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Implementation of APSO and Improved APSO on Non-Cascaded and Cascaded Short Term Hydrothermal Scheduling

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ABSTRACT Short-term hydrothermal scheduling (STHTS) is a highly non-linear, multi-model, non-convex, and multi-dimensional optimization problem that has been worked upon for about 5 decades. Many research articles have been published in solving different test cases of STHTS problem, while establishing the superiority of one type of optimization algorithm over the type, in finding the near global best solution of these complex problems. This paper presents the implementation of an improved version of a variant of the Particle Swarm Optimization algorithm (PSO), known as Accelerated Particle Swarm Optimization (APSO) on three benchmark test cases of STHTS problems. The adaptive and variable nature of the local and global search coefficients for the proposed APSO significantly improve its performance in obtaining the optimal solution for the STHTS test cases. Two of these cases are non-cascaded cases of STHTS problem (NCSTHTS) and one case is cascaded case of STHTS problem (CSTHTS). The results are compared with the results of the previous implementations of the other algorithms as presented in the literature. Due to the stochastic nature of the meta-heuristic algorithms, the parametric and non-parametric statistical tests have been implemented to establish the superiority of results of one type of algorithm over the results of the other type of algorithms.

INDEX TERMS Short Term Hydro Thermal Scheduling (STHTS), Non-Cascaded Short Term Hydro Thermal Scheduling (NCSTHTS), Cascaded Short-Term Hydro-Thermal Scheduling (CSTHTS), Parametric tests, Non-Parametric Tests, Improved APSO

I. INTRODUCTION

STHTS is a highly non-linear, multi-modal, non-convex and multi-dimensional optimization problem in which combined economic dispatch of the hydel and thermal generating units is implemented using an optimization algorithm. Extracting from reference [1], this problem can be defined mathematically in generic form by equations (1) to (9).

$$min(f) = \sum_{m=1}^{N} n_m F_m \tag{1}$$

$$\sum_{i=1}^{N_s} P_{th_{i,m}} + \sum_{j=1}^{N_s} P_{hyd_{j,m}} = P_{Demand} + P_{losses} \tag{2}$$

where,

$$P_{hyd_{j,m}} = f(V_{hyd_{j,m}}, Q_{hyd_{j,m}}) \tag{3}$$

$$P_{hyd_j}^{min} \leq P_{hyd_{j,m}} \leq P_{hyd_j}^{max} \tag{4}$$

$$P_{th_i}^{min} \leq P_{th_{i,m}} \leq P_{th_i}^{max} \tag{5}$$

$$V_{hyd_j}^{min} \leq V_{hyd_{j,m}} \leq V_{hyd_j}^{max} \tag{6}$$

$$Q_{hyd_j}^{min} \leq Q_{hyd_{j,m}} \leq Q_{hyd_j}^{max} \tag{7}$$

$$\sum_{m=1}^{N} Q_{j,m} = Q_{j,total} \tag{8}$$

$$V_{hyd_{j,(m+1)}} = V_{hyd_{j,m}} + I_{hyd_{j,m}} - Q_{hyd_{j,m}} - S_{hyd_{j,m}} + \sum_{i=1}^{R_{u,j}} (Q_{hyd_{m-t_{(i,j)}}} + S_{hyd_{m-t_{(i,j)}}}) \quad (9)$$

where, m is equal to the number of scheduling hours and j is the number of reservoirs. $R_{u,j}$ is the number of upstream reservoirs of the j^{th} reservoir.

Equation (1) is the main objective of the STHTS problem, i.e., to minimize the cost of the scheduling of hydro and thermal generators. The equation (2), which is an equality constraint of the problem, assures that the sum of powers produced by both thermal and hydel plants is equal to the sum of power demand and the transmission losses of power system. Equation (3) presents hydel power at scheduling interval m of reservoir j as function of the volume of j^{th} water reservoir and the water discharge rate of j^{th} interval. Inequalities (4) and (5) give respectively the lowest and highest limits of the hydel and Thermal powers of unit j and i at the scheduling interval m. The inequalities (6) and (7) and are related to the operation of the water reservoir, whereas equation (8) gives the allowed value of the water discharged by the j^{th} reservoir for total time of N intervals. The reservoir's volume and the discharges are balanced by the continuity equation (9). The cost F_m is the function of the power of thermal power generator which is in fact the function of the fuel cost. This relation of cost and thermal power is given by equation (10) as.

$$F_m = a + bP_{th_m} + cP_{th_m}^2 \quad (10)$$

Which is a quadratic function of thermal power at m^{th} scheduling interval and a, b and c are coefficients of scheduling equation. Depending on the thermal generator, this equation can be of higher orders as well which increase the non-linearity of the objective function. Moreover, since the scheduling problem has many scheduling intervals, the STHTS problem becomes a multi-dimensional optimization problem, where each scheduling interval is considered as one dimension of the problem. The multi-dimensional function makes STHTS problem highly multi-model, i.e., a problem with objective function having multiple peaks. This article specifically discusses two benchmark cases of NCSTHTS problem and one benchmark case of CSTHTS problem. According to reference [1], NCSTHTS problem deals with the economic dispatch of one water reservoir based hydel power plant and an equivalent or composite of many thermal power plants. Mathematically, the NCSTHTS can be defined by the above equations and inequalities if $j = 1$. According to reference [2], NCSTHTS problem was solved by using improved fast evolutionary programming algorithm. reference [3], has implemented a variant of improved fast evolutionary programming to solve NCSTHTS problem. Reference [1], has implemented gradient search algorithm a conventional optimization algorithm based on derivatives, to solve NCSTHTS problem. Reference [4], has implemented simulated annealing algorithm to solve NCSTHTS problem. Reference [5], has implemented famous Cuckoo search algorithm, to solve NCSTHTS problem. Reference [6], has implemented two variants of Cuckoo search algorithm, based on Cauchy mutation and Levy flights. Reference [7], has implemented the canonical version of particle swarm optimization to solve a modified case of NCSTHTS problem. Reference [8], has implemented an improved version of canonical Particle swarm optimization (PSO) to solve NCSTHTS problem.

