

## Implementation the Technique of Orthogonal Frequency Division Multiplexing Using 16-Point Fast Fourier Transform and Inverse Fast Fourier Transform

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**Abstract:** Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier modulation technique which divides the available spectrum into many carriers. OFDM uses the spectrum efficiently compared to FDMA by spacing the channels much closer together and making all carriers orthogonal to one another to prevent interference between the closely spaced carriers. The main advantage of OFDM is its robustness to channel fading in wireless environment. The objective of this work is to design and implement an OFDM transmitter and receiver using MATLAB. The work concentrates on developing Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT). The work also includes designing a mapping module, serial to parallel and parallel to serial converter module. The design uses 16-point FFT and IFFT for the processing module which indicates that the processing block contains 16 input data. All modules are implemented using MATLAB.

**Keywords:** Orthogonal Frequency Division Multiplexing (OFDM), Fast Fourier Transform (FFT), Inverse Fast Fourier Transform (IFFT), Cyclic Prefix (CP), Quadrature Amplitude Modulation (QAM), Quadrature Amplitude Demodulation (QAD), Encoder, Decoder

### I. Introduction

The increasing role of multimedia and computer applications in communications has intensified the research in the field of Wireless Broadband Multimedia Communication Systems (WBMCS). Broadband systems refer to systems with very high data rates. Over the last few years, Ultra-Wideband (UWB) communications systems have gained significant interest from industry [1]-[2]. The reason behind this is that the technology promises to deliver very high data rates ranging from 110 Mbps at a distance of 10 meters up to 480 Mbps at a distance of 2 meters in a real multipath channel, while consuming very little power as well as having a small size and insignificant weight. The transmission of such high data rates over real channels in a multipath environment imposes large bandwidths, thus pushing carrier frequencies to very high levels [3]. Transmitting such high data rates that can resist all radio channel impairments requires a careful choice of modulation technique. Orthogonal Frequency Division Multiplexing (OFDM) seems to be a very good choice. OFDM is known for its high spectral efficiency, it also offers inherent resistance to narrowband interference. Orthogonal Frequency Division Multiplexing (OFDM) multicarrier modulation is shown in Figure 1.1. The idea of OFDM is to distribute the high data stream into many low-rate data streams that are transmitted in parallel way over many subchannels within the same bandwidth [4]. Thus in each subchannel the symbol duration is low as compared to the maximum delay of the channel and ISI can be handled [5].

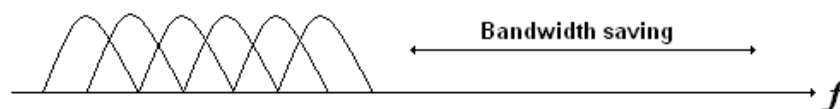


Figure 1.1: Orthogonal Frequency Division Multiplexing (OFDM) multicarrier modulation

In order to eliminate further the effect of ISI and ISI, OFDM introduces what is called the Cyclic Prefix (CP), which is a guard interval between two consecutive OFDM symbols [6]. The CP is made by copying the last part of the time domain OFDM symbol and appending it to the beginning of the OFDM symbol in the guard interval, it is a cyclic extension of the OFDM symbol [7]. The procedure is shown in Figure 1.2.

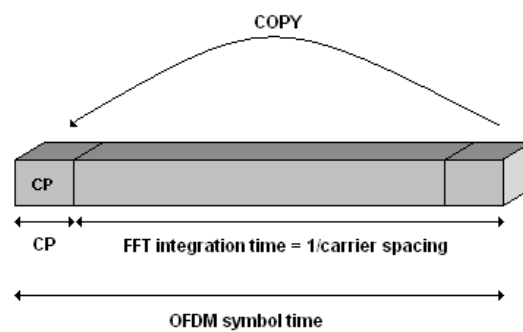


Figure 1.2: OFDM Symbol with Cyclic Prefix

In a single OFDM transmission, all the subcarriers are synchronized to each other, restricting the transmission to digital modulation schemes. OFDM is symbol based, and can be thought of as a large number of low bitrate carrier streams transmitting in parallel. Since these multiple carriers form a single OFDM transmission, they are commonly referred to as subcarriers, with the term of carrier reserved for describing the RF carrier mixing the signal from baseband [8]. The overall OFDM model is given in Figure 1.3.

