Econ Theory (2009) 39:307–330
DOI 10.1007/s00199-008-0335-y

RESEARCH ARTICLE

Implementing cooperative solution concepts: a generalized bidding approach

Yuan Ju · David Wettstein

Received: 13 December 2006 / Accepted: 30 January 2008 / Published online: 15 February 2008
© Springer-Verlag 2008

Abstract This paper provides a framework for implementing and comparing several solution concepts for transferable utility cooperative games. We construct bidding mechanisms where players bid for the role of the proposer. The mechanisms differ in the power awarded to the proposer. The Shapley, consensus and equal surplus values are implemented in subgame perfect equilibrium outcomes as power shifts away from the proposer to the rest of the players. Moreover, an alternative informational structure where these solution concepts can be implemented without imposing any conditions of the transferable utility game is discussed as well.

The authors thank Yukihiko Funaki, Andreu Mas-Colell, David Pérez-Castrillo and Jana Vyrastekova for helpful discussions. We also appreciate the comments from the seminar and conference participants at Keele University, Tilburg University, University of Haifa, University of Warwick, CORE at Louvain-la-Neuve, Catholic University Leuven, Pablo de Olavide University in Seville, University of Vigo, the 2006 Annual Conference of the Israeli Mathematical Union in Neve Ilan, Israel, and the 61st European Meeting of the Econometric Society in Vienna, Austria in 2006. In particular, we are grateful to the associate editor and an anonymous referee. Their valuable comments and constructive suggestions contributed to a significant improvement of the paper. Wettstein acknowledges the financial support of the Pinchas Sapir Center for Development in Tel Aviv University.

Y. Ju
Department of Economics and Related Studies, University of York,
Heslington, York YO10 5DD, UK
e-mail: yj506@york.ac.uk

Y. Ju
Extramural fellow of CentER for Economic Research, Tilburg University,
Tilburg, The Netherlands

D. Wettstein (✉)
Department of Economics, Ben-Gurion University of the Negev,
Beer-Sheva 84105, Israel
e-mail: wettstn@bgu.ac.il

Keywords Implementation · Bidding mechanism · Shapley value · Consensus value · Equal surplus value

JEL Classification C71 · C72 · D62

1 Introduction

Cooperation among individuals, firms or countries generates benefits to be shared and costs to be imputed. The analysis of these problems proceeded both axiomatically, studying the implications of normative issues and strategically, deriving the likely outcomes of maximizing behavior by the parties involved. The merging of both approaches lies at the core of the Nash (1953) program calling for a non-cooperative (strategic) foundation to cooperative (normative) solution concepts.

We provide a non-cooperative foundation to several cooperative solution concepts by using a class of bidding mechanisms that differ in the power awarded to the proposer chosen through a bidding process. The mechanisms constructed are related to the bidding mechanisms first constructed by Pérez-Castrillo and Wettstein (2001, 2002). The bidding for the role of the proposer is the same as in the previous mechanisms, however the role itself varies from one mechanism to another. Whereas previously the proposer was the only player allowed to make offers and once declined she was removed from the game, we now allow for a second round of offers. In this manner we are able to implement a continuum of cooperative solution concepts.

We construct explicit mechanisms implementing the Shapley (1953) value, the equal surplus value (cf. Driessen and Funaki 1991; Moulin 2003) and the consensus value (Ju et al. 2007). In all mechanisms, the players first participate in a bidding procedure to determine a proposer. The proposer announces an offer to all the other players. If the offer is accepted, the proposer pays out according to it and collects the value generated by the grand coalition. If the offer is rejected the other players engage again in the same game. The difference between the mechanisms is in what happens when the other players have finished the game. In all the mechanisms we construct the proposer and the other players have the right to make, accept and reject a second set of offers. The precise rules as to who makes the offer and who has a right to reject or accept vary according to the solution implemented.

The Shapley value is implemented when the proposer chosen first can make a second offer to the other players. The equal surplus value emerges as an equilibrium outcome when the other players can make the proposer (who was “left out”) an offer to join them. The consensus value is the equilibrium outcome when the proposer and the rest of the players bid for the right to make another offer. These, and actually a whole continuum of values, are the equilibrium outcomes of the variants of one basic bidding mechanism. This approach does not use the structure of any specific value to generate a specific mechanism tailored for it. The mechanism, through the bidding, allows players to consider the payoffs to all possible sub-coalitions, unlike a mechanism where only the grand coalition or singletons matter, which would, not surprisingly, implement the equal surplus value. The emergence of a solution concept, not directly related to the mechanism, serves to highlight intriguing features of the solution concept. The consensus value, for example, is “the result” of having players compete for the right to make a second offer rather than arbitrarily assigning it to a particular player.

This option of "re-entering" the game after being rejected is very reasonable. Even in the absence of such an explicit option, players in any "real-life" situation may try to exercise it through a mutual agreement, given the existence of potential benefits. This argument leads to the study of implementation with renegotiation (Maskin and Moore 1999; Baliga and Brusco 2000). Clearly, suitably modified versions of the general constructions in these papers as well as those in the usual implementation literature using sequential mechanisms (Moore and Repullo 1988; Maniquet 2003) would provide a non-cooperative foundation to the solution concepts we discuss. However, these mechanisms appropriate for general environments would be highly complex, requiring the transmission of large amounts of information, compared to our, as well as, previous mechanisms constructed to realize cooperative solution concepts.

Furthermore, following the same spirit as in Serrano (1995) and Dagan et al. (1997), we offer an alternative specification of the cooperative environment, where a coalition can, if necessary, prove what is the amount it can generate for its members to share. One such instance is the situation where the players have to share among themselves a given estate with well documented claims on the part of every coalition (the strategic analysis with such an informational structure has been surveyed by Thomson 2003). In this setting we show that suitably defined generalized bidding mechanisms implement the solution concepts, previously discussed, for any transferable utility (TU) game.

Several previous papers have indeed dealt with providing non-cooperative foundations to cooperative solution concepts. Gul (1989, 1999) suggested a bargaining procedure that leads to the Shapley value. Hart and Mas-Colell (1996) constructed a bargaining procedure that leads to the Shapley value in TU games and the Nash bargaining solution for pure bargaining problems. Krishna and Serrano (1995) provided further results regarding this procedure. Vidal-Puga (2005) generalized this procedure to allow for a coalition structure among players, which led to the Owen (1977) value in TU games and a generalization of the Nash bargaining solution. Hart and Moore (1990), Winter (1994), Dasgupta and Chiu (1998) and Vidal-Puga (2004) constructed games that lead to the Shapley value.[1] Vidal-Puga and Bergantiños (2003) introduced a coalitional bidding mechanism, as an extension of the bidding mechanism defined by Pérez-Castrillo and Wettstein (2001), and implemented the Owen (1977) value . By considering the possibility of the breakdown of negotiations when rejecting an offer, Ju et al. (2007) designed a two-level bidding mechanism and provided an implementation of the consensus value.

The generalized bidding approach, using the same basic game with different "end-games" appended to it to implement a variety of values, highlights the different "non-cooperative" rationales underlying the various values. This approach provides a structured algorithm to design mechanisms for implementing cooperative solution concepts. It should be noted that the generalized bidding mechanisms introduced in this

[1] An extensive discussion of these implementations of the Shapley value can be found in Pérez-Castrillo and Wettstein (2001) which implements the Shapley value via a bidding mechanism. For the implementations of other cooperative solutions and a general view of the research area, we refer to Serrano (2005).

paper yield the actual values implemented rather than implementing them in expected terms.

Moreover, this approach can be used to implement solution concepts in other cooperative environments such as partition function form games, games with a coalition structure and primeval games (cf. Ju and Borm 2008). Being able to apply the same extensive form to varied domains of cooperative games is one of the objectives of the Nash program as stated in Hart and Mas-Colell (1996) and Serrano (2005).

In the next section, we present the environment and the solution concepts to be implemented. In Sect. 3, we describe the basic mechanism and show that suitably defined variants of it implement the different value concepts. Section 4 presents the alternative interpretation of the environment and the modified mechanisms. The last section concludes by discussing several possible extensions and applications of the approach, which suggests further directions of research.

2 The cooperative model and the values

We denote by $N = \{1, \ldots, n\}$ the set of players, and let $S \subseteq N$ denote a coalition of players. A cooperative game in characteristic form is denoted by (N, v) where $v : 2^N \to \mathbb{R}$ is a characteristic function satisfying $v(\emptyset) = 0$. Throughout the paper, $|S|$ denotes the cardinality of S, and in particular, when no confusion arises, let $|N| = n$. For a coalition S, $v(S)$ is the total payoff that the members in S can obtain if S forms. For notational simplicity, given $i \in N$, we use $v(i)$ instead of $v(\{i\})$ to denote the stand-alone payoff of player i. A *value* is a mapping f which associates with every game (N, v) a vector in $\mathbb{R}^n$. A value determines the payoffs for every player in the game.

Given a cooperative game (N, v) and a subset $S \subseteq N$, we define the subgame $(S, v|_S)$ by assigning the value $v|_S(T) \equiv v(T)$ for any $T \subseteq S$.

We denote by ϕ the Shapley value for game (N, v) which is defined by

$$\phi_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)]$$

for all $i \in N$. It is the unique value that satisfies efficiency, additivity, symmetry and the null player property.

The equal surplus value, denoted by ϕ^{es}, is a more straightforward value and allocates to each player, besides her stand-alone payoff generated by her singleton coalition, an equal share of the surplus (in excess of the sum of all players' stand-alone payoffs) generated by the grand coalition. Formally, it is defined by

$$\phi_i^{es}(N, v) = v(i) + \frac{1}{n} \left(v(N) - \sum_{j \in N} v(j) \right)$$

for all $i \in N$. The equal surplus value fails to satisfy the null player property. However, this solution concept can be well motivated from an egalitarian perspective. For axiomatizations of the equal surplus value, we refer to van den Brink and Funaki (2004).