According to reference [1], The CSTHTS problem deals with the combined economic operation of a chain of multiple reservoirs present on the same stream in series, i.e., one reservoir-based power plant is downhill the other reservoir-based power plant. In such problems, there can also be several thermal generating units, but are usually presented as individual units, or as an equivalent thermal unit. Mathematically, the CSTHTS can be defined by the above equations and inequalities if $j > 1$. Reference [9],has implemented three variants of evolutionary programming algorithm, known as FEP, CEP and IFEP to solve CSTHTS problem. Reference [10] , has solved CSTHTS problem using genetic algorithm. Reference [11], has implemented improved particle swarm optimization algorithm to solve CSTHTS problem. Reference [12], has implemented adaptive and modified adaptive PSO to solve CSTHTS problem. Reference [13], has solved CSTHTS problem using differential evolution algorithm. Reference [14], has implemented real coded genetic algorithm to solve CSTHTS problem. Reference [15], has implemented PSO, enhanced PSO and enhanced genetic algorithm to solve CSTHTS problem. Reference [16], has solved CSTHTS problem using modified differential evolution algorithm. Reference [17], has implemented teaching learning based algorithm to solve CSTHTS problem. Reference [18], has implemented a hybrid of real coded genetic algorithm and artificial fish swarm algorithm to solve CSTHTS problem. Reference [19], solved CSTHTS problem using moth-flame optimization, grey wolf optimization, PSO-ALNS algorithm, and a combination of grey wolf and dragon fly algorithm. Reference [20], solved CSTHTS problem using small population based PSO, which proved to be the best algorithm in solving CSTHTS problem. This paper will present an improved variant of APSO algorithm, which was previously suggested and presented by reference [21], to solve optimization problems in mechanical engineering. Reference [22], has presented a detailed comparative study of many technical articles on the topic of STHTS problems. The reference [23] discussed the cascaded hydro thermal scheduling problem using improved PSO with adaptive cognitive and social components. However, the suggested improved version depends upon the two update equations which increases the complexity of the algorithm for large-scale optimization problems. The reference [24] suggested the novel PSO technique by introducing the adaptive inertia weight constant to solve the cascaded hydro thermal scheduling. However, the suggested technique does not consider the social and cognitive coefficients for the PSO technique. The reference

[25] suggested the diversified PSO technique for the cascaded hydro thermal scheduling problem. However, the suggested technique depends upon the population size and the optimal selection of the optimal number of particles.

According to the no free lunch theorems presented in reference [26], the established superiority of one algorithm over other algorithms in solving one case of optimization problem does not establish its superiority for all the cases of all the optimization problems. Therefore, it is needed to find an appropriate optimization algorithm for every type of optimization problem if taken individually. Out of several cases of NCSTHTS and CSTHTS, this paper presents solution of two cases of NCSTHTS problem and one case of CSTHTS problem by implementing accelerated particle swarm optimization (APSO) algorithm and a new improvement in APSO algorithm. The results will be compared with previously found solutions of these test cases by other conventional and metaheuristic optimization algorithms. APSO algorithms are one of the easiest kinds of metaheuristic optimization algorithms, in terms of their solution update process and were first formulated by the authors of reference [26]. The motivations behind writing this research article are as follows:

1) The problem of consideration is multi-dimensional and highly multi-modal and non-linear in nature. Therefore, it cannot be stated with absolute strength that whether these algorithms have achieved the true global optima of the objective function. And finding better and better solutions of these problems is the main and perpetual research gap, for which the researchers have been solving these and similar other problems of STHTS for almost five decades as was discussed in the latest comprehensive review article, reference [22]. It is therefore needed to find an algorithm that helps finding better approximates of global optimum solution with fast convergence rates.
2) APSO algorithms are one of the easiest kinds of metaheuristic optimization algorithms, in terms of their solution update process and were first formulated by the authors of reference [26]. One of the aims of this research paper is to establish that this very easy algorithm if properly tuned, by making some variations, can outperform many conventional and other intensive and modern meta heuristic optimization algorithms by providing good approximates of the global optimum solutions of the mentioned cases of NCSTHTS and CSTHTS.
3) The other aim of this research paper is to emphasis on establishing the superiority of these algorithms over the others by performing true statistical hypothesis testing, like parametric and non-parametric tests, owing to the stochastic nature of these metaheuristic algorithms, which most of the references have not implemented to establish the strength of their implementations of optimization algorithms on STHTS problems. References [27], [28], has already emphasized on this while solving different cases of STHTS problems in detail.

II. APSO AND ITS IMPROVEMENT

APSO algorithm is a very promising variant of the canonical PSO [26], made by the authors of reference [26]. The beauty of this algorithm is that it has a single update equation for the particles without utilizing the velocity update equation, as compared to the original PSO algorithm as presented by equation (11) and taken from reference [26].

$$x_i^{t+1} = (1-\beta)x_i^t + \beta g + \alpha\epsilon \tag{11}$$

where, the typical values of α and β are usually taken as 0.2 and 0.5 respectively in the canonical version. References [29] has already established the superiority of APSO and its variants based on dynamic search space squeezing on some of the test cases of NCSTHTS problem over other metaheuristic optimization algorithms, with the help of hypothesis testing by implementing independent sample t tests. This article has applied another improved version of APSO algorithm, which again has a single step update equation without utilizing velocity update equation as given in equation (12). However, the improvement is in terms of updating the exploration coefficient α given in equation (13) and the exploitation coefficient β given in equation (14) in the update process, given by the following equations as presented by reference [21]. Another modification in Equation (11) as suggested by reference [21], is the use of local best p_i^t of each particle at any iteration t, instead of the particles current position at time t.

$$x_i^{t+1} = (1-\beta(t))p_i^t + \beta(t)g^t + \alpha(t)R_i^t \tag{12}$$

$$\alpha(t) = \alpha_{max} - \left(\frac{\alpha_{max}-\alpha_{min}}{t_{max}}\right)t \tag{13}$$

$$\beta(t) = \beta_{min} + (\beta_{max}-\beta_{min})\sin\left\{\frac{\pi t}{2t_{max}}\right\} \tag{14}$$

There values are kept in between 0 and 1 and there practically best range of performance is between the mentioned max and min limits as mentioned in reference [26]. Increasing them greater than 1 diverges the results of implementation of APSO algorithm. In the canonical version, the values of α and β are taken as fixed, as mentioned in reference [26]. The improvements in the canonical APSO are usually made by modifying α and β coefficients. The value of α is varied from 0.6 to 0.2 and β value is varied from 0.7 to 0.1 doing this research. Authors have found these ranges of α and β to give the best results while implementing improved APSO on the three test cases of STHTS.