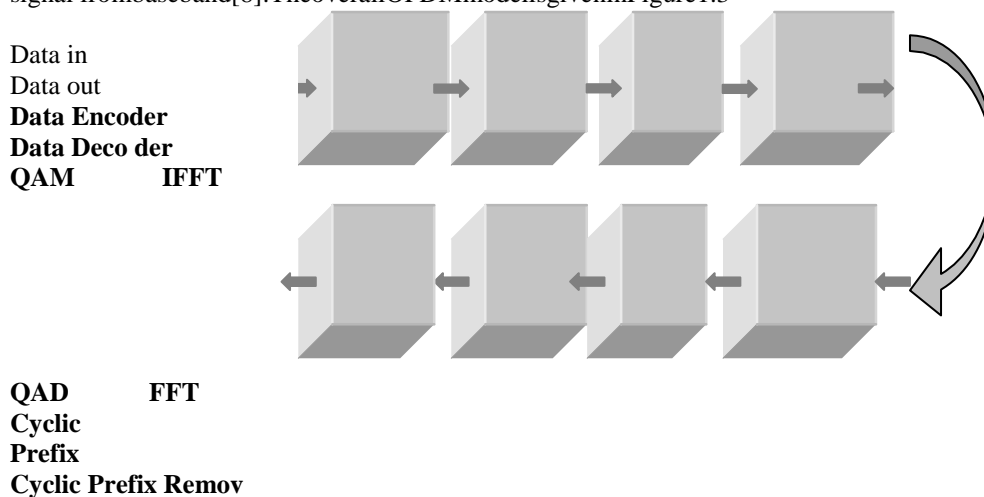


Figure 1.3: OFDM Model

## II. Methodology

In multimedia communication, a demand emerges for high-speed, high-quality digital mobile portable reception and transmission. A receiver has to cope with a signal that is often weaker than desirable and that contains many echoes. Simple digital systems do not work well in the multipath environment.

For a given overall data rate, increasing the number of carriers reduces the data rate that each individual carrier must convey, and hence lengthens the symbol period. This means that the inter-symbol interference affects a smaller percentage of each symbol as the number of carriers and hence the symbol period increases. In a single carrier system, the responses of individual bits are overlapping, thus creating ISI. In a conventional serial data system, the symbols are transmitted sequentially, with the frequency spectrum of each data symbol allowed to occupy the entire available bandwidth. In a parallel data transmission system several symbols are transmitted at the same time, what offers possibilities for alleviating many of the problems encountered with serial systems. In OFDM, the data is divided among a large number of closely spaced carriers. The entire bandwidth is filled from a single source of data. Instead of transmitting in serial way, data is transferred in a parallel way [5]. Only a small amount of the data is carried on each carrier and by this slowing of the bitrate per carrier (not the total bitrate), the influence of inter-symbol interference is significantly reduced. An important part of the OFDM system design is that the bandwidth occupied is greater than the correlation bandwidth of the fading channel. Although some of the carriers are degraded by multipath fading, the majority of the carriers should still be adequately received [9].

It is possible, however, to arrange the carriers in an OFDM signal so that the sidebands of the individual carriers overlap and the signals can still be received without adjacent carrier interference [10]. In order to do this the carriers must be mathematically orthogonal [8]. The receiver acts as a bank of demodulators, translating each carrier down to DC, the resulting signal then being integrated over a symbol period to recover the raw data. If the other carriers all beat down to frequencies which, in the time domain, have a whole number of cycles in the symbol

period, then the integration process results in zero contribution from all these carriers. Thus the carriers are linearly independent (i.e. orthogonal).

Mathematically, suppose we have a set of signals  $\psi_p$ ,

$$\int_a^b \Psi_p(t) \cdot \Psi_q^*(t) dt = \begin{cases} 1 & \text{for } p=q \\ 0 & \text{for } p \neq q \end{cases}$$

Where the \* indicates the complex conjugate and interval  $[a, b]$  is a symbol period. A fairly simple mathematical proof exists, that the series  $\sin(\pi mx)$  for  $m=1, 2, \dots$  is orthogonal over the interval.

In order to avoid a large number of modulators and filters at the transmitter and complementary filters and demodulators at the receiver, it is desirable to be able to use modern digital signal processing techniques, such as fast Fourier transform (FFT). Mathematically, each carrier can be described as a complex wave.

$$s_n(t) = A_n(t) \cdot e^{j[\omega_c t + \phi_n(t)]} \quad (5.1)$$

The values of the parameters are constant over the symbol duration period. OFDM consists of many carriers. Thus the complex signals  $S_N(t)$  are represented by:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n(t) \cdot e^{j[\omega_n t + \phi_n(t)]} \quad (5.2)$$

Where

$$\omega_n = \omega_0 + n\Delta\omega$$

This is of course a continuous signal. If we consider the waveforms of each component of the signal over one symbol period, then the variables take on fixed values, which depend on the frequency of that particular carrier, and so can be written:

$$\begin{aligned} \phi_n(t) &= \phi_n \\ A_n(t) &= A_n \end{aligned}$$

If the signal is sampled using a sampling frequency of  $1/T$ , then the resulting signal is represented by:

$$s(kT) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n \cdot e^{j[(\omega_0 + n\Delta\omega)kT + \phi_n]} \quad (5.3)$$

$$\tau = NT$$

If we now simplify eq. 5.3, without a loss of generality, then the signal becomes:

$$s(kT) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} A_n \cdot e^{j\phi_n} \cdot e^{j(n\Delta\omega)kT} \quad (5.4)$$

Now Eq. 5.4 can be compared with the general form of the inverse Fourier transform:

$$g(kT) = \frac{1}{N} \sum_{n=0}^{N-1} G\left(\frac{n}{NT}\right) e^{j\frac{2\pi n k}{N}} \quad (5.5)$$

In eq.5.4, the function  $A_n e^{j\theta_n}$  is no more than a definition of the signal in the sampled frequency domain, and  $s(kT)$  is the time domain representation. Eqs.5.4 and 5.5 are equivalent if:

$$\Delta f = \frac{\Delta \omega}{2\pi} = \frac{1}{NT} = \frac{1}{\tau} \quad (5.6)$$

This is the same condition that was required for orthogonality. Thus, one consequence of maintaining orthogonality is that the OFDM signal can be defined by using Fourier transform procedures.