Ju et al. (2007) proposed a recursive two-sided negotiation procedure to establish cooperation and share the payoff of the grand coalition. This procedure leads to a new value, the consensus value, denoted by ψ. It is shown that the consensus value equals the middle point between the Shapley value and the equal surplus value. That is,

$$\psi_i(N, v) = \frac{1}{2}\phi_i(N, v) + \frac{1}{2}\phi_i^{es}(N, v)$$

for all $i \in N$. The consensus value is the unique solution concept that satisfies efficiency, additivity, symmetry and the neutral null player property. Alternative characterizations for this value using an equal welfare loss property or by means of individual rationality and a type of monotonicity can be found in Ju et al. (2007) and van den Brink et al. (2005), respectively.

From a cooperative (normative) point of view, the applications and suitability of these solution concepts in different contexts can be further elaborated on based upon the four fundamental principles of distributive justice discussed in Moulin (2003): compensation, reward, exogenous rights, and fitness.

3 The generalized bidding mechanisms

In this section, we construct the family of bidding mechanisms that will implement the various cooperative solutions. These mechanisms provide a convenient benchmark to evaluate and compare these values from a non-cooperative perspective.

The basic *bidding mechanism* can be described informally as follows: At stage 1 the players bid to choose a proposer. Each player bids by submitting an $(n-1)$-tuple of numbers (positive or negative), one number for each player (excluding herself). The player for whom the net bid (the difference between the sum of bids made by the player and the sum of bids the other players made to her) is the highest, is chosen as the proposer. Before moving to stage 2, the proposer pays to each player the bid she made. So at this stage, the net bids are used to measure players' willingness to become the proposer. As a reward to the chosen proposer for her effort (represented by her net bid), she has the right to make a scheme how to split $v(N)$ among all the players at the next stage.

At stage 2 the proposer offers a vector of payments to all other players in exchange for joining her to form the grand coalition. The offer is accepted if all the other players agree. In case of acceptance the grand coalition indeed forms and the proposer receives $v(N)$ out of which she pays out the offers made. In case of rejection the proposer "waits" while all the other players go again through the same game.

The mechanism described thus far implements the Shapley value[2] as shown in Pérez-Castrillo and Wettstein (2001). We now add further bidding and offer stages, in case of rejection of one proposer but followed by acceptance of the offer made by

[2] In the case where the rejected proposer gets her stand-alone payoff instead of "waiting".

the next proposer, to obtain what we term a *generalized bidding mechanism*. In these additional stages the two proposers (the one rejected and the one following her but whose offer was accepted) bid and accept further offers (note that these stages are also present in the game played by any remaining group of players).

The first variant implementing the Shapley value has the rejected proposer (denoted for simplicity by a) make an offer to the proposer whose offer was accepted (denoted for simplicity by b). The offer is for a to form the coalition rather than b. If the offer is accepted the coalition forms, a receives the value of the coalition and pays the offer, b receives the offer from a and pays all the commitments made by him, and all the other players in the colaition receive what they were promised. In this variant a retains the right to make offers.

The second variant implementing the equal surplus value has b make an offer to a. If the offer is accepted the coalition forms, a receives the offer, b receives the value of the coalition and pays the offer to a as well as what he owes to the remaining players. In this variant a loses the right to make offers.

In the third variant implementing the consensus value a and b bid for the right to make an offer. If a wins the game proceeds as in the first variant and if b wins the second variant goes into effect.

We now formally describe the bidding games and start by describing the mechanism implementing the Shapley value.[3]

Mechanism A1 If there is only one player $\{i\}$, she simply receives $v(i)$. When the player set $N = \{1, \ldots, n\}$ consists of two or more players, the mechanism is defined recursively.

Stages 1 to 3 provide for any set of (active) players S a proposer in S, chosen via a bidding procedure (stage 1), an offer made by the proposer to the rest of the players in S (stage 2), and an acceptance or rejection (stage 3). If stage 3 ends with a rejection, all players in S other than the rejected proposer proceed again through stages 1 to 3 where the set of active players is reduced by excluding the rejected proposer. If stage 3 ends with acceptance, for $S = N$ the game ends; but for a coalition S smaller than N, the game moves to stage 4 and then ends with stage 5. At stage 4 the last rejected proposer makes an offer to the accepted proposer, and at stage 5, the offer is either accepted or rejected and final payoffs are realized.

The mechanism starts with $S = N$.

Stage 1: Each player $i \in S$ makes $s-1$ (where $s = |S|$) bids $b^i_j \in \mathbb{R}$ with $j \in S\backslash\{i\}$. For each $i \in S$, define the *net bid* of player i by $B^i = \sum_{j\in S\backslash\{i\}} b^i_j - \sum_{j\in S\backslash\{i\}} b^j_i$. Let $i_s = \text{argmax}_{i\in S}(B^i)$ where an arbitrary tie-breaking rule is used in case of a non-unique maximizer. Once the winner i_s has been chosen, player i_s pays every player $j \in S\backslash\{i_s\}$ her bid $b^{i_s}_j$.

Stage 2: Player i_s makes a vector of offers $x^{i_s}_j \in \mathbb{R}$ to every player $j \in S\backslash\{i_s\}$. (This offer is additional to the bids paid at stage 1.)

[3] The presentation of the mechanisms and the corresponding proofs of the relevant theorems have been substantially improved following the referees' very helpful comments.

Stage 3: The players in S other than i_s, sequentially, either accept or reject the offer. If at least one player rejects it, then the offer is rejected. Otherwise, the offer is accepted.
If the offer made at stage 3 is rejected, all players in S other than i_s proceed again through the mechanism from stage 1 where the set of active players is $S\backslash\{i_s\}$.[4] Meanwhile, player i_s waits for the negotiation outcome of $S\backslash\{i_s\}$. Dependent upon whether or not player i_{s-1}, the proposer of $S\backslash\{i_s\}$, can have his offer be accepted within $S\backslash\{i_s\}$, player i_s will either be called for renegotiation with i_{s-1} or be left alone. The renegotiation will follow a similar process as specified below in stages 4 and 5. If being left alone, player i_s will receive payoff $v(i_s)$ at this stage and her final payoff will be $v(i_s) - \sum_{j\in S\backslash\{i_s\}} b_j^{i_s} + \sum_{t=s+1}^{n} b_{i_s}^{i_t}$.
If the offer is accepted, we have to distinguish between two cases where $S = N$ and $S \neq N$. In the case where $S = N$, which means that all players agree with the proposer on the scheme of sharing $v(N)$, the game ENDS. Then, at this stage, each player $j \in N\backslash\{i_n\}$ receives $x_j^{i_n}$, and player i_n receives $v(N) - \sum_{j\in N\backslash\{i_n\}} x_j^{i_n}$. Hence, the final payoff to player $j \neq i_n$ is $x_j^{i_n} + b_j^{i_n}$ while player i_n receives $v(N) - \sum_{j\in N\backslash\{i_n\}} x_j^{i_n} - \sum_{j\in N\backslash\{i_n\}} b_j^{i_n}$. In the case where $S \neq N$, stages 4 and 5 are reached.

Stage 4: The rejected proposer preceding i_s, who is denoted by i_{s+1}, makes an offer $\tilde{x}_{i_s}^{i_{s+1}}$ in $\mathbb{R}$, to player i_s. (The offer is to let i_{s+1} form the coalition $S \cup \{i_{s+1}\}$.)

Stage 5: Player i_s accepts or rejects the offer and the game ENDS.
The payoffs to all the players who proposed before i_{s+1} and to all the players in $S\backslash\{i_s\}$ are the same independent of whether there was a rejection or an acceptance at stage 5.
All proposers before i_{s+1} receive their stand-alone payoffs in addition to all the payments received and paid out in the bidding stages they participated in. Hence the final payoff to player i_m for $m > s+1$ is $v(i_m) - \sum_{l\in N\backslash(\cup_{k=m}^{n} i_k)} b_l^{i_m} + \sum_{t=m+1}^{n} b_{i_m}^{i_t}$.
Every player $j \in S\backslash\{i_s\}$ receives $x_j^{i_s}$ and the overall payoff to the player is derived by adding to it all the bids received, which were made by all previously rejected proposers. Hence, the final payoff to player $j \in S\backslash\{i_s\}$ is $x_j^{i_s} + \sum_{t=s}^{n} b_j^{i_t}$.
The payoffs to the players i_s and i_{s+1} depend on whether or not there was an acceptance at stage 5.
If the offer at stage 5 is accepted then at this stage player i_s receives $\tilde{x}_{i_s}^{i_{s+1}}$ minus the bids and offers she made to the players in S, while player i_{s+1} receives $v(S\cup\{i_{s+1}\}) - \tilde{x}_{i_s}^{i_{s+1}}$. The overall payoffs to these two players are given by adding to these amounts the sum of bids received and made in all the preceding stages, respectively. Hence, the final payoff to player i_s is $\tilde{x}_{i_s}^{i_{s+1}} - \sum_{j\in S\backslash\{i_s\}} b_j^{i_s} - \sum_{j\in S\backslash\{i_s\}} x_j^{i_s} + \sum_{t=s+1}^{n} b_{i_s}^{i_t}$, and the final payoff to player i_{s+1} is $v(S \cup \{i_{s+1}\}) - \tilde{x}_{i_s}^{i_{s+1}} - \sum_{j\in S} b_j^{i_{s+1}} + \sum_{t=s+2}^{n} b_{i_{s+1}}^{i_t}$.

[4] To make it clearer, here we explicitly explain the amount of payoff that players in $S\backslash\{i_s\}$ will bargain for, although it is incorporated in the description of the following stages provided below. Because of the chance of renegotiation at stages 4 and 5, players in $S\backslash\{i_s\}$ bid for becoming the proposer i_{s-1} so as to win the offer $\tilde{x}_{i_{s-1}}^{i_s}$ made by i_s at stage 4. In equilibrium, it equals $v(S\backslash\{i_s\})$.

If the offer at stage 5 is rejected then at this stage player i_s receives $v(S)$ minus the bids and the offers she made to the players in S, while player i_{s+1} receives his stand-alone payoff $v(i_{s+1})$. The overall payoff to these two players are given by adding to these amounts the sum of bids received and made in all the preceding stages, respectively. Hence, the final payoff to player i_s is $v(S) - \sum_{j \in S \setminus \{i_s\}} b_j^{i_s} - \sum_{j \in S \setminus \{i_s\}} x_j^{i_s} + \sum_{t=s+1}^{n} b_{i_s}^{i_t}$, and the final payoff to player i_{s+1} is $v(i_{s+1}) - \sum_{j \in S} b_j^{i_{s+1}} + \sum_{t=s+2}^{n} b_{i_{s+1}}^{i_t}$.

