6	4	0.3679306D-33
3	2	169	6	1	0.1712667D-08
3	2	9	1	1	0.3744062D-07
4	4	86	2	1	0.3985528D-10
4	4	202	8	1	0.3771385D-09
4	4	386	35	1	0.3476459D-10
5	3	18	3	1	0.2753443D-12
5	3	20	4	1	0.2250996D-09
5	3	37	8	1	0.7202777D-09
6	6	60	6	1	0.3512552D-12
6	6	140	6	1	0.2662996D-13
6	9	92	4	1	0.5193497D-11
6	9	274	12	4	0.3888432D-01
7	5	10	1	1	0.6046389D-12
7	5	134	24	1	0.1936649D-09
7	5	222	47	1	0.1046833D-09
7	6	13	2	1	0.1434344D-10
7	6	91	13	1	0.7219636D-10
7	6	129	21	1	0.1500995D-10
7	7	11	1	1	0.1935816D-09
7	7	270	43	1	0.1718097D-08
7	7	64	11	4	0.6757599D+12
7	8	56	8	4	0.6440512D-01
7	9	23	2	1	0.4208829D-09
8	10	11	2	1	0.6195651D-14
8	10	11	2	1	0.2948663D-14
8	10	27	5	1	0.3262082D-13
8	30	22	3	1	0.5471298D-13
8	40	12	2	1	0.4314698D-12
9	10	6	1	1	0.2525050D-14
9	10	9	1	1	0.1725822D-12
9	10	42	1	1	0.4174172D-09
10	1	6	1	1	0.2775558D-16
10	1	8	1	1	0.5551115D-16
10	1	15	1	1	0.1387779D-16
10	10	6	1	1	0.5005860D-14
10	10	9	1	1	0.2188228D-12
10	10	19	2	1	0.3039706D-14
11	10	60	7	4	0.5296392D-02
11	10	34	5	1	0.5916591D-10
11	10	44	4	1	0.1511697D-08
12	10	21	1	1	0.5353163D-11
12	10	25	1	1	0.1506312D-09
12	10	39	2	1	0.0
13	10	11	1	1	0.1493879D-07
13	10	49	1	1	0.5081991D-08
13	10	22	2	1	0.9878839D-10
14	10	20	1	1	0.2057900D-08
14	10	25	2	1	0.7953613D-08
14	10	38	2	1	0.4526430D-09

SUMMARY OF 53 CALLS TO LMDER1

NPROB	N	M	NFEV	NJEV	INFO	FINAL L2 NORM
1	5	10	3	2	3	0.2236068D+01
1	5	50	3	2	3	0.6708204D+01
2	5	10	3	2	1	0.1463850D+01
2	5	50	3	2	1	0.3482630D+01
3	5	10	3	2	1	0.1909727D+01
3	5	50	3	2	1	0.3691729D+01
4	2	2	22	16	2	0.1394700D-15
4	2	2	8	5	2	0.1387779D-15
4	2	2	6	4	2	0.1387779D-15
5	3	3	11	8	2	0.9936519D-16
5	3	3	20	15	2	0.1044677D-18
5	3	3	19	16	2	0.1883267D-28
6	4	4	62	59	4	0.4508813D-33
6	4	4	68	64	4	0.1894894D-34
6	4	4	75	68	4	0.4699954D-33
7	2	2	14	8	1	0.6998875D+01
7	2	2	19	12	1	0.6998875D+01
7	2	2	24	17	1	0.6998875D+01
8	3	15	6	5	1	0.9063596D-01
8	3	15	37	36	1	0.4174769D+01
8	3	15	14	13	1	0.4174769D+01
9	4	11	18	16	1	0.1753584D-01
9	4	11	78	70	1	0.3205219D-01
9	4	11	500	380	1	0.1753584D-01
10	3	16	126	116	2	0.9377945D+01
10	3	16	400	348	5	0.8006049D+03
11	6	31	8	7	1	0.4782959D-01
11	6	31	14	13	1	0.4782959D-01
11	6	31	15	14	1	0.4782959D-01
11	9	31	8	7	2	0.1183115D-02
11	9	31	19	15	1	0.1183115D-02
11	9	31	18	15	1	0.1183115D-02
11	12	31	10	9	3	0.2173104D-04
11	12	31	13	12	3	0.2173104D-04
11	12	31	34	28	3	0.2173104D-04
12	3	10	7	6	2	0.1331111D-15
13	2	10	21	12	1	0.1115178D+02
14	4	20	255	237	1	0.2929543D+03
14	4	20	53	42	1	0.2929543D+03
14	4	20	237	221	1	0.2929543D+03
15	1	8	1	1	4	0.1886238D+01
15	1	8	29	28	1	0.1884248D+01
15	1	8	47	46	1	0.1884248D+01
15	8	8	39	20	1	0.5930324D-01
15	9	9	12	9	2	0.2373103D-15
15	10	10	25	12	1	0.8064710D-01
16	10	10	14	12	2	0.2831409D-14
16	10	10	13	8	2	0.5998352D-14
16	10	10	44	42	2	0.4081981D-14
16	30	30	19	14	2	0.9132593D-13
16	40	40	19	14	2	0.4212873D-12
17	5	33	18	15	1	0.7392493D-02
18	11	65	16	12	1	0.2003440D+00