### 2.1 The use of the FFT in OFDM

At the transmitter, the signal is defined in the frequency domain. It is a sampled digital signal, and it is defined such that the discrete Fourier spectrum exists only at discrete frequencies. Each OFDM carrier corresponds to one element of this discrete Fourier spectrum. The amplitudes and phases of the carriers depend on the data to be transmitted. The data transitions are synchronized at the carriers, and can be processed together, symbol by symbol [11]. The definition of the (N-point) discrete Fourier transform (DFT) is:

$$X_p[k] = \sum_{n=0}^{N-1} x_p[n] e^{-j(2\pi / N)kn} \quad (5.7)$$

And the (N-point) inverse discrete Fourier transform (IDFT):

$$x_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_p[k] e^{j(2\pi / N)kn} \quad (5.8)$$

A natural consequence of this method is that it allows us to generate carriers that are orthogonal. The members of an orthogonal set are linearly independent. Consider a data sequence  $(d_0, d_1, d_2, \dots, d_{N-1})$ , where each  $d_n$  is a complex number  $d_n = a_n + jb_n$ .

$$D_m = \sum_{n=0}^{N-1} d_n e^{-j(2\pi n m / N)} = \sum_{n=0}^{N-1} d_n e^{-2j\pi f_n t_m} \quad (5.9)$$

There are parts of the vector  $D$  has components;

$$Y_m = \text{Re}\{D_m\} = \sum_{n=0}^{N-1} [a_n \cos(2\pi f_n t_m) + b_n \sin(2\pi f_n t_m)] \quad ; k=0,1,2,\dots,N-1 \quad (5.10)$$

If these components are applied to a low-pass filter at time intervals, a signal is obtained that closely approximates the frequency division multiplexed signal

$$y(t) = \sum_{n=0}^{N-1} [a_n \cos(2\pi f_n t_m) + b_n \sin(2\pi f_n t_m)] \quad ; 0 \leq t \leq N\Delta t \quad (5.11)$$

ISI takes place when echoes on different-length propagation paths result in overlapping received symbols [12]. Problems can occur when one OFDM symbol overlaps with the next one. There is no correlation between two consecutive OFDM symbols and therefore interference from one symbol with the other will result in a disturbed signal [13]. In addition, once the incoming signal is split into the respective transmission sub-carriers, a guard interval is added between each symbol. Each symbol consists of useful symbol duration  $T_s$  and guard interval  $\Delta t$ , in which a signal of  $T_s$  is cyclically repeated. This is shown in Figure 1.4.

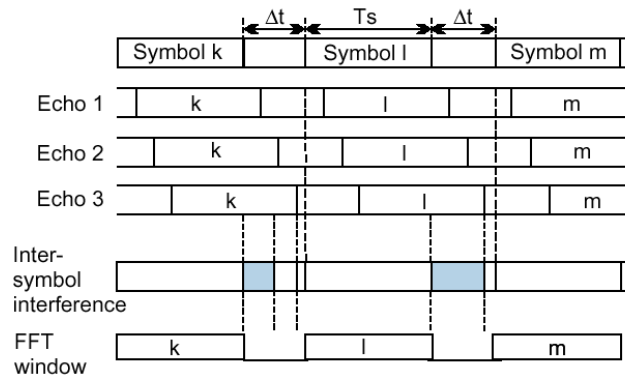


Figure 1.4: Combating ISI using guard interval

### 2.2 OFDM Generation and Reception

OFDM signals are typically generated digitally due to the difficulty in creating large banks of phase-locked oscillators and receivers in the analog domain. Figure 1.5 shows the block diagram of a typical OFDM transceiver. The transmitter section converts digital data to be transmitted, into a mapping of sub-carrier amplitude and phase. It then transforms this spectral representation of the data into the time domain using an Inverse Discrete Fourier Transform (IDFT). The Inverse Fast Fourier Transform (IFFT) performs the same operations as an IDFT, except that it is much more computationally efficient, and so is used in all practical systems [11]. In order to transmit the OFDM signal, the calculated time domain signal is then mixed up to the required frequency.

The receiver performs the reverse operation of the transmitter, mixing the RF signal to baseband for processing, then using a Fast Fourier Transform (FFT) to analyze the signal in the frequency domain. The amplitude and phase of the sub-carriers are then picked out and converted back to digital data.