We note that in the case the mechanism reaches the situation where the set of active players consists of one player only, i.e. $|S| = 1$, the corresponding stages 1 to 3 are redundant and this single player is considered as the proposer for herself whose offer is accepted immediately and the game moves to stages 4 and 5 where she will renegotiate with the previously rejected proposer i_2.

We will show that for any zero-monotonic game (N, v) (i.e., $v(S) \geq v(S \setminus \{i\}) + v(\{i\})$ for all $S \subseteq N$ and $i \in S$), the subgame perfect equilibrium (SPE) outcomes of Mechanism A1 coincide with the payoff vector $\phi(N, v)$ as prescribed by the Shapley value.

Theorem 3.1 *Mechanism A1 implements the Shapley value of a zero-monotonic game (N, v) in SPE.*

Proof [5] Let (N, v) be a zero-monotonic game. The proof proceeds by induction on the number of players n. The induction assumption precisely stated is that whenever the mechanism is used by n players with a given characteristic function (satisfying zero-monotonicity), it implements the Shapley value corresponding to this characteristic function. It is easy to see that the theorem holds for $n = 1$. We assume that it holds for all $m \leq n - 1$ and show that it is satisfied for n.

First we show that the Shapley value is an SPE outcome. We explicitly construct an SPE that yields the Shapley value as an SPE outcome. Consider the following strategies, which the players would follow in any game they participate in (we describe it for the whole set of players, N, but these are also the strategies followed by any player in a subset S that is called upon to play the game, with S replacing N):

At stage 1, each player $i \in N$ announces $b_j^i = \phi_j(N, v) - \phi_j(N \setminus \{i\}, v|_{N \setminus \{i\}})$ for every $j \in N \setminus \{i\}$.

At stage 2, a proposer, player i_n, offers $x_j^{i_n} = \phi_j(N \setminus \{i_n\}, v|_{N \setminus \{i_n\}})$ to every $j \in N \setminus \{i_n\}$.

At stage 3, any player $j \in N \setminus \{i_n\}$ accepts any offer which is greater than or equal to $\phi_j(N \setminus \{i_n\}, v|_{N \setminus \{i_n\}})$ and rejects any offer strictly less than $\phi_j(N \setminus \{i_n\}, v|_{N \setminus \{i_n\}})$.

At stage 4, player i_n makes an offer $\widetilde{x}_{i_{n-1}}^{i_n} = v(N \setminus \{i_n\})$ to any selected proposer $i_{n-1} \in N \setminus \{i_n\}$.

At stage 5, player i_{n-1}, the proposer of the set of players $N \setminus \{i_n\}$, accepts any offer greater than or equal to $v(N \setminus \{i_n\})$ and rejects any offer strictly less than it.

[5] The proof is similar to the proof of Theorem 1 in Pérez-Castrillo and Wettstein (2001). We write it out explicitly since the strategies and types of equilibria, while also leading to the Shapley payoffs vector, are different. The proofs of the other theorems of the paper can be constructed along similar lines.

Clearly the combination of these strategies of all players in N leads to acceptance at stage 3, which yields the Shapley value for any player who is not the proposer, since the game ends following the acceptance at stage 3 and $b_j^{i_n} + x_j^{i_n} = \phi_j(N, v)$, for all $j \neq i_n$. Moreover, given that following the strategies the offer is accepted by all players and the grand coalition is formed, the proposer also obtains her Shapley value.

To check that the above strategies constitute an SPE, note first that the given strategy profile yields subgame perfection at stages 2, 3, 4 and 5 because the corresponding actions are best responses by zero-monotonicity: In case of rejection at stage 3, proposer i_n can obtain $v(N) - v(N \backslash \{i_n\})$ in the end (it pays her to make an offer that is accepted at stage 4, by zero-monotonicity). Also following the rejection at stage 3, it is obvious that players within $N \backslash \{i_n\}$ will obtain $v(N \backslash \{i_n\})$ by accepting the offer at stage 4. This subgame, with respect to $N \backslash \{i_n\}$ after stage 3 with i_n waiting outside, is equivalent to a game with player set $N \backslash \{i_n\}$ bargaining over $v(N \backslash \{i_n\})$ using the same rule as specified in the mechanism, without i_n being involved. Then we can apply the induction hypothesis, by which we have the Shapley value as the outcome of this game. That is, each player $j \in N \backslash \{i_n\}$ gets $\phi_j(N \backslash \{i_n\}, v|_{N \backslash \{i_n\}})$. To verify the actions at stage 1, i.e. the bids, complete an SPE, note that all net bids equal zero by the balanced contributions property for the Shapley value (Myerson 1980). To show that a change in the bids made by a player i cannot increase that player's payoff we consider the following two cases: First, player i may change the vector of her bids so that another player becomes the proposer, this will not change her payoff, which would still equal her Shapley value. Second, if she changes the vector of her bids and following it she is still the proposer, it must be that her total bid $(\sum_{j \in N \backslash \{i\}} b_j^i)$ did not decline, which again means her payoff cannot improve. That is, any deviation of the bidding strategy of player i specified at stage 1 cannot improve the payoff of player i. Hence, no player has an incentive to change its bid, showing that the given strategy profile is an SPE.

The proof that any SPE yields the Shapley value proceeds by a series of claims.

Claim (a). If $v(N) > v(N \backslash \{i_n\}) + v(i_n)$ then the only SPE strategies at stages 4 and 5 are as follows: At stage 5, any player i_{n-1} (the proposer from the set of players $N \backslash \{i_n\}$) accepts any offer greater than or equal to $v(N \backslash \{i_n\})$ and rejects any offer strictly less than it, and at stage 4 player i_n offers exactly $v(N \backslash \{i_n\})$ to player i_{n-1}. If $v(N) = v(N \backslash \{i_n\}) + v(i_n)$ then there exists another SPE strategy configuration (besides the above one): At stage 5, any player i_{n-1} accepts any offer strictly greater than $v(N \backslash \{i_n\})$ and rejects any offer less or equal to it, and at stage 4 player i_n makes an offer to player i_{n-1} that is less than or equal to $v(N \backslash \{i_n\})$. Hence, in any SPE, at stages 4 and 5, players i_n and i_{n-1} will end up with receiving $v(N) - v(N \backslash \{i_n\})$ and $v(N \backslash \{i_n\})$, respectively.

The two types of SPE specified in the claim can be readily verified due to zero-monotonicity.

Claim (b). In any SPE, at stage 3, all players other than the proposer i_n accept the offer if $x_j^{i_n} > \phi_j(N \backslash \{i_n\}, v|_{N \backslash \{i_n\}})$ for every $j \neq i_n$. Otherwise, if $x_j^{i_n} < \phi_j(N \backslash \{i_n\}, v|_{N \backslash \{i_n\}})$ for at least some $j \neq i_n$, then the offer is rejected.

Note that if an offer made by the proposer i_n is rejected at stage 3, all other players $N\backslash\{i_n\}$ will, by Claim (a), bid over the right to share exactly $v(N\backslash\{i_n\})$. Consequently, in case of rejection at stage 3, by the induction hypothesis, the payoff to a player $j \neq i_n$ is $\phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$. We denote the last player that has to decide whether to accept or reject the offer by β. If the game reaches β, i.e. there has been no previous rejection, her optimal strategy involves accepting any offer higher than $\phi_\beta(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ and rejecting any offer lower than $\phi_\beta(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$. The second to last player, denoted by $\beta - 1$, anticipates the reaction of player β. So, $\beta - 1$ will accept the offer when the game reaches him with $x^{i_n}_{\beta-1} > \phi_{\beta-1}(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ and $x^{i_n}_{\beta} > \phi_\beta(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$. If $x^{i_n}_{\beta-1} < \phi_{\beta-1}(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ and $x^{i_n}_{\beta} > \phi_\beta(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$, player $\beta - 1$ will reject the offer. If $\beta - 1$ observes $x^{i_n}_{\beta} < \phi_\beta(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$, he will be indifferent to accepting or rejecting any offer $x^{i_n}_{\beta-1}$. Following this argument till the first player, Claim (b) is proved.

Claim (c). For the game that starts at stage 2 there exist several types of SPE, which, however, are all equivalent in terms of payoffs to the players. An obvious SPE restricted to stages 2 and 3 is as follows: At stage 2, player i_n offers $x^{i_n}_j = \phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ to all $j \neq i_n$; at stage 3, every player $j \neq i_n$ accepts any offer $x^{i_n}_j \geq \phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ and rejects the offer otherwise. In addition, there are other equilibrium configurations where some player(s) j in $N\backslash\{i_n\}$ reject any offer $x^{i_n}_j \leq \phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ and player i_n offers to some of these players j in $N\backslash\{i_n\}$ something less than or equal to $\phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$. In all of these equilibria, player i_n "recovers" the amount $v(N) - v(N\backslash\{i_n\})$ at stages 4 and 5, whereas each player j in $N\backslash\{i_n\}$ receives $\phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ from the stage 3 play. We note that we need not distinguish here between the cases $v(N) > v(N\backslash\{i_n\}) + v(i_n)$ and $v(N) = v(N\backslash\{i_n\}) + v(i_n)$, since the fear of rejection at stage 2, which does not matter in the case of equality, does not play a role even in the case of strict inequality since i_n can recover $v(N) - v(N\backslash\{i_n\})$ at stages 4 and 5. Hence, the payoffs (ignoring the stage 1 bids) resulting from stages $2 - 5$ are $v(N) - v(N\backslash\{i_n\})$ to the proposer i_n and $\phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}})$ to every $j \in N\backslash\{i_n\}$.