SUMMARY OF 53 CALLS TO LMSTR1

NPROB	N	M	NFEV	NJEV	INFO	FINAL L2 NORM
1	5	10	3	2	2	0.2236068D+01
1	5	50	3	2	2	0.6708204D+01
2	5	10	3	2	1	0.1463850D+01
2	5	50	3	2	1	0.3482630D+01
3	5	10	3	2	1	0.1909727D+01
3	5	50	3	2	1	0.3691729D+01
4	2	2	21	16	4	0.0
4	2	2	8	5	2	0.1387779D-15
4	2	2	6	4	2	0.0
5	3	3	11	8	2	0.9936521D-16
5	3	3	20	15	2	0.1044678D-18
5	3	3	19	16	2	0.2354083D-28
6	4	4	65	60	4	0.4433152D-33
6	4	4	67	64	4	0.7945177D-35
6	4	4	77	73	4	0.6418465D-37
7	2	2	14	8	1	0.6998875D+01
7	2	2	19	12	1	0.6998875D+01
7	2	2	24	17	1	0.6998875D+01
8	3	15	6	5	1	0.9063596D-01
8	3	15	37	36	1	0.4174769D+01
8	3	15	14	13	1	0.4174769D+01
9	4	11	18	16	1	0.1753584D-01
9	4	11	78	70	1	0.3205219D-01
9	4	11	488	377	1	0.1753584D-01
10	3	16	126	116	3	0.9377945D+01
10	3	16	400	344	5	0.7997702D+03
11	6	31	8	7	1	0.4782959D-01
11	6	31	14	13	1	0.4782959D-01
11	6	31	15	14	1	0.4782959D-01
11	9	31	8	7	2	0.1183115D-02
11	9	31	19	15	1	0.1183115D-02
11	9	31	19	16	2	0.1183115D-02
11	12	31	10	9	3	0.2173104D-04
11	12	31	13	12	2	0.2173104D-04
11	12	31	34	28	2	0.2173104D-04
12	3	10	7	6	2	0.1331111D-15
13	2	10	21	12	1	0.1115178D+02
14	4	20	254	236	1	0.2929543D+03
14	4	20	53	42	1	0.2929543D+03
14	4	20	237	221	1	0.2929543D+03
15	1	8	1	1	4	0.1886238D+01
15	1	8	29	28	1	0.1884248D+01
15	1	8	47	46	1	0.1884248D+01
15	8	8	39	20	1	0.5930324D-01
15	9	9	12	9	2	0.4204808D-15
15	10	10	25	12	1	0.8064710D-01
16	10	10	14	12	2	0.4147068D-14
16	10	10	13	8	2	0.1426985D-14
16	10	10	29	26	2	0.1426985D-14
16	30	30	19	14	2	0.1157636D-12
16	40	40	19	14	2	0.1745470D-12
17	5	33	18	15	1	0.7392493D-02
18	11	65	16	12	1	0.2003440D+00