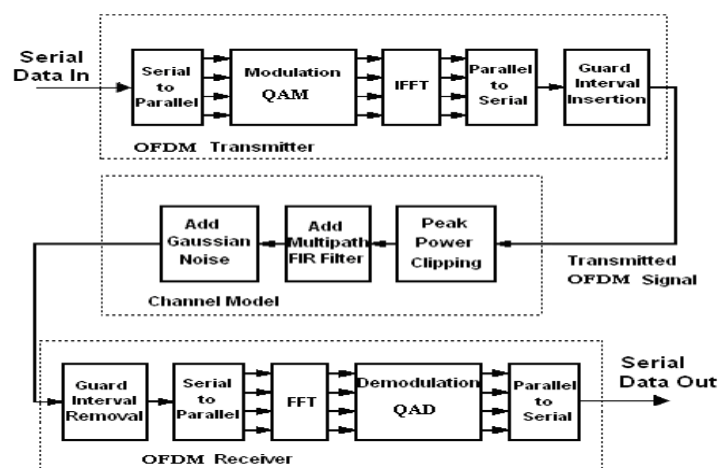


Figure 1.5: Complete OFDM Model, with channel characteristics

### 2.3 Serial to Parallel Conversion

Data to be transmitted is typically in the form of a serial data stream. In OFDM, each symbol typically transmits 40- 4000 bits, and so a serial to parallel conversion stage is needed to convert the input serial bit stream to the data to be transmitted in each OFDM symbol. The data allocated to each symbol depends on the modulation scheme used and the number of sub-carriers. For a sub-carrier modulation of 16-QAM each sub-carrier carries 4 bits of data, and so for a transmission using 100 sub-carriers the number of bits per symbol would be 400 [14].

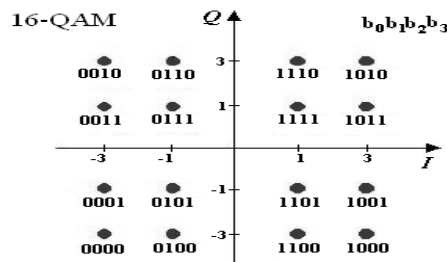
**2.4 Subcarrier Modulation Mapping**

The OFDM subcarriers can be modulated by using 16-QAM [15]. The conversion can be performed according to the gray code constellation mappings, with the input bit  $b_0$  being the earliest in the stream. The output values are formed by multiplying the resulting value  $(I+jQ)$  by a normalization factor  $K_{MOD}$  as described as  $d = I + jQ$ . The normalization factor  $K_{MOD}$  for 16-QAM is  $1/\sqrt{10}$ . In 16-QAM  $b_0 b_1$  determines the  $I_{out}$  value and  $b_2 b_3$  determines the  $Q$  value, as illustrated in Table 1.1.

Table 1.1: 16-QAM encoding Table.

Input bits ( $b_0 b_1$ )	I-out	Input bits ( $b_2 b_3$ )	Q-out
00	-3	00	-3
01	-1	01	-1
11	1	11	1
10	3	10	3

Once each subcarrier has been allocated bits for transmission, they are mapped using a modulation scheme to a subcarrier amplitude and phase, which is represented by a complex In-phase and Quadrature-phase (IQ) vector. Figure 1.6 shows an example of subcarrier modulation mapping. This example shows 16-QAM, which maps 4 bits for each symbol.



In the receiver, mapping the received IQ vector back to the data word performs subcarrier demodulation. During transmission, noise and distortion become added to the signal due to thermal noise, signal power reduction and imperfect channel equalization. Figure 1.7 shows an example of a received 16-QAM signal with a SNR of 18dB. Each of the IQ points is blurred in location due to the channel noise. For each received IQ vector the receiver has to estimate the most likely original transmission vector. This is achieved by finding the transmission vector that is closest to the received vector. Errors occur when the noise exceeds half the spacing between the transmission IQ points, making it cross over a decision boundary.

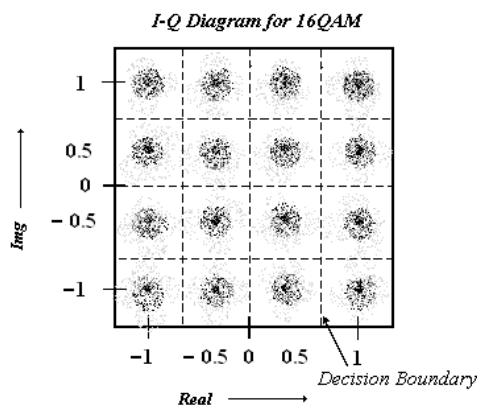
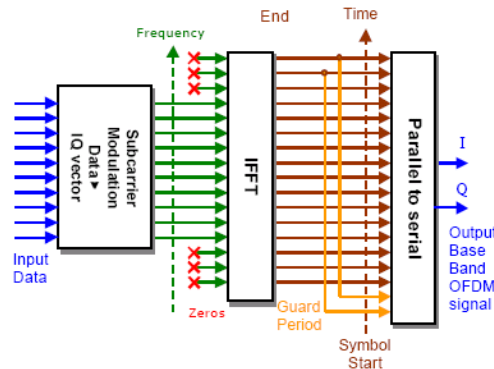


Figure 1.7: IQ plot for 16-QAM data with added noise.