One can readily see that the proposed strategies constitute SPE. For all the candidate SPE the final payoffs to the proposer i_n and every other player $j \neq i_n$ are $v(N) - v(N\backslash\{i_n\}) - \sum_{j\in N\backslash\{i_n\}} b^{i_n}_j$ and $\phi_j(N\backslash\{i_n\}, v|_{N\backslash\{i_n\}}) + b^{i_n}_j$, respectively. This implies that the proposer has no incentive to increase any offer. If the new offers were accepted it will diminish the proposer's payoff; if they were rejected it will leave the proposer's payoff unchanged. Decreasing an offer, which leads to rejection will again not change the payoff to the proposer. Moreover, we like to note that in the case where $v(N) > v(N\backslash\{i_n\}) + v(i_n)$ it cannot be part of SPE that an offer is rejected at stage 3 and, furthermore, the offer made at stage 4 is also rejected. If this were to happen, the player who made an offer at stage 4 can obtain, due to zero-monotonicity, a better outcome by making instead an offer that must be accepted.

Claim (d). In any SPE, $B^i = B^j$ for all $i, j \in N$, and hence $B^i = 0$ for all $i \in N$.

Denote $\Omega = \{i \in N | B^i = \max_{j\in N}(B^j)\}$. If $\Omega = N$ the claim is satisfied since $\sum_{i\in N} B^i = 0$. Otherwise, we can show that any player i in Ω has the incentive to change her bids so as to decrease the sum of payments in case she wins. Furthermore, these changes can be made without altering the set Ω. Hence, the player maintains the same probability of winning and obtains a higher expected payoff. Take some player $j \notin \Omega$. Let player $i \in \Omega$ change her strategy by announcing ${b'}_k^i = b_k^i + \epsilon$ for all $k \in \Omega \backslash \{i\}$, and ${b'}_j^i = b_j^i - |\Omega|\epsilon$ for j, and $b'^i_l = b^i_l$ for all $l \notin \Omega \cup \{j\}$. Then, the new net bids are $B'^i = B^i - \epsilon$, $B'^k = B^k - \epsilon$ for all $k \in \Omega \backslash \{i\}$, $B'^j = B^j + |\Omega|\epsilon$ and $B'^l = B^l$ for all $l \notin \Omega \cup \{j\}$. If ϵ is small enough so that $B^j + |\Omega|\epsilon < B^i - \epsilon$, then $B'^l < B'^i = B'^k$ for all $l \notin \Omega$ (including j) and for all $k \in \Omega$. Therefore, Ω does not change. However, $\sum_{h \in N \backslash \{i\}} b^i_h - \epsilon < \sum_{h \in N \backslash \{i\}} b^i_h$.

Claim (e). In any SPE, each player's final payoff is the same regardless of whom is chosen as the proposer.

This claim can be readily proved by contradiction. If some player can get extra payoff given a specific identity of the proposer, then this player will have incentive to adjust her bids accordingly, which contradicts Claim (d).

Claim (f) In any SPE, the final payment received by each of the players coincides with each player's Shapley value.

We know that if player i is the proposer, her final payoff will be $v(N) - v(N \backslash \{i\}) - \sum_{j \in N \backslash \{i\}} b^i_j$. In case of player $j \neq i$ becoming the proposer, player i's final payoff will be $\phi_i(N \backslash \{j\}, v|_{N \backslash \{j\}}) + b^j_i$. Then, the sum of payoffs to player i over all possible choices of the proposer is (note that all net bids are zero)

$$\begin{aligned} & v(N) - v(N \backslash \{i\}) - \sum_{j \in N \backslash \{i\}} b^i_j + \sum_{j \in N \backslash \{i\}} \left(\phi_i(N \backslash \{j\}, v|_{N \backslash \{j\}}) + b^j_i \right) \\ & = v(N) - v(N \backslash \{i\}) + \sum_{j \in N \backslash \{i\}} \phi_i(N \backslash \{j\}, v|_{N \backslash \{j\}}) \\ & = n \cdot \phi_i(N, v). \end{aligned}$$

Since the payoffs are the same regardless of who is the proposer (by Claim (e)), we see that the payoff of each player in any equilibrium must coincide with her Shapley value. □

In order to arrive at the Shapley value the proposer chosen through bidding at stage 1 has the power to make another offer, following the rejection of her initial offer, before the conclusion of the game. An equally plausible scenario is that the proposer chosen at stage 1 forfeits the right to make another offer once rejected. It is the proposer chosen in the following stage who has the right to make a second offer before the game ends. Hence we have a new generalized bidding mechanism, described in what follows, which is shown to implement the equal surplus value.

Mechanism A2 The mechanism is identical structure-wise to Mechanism A1. Stages 1, 2 and 3 are in effect the same as in Mechanism A1. Below we will mainly describe stages 4 and 5 where the difference from Mechanism A1 lies in. The mechanism starts with $S = N$.

Stages 1, 2, and 3: Same as in Mechanism A1.
If the offer made at stage 3 is rejected, all players in S other than i_s proceed again through stages 1 to 3 where the set of active players is $S\backslash\{i_s\}$.[6] Meanwhile, player i_s waits for the negotiation outcome of $S\backslash\{i_s\}$. Dependent upon whether or not the player i_{s-1}, the proposer of $S\backslash\{i_s\}$, can make his offer be accepted within $S\backslash\{i_s\}$, player i_s will either be called for renegotiation with i_{s-1} or be left alone. The renegotiation will follow the rules as specified below in stages 4 and 5 of the current mechanism.
If the offer is accepted, we have to distinguish between two cases where $S = N$ and $S \neq N$. In the case where $S = N$, the game ends as in Mechanism A1. In the case where $S \neq N$, stages 4 and 5 are reached.
Stage 4: Proposer i_s makes an offer $\widetilde{x}^{i_s}_{i_{s+1}}$ in $\mathbb{R}$ to the previously rejected proposer i_{s+1}. (The offer is to let i_s form the coalition $S \cup \{i_{s+1}\}$.)
Stage 5: Player i_{s+1} accepts or rejects the offer and the game ENDS.
The payoffs to all the players who proposed before i_{s+1} and to all the players in $S\backslash\{i_s\}$ are the same independent of whether there was a rejection or an acceptance at stage 5, and are identical to the payoffs in Mechanism A1.
The payoffs to the players i_s and i_{s+1} depend on whether or not there was an acceptance at stage 5.
If the offer at stage 5 is accepted then at this stage player i_s receives $v(S \cup \{i_{s+1}\}) - \widetilde{x}^{i_s}_{i_{s+1}}$ minus the bids and offers she made to the players in S, while player i_{s+1} receives $\widetilde{x}^{i_s}_{i_{s+1}}$. The overall payoffs to these two players are given by adding to these amounts the sum of bids received and made in all the preceding stages, respectively. Hence, the final payoff to player i_s is $v(S \cup \{i_{s+1}\}) - \widetilde{x}^{i_s}_{i_{s+1}} - \sum_{j\in S\backslash\{i_s\}} b^{i_s}_j - \sum_{j\in S\backslash\{i_s\}} x^{i_s}_j + \sum_{t=s+1}^n b^{i_t}_{i_s}$, and the final payoff to player i_{s+1} is $\widetilde{x}^{i_s}_{i_{s+1}} - \sum_{j\in S} b^{i_{s+1}}_j + \sum_{t=s+2}^n b^{i_t}_{i_{s+1}}$.
If the offer at Stage 5 is rejected then at this stage player i_s receives $v(S)$ minus the bids and the offers she made to the players in S, while player i_{s+1} receives his stand-alone payoff $v(i_{s+1})$. The overall payoff to these two players are given by adding to these amounts the sum of bids received and made in all the preceding stages, respectively. Hence, the final payoff to player i_s is $v(S) - \sum_{j\in S\backslash\{i_s\}} b^{i_s}_j - \sum_{j\in S\backslash\{i_s\}} x^{i_s}_j + \sum_{t=s+1}^n b^{i_t}_{i_s}$, and the final payoff to player i_{s+1} is $v(i_{s+1}) - \sum_{j\in S} b^{i_{s+1}}_j + \sum_{t=s+2}^n b^{i_t}_{i_{s+1}}$.

Theorem 3.2 *Mechanism A2 implements the equal surplus value of a zero-monotonic game (N, v) in SPE.*

Proof The proof is similar to that of Theorem 3.1. The differences are in the construction of the SPE strategies and in Claim (f). Hence, we explicitly construct an SPE that yields the equal surplus value as an SPE outcome and show that the counterpart of Claim (f) (that payoffs must coincide with the equal surplus value) holds as well.

To construct an SPE, consider the following strategies, which the players would follow in any game they participate in (we describe it for the whole set of players, N,

[6] In this mechanism, players in $S\backslash\{i_s\}$ actually compete for becoming the proposer i_{s-1} so as to win the right of making offer $\widetilde{x}^{i_{s-1}}_{i_s}$ to i_s at stage 4. Hence, in equilibrium, the amount of payoff that players in $S\backslash\{i_s\}$ bargain for equals $v(S) - v(i_s)$.

but these are also the strategies followed by any player in a subset S that is called upon to play the game, with S replacing N):

At stage 1, each player $i \in N$, announces $b^i_j = \phi^{es}_j(N, v) - \phi^{es}_j(N\backslash\{i\}, v^{-i})$, for every $j \in N\backslash\{i\}$. Note that the game $(N\backslash\{i\}, v^{-i})$ is defined by $v^{-i}(N\backslash\{i\}) = v(N) - v(i)$ and $v^{-i}(S) = v(S)$, for any $S \subset N\backslash\{i\}$.
At stage 2, a proposer, player i_n, offers $x^{i_n}_j = \phi^{es}_j(N\backslash\{i_n\}, v^{-i_n})$ to every $j \in N\backslash\{i_n\}$.
At stage 3, any player $j \in N\backslash\{i_n\}$ accepts any offer which is greater than or equal to $\phi^{es}_j(N\backslash\{i_n\}, v^{-i_n})$ and rejects any offer strictly less than $\phi^{es}_j(N\backslash\{i_n\}, v^{-i_n})$.
At stage 4, a proposer within $N\backslash\{i_n\}$, player i_{n-1} makes an offer $\widetilde{x}^{i_{n-1}}_{i_n} = v(i_n)$ to i_n.
At stage 5, player i_n, the rejected and "waiting" proposer for the set of players N, accepts any offer greater than or equal to $v(i_n)$ and rejects any offer strictly less than it.