SUMMARY OF 53 CALLS TO LMDIF1

NPROB	N	M	NFEV	NJEV	INFO	FINAL L2 NORM
1	5	10	3	2	1	0.2236068D+01
1	5	50	3	2	1	0.6708204D+01
2	5	10	4	2	1	0.1463850D+01
2	5	50	4	2	1	0.3482630D+01
3	5	10	3	2	1	0.1909727D+01
3	5	50	4	2	1	0.3691729D+01
4	2	2	22	16	2	0.0
4	2	2	8	5	2	0.6938894D-15
4	2	2	7	5	2	0.0
5	3	3	12	9	2	0.5165626D-24
5	3	3	20	15	2	0.4447791D-17
5	3	3	19	16	2	0.1045223D-21
6	4	4	234	192	5	0.9887847D-21
6	4	4	232	192	5	0.7654726D-19
6	4	4	230	193	5	0.8369304D-19
7	2	2	14	8	1	0.6998875D+01
7	2	2	19	12	1	0.6998875D+01
7	2	2	24	17	1	0.6998875D+01
8	3	15	6	5	1	0.9063596D-01
8	3	15	36	35	1	0.4174769D+01
8	3	15	14	13	1	0.4174769D+01
9	4	11	18	16	1	0.1753584D-01
9	4	11	77	69	1	0.3205219D-01
9	4	11	232	193	5	0.3070894D-01
10	3	16	126	116	3	0.9377945D+01
10	3	16	4	3	4	0.6237599D+05
11	6	31	9	8	1	0.5104662D-01
11	6	31	14	13	1	0.4782959D-01
11	6	31	15	14	1	0.4782959D-01
11	9	31	9	8	1	0.1183214D-02
11	9	31	31	16	2	0.1183115D-02
11	9	31	28	16	2	0.1183115D-02
11	12	31	23	9	2	0.2173105D-04
11	12	31	23	12	2	0.2173207D-04
11	12	31	52	32	2	0.2173204D-04
12	3	10	7	6	2	0.1422050D-15
13	2	10	21	12	1	0.1115178D+02
14	4	20	212	197	5	0.2929563D+03
14	4	20	55	43	1	0.2929543D+03
14	4	20	212	197	5	0.2929544D+03
15	1	8	2	1	1	0.1886238D+01
15	1	8	29	28	1	0.1884248D+01
15	1	8	47	46	1	0.1884248D+01
15	8	8	39	20	1	0.5930324D-01
15	9	9	12	9	2	0.1989012D-15
15	10	10	25	12	1	0.8064710D-01
16	10	10	7	5	1	0.1000000D+01
16	10	10	13	8	2	0.1461391D-14
16	10	10	28	26	3	0.1000000D+01
16	30	30	4	2	3	0.1000000D+01
16	40	40	11	10	2	0.2805404D-12
17	5	33	18	15	1	0.7392493D-02
18	11	65	16	12	1	0.2003440D+00

SUMMARY OF 14 TESTS OF CHKDER

NPROB	N	STATUS	ERRMIN	ERRMAX
1	2	F	0.3644165D-01	0.1000000D+01
2	4	F	0.5678432D-01	0.1000000D+01
3	2	F	0.0	0.0
4	4	T	0.9801855D+00	0.1000000D+01
5	3	F	0.0	0.1000000D+01
6	9	F	0.0	0.1062543D+00
7	7	F	0.0	0.0
8	10	T	0.1000000D+01	0.1000000D+01
9	10	F	0.1652737D-01	0.1620568D+00
10	10	F	0.0	0.5761877D-01
11	10	F	0.0	0.0
12	10	F	0.2027908D-01	0.2027917D-01
13	10	T	0.8844429D+00	0.1000000D+01
14	10	F	0.0	0.0

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