**2.5 Frequency to Time Domain Conversion**

After the subcarrier modulation stage each of the data subcarriers is set to an amplitude and phase based on the data being sent and the modulation scheme; all unused subcarriers are set to zero. This sets up the OFDM signal in the frequency domain. An IFFT is then used to convert this signal to the time domain, allowing it

to be transmitted [16]. Figure 1.8 shows the IFFT section of the OFDM transmitter. In the frequency domain, before applying the IFFT, each of the discrete samples of the IFFT corresponds to an individual subcarrier. Most of the subcarriers are modulated with data. The outer subcarriers are un-modulated and set to zero amplitude. These zero subcarriers provide a frequency guardband before the Nyquist frequency and effectively act as an interpolation of the signal and allows for a realistic rolloff in the analog anti-aliasing reconstruction filters.



### III. Simulation results

Data encoder produces binary codes for any applied signal that is we can say that it performs Analog to Digital conversion. As performed the digital processing, for this we required digital data to be applied. The applied data was not in digital format so data encoder is used for converting non-digital data to digital form. The encoders used are different for all three types of input data. The encoder output for Text and image is shown in Figure 1.9 and 1.10 respectively.

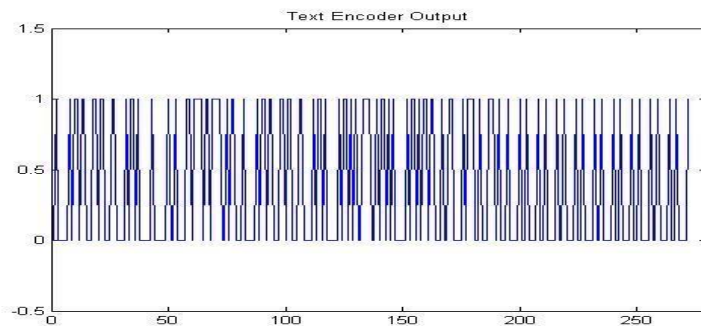


Figure 1.9: Text Encoder Output

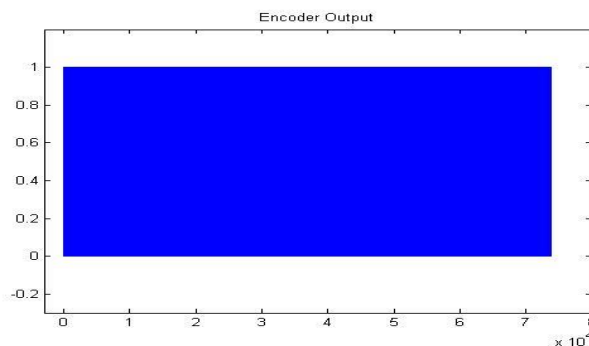


Figure 1.10: Image Encoder Output

#### 3.1. SubCarrier Modulation (QAM)

Once each subcarrier has been allocated bits for transmission, they are mapped using a modulation scheme to a subcarrier amplitude and phase, which is represented by a complex In-phase and Quadrature-phase (IQ) vector. Figure 1.11 shows an example of subcarrier modulation mapping. This example shows 16-QAM, which maps 4 bits for each symbol.

In the receiver, mapping the received IQ vector back to the data word performs subcarrier demodulation. During transmission, noise and distortion becomes added to the signal due to thermal noise, signal power reduction and imperfect channel equalization. Figure 1.12 shows an example of a received 16-QAM signal with a SNR of 18dB. Each of the IQ points is blurred in location due to the channel noise. For each received IQ vector the receiver has to estimate the most likely original transmission vector. This is achieved by finding the transmission vector that is closest to the received vector. Errors occur when the noise exceeds half the spacing between the transmission IQ points, making it cross over a decision boundary. The simulated result of QAM is shown in Figure 1.13.

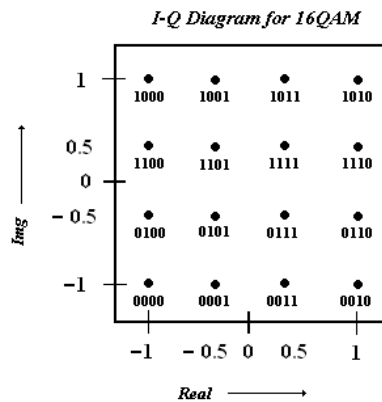


Figure 1.11: Example IQ modulation constellation (16-QAM, with gray coding of the data to each location)

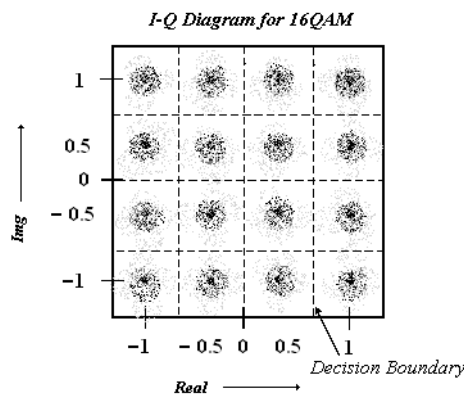


Figure 1.12: IQ plot for 16-QAM data with added noise.