Where, R_i^t is an $N \times d$ dimensional matrix of uniform random numbers as given in reference [21], where N is the number of particles and d is the dimension or number of scheduling intervals of STHTS problem. Higher values of alpha increase the randomization of an algorithm helping the algorithm to avoid premature convergence to local optima. When the algorithm proceeds to the end of iterations, the

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particles converge to the global optimum solution, while avoiding the local peaks in the previous iterations, and therefore it is required that the particle do not oscillate from the converged area. Therefore, alpha is usually modified to reduce from a high value to low value. The reduction can be an exponential decay or can be linear. It varies from problem to problem that any of the method can work. It was found that linear decay as suggested by reference [21] works very well for these STHTS problems, and therefore we selected this variation.

As far as variation in beta is concerned, beta decides the weight of either the global best of the iteration ''g'' and current position or local best x_i^t / p_i^t of any particle. There is concept of chaotic maps that presents various sinusoidal maps or models of modifying this beta variable to keep alive the diversity of the solution space, rather than keeping it constant or linearly increased or decreased. Its similar in approach to the concept of levy flight in cuckoo search and firefly algorithm. There are many chaotic maps, and it was found that this chaotic map as given in Equation (14), as was suggested in reference [21], works very well for these STHTS problems.

To conclude, the high value of α gives particle a chance to have more exploration, and thus increase the global search ability by having increased diversity, however, the step sized reduction of alpha with increasing number of iterations guaranties the convergence of particles to the good approximate of the global best solution. The high value of β allows particles to have more influence of the global best particle of each iteration whereas, the smaller values of β give good weight to the local best position p_i^t of particle as present in its iterative memory. Great results have been taken using this simple modification in the original APSO algorithm, on the test cases of NCSTHTS and CSTHTS problems.

III. METHODOLOGY TO SOLVE STHTS PROBLEMS

To solve the test cases of STHTS problem by applying APSO variant, the following are the steps that were applied.

1) Randomly initialize the discharge rate vectors (particles) within the given discharge rate constraints. In this paper, uniform random number generators have been used.
2) Calculate the Volume vectors, hydro power, and thermal power using the discharge rate vectors of Step 1.
3) Check the constraints. If the limits are violated, restart from Step 1. If the limits are intact, proceed to Step 4.
4) Find the total cost using the thermal power values found in Step 2, against each vector of particles.
5) Take the minimum cost value and its corresponding discharge rate vector. That discharge rate vector will be the global best particle.
6) Update all the particles using APSO/improved APSO updating "(12)".
7) Iterate from Steps 2–6 till the stopping criterion (maximum number of iterations) is reached. Get the results.

Due to the stochastic nature of algorithms, the superiority of one algorithm over the other algorithm must be established by applying statistical tests to compare the means. The two classes of tests are known as the parametric tests and non-parametric tests. For each test case of STHTS problems, solved in this article, the algorithms were applied for 50 trials and the minimum values of the objective function (cost) were collected. The data sets for each test case for the improved version of APSO was compared with original APSO algorithm based on parametric and non-parametric statistical tests. Results of both the algorithms were also compared to the results of the other algorithms implemented on the same test cases, as available in other published research articles. SPSS software was used to perform the parametric and non-parametric tests.

IV. RESULTS AND DISCUSSIONS

Reference [27]–[29], from the researchers of this article have already established the superiority of APSO algorithm both in its canonical form and in its improved forms on several test cases of STHTS problems. It is also already established the APSO algorithm and its variants perform very well to give good approximates to global optimum solutions to several multi-dimensional benchmark functions, like Michaelwicz 3-D function, Rosenbrook function, egg crate function and many more alike. This article specifically presents the results of the implementations of APSO and its proposed improved version on the three test cases of STHTS problems and their comparison with other algorithms implemented on the same problems. It is important to state that according to the no free lunch theorems [26], the good results of the proposed algorithm do not guarantee that the algorithm will perform the best for each test case of STHTS problem. However, it will guarantee that the types of STHTS problems, having structure like the three test cases, solved in this paper, will be solved with good results using APSO and the proposed improved APSO algorithm.

A. CASE 1 OF STHTS

This section discusses the results of the implementation of APSO and proposed improved APSO algorithm on the benchmark case of NCSTHTS problem. The problem taken is the same as that discussed in references [1]–[6]. The problem was solved using canonical APSO algorithm and Improved APSO for 50 trials each. The convergence characteristics of the best results of proposed improved APSO has been presented in Figure 4. The results of the discharge rate, volume, hydel power, thermal power, individual period's cost, and total cost for the implementation of proposed improved APSO have been presented in Table 16. Tables 1 and 2 give the statistical comparison of the implementations of APSO and improved APSO on the case 1 of STHTS problem with the help of non-parametric Mann Whitney's U test. The rank value of Variant APSO (proposed improved APSO) is less than canonical (Simple) APSO, which tells that the improved APSO is performing better than canonical APSO in finding

TABLE 1. Non-parametric Mann Whitney U test for case 1

	Test Statistics	
Case 1	Mann-Whitney U	180.90
	Wilcoxon W	1455.00
	Z	−7.377
	Asymp. Sig. (2-tailed)	.000