One can readily verify that these strategies yield the equal surplus value for any player and constitute an SPE. Note that the induction assumption used here is basically the same as that in Theorem 1: whenever the mechanism is used by n players with a given characteristic function (satisfying zero-monotonicity) it implements the equal surplus value corresponding to this characteristic function. Following a rejection at stage 3, all other players will eventually get $v(N) - v(i_n)$, rather than $v(N\backslash\{i_n\})$, as this entity is what the chosen proposer among them will obtain at stages 4 and 5 by making the appropriate renegotiation offer to i_n. Hence, the corresponding subgame is equivalent to a game $(N\backslash\{i_n\}, v^{-i_n})$ with player set $N\backslash\{i_n\}$ bargaining over $v(N) - v(i_n)$ whereas all other coalitional values are unchanged: $v^{-i_n}(S) = v(S)$ for all $S \subset N\backslash\{i_n\}$. Then, the induction hypothesis implies that each player $j \in N\backslash\{i_n\}$ gets the equal surplus value, $v(j) + \frac{v(N)-v(i_n)-\sum_{k\in N\backslash\{i_n\}} v(k)}{n-1}$, as the outcome of this game.

To show that in any SPE the final payment received by each of the players coincides with each player's equal surplus value, we note that if i is the proposer, her final payoff will be $v(N) - (v(N) - v(i)) - \sum_{j\neq i} b^i_j$, whereas if $j \neq i$ is the proposer, i will get final payoff $\phi^{es}_i(N\backslash\{j\}, v^{-j}) + b^j_i = (v(i) + \frac{v(N)-v(j)-\sum_{k\neq j} v(k)}{n-1}) + b^j_i$. Hence the sum of the payoffs to player i over all possible choices is (recall that all net bids are zero)

$$\begin{aligned} &v(N) - (v(N) - v(i)) - \sum_{j\neq i} b^i_j + \sum_{j\neq i}\left(v(i) + \frac{v(N) - v(j) - \sum_{k\neq j} v(k)}{n-1} + b^j_i\right) \\ &= nv(i) + \left(v(N) - \sum_{l\in N} v(l)\right) \\ &= n \cdot \phi^{es}_i(N, v). \end{aligned}$$

Since the payoffs are the same regardless of who is the proposer (by the same reason as discussed in Claim (e) of the proof for Theorem 3.1) we see that the payoff of each player in any equilibrium must coincide with the equal surplus value. □

The fact that Mechanism A2 implements the equal surplus value is quite surprising since in this mechanism the payoffs to all sub-coalitions play a role. Moreover, the change in the roles of the rejected proposer and the current proposer between mechanisms A1 and A2 led to a striking difference in the equilibrium outcome from the Shapley to the equal surplus value. To gain some intuition for the reason to this change,[7] we offer the following observation. The main difference between the two values is that the Shapley value assigns zero payoff to a null-player, whereas the equal surplus value does not. This is reflected in the fact that in Mechanism A1, a null player can by no means obtain a positive payoff no matter whether he is a proposer or a non-proposer who chooses to reject an offer. On the other hand in Mechanism A2 a null player can obtain the role of a proposer when rejecting the first chosen proposer, in that capacity he may be able to extract some surplus from the previously rejected proposer. More precisely, the null player can extract the marginal contribution of a non-null player by rejecting the latter's unfavorable offer and then offering him the stand-alone payoff in renegotiation. This implies that the offer made to the null-player should be strictly positive. Thus, a null player might obtain a strictly positive payoff in Mechanism A2.

One might, of course, consider a more "direct" mechanism that simply requires the game to breakdown so that each player obtains her stand-alone value in case of the rejection of the offer made by the proposer. This will also implement the equal surplus value. However, this result seems to be "dictated" by the rules of the mechanism by focusing only on the grand coalition and the singletons. The use of Mechanism A2 allows us to compare the implementation of the equal surplus value to that of other values, within the same framework where all sub-coalitions are involved, and shows it hinges upon leaving much less power at the hands of the "first" proposer chosen by the bidding.

The Shapley and equal surplus values resulted from a "zero-one" decision, either the first stage proposer or the subsequently chosen proposer have the right to make a second offer. It is also of interest to know what happens if the power to make a second offer is somehow shared between the two. One could randomize giving each an equal probability to have the right to make another offer. Alternately the two could bargain via a Rubinstein alternating offer game (Rubinstein 1982). We adopt again a bidding approach letting the two bid for the right to make a further offer. The mechanism is formally described in what follows and is shown to implement the consensus value.

Mechanism A3 The rules of stages 1, 2 and 3 are the same as before. Below we will mainly describe stages 4 and 5 where the difference from mechanisms A1 and A2 lies in. The mechanism starts with $S = N$.

Stages 1, 2, and 3: Same as in Mechanism A1.
Here we like to note that if the offer made at stage 3 is rejected, all players in S other than i_s proceed again through stages 1 to 3 where the set of active players is $S\backslash\{i_s\}$. In the current mechanism, players in $S\backslash\{i_s\}$ actually compete for becoming the proposer i_{s-1} so as to win the right of renegotiating with i_s. The renegotiation

[7] We are grateful to Andreu Mas-Colell for an insightful discussion and comments regarding this issue.

between i_{s-1} and i_s is in fact a 2-player bidding game. That is, both of them simultaneously make bids at stage 4 and the winner will have the right to make a new offer to the other player at stage 5 while the other player accepts or rejects the offer. Hence, in equilibrium, the amount of payoff that players in $S\backslash\{i_s\}$ bargain for equals $\frac{1}{2}\left(v(S\backslash\{i_s\}) + (v(S) - v(i_s))\right)$.

Stage 4: Players i_s and i_{s+1} bid for the right to take the role of the proposer (hence, the game played, in fact, coincides with the stage 1 game for 2 players). Players i_s and i_{s+1} simultaneously submit bids $\widetilde{b}^{i_s}_{i_{s+1}}$ and $\widetilde{b}^{i_{s+1}}_{i_s}$ in $\mathbb{R}$. The player with the larger net bid pays the bid to the other player and assumes the role of the proposer. In case of identical bids the proposer is chosen randomly.

Stage 5: Depending on whether the proposer is i_{s+1} or i_s, the game proceeds as in Mechanism A1 (when i_{s+1} is the proposer) or Mechanism A2 (when i_s is the proposer).

The payoffs to all the players who proposed before i_{s+1} and to all the players in $S\backslash\{i_s\}$ are the same independent of whether there was a rejection or an acceptance at stage 5, and are the same as in mechanism A1.

In the case where the proposer is i_{s+1}, in stage 5, the payoffs of i_s and i_{s+1} are derived by adding to the payoffs in Mechanism A1 the bid from stage 4. Hence, if the offer at stage 5 is accepted, the final payoff to player i_s is $\widetilde{b}^{i_{s+1}}_{i_s} + \widetilde{x}^{i_{s+1}}_{i_s} - \sum_{j\in S\backslash\{i_s\}} b^{i_s}_j - \sum_{j\in S\backslash\{i_s\}} x^{i_s}_j + \sum_{t=s+1}^{n} b^{i_t}_{i_s}$, and the final payoff to player i_{s+1} is $v(S\cup\{i_{s+1}\}) - \widetilde{b}^{i_{s+1}}_{i_s} - \widetilde{x}^{i_{s+1}}_{i_s} - \sum_{j\in S} b^{i_{s+1}}_j + \sum_{t=s+2}^{n} b^{i_t}_{i_{s+1}}$. If the offer at stage 5 is rejected, the final payoff to player i_s is $\widetilde{b}^{i_{s+1}}_{i_s} + v(S) - \sum_{j\in S\backslash\{i_s\}} b^{i_s}_j - \sum_{j\in S\backslash\{i_s\}} x^{i_s}_j + \sum_{t=s+1}^{n} b^{i_t}_{i_s}$, and the final payoff to player i_{s+1} is $v(i_{s+1}) - \widetilde{b}^{i_{s+1}}_{i_s} - \sum_{j\in S} b^{i_{s+1}}_j + \sum_{t=s+2}^{n} b^{i_t}_{i_{s+1}}$.

In the case where the proposer is i_s, in stage 5, the payoffs of i_s and i_{s+1} are derived by adding to the payoffs in Mechanism A2 the bid from stage 4. Hence, if the offer at stage 5 is accepted, the final payoff to player i_s is $v(S\cup\{i_{s+1}\}) - \widetilde{b}^{i_s}_{i_{s+1}} - \widetilde{x}^{i_s}_{i_{s+1}} - \sum_{j\in S\backslash\{i_s\}} b^{i_s}_j - \sum_{j\in S\backslash\{i_s\}} x^{i_s}_j + \sum_{t=s+1}^{n} b^{i_t}_{i_s}$, and the final payoff to player i_{s+1} is $\widetilde{b}^{i_s}_{i_{s+1}} + \widetilde{x}^{i_s}_{i_{s+1}} - \sum_{j\in S} b^{i_{s+1}}_j + \sum_{t=s+2}^{n} b^{i_t}_{i_{s+1}}$. If the offer at stage 5 is rejected, the final payoff to player i_s is $v(S) - \widetilde{b}^{i_s}_{i_{s+1}} - \sum_{j\in S\backslash\{i_s\}} b^{i_s}_j - \sum_{j\in S\backslash\{i_s\}} x^{i_s}_j + \sum_{t=s+1}^{n} b^{i_t}_{i_s}$, and the final payoff to player i_{s+1} is $v(i_{s+1}) + \widetilde{b}^{i_s}_{i_{s+1}} - \sum_{j\in S} b^{i_{s+1}}_j + \sum_{t=s+2}^{n} b^{i_t}_{i_{s+1}}$.

Theorem 3.3 *Mechanism A3 implements the consensus value of a zero-monotonic game (N, v) in SPE.*

Proof The proof is again similar to that of Theorem 3.1. The differences are once more in the construction of the SPE strategies and in Claim (f). Hence, we explicitly construct an SPE that yields the consensus value and show that Claim (f) (that payoffs must coincide with the consensus value) also holds.