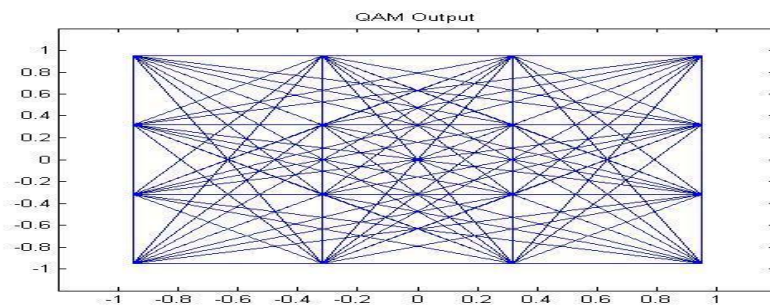


Figure 1.13: 16-QAM Output

### 3.2 Inverse Fast Fourier Transform (IFFT)

In this stage the Inverse Fast Fourier Transform takes place. The signal is transformed from frequency domain to its time domain and different orthogonal subcarriers are set up. It is the most important stage in the OFDM transmitter; it is the stage which gives us the concept of orthogonality. The IFFT for text and image are shown in Figure 1.14 and Figure 1.15 respectively.



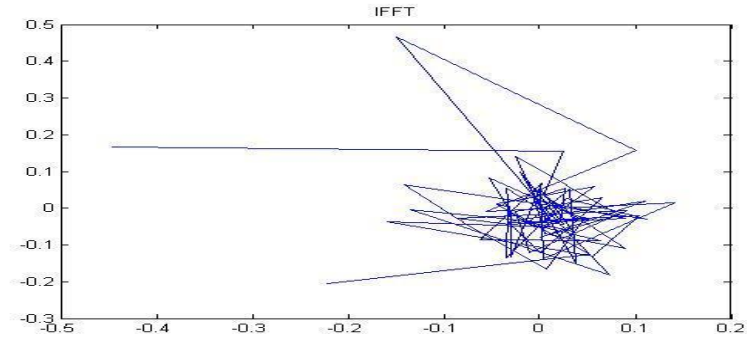


Figure 1.14: IFFT for text data

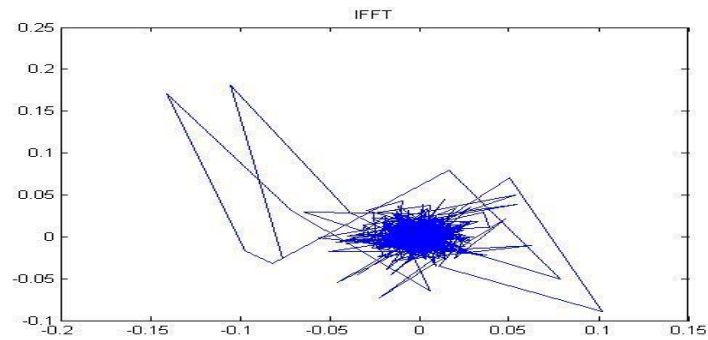


Figure 1.15: IFFT for image data

### 3.3 Fast Fourier Transform (FFT)

At the receiving end the signal is again transformed from time domain to frequency domain. The FFT for text and image are shown in Figure 1.16 and 1.17 accordingly.