TABLE 2. Rank statistics of non-parametric Mann Whitney U test for case 1

	Ranks			
	Algorithm	**N**	**Mean Rank**	**Sum of Ranks**
Case 1	Simple APSO	50	71.90	3595.00
	Variant APSO	50	29.10	1455.00
	Total	100		

TABLE 3. Group statistics of independent sample t test for case 1

	Group Statistics				
	Algorithm	**N**	**Mean**	**Std. deviation**	**Std. error mean**
Case 1	Simple APSO	50	697906.4424	3990.589238	564.3545423
	Variant APSO	50	694548.7267	2809.224406	397.2843255

TABLE 4. Independent sample t test for equality of means for case 1

Independent Samples Test									
Case 1	**Independent Samples Test**		**t**	**df**	**Sig. (2-tailed)**	**t-test for Equality of Means**		**95 % Confidence Interval of the Difference**	
	F	**Sig.**				**Mean difference**	**Std. Error Difference**	**Lower**	**Upper**
Equal Variances assumed	45.6	.000	4.865	98	.000	3357.715646	690.1672874	1988.10113	4727.33017
Equal Variances not assumed			4.865	87.99	.000	3357.715646	690.1672874	1986.15100	4729.28029

TABLE 5. Comparison of cost obtained by implementation of different algorithms on case 1

Algorithm	Minimum ($)	Computation time (sec)
PSO [7]	693428.5	NA
Improved PSO [8]	693428.5	NA
Canonical APSO	693432.582	NA
Proposed Improved APSO	**693427.081**	0.017

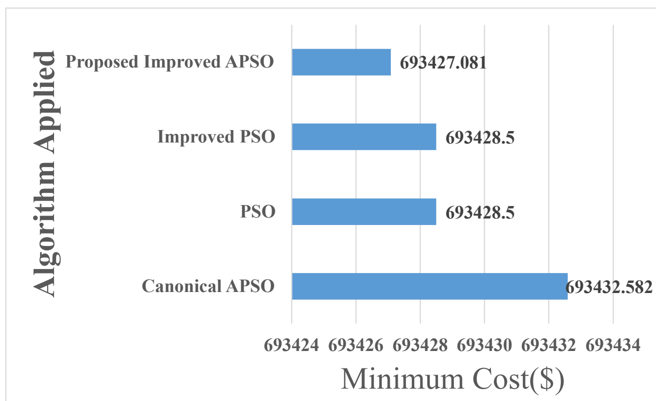

FIGURE 1. Cost comparison of different algorithms for case 1

global approximate (minimum cost) of case 1. Tables 3 and 4, give the statistical comparison of canonical and proposed improved APSO with the help of parametric independent sample t test to compare means. the 2-tailed significance value of the test is equal to 0.000%, which is less than 0.05% to tell that the performance of both the algorithms is significantly different from each other. Moreover, Table 5 and Figure 1, establish that the proposed improved APSO algorithms achieved the minimum cost for case 1 of STHTS problem, i.e. better than costs achieved by all the previously implemented algorithms and canonical APSO algorithm.

B. CASE 2 OF STHTS

This section discusses the results of the implementation of APSO and proposed improved APSO algorithm on the benchmark case of NCSTHTS problem. The problem taken is the same as that discussed in references [7], [8]. The problem was solved using canonical APSO algorithm and Improved APSO for 50 trials each. The convergence characteristics of the best results of proposed improved APSO has been presented in Figure 5. The results of the discharge rate, volume, hydel power, thermal power, individual period's cost, and total cost for the implementation of proposed improved APSO have been presented in Table 17.

Tables 6 and 7 give the statistical comparison of the implementations of APSO and improved APSO on the case 2 of STHTS problem with the help of non-parametric Mann

TABLE 6. Non-parametric Mann Whitney U test for case 2

	Test Statistics	
Case 2	Mann-Whitney U	0.000
	Wilcoxon W	1275.000
	Z	−8.912
	Asymp. Sig. (2-tailed)	.000

TABLE 7. Rank statistics of non-parametric Mann Whitney U test for case 2

	Ranks			
	Algorithm	**N**	**Mean Rank**	**Sum of Ranks**
Case 2	Simple APSO	50	75.50	3775.00
	Variant APSO	50	25.50	1275.00
	Total	100		

TABLE 8. Group statistics of independent sample t test for case 2

	Group Statistics				
	Algorithm	**N**	**Mean**	**Std. deviation**	**Std. error mean**
Case 2	Simple APSO	50	709862.2128	0.0868824805	0.0122870382
	Variant APSO	50	709862.0490	0.0000404060	0.0000057143

TABLE 9. Independent sample t test for equality of means for case 2

Independent Samples Test									
Case 2	**Independent Samples Test**		**t**	**df**	**Sig. (2-tailed)**	**t-test for Equality of Means**		**95 % Confidence Interval of the Difference**	
	F	**Sig.**				**Mean difference**	**Std. Error Difference**	**Lower**	**Upper**
Equal Variances assumed	124.071	.000	13.334	98	.000	0.1638300004	0.0122870395	0.1394467703	0.1882132305
Equal Variances not assumed			13.334	49	.000	0.1638300004	0.0122870395	0.1391382703	0.1885217306

TABLE 10. Comparison of cost obtained by implementation of different algorithms on case 2

Algorithm	Minimum ($)	Computation times (sec)
IFEP [2]	709862.05	59.7
RIFEP [3]	709862.05	NA
GS [1]	709877.38	NA
SA [4]	709874.36	901
CSA [5]	709862.05	4.54
ORCSA-Lévy flight [6]	709862.05	0.18
ORCSA-Cauchy [6]	709862.05	0.18
Canonical APSO	709862.084	0.017
Proposed Improved APSO	**709862.0489**	0.017