To construct an SPE yielding the consensus value consider the following strategies, which the players would follow in any game they participate in (we describe it for the whole set of players, N, but these are also the strategies followed by any player in a subset S that is called upon to play the game, with S replacing N):

At stage 1, each player $i \in N$ announces $b^i_j = \psi_j(N, v) - \psi_j(N\backslash\{i\}, \widehat{v}^{-i})$, for every $j \in N\backslash\{i\}$. Note that the game $(N\backslash\{i\}, \widehat{v}^{-i})$ is defined by $\widehat{v}^{-i}(N\backslash\{i\}) = v(N\backslash\{i\}) + \frac{v(N)-v(N\backslash\{i\})-v(i)}{2}$ and $\widehat{v}^{-i}(S) = v(S)$, for all $S \subset N\backslash\{i\}$.
At stage 2, a proposer, player i_n, offers $x^{i_n}_j = \psi_j(N\backslash\{i_n\}, \widehat{v}^{-i_n})$ to every $j \in N\backslash\{i_n\}$.
At stage 3, any player $j \in N\backslash\{i_n\}$ accepts any offer which is greater than or equal to $\psi_j(N\backslash\{i_n\}, \widehat{v}^{-i_n})$ and rejects any offer strictly less than $\psi_j(N\backslash\{i_n\}, \widehat{v}^{-i_n})$.
At stage 4, player i_n announces $\widetilde{b}^{i_n}_{i_{n-1}} = v(N\backslash\{i_n\}) + \frac{v(N)-v(N\backslash\{i_n\})-v(i_n)}{2} - v(N\backslash\{i_n\}) = \frac{v(N)-v(N\backslash\{i_n\})-v(i_n)}{2}$ while player i_{n-1} announces $\widetilde{b}^{i_{n-1}}_{i_n} = v(i_n) + \frac{v(N)-v(i_n)-v(N\backslash\{i_n\})}{2} - v(i_n) = \frac{v(N)-v(i_n)-v(N\backslash\{i_n\})}{2}$.
At stage 5, player i_n makes an offer $\widetilde{x}^{i_n}_{i_{n-1}} = v(N\backslash\{i_n\})$ to i_{n-1} and player i_{n-1} makes an offer $\widetilde{x}^{i_{n-1}}_{i_n} = v(i_n)$ to i_n. Moreover, i_n accepts any offer greater than or equal to $v(i_n)$ and rejects any offer strictly less than it. Similarly, i_{n-1} accepts any offer greater than or equal to $v(N\backslash\{i_n\})$ and rejects any offer strictly less than it.

One can readily verify that these strategies yield the consensus value for any player and constitute an SPE. Similar to the previous two theorems, the induction assumption used here is that whenever the mechanism is used by n players with a given characteristic function (satisfying zero-monotonicity) it implements the consensus value corresponding to this characteristic function. In order to apply the induction hypothesis, we observe that following the rejection at stage 3, all other players will get $v(N\backslash\{i_n\}) + \frac{v(N)-v(N\backslash\{i_n\})-v(i_n)}{2}$ at stages 4 and 5 because the two parties have equal power of making the renegotiation offer. Hence, the corresponding subgame is equivalent to a game $(N\backslash\{i_n\}, \widehat{v}^{-i_n})$ defined by $\widehat{v}^{-i_n}(N\backslash\{i_n\}) = v(N\backslash\{i_n\}) + \frac{v(N)-v(N\backslash\{i_n\})-v(i_n)}{2}$ and $\widehat{v}^{-i_n}(S) = v(S)$, for all $S \subset N\backslash\{i_n\}$. Then, the induction hypothesis implies that each player $j \in N\backslash\{i_n\}$ gets the consensus value of the game $\widehat{v}^{-i_n}$.

To show that in any SPE each player's final payoff coincides with her consensus value, we note that if i is the proposer her final payoff is given by $v(N) - (v(N\backslash\{i\}) + \frac{v(N)-v(N\backslash\{i\})-v(i)}{2}) - \sum_{j\neq i} b^i_j$ whereas if $j \neq i$ is the proposer, the final payoff of i is $\psi_i(N\backslash\{j\}, \widehat{v}^{-j}) + b^j_i$. Hence the sum of payoffs to player i over all possible choices of the proposer is (note that all net bids are zero)

$$\begin{aligned}
& v(N) - \left(v(N\backslash\{i\}) + \frac{v(N)-v(N\backslash\{i\})-v(i)}{2}\right) \\
& \quad - \sum_{j\neq i} b^i_j + \sum_{j\neq i}\left(\psi_i(N\backslash\{j\}, \widehat{v}^{-j}) + b^j_i\right) \\
& = \frac{v(N)-v(N\backslash\{i\})+v(i)}{2} + \sum_{j\neq i}\left(\frac{1}{2}\phi_i(N\backslash\{j\}, \widehat{v}^{-j}) + \frac{1}{2}\phi^{es}_i(N\backslash\{j\}, \widehat{v}^{-j})\right) \\
& = \frac{v(N)-v(N\backslash\{i\})+v(i)}{2} \\
& \quad + \frac{1}{2}\sum_{j\neq i}\left(\phi_i(N\backslash\{j\}, v|_{N\backslash\{j\}}) + \frac{\frac{v(N)-v(N\backslash\{j\})-v(j)}{2}}{n-1}\right)
\end{aligned}$$

$$+\frac{1}{2}\sum_{j\neq i}\left(v(i)+\frac{\frac{v(N)+v(N\backslash\{j\})-v(j)}{2}-\sum_{k\in N\backslash\{j\}}v(k)}{n-1}\right)$$
$$=\frac{1}{2}\left(v(N)-v(N\backslash\{i\})+\sum_{j\neq i}\phi_i(N\backslash\{j\},v|_{N\backslash\{j\}})\right)$$
$$+\frac{1}{2}\left(nv(i)+\left(v(N)-\sum_{l\in N}v(l)\right)\right)$$
$$=n\left(\frac{1}{2}\phi_i(N,v)+\frac{1}{2}\phi_i^{es}(N,v)\right)$$
$$=n\psi_i(N,v).$$

Since the payoffs are the same regardless of who is the proposer, the payoff of each player in any equilibrium must coincide with the consensus value. □

As discussed earlier Mechanism A3 requires both proposers to compete for the right to make a further proposal and *a priori* both have equal power. However, what happens if the mechanism treats the players asymmetrically: bids made by one player are "worth more" than those made by the other. Such a mechanism implements the α-consensus value (cf. Ju et al. 2007) of a zero-monotonic game in SPE. We also note that the bidding stage for the right to make a further proposal can be replaced by a random stage where the rejected proposer has probability α of making the further offer while the other proposer has probability $1-\alpha$ to propose, which will also implement the α-consensus value.[8]

The mechanisms constructed can be adapted in several ways. One option is to vary the treatment of a proposer in case she makes an offer that is rejected. We could make it less attractive to make an offer that is rejected, steering the players to end the game sooner rather than later. In the mechanisms to implement the Shapley value, the new rule would allow for any arbitrary payoff $\theta^{i_s}\leq v(i_s)$ to be given to the proposer i_s at stage 5 in case no agreement is reached, whereas the rest of the players still obtain $v(S\backslash\{i_s\})$ if coalition $S\backslash\{i_s\}$ forms. The difference between $v(i_s)$ and θ^{i_s} may be interpreted as a punishment. This mechanism would encourage the players to make acceptable offers and lead to larger coalitions similar to Moldovanu and Winter (1994) where it is stated that "we assume that each player prefers to be a member of large coalitions rather than smaller ones provided that he earns the same payoff in the two agreements" and Hart and Mas-Colell (1996) "both proposers and respondents break ties in favor of quick termination of the game".

The extreme case is where the proposer receives zero in case an offer is rejected and stages 4 and 5 are the same as in Mechanism A2. This mechanism implements the egalitarian solution.[9] Moreover, one can implement any convex combination of

[8] We thank a referee for suggesting this random version. This idea is further applied in Sect. 4.

[9] For a TU game (N,v), the egalitarian solution, denoted by ϕ^{eg}, is defined by $\phi_i^{eg}(N,v)=\frac{v(N)}{n}$ for all $i\in N$.

the egalitarian solution and the Shapley value using a construction similar to that implementing the α-consensus value.

Moreover, we like to stress that one can further generalize the renegotiation idea to design alternative mechanisms to implement these values. One can observe that in the current mechanisms described above, following a rejection of the offer made by i_s, although i_{s-1} can renegotiate with i_s after the offer made by i_{s-1} was immediately accepted within $S\backslash\{i_s\}$, neither i_{s-1} nor i_s can renegotiate with the previously rejected proposer i_{s+1}. Then, one may think about another scenario: so long as a coalition can be formed either through immediate acceptance of the offer or via renegotiation, the corresponding proposer or the one making renegotiation acceptable becomes the representative of the coalition. This representative player can then "move on" to renegotiate with the previously rejected proposer. Once an agreement is made in the renegotiation, the new coalition is formed and the (possibly new) representative player can then go on renegotiating in the same fashion until either the grand coalition is formed or a renegotiation fails. Dependent upon which player makes an offer in renegotiation, various mechanisms parallel to those described in the paper can be constructed and will implement the corresponding solution concepts as well. This further shows that our implementation results are robust with respect to changes in the renegotiation protocol.

The addition of the renegotiation stage provides the players, bargaining over the division of the benefits, with more options. Hence, these bidding mechanisms might be more attractive to participants than simple take-it-or-leave-it mechanisms. While the addition of one stage of renegotiation makes a great difference, adding more stages is redundant in that it leads basically to the same results.

4 Implementation in "verifiable" environments

The literature of non-cooperative implementation of cooperative solutions generally imposes technical assumptions on the game environments in order to provide incentives to players to behave in a desirable way, e.g. monotonicity being adopted in Hart and Mas-Colell (1996) and zero-monotonicity in Pérez-Castrillo and Wettstein (2001). The mechanisms discussed in the above section follow the same spirit and require the corresponding games are zero-monotonic. Then, it is natural to ask: Can we design non-cooperative games to generate these solutions as equilibrium outcomes without imposing any technical condition on the games?