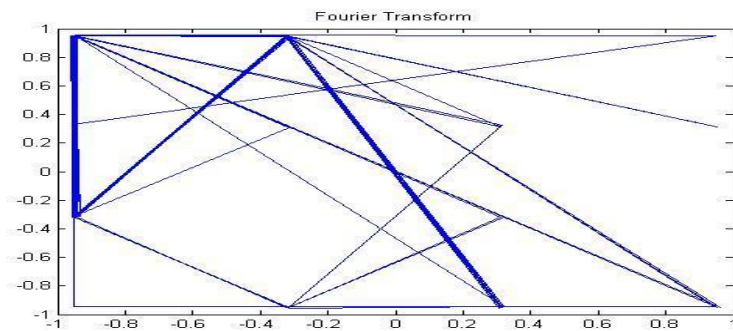


Figure 1.16: FFT for text data

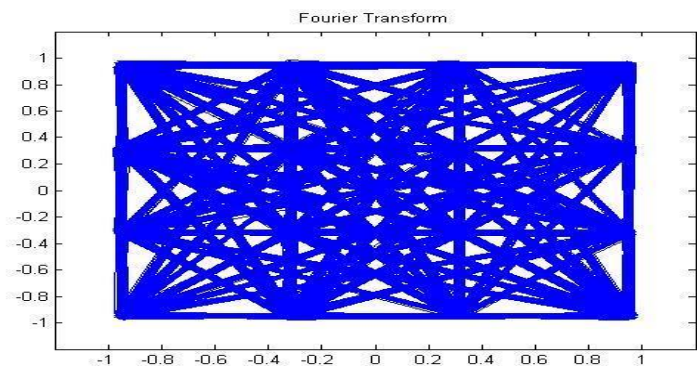


Figure 1.17: FFT for image data

### 3.4 Subcarrier demodulation (QAD)

Quadrature Amplitude Demodulation (QAD) is used to demodulate the signal from subcarriers. For this use, it is the exact reverse of QAM as shown in Figure 1.18 and 1.19 respectively.

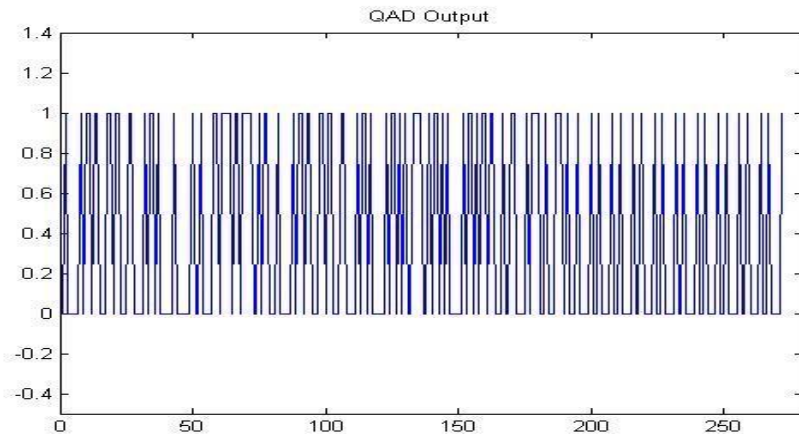


Figure 1.18: QAD demodulation for Text data

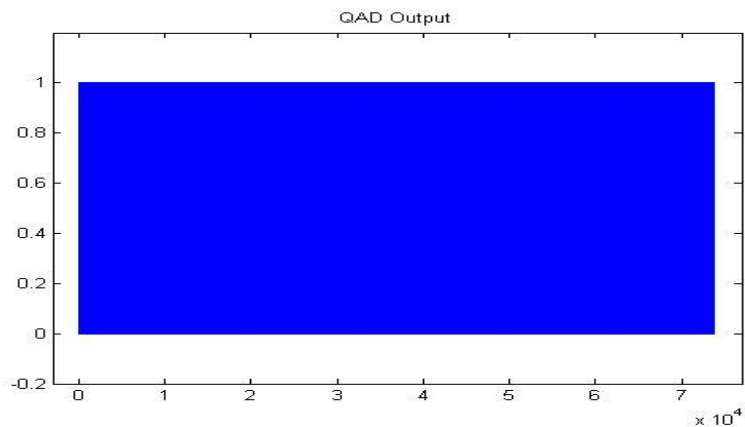
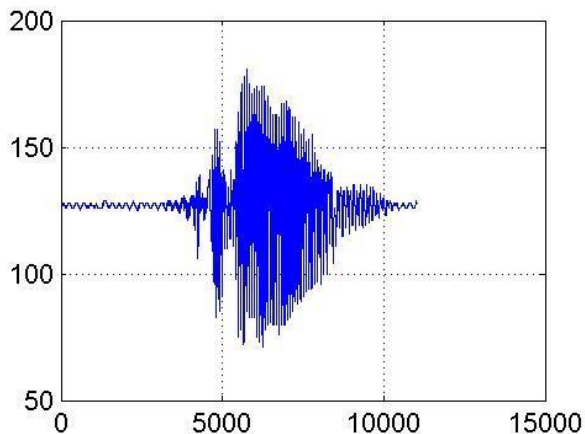


Figure 1.19: QAD demodulation for Image data

### 3.5 Data Decoder

The demodulated signal is then applied to the data decoder, which converts the binary encoded data into normal form. This is our required output which is shown in Figure 1.20.



(a)



(b) (c) Allah is the Supreme Power.