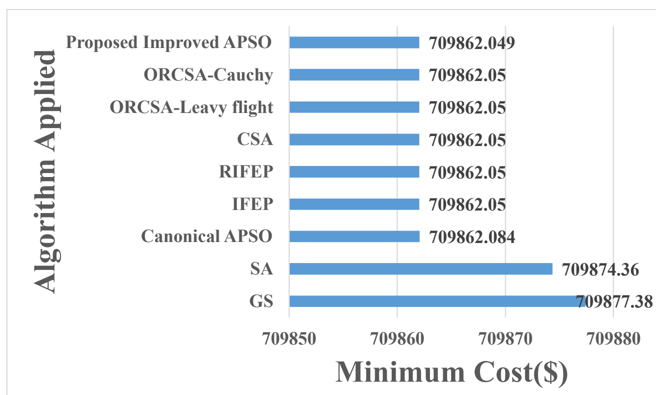

FIGURE 2. **Cost comparison of different algorithms for case 2**

Whitney's U test. The rank value of Variant APSO (proposed improved APSO) is less than canonical (Simple) APSO, which tells that the improved APSO is performing better than canonical APSO in finding global approximate (minimum cost) of case 2. Tables 8 and 9, give the statistical comparison of canonical and proposed improved APSO with the help of parametric independent sample t test to compare means. the 2-tailed significance value of the test is equal to 0.000%, which is less than 0.05% to tell that the performance of both the algorithms is significantly different from each other. Moreover, Table 10 and Figure 2, establish that the proposed improved APSO algorithms achieved the minimum cost for case 1 of STHTS problem, i.e. better than costs achieved by all the previously implemented algorithms and canonical APSO algorithm.

C. CASE 3 OF STHTS

This section discusses the results of the implementation of APSO and proposed improved APSO algorithm on the benchmark case of CSTHTS problem. The problem taken discusses the economic dispatch of four hydel units in cascade and present on same stream, and one equivalent thermal

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TABLE 11. Non-parametric Mann Whitney U test for case 3

	Test Statistics	
Case 3	Mann-Whitney U	.000
	Wilcoxon W	1275.000
	Z	−8.617
	Asymp. Sig. (2-tailed)	.000

TABLE 12. Rank statistics of non-parametric Mann Whitney U test for case 3

	Ranks			
Case 3	**Algorithm**	**N**	**Mean Rank**	**Sum of Ranks**
	Simple APSO	50	75.50	3775.00
	Variant APSO	50	25.50	1275.00
	Total	100		

TABLE 13. Group statistics of independent sample t test for case 3

	Group Statistics				
Case 3	**Algorithm**	**N**	**Mean**	**Std. deviation**	**Std. error mean**
	Simple APSO	50	923337.1414	482.7482852	68.27091722
	Variant APSO	50	922351.7587	14.31815842	2.024893383

TABLE 14. Independent sample t test for equality of means for case 3

Independent Samples Test									
Case 3	**Independent Samples Test**		**t**	**df**	**Sig. (2-tailed)**	**t-test for Equality of Means**		**95 % Confidence Interval of the Difference**	
	F	**Sig.**				**Mean difference**	**Std. Error Difference**	**Lower**	**Upper**
Equal Variances assumed	50.650	.000	14.427	98	.000	985.3826840	68.30093946	849.8416925	1120.923675
Equal Variances not assumed			14.427	49.086	.000	985.3826840	68.30093946	848.1329047	1122.632463

TABLE 15. Comparison of cost obtained by implementation of different algorithms on case 3

Algorithm	Minimum cost ($)	Average cost ($)	Maximum cost ($)	Computation time (sec)
FEP [9]	930267.92	930897.44	931396.81	NA
CEP [9]	930166.25	930373.23	930927.01	NA
IFEP [9]	930129.82	930290.13	930881.92	NA
GA [10]	932734	936969	939734	NA
IPSO [11]	922553.49	NA	NA	NA
APSO [12]	926151.54	NA	NA	NA
MAPSO [12]	922421.66	922544	923508	NA
DE [13]	923991.08	NA	NA	NA
BCGA [14]	926922.71	927815.35	929451.09	NA
RCGA [14]	925940.03	926120.26	926538.81	NA
EGA [15]	934727	936058	937339	NA
PSO [15]	928878	933085	938012	NA
EPSO [15]	922904	923527	924808	NA
MDE [16]	922555.44	NA	NA	NA
TLBO [17]	922373.39	922462.24	922873.81	NA
RCGA-AFSA [18]	922340	922362	922346	NA
MFO [19]	924455	925431	924836	NA
GWO [19]	924259	925210	924784	NA
PSO-ALNS [19]	923542	924025	923755	NA
CGWO-DA [19]	923259	923711	923444	67
SPPSO [20]	922336.31	NA	NA	16
Canonical APSO	922615.3048	923322.9877	924967.5195	80
Proposed Improved APSO	**922335.6037**	922351.7587	922443.6	100

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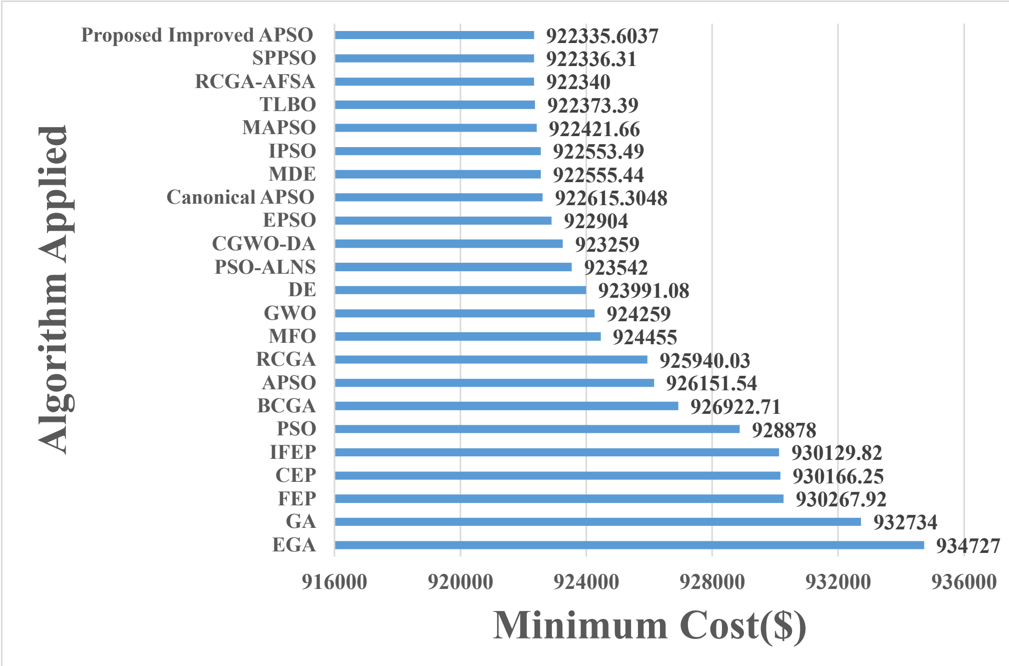