To attain this target, we introduce a different informational structure, similar to that Serrano (1995) used when implementing the nucleolus (Schmeidler 1969) for bankruptcy problems. Such an informational structure is also adopted in Dagan et al. (1997) to implement consistent and monotonic bankruptcy rules. In the previous section the players were fully informed as regards the characteristic function v, whereas the "designer" of the mechanism had no knowledge of what different coalitions can achieve. In this section, the different informational structure requires that the players in addition to being fully informed with respect to the characteristic function, can also, *if necessary*, prove what each coalition of players can obtain. Put differently, the value of each coalition cannot only be observed but also verified by an outside authority *if needed*. One such conceivable scenario is where a set of players (heirs), $N = \{1, \ldots, n\}$, have to divide a sum (estate) of known size, $v(N)$. Furthermore, each subset of the players can prove what part of the sum they are entitled to (have documented claims regarding their part of the estate).[10] This informational structure and the relevant strategic analysis of the claim problems have been discussed and surveyed by Thomson (2003). For a more recent study, we refer to García-Jurado et al. (2006).

Introducing such an informational structure serves two purposes. The first is that it provides a much wider scope for applying our mechanisms and shows how they can be easily adapted to handle versatile environments. The second is that as more information is made potentially available, the solution concepts we discussed can be implemented without imposing any further conditions, such as zero-monotonicity, on the environment.

The bidding mechanism we now construct, can be informally described as follows: Stages 1, 2 and 3 are the same as in previous mechanisms up to the point where an offer is rejected at stage 3. In case of rejection all the players other than the proposer play a similar game with one player less. The mechanism can implement different solution concepts by introducing a parameter that will affect the size of the pie to be shared within this reduced game (and of course, the effect will be carried on to all the following games if the corresponding offers are rejected).

While it is sufficient to provide a parameterized mechanism to implement any α-consensus value that is a convex combination of the Shapley value and equal surplus value, below we explain the three focal cases to highlight the motivation and justification.

In the first case, yielding the Shapley value, the players other than the rejected proposer i_n bargain over their prescribed coalitional payoff $v(N\backslash\{i_n\})$, and the rejected proposer receives what remains in $v(N)$, i.e. $v(N) - v(N\backslash\{i_n\})$.

As one can see, the key feature of the first case is that it specifies a rule such that in case of the offer made by i_n being rejected, the rest of the players are guaranteed to bargain over $v(N\backslash\{i_n\})$, which is the payoff they can achieve without i_n, whereas i_n receives what remains in $v(N)$. If such a rule can be justified, then an opposite choice can be supported as well, which results in the second case: In return to the highest net bid made by proposer i_n, she should be guaranteed with her stand-alone payoff $v(i_n)$ in case of the offer rejected so that the remaining players bargain over the residual, i.e. $v(N) - v(i_n)$.

The third case takes, as before, a less extreme approach and shares the benefits generated by the grand coalition between the rejected proposer and the other players. Once an offer is rejected, we move from the status-quo outcome where proposer i_n gets $v(i_n)$ and the remaining players bargain over $v(N\backslash\{i_n\})$ to a new starting point where the rejected proposer receives $v(i_n) + \frac{1}{2}\left(v(N) - v(i_n) - v(N\backslash\{i_n\})\right)$, and the remaining players bargain over $v(N\backslash\{i_n\}) + \frac{1}{2}\left(v(N) - v(i_n) - v(N\backslash\{i_n\})\right)$. Hence each obtains half of the surplus generated by the grand coalition.

[10] If we define the coalitional claims simply as the sum of the corresponding individual claims, then we obtain the conventional claim problems, also known as bankruptcy problems.

Below we formally describe the bidding game (parameterized by α), focusing only on the rules in case where the offer made by a proposer (chosen in the bidding stage) has been rejected.

Mechanism B If there is only one player $\{i\}$, she simply receives $v(i)$. When the player set $N = \{1, \ldots, n\}$ consists of two or more players, the mechanism is defined recursively, and starts with the set of active players $S = N$. Goto stage 1.

Stages 1 and 2: Same as in Mechanism A1.
Stage 3: The players in S other than i_s, sequentially, either accept or reject the offer. If at least one player rejects it, then the offer is rejected. Otherwise, the offer is accepted. The whole game ENDS whenever an offer made by a proposer is accepted. If the offer made at stage 3 is rejected, all players in S other than i_s proceed again through the mechanism from stage 1 to stage 3 with the set of active players being $S\backslash\{i_s\}$ and bargain over $v^\alpha(S\backslash\{i_s\}) = v(S\backslash\{i_s\}) + (1-\alpha)(v^\alpha(S) - v(S\backslash\{i_s\}) - v(i_s))$, where $v^\alpha(N) = v(N)$ and $\alpha \in [0, 1]$. As the rejected proposer, player i_s leaves the game with $v(i_s) + \alpha(v^\alpha(S) - v(S\backslash\{i_s\}) - v(i_s))$. Hence, the final payoff of i_s is $v(i_s) + \alpha(v^\alpha(S) - v(S\backslash\{i_s\}) - v(i_s)) - \sum_{j\in S\backslash\{i_s\}} b_j^{i_s} + \sum_{t=s+1}^n b_{i_s}^{i_t}$.
If the offer is accepted, we have to distinguish between two cases where $S = N$ and $S \neq N$. In the case where $S = N$, which means that all players agree with the proposer on the scheme of sharing $v(N)$, the game ENDS. Then, at this stage, each player $j \in N\backslash\{i_n\}$ receives $x_j^{i_n}$, and player i_n receives $v(N) - \sum_{j\in N\backslash\{i_n\}} x_j^{i_n}$. Hence, the final payoff to player $j \neq i_n$ is $x_j^{i_n} + b_j^{i_n}$ while player i_n receives $v(N) - \sum_{j\in N\backslash\{i_n\}} x_j^{i_n} - \sum_{j\in N\backslash\{i_n\}} b_j^{i_n}$. In the case where $S \neq N$, the game also ENDS while the final payoff of the proposer i_s is $v^\alpha(S) - \sum_{j\in S\backslash\{i_s\}} x_j^{i_s} - \sum_{j\in S\backslash\{i_s\}} b_j^{i_s} + \sum_{t=s+1}^n b_{i_s}^{i_t}$, and the final payoff to player $j \in S\backslash\{i_s\}$ is $x_j^{i_s} + \sum_{t=s}^n b_j^{i_t}$.

Note that in case the mechanism reaches the situation where the set of active players consists of one player only, the corresponding stages 1 to 3 are redundant and this single player becomes the proposer for herself whose offer is accepted and the game ends.

Theorem 4.1 *Mechanism B implements the α-consensus value ψ^α of an arbitrary cooperative game (N, v) in SPE.*

Proof Since the proof follows the same line as that of Theorem 3.1, we will skip most of it and stress just two aspects to illustrate the way the proof proceeds. First, to construct an SPE yielding the α-consensus value, consider the following strategies, which the players would follow in any game they participate in (we describe it for the whole set of players, N, but these are also the strategies followed by any player in a subset S that is called upon to play the game, with S replacing N):

At stage 1, each player $i \in N$ announces $b_j^i = \psi_j^\alpha(N, v) - \psi_j^\alpha(N\backslash\{i\}, v^\alpha|_{-i})$, for every $j \in N\backslash\{i\}$. Note that the game $(N\backslash\{i\}, v^\alpha|_{-i})$ is defined by $v^\alpha|_{-i}(N\backslash\{i\}) = v(N\backslash\{i\}) + (1-\alpha)(v(N) - v(N\backslash\{i\}) - v(i))$ and $v^\alpha|_{-i}(S) = v(S)$, for all $S \subset N\backslash\{i\}$.
At stage 2, a proposer, player i_n, offers $x_j^{i_n} = \psi_j^\alpha(N\backslash\{i\}, v^\alpha|_{-i_n})$ to every $j \in N\backslash\{i_n\}$.

At stage 3, any player $j \in N\backslash\{i_n\}$ accepts any offer which is greater than or equal to $\psi_j^\alpha(N\backslash\{i_n\}, v^\alpha|_{-i_n})$ and rejects any offer strictly less than $\psi_j^\alpha(N\backslash\{i_n\}, v^\alpha|_{-i_n})$.

Second, we explicitly provide Claim (c) below to describe the full set of SPE and show that the Claim (f) (that payoffs must coincide with the α-consensus value) also holds. *Claim (c)*. For the game that starts at stage 2 there exist two types of SPE. One is that at stage 2 player i_n offers $x_j^{i_n} = \psi_j^\alpha(N\backslash\{i_n\}, v^\alpha|_{-i_n})$ to all $j \neq i_n$ and, at stage 3, every player $j \neq i_n$ accepts any offer $x_j^{i_n} \geq \psi_j^\alpha(N\backslash\{i_n\}, v^\alpha|_{-i_n})$ and rejects the offer otherwise. The other is that at stage 2 the proposer offers $x_j^{i_n} \leq \psi_j^\alpha(N\backslash\{i_n\}, v^\alpha|_{-i_n})$ to some players $j \neq i^*$ and, at stage 3, any player $j \in N\backslash\{i^*\}$ rejects any offer $x_j^{i_n} \leq \psi_j^\alpha(N\backslash\{i_n\}, v^\alpha|_{-i_n})$.