Figure 1.20: Received information, (a) Audio Signal, (b) Image, (c) Text

### 3.6 Cyclic Prefix

All the cyclic prefix and cyclic prefix removal for text and image are shown in Figure 1.21

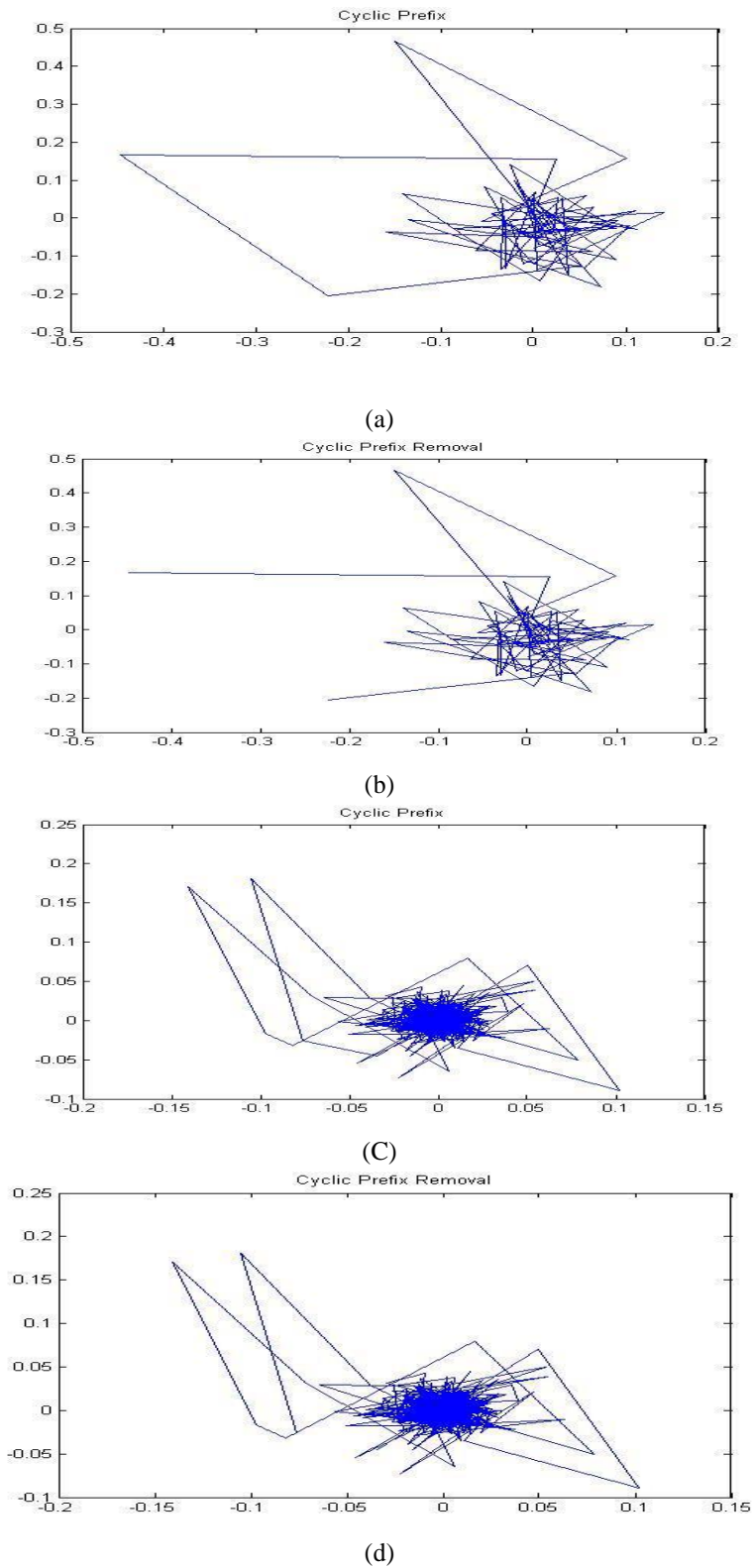


Figure 1.21: (a) cyclic prefix for text data (b) cyclic prefix removal for text data (c) Cyclic prefix for image data (d) Cyclic prefix removal for image data

#### IV. Conclusion

The purpose of this work is to give some insight into the power of the OFDM transmission scheme. Here not only the transmission scheme is itself discussed, but also some of the problems that were present in communication as well as the technique to correct them.

This model fulfills almost all the basic building blocks of a commercial OFDM system for both the transmitting side and the receiving side operations using MATLAB. We performed a real time simulation of OFDM model, by applying three different types of data, which are Text, Image and Audio. Text was fed directly through keyboard and was stored in a character typed data array. We got different images from system memory and applied to our model. The model is supporting only gray scaled images, for this we converted the RGB image to gray scaled image. We got audio data from external side using a microphone. OFDM prove to revolutionize mobile communications by allowing it to be more reliable and robust while maintaining the high data rate that digital communication demands. To achieve high spectrum efficiency, parameters such as cyclic prefix, number of carriers, and constellation must adapt to the current situation of the communication channel. Here it is shown that such flexibility can be obtained with a reasonable amount of extra hardware. The flexibility also contributes to a larger set of possible applications and thus larger fabrication volumes and lower price. The only extra source of power consumption due to the flexibility is the leakage power in the unused parts of the design. The OFDM model has been implemented by simulation and implemented in real time operation by using Digital Signal processor Kit (DSK). This simulation on OFDM model can be extended by introducing multipath effects and channel's and system's nonlinearities which make the OFDM simulation a full functional practice.

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