FIGURE 3. Cost comparison of different algorithms for case 3

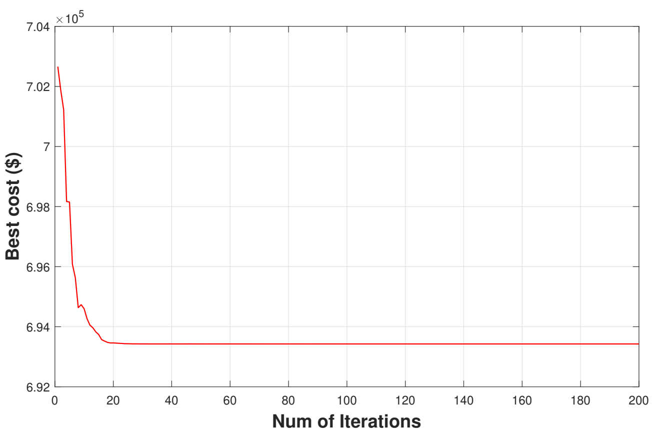

FIGURE 4. Convergence characteristics of improved APSO for case 1

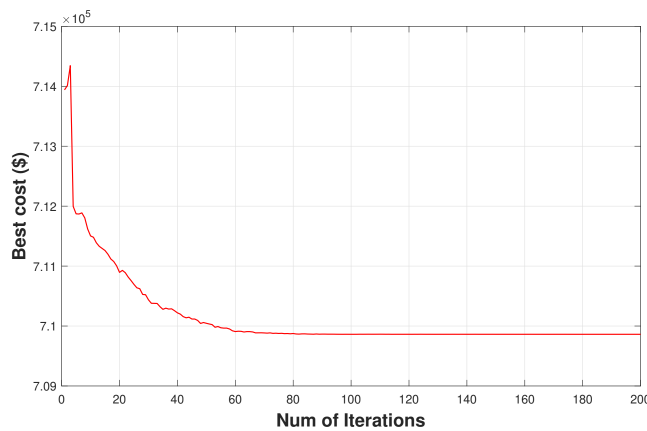

FIGURE 5. Convergence characteristics of improved APSO for case 2

unit for twenty four scheduling intervals of one hour each, and is the same as that discussed in references [9]–[20]. The problem was solved using canonical APSO algorithm and Improved APSO for 50 trials each. The convergence characteristics of the best results of proposed improved APSO has been presented in Figure 6.

The results of the discharge rate, volume, hydel power, thermal power, individual period's cost, and total cost for the implementation of proposed improved APSO have been presented in Tables 18 and 19. Tables 11 and 12 give the statistical comparison of the implementations of APSO and improved APSO on the case 3 of STHTS problem with the help of non-parametric Mann Whitney's U test. The rank

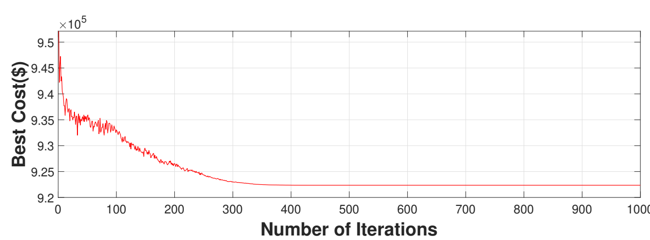

FIGURE 6. Convergence characteristics of improved APSO for case 3

value of Variant APSO (proposed improved APSO) is less than canonical (Simple) APSO, which tells that the improved APSO is performing better than canonical APSO in finding

TABLE 16. Power flow and cost optimization with improved APSO algorithm implementation on case 1

Interval	Thermal power (MW)	Hydel power (MW)	Discharge rate (acre-ft/hr)	Volume (acre-ft)	Individual cost ($)	Total cost ($)
1	806.37665	393.623350	2286.30803415119	96564.30359018570	9190.112900	693427.08130
2	806.38129	693.618715	3777.28501319845	75236.88343180440	9190.169300	
3	1100.00000	0.000000	0.00000000000	99236.88343180400	12921.400000	
4	806.08850	993.911510	5269.74020185221	60000.00100957780	9186.606840	
5	950.00000	0.000000	0.00000000000	84000.00100957780	10975.600000	
6	561.56940	738.430600	4000.00008413000	60000.00000000000	6321.701230	

TABLE 17. Power flow and cost optimization with improved APSO algorithm implementation on case 2

Interval	Demand (MW)	Thermal power (MW)	Hydel power (MW)	Discharge rate (acre-ft/hr)	Volume (acre-ft)	Individual cost ($)	Total cost ($)
1	1200	896.317426202005	303.682573797995	1839.30239177604	101928.371298688	10299.3485895231	709862.048945468
2	1500	896.319947586406	603.680052413594	3330.28986049556	85964.892972741	10299.3801029269	
3	1100	896.288281814478	203.711718185522	1342.44723938204	93855.526100156	10298.9843314702	
4	1800	896.319146543321	903.680853456679	4821.29384167969	60000.000000000	10299.3700911252	
5	950	788.983933021700	161.016066978300	1130.24985288215	70437.001765414	8979.0441734818	
6	1300	788.983873819345	511.016126180654	2869.75014711785	60000.000000000	8979.0434569284	

global approximate (minimum cost) of case 3. Tables 13 and 14, give the statistical comparison of canonical and proposed improved APSO with the help of parametric independent sample t test to compare means. the 2-tailed significance value of the test is equal to 0.000%, which is less than 0.05% to tell that the performance of both the algorithms is significantly different from each other. Moreover, Table 15 and Figure 3, establish that the proposed improved APSO algorithms achieved the minimum cost for case 1 of STHTS problem, i.e. better than costs achieved by all the previously implemented algorithms and canonical APSO algorithm.

The best results achieved by proposed improved version of APSO was possible in case 3 with large number of particles, i.e. 5000 particles and the program was run for 5000 iterations. Though, the number of particles and number of iterations were large, still, owing to its simplicity, the best results were achieved in 243 seconds on an average. If the particles were reduced, say to 150 particles, and the program was run for 1000 iterations, the minimum cost achieved was still better than all the results achieved by other algorithms, except, the minimum cost achieved was almost equal to the minimum cost achieved by small population based PSO as given in reference [20], but with very small time of about 35 seconds on an average.

V. RESULTS ANALYSIS

The results in the previous section of the implementations of Improved APSO on the three test cases and their comparison with the results of the previously made and presented implementations of other algorithms, as found in literature establishes the following points.

1) APSO algorithm either in its canonical form or in its improved form are easiest to program and implement comprehensive and smart metaheuristic optimization algorithms, which provide nearest approximates to global optimum solution of the three types of STHTS problems as discussed in this article and in fast convergence time.
2) In single step update equation, though including the updating steps of the tuning parameters alpha and beta, the algorithm has good ability of local search, as epsilon vector, a vector of uniform random numbers with zero mean and standard deviations taken the influence from the local best p_i^t of each particle x_i^t, at each iteration. Moreover, the starting high value of alpha and ultimate decrease of alpha linearly at the end of iterations, leads to a smooth shift from local search to global search, the time when local search concurs with global search. The chaotic change of beta helps in each iteration to give some weight to local search as well as the global search, while oscillating the weight on each iteration from global search to local search helps maintaining the diversity of search space till the end of iterations, that helps avoiding sticking to local optima of the objective function.
3) The other metaheuristic algorithms, many of which have multiple step particle update equations, and each equation has tuning parameters like alpha and beta that further require updates, make the convergence behavior quite slow. Although SPPSO in reference [20], has provided very promising results and less computation time, however, improved APSO algorithm, being very less complex algorithm has provided better result than SPPSO, though in high convergence time. The simple and single step update process of APSO and improved APSO allows to choose a bigger particle space to achieve results in small time. Further improvements

can be made and are worked upon to further reduce the particle size and increasing even further the convergence rate of APSO and its variants.

4) True statistical hypothesis testing is required to establish the superiority of one type of metaheuristic algorithm over the other type of metaheuristic algorithm. The types of tests are parametric (independent sample t test) and non-parametric (Mann Whitney's U test), which have not been provided in literature written on STHTS problems. This article provides a comparison of simple APSO algorithm and improved APSO algorithm with the help of statistical tests to establish that the performance of improved APSO is not only better than simple APSO but statistically different to the performance of APSO as well. And this is how the future research on these STHTS problems, which still can be solved for better results, can be made using metaheuristic optimization algorithms.

VI. CONCLUSION

This article has presented the implementation of APSO and improved APSO algorithms to solve three benchmark test cases of STHTS problem. The results were compared with the results of the implementations of other very promising optimization algorithms. It has been statistically established that the APSO and improved APSO algorithms are one of the easiest types of metaheuristic optimization algorithms and they surpass in performance to many complex metaheuristic and conventional optimization algorithms. It is also established that owing to the stochastic nature of these algorithms, the superiority of one type of algorithm is to be established over the other type of algorithm with proper hypothesis testing, with tests like independent sample t test (parametric) and Mann Whitney's test (non-parametric). These tests statistically proved that improved APSO algorithm performed better than original APSO algorithm.

VII. ACKNOWLEDGMENTS

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TABLE 18. Power flow and cost optimization with improved APSO algorithm implementation on case 3 part (a)

Interval	Volume 1 (acre-ft)	Volume 2 (acre-ft)	Volume 3 (acre-ft)	Volume 4 (acre-ft)	Q1 (acre-ft/hr)	Q2 (acre-ft/hr)	Q3 (acre-ft/hr)	Q4 (acre-ft/hr)
1	99.9176207300154	80.6954495584646	148.100000000000	109.8000000000000	10.08237926998460	7.30455044153538	30.0000000000000	13.000000000000
2	98.6045504470205	82.5350269120885	126.300000000000	99.2000000000000	10.31307028299490	6.16042264637610	29.9999999999999	13.000000000000
3	97.5716288119933	85.5334311201366	110.382379269985	87.7999999999999	9.03292163502719	6.00159579195193	29.9999999999999	13.00000000000
4	96.0246696181043	88.5050936438670	100.000000000000	74.7999999999999	8.54695919388902	6.02833747626955	29.9999999945151	13.000000000000
5	93.8832508468149	90.3671406236496	100.000000000090	91.7999999999998	8.14141877128944	6.13795302021746	18.1933442813131	13.000000000000
6	92.7992335659956	91.2540112264871	100.211755705654	108.8000000000000	8.08401728081928	6.11312939716252	18.3367992802775	13.000000000000
7	92.5253196726685	90.5753817963592	100.378443820421	125.7999999999990	8.27391389332708	6.67862943012784	17.0030681327912	13.000000000000
8	93.2190090453187	90.3414574153504	100.477745496244	142.7999999917920	8.30631062734985	7.23392438100885	16.1226686252139	13.000000002723
9	94.5298404148569	90.4373635581088	100.809256171346	147.9933442683890	8.68916863046172	7.90409385724159	15.0555326153882	13.000000004716
10	96.8546638353716	91.2294582074605	101.593947572694	153.3301434955610	8.67517657948532	8.20790535064824	15.2002486561294	13.000000053105
11	100.1356514336150	92.1440143311741	103.025454602135	157.3332116275590	8.71901240175665	8.08544387628640	15.4915859820301	13.000000000793
12	101.4349073763460	91.9115734783342	105.914450424069	159.9805546232280	8.70074405726854	8.23244085283994	15.6902746147923	13.475325629545
13	103.9201130310630	91.3319412749414	110.716631645545	160.0000000000000	8.51479434528296	8.57963220339281	16.1247365309289	15.036087238616
14	107.5194510314470	91.6145211984701	113.908008421611	160.0000000000000	8.40066199961623	8.71742007647125	16.5948111574894	15.200248656130
15	110.2229484161740	91.7510246987457	116.817182222847	160.0000000000000	8.29650261527279	8.86349649972442	16.8380613968870	15.491585982030
16	112.1210319538940	90.8158170615657	118.587779192313	160.0000000000000	8.10191646228028	8.93520763718000	17.2096972335429	15.690274614792
17	113.1298625227730	88.4420648239842	120.986008284328	160.0000000000000	7.99116943112081	9.37375223758155	16.6156935997285	16.124736530928
18	113.3912171528490	84.7916663983478	124.268375901927	160.0000000000000	7.73864536992443	9.65039842563635	15.6830453444063	16.594811157489
19	112.6454661804540	81.6121205832668	127.594347812156	160.0000000000000	7.74575097239470	10.17954581508100	14.6004051580720	16.838061396887
20	111.0511820906450	78.6077624039752	132.144601003217	160.0000000000000	7.59428408980904	11.00435817929170	13.5621444164449	17.209697233543
21	110.5056425324080	75.8756378678693	141.540744406893	158.4896234988360	7.54553955823733	11.73212453610590	10.0000059943550	18.126070100892
22	111.0703445590960	75.3460855263253	151.314574259401	154.4427644850580	7.43529797331196	9.52955234154399	10.0000000523818	19.729904358184
23	115.0271730409160	72.9293863910206	160.864470982060	148.0936226985660	5.04317151818033	10.41669913530470	10.0000010148704	20.949546944564
24	120.0000000000000	70.0000000000000	170.000000000000	140.0000000000000	5.02717304091559	10.92938639102060	10.0318934914775	21.655767115011

TABLE 19. Power flow and cost optimization with improved APSO algorithm implementation on case 3 part (b)

Interval	Power Demand (MW)	P Hydel 1 (MW)	P Hydel 2 (MW)	P Hydel 3 (MW)	P Hydel 4 (MW)	P Thermal (MW)	Individual Cost ($)	Total Cost ($)
1	1370	86.3462057472339	58.1737419531172	0.0000000000000	200.093680000000	1025.386372299650	26790.25277314890	922335.603737623
2	1390	86.8753112145199	51.6073407298694	0.0000000000000	187.755280000000	1063.762068055610	27687.41118153560	
3	1360	80.3301102698723	52.1537107330013	0.0000000000000	173.733280000001	1053.782898997130	27453.54845718240	
4	1290	77.1050516300032	53.9332291494027	0.0000000000000	156.791680000000	1002.170039220590	26250.35432805820	
5	1290	73.9815933454771	55.6819123573198	25.2347426995877	178.742080000000	956.359671597615	25191.35333759080	
6	1410	73.2485098330200	55.9517321831727	24.7554456540876	198.958480000000	1057.085832329720	27530.90889455510	
7	1650	74.2701829302151	59.5798795220015	29.7667711497770	217.440880000000	1268.942286398010	32584.12095125970	
8	2000	74.7144900403963	63.1691778475106	32.4816321658843	234.189280019002	1595.445419927210	40723.44423853580	
9	2240	77.3686797108313	67.4517904259192	35.2381103903480	238.959989788317	1820.981429684580	46594.79018445620	
10	2320	78.1197845304546	69.7065553071625	35.2691884286772	243.693888822735	1893.210582910970	48518.13581438280	
11	2230	79.4618109692712	69.4568736971882	35.2325488234816	247.132563589913	1798.716202920150	46006.11105336180	
12	2310	79.7664246103062	70.2143566768527	36.0110249541032	254.160391477977	1869.847802280760	47893.73941117900	
13	2230	79.4136358048605	71.9297153422320	36.9588550391690	268.975103182794	1772.722690630940	45321.36713586970	
14	2200	79.6774757956589	72.8647730740115	37.0084302272856	270.443812025760	1740.005508877280	44463.34411229040	
15	2130	79.6639946484955	73.7564350046365	37.5063363345808	273.009191956816	1666.064042055470	42539.96839192550	
16	2270	78.8119808791123	73.6449455660785	37.0956483381414	274.728568521840	1605.718856694830	40986.46814203140	
17	2130	78.2759398910106	74.6619455771802	39.8277847030414	278.402963507136	1658.831366321630	42353.00523716030	
18	2140	76.6092100221988	73.9178215019351	43.3934679970380	282.246735625858	1663.832764852970	42482.26802397360	
19	2240	76.5217403351308	74.4760187115521	46.5600707174740	284.181988193315	1758.260182042530	44941.55323072900	
20	2280	75.1708750037456	76.0841805192102	49.2425818488125	287.067812868739	1792.434549759490	45840.38658572530	
21	2240	74.7241570088724	76.9848732265781	50.6090707981925	292.346726313351	1745.335172653010	44602.82504473710	
22	2120	74.0585775203512	67.2436609653530	52.7732957104756	298.657916559740	1627.266549244080	41539.51059006380	
23	1850	55.1058683828840	69.6663265534025	54.5926819168372	298.466766455987	1372.168356690890	35111.32444667260	
24	1590	55.2751164674625	69.6695803599994	56.1196546055382	292.458142009623	1116.477506557380	28929.41217119880	