To show Claim (f), we note that if i is the proposer her final payoff is given by $v(N) - (v(N\backslash\{i\}) + (1-\alpha)(v(N) - v(N\backslash\{i\}) - v(i))) - \sum_{j\neq i} b_j^i$ whereas if $j \neq i$ is the proposer, the final payoff of i is $\psi_i^\alpha(N\backslash\{j\}, v^\alpha|_{-j}) + b_i^j$. Hence the sum of payoffs to player i over all possible choices of the proposer is (note that all net bids are zero)

$$
\begin{aligned}
&v(N) - (v(N\backslash\{i\}) + (1-\alpha)(v(N) - v(N\backslash\{i\}) - v(i))) - \sum_{j\neq i} b_j^i \\
&\quad + \sum_{j\neq i}\left(\psi_i^\alpha(N\backslash\{j\}, v^\alpha|_{-j}) + b_i^j\right) \\
&= \alpha(v(N) - v(N\backslash\{i\})) + (1-\alpha)v(i) \\
&\quad + \sum_{j\neq i}\left(\alpha\phi_i(N\backslash\{j\}, v^\alpha|_{-j}) + (1-\alpha)\phi_i^{es}(N\backslash\{j\}, v^\alpha|_{-j})\right) \\
&= \alpha(v(N) - v(N\backslash\{i\})) + (1-\alpha)v(i) \\
&\quad + \alpha\sum_{j\neq i}\left(\phi_i(N\backslash\{j\}, v|_{N\backslash\{j\}}) + \frac{(1-\alpha)(v(N) - v(N\backslash\{j\}) - v(j))}{n-1}\right) \\
&\quad + (1-\alpha)\sum_{j\neq i}\left(v(i) + \frac{\alpha v(N\backslash\{j\}) + (1-\alpha)(v(N) - v(j)) - \sum_{k\in N\backslash\{j\}} v(k)}{n-1}\right) \\
&= \alpha\left(v(N) - v(N\backslash\{i\}) + \sum_{j\neq i}\phi_i(N\backslash\{j\}, v|_{N\backslash\{j\}})\right) \\
&\quad + (1-\alpha)\left(nv(i) + \left(v(N) - \sum_{l\in N} v(l)\right)\right) \\
&= n\left(\alpha\phi_i(N, v) + (1-\alpha)\phi_i^{es}(N, v)\right) \\
&= n\psi_i^\alpha(N, v).
\end{aligned}
$$

Since the payoffs are the same regardless of who is the proposer, the payoff of each player in any equilibrium must coincide with the α-consensus value. □

One can readily see that Mechanism B implements the Shapley value, the equal surplus value and the consensus value when α equals 1, 0, and 0.5, respectively.

Note that the relationship between mechanisms and the solution concepts to be implemented is not straightforward. For example, consider a very close variation of Mechanism B as follows. If the offer made by i_n is rejected at stage 3, player i_n leaves the game and receives $v(N) - v(N\backslash\{i_n\})$ from this stage, whereas all other players proceed to play a similar game with player set $N\backslash\{i_n\}$ bargaining over $v(N\backslash\{i_n\})$. So far the rules are the same as in Mechanism B where $\alpha = 1$. However, the difference lies in the game played by $N\backslash\{i_n\}$. Here it requires the next rejected proposer, i_{n-1}, also obtains the marginal contribution to the grand coalition, i.e., $v(N) - v(N\backslash\{i_{n-1}\})$, whereas $N\backslash\{i_n, i_{n-1}\}$ will bargain over $v(N\backslash\{i_n\}) - (v(N) - v(N\backslash\{i_{n-1}\}))$. That is, any future rejected proposer still get his marginal contribution with respect to the grand coalition. One can check that such a mechanism implements the ENSC (egalitarian nonseparable contribution) value (cf. Driessen and Funaki 1991), which is regarded as a simple version of the well known SCRB (separable contributions remaining benefits) method that is widely used in cost allocation in water field resources (cf. Young et al. 1982). Formally, it is defined by

$$\phi^{ensc} = (v(N) - v(N\backslash\{i\})) + \frac{1}{n}\left(v(N) - \sum_{j\in N}(v(N) - v(N\backslash\{j\}))\right)$$

for all $i \in N$.

It is noted that, by suitable modifications, other results in Sect. 3 can be obtained in this environment as well.

5 Conclusion

In this paper we provided a unified framework to implement and study values for transferable utility environments. The main building block is a bidding mechanism that starts by having the players bid for the role of the proposer. The proposer makes an offer to all the remaining players, if the offer is accepted the game ends. In case of rejection the remaining players play the same game again. Once this process ends, the first proposer "re-enters" the game, to play against the proposer ("second proposer") chosen from the remaining players. From here onwards the mechanisms differ. In order to implement the Shapley value the original proposer has the right to make another offer before the game ends. To achieve the equal surplus value the second proposer is awarded that right. The consensus value is implemented when the two proposers bid for the right to make another offer. In effect, any average of the Shapley and equal surplus values can be achieved by suitably adjusting the rules of the mechanism for the two proposers' interaction. These results are valid for any transferable utility game satisfying zero-monotonicity. We also showed that in the case where the payoffs that different coalitions can obtain are verifiable by an outside party, the mechanism can be modified to implement the above solution concepts in any transferable utility environment. The usefulness of the generalized bidding approach is further illustrated by discussing the mechanisms to implement other solution concepts like the egalitarian value and the ENSC value.

The design of a single basic mechanism to implement several cooperative solution concepts serves twin purposes. On one hand it provides a robust non-cooperative foundation for the application of various solutions and on the other hand it makes it possible to examine them critically by the rules needed to implement them. This might provide important insights as the rules of the game are "quite detached" from the axioms generating these values.

There are several possible extensions of the "generalized bidding" approach to other cooperative environments and solution concepts. For games in partition function form, the use of similar mechanisms can complement results obtained by Maskin (2003) and Macho-Stadler et al. (2006) by implementing values proposed by Pham Do and Norde (2007) and Ju (2007). For games with a coalition structure, these mechanisms can serve as an alternative way of implementing the Owen (1977) value which was implemented by Vidal-Puga and Bergantiños (2003) for strictly superadditive games. Recently, Ju and Borm (2008) introduced a new class of games, primeval games, to model inter-individual externalities and analyze compensation rules from a normative point of view. The implementability of these compensation rules via generalized bidding mechanisms is another interesting direction of research. Moreover, with the same bidding design but varying the other details of the bargaining protocols, one can expect alternative equilibrium outcomes, which may result in new cooperative solution concepts.

The class of mechanisms suggested also possesses several features that render them appealing for experimental studies. The bidding stage gives the subjects an added incentive to carefully consider their decisions. They have to weigh the effects of bids they make in determining who will be the proposer and how much they will have to pay. The presence of the renegotiation stage makes it possible to correct previous mistakes and incorporate insights obtained in previous stages.

These features not present in previous mechanisms also serve to enhance the attractiveness of our mechanism and make it easier to convince potential players to use it in order to reach the values discussed.

Moving away from general cooperative environments, the mechanisms constructed in this paper can also resolve distributional problems in many concrete settings such as cost-sharing environments, bankruptcy disputes and dissolution of partnerships.

References

Baliga, S., Brusco, S.: Collusion, renegotiation and implementation. Soc Choice Welf **17**, 69–83 (2000)

van den Brink, R., Funaki, Y.: Axiomatizations of a class of equal surplus sharing solutions for TU-games. Tinbergen Institute Discussion Paper, TI 2004-136/1, Tinbergen Institute, The Netherlands (2004). Theory and Decision (forthcoming)

van den Brink, R., Funaki, Y., Ju, Y.: Axiomatizations of the convex combinations of a class of equal surplus solutions and the Shapley value. Manuscript, Free University Amsterdam, The Netherlands (2005)

Dagan, N., Serrano, R., Volij, O.: A non-cooperative view of consistent bankruptcy rules. Games Econ Behav **18**, 55–72 (1997)

Dasgupta, A., Chiu, Y.S.: On implementation via demand commitment games. Int J Game Theory **27**, 161–189 (1998)

Driessen, T.S.H., Funaki, Y.: Coincidence of and collinearity between game theoretic solutions. OR Spektrum **13**, 15–30 (1991)
García-Jurado, I., González-Díaz, J., Villar, A.: A non-cooperative approach to bankruptcy problems. Spanish Econ Rev **8**, 189–197 (2006)
Gul, F.: Bargaining foundations of Shapley value. Econometrica **57**, 81–95 (1989)
Gul, F.: Efficiency and immediate agreement: a reply to Hart and Levy. Econometrica **67**, 913–918 (1999)
Hart, S., Mas-Colell, A.: Bargaining and value. Econometrica **64**, 357–380 (1996)
Hart, O., Moore, J.: Property rights and the theory of the firm. J Polit Econ **98**, 1119–1158 (1990)
Ju, Y.: The consensus value for games in partition function form. Int Game Theory Rev **9**, 437–452 (2007)
Ju, Y., Borm, P.: Externalities and compensation: primeval games and solutions. J Math Econ **44**, 367–382 (2008)
Ju, Y., Borm, P., Ruys, P.: The consensus value: a new solution concept for cooperative games. Soc. Choice Welf **28**, 685–703 (2007)
Krishna, V., Serrano, R.: Perfect equilibria of a model of *n*-person non-cooperative bargaining. Int J Game Theory **27**, 259–272 (1995)
Macho-Stadler, I., Pérez-Castrillo, D., Wettstein, D.: Efficient bidding with externalities. Games Econ Behav **57**, 304–320 (2006)
Maniquet, F.: Implementation under perfect information in economic environments. Soc Choice Welf **21**, 323–346 (2003)
Maskin, E.: Bargaining, coalitions and externalities. Presidential Address of the Econometric Society (2003)
Maskin, E., Moore, J.: Implementation and renegotiation. Rev Econ Stud **66**, 39–56 (1999)
Moldovanu, B., Winter, E.: Core implementation and increasing returns to scale for cooperation. J Math Econ **23**, 533–548 (1994)
Moore, J., Repullo, R.: Subgame perfect implementation. Econometrica **58**, 1083–1099 (1988)
Moulin, H.: Fair Division and Collective Welfare. Cambridge, MA: MIT Press (2003)
Myerson, R.B.: Conference structures and fair allocation rules. Int J Game Theory **9**, 169–182 (1980)
Nash, J.F.: Two person cooperative games. Econometrica **21**, 128–140 (1953)
Owen, G: Values of games with a priori unions. In: Henn, R., Moeschlin, O. (eds.) Essays in Mathematical Economics and Game Theory., pp. 76–88. Berlin: Springer (1977)
Pérez-Castrillo, D., Wettstein, D.: Bidding for the surplus: a non-cooperative approach to the Shapley value. J Econ Theory **100**, 274–294 (2001)
Pérez-Castrillo, D., Wettstein, D.: Choosing wisely: a multi-bidding approach. Am Econ Rev **92**, 1577–1587 (2002)
Pham Do, K.H., Norde, H.: The Shapley value for partition function form games. Int Game Theory Rev **9**, 353–360 (2007)
Rubinstein, A.: Perfect equilibrium in a bargaining model. Econmetrica **50**, 97–109 (1982)
Schmeidler, D.: The Nucleolus of a characteristic function game. SIAM J Appl Math **17**, 1163–1170 (1969)
Serrano, R.: Strategic bargaining, surplus sharing problems and the Nucleolus. J Math Econ **24**, 319–329 (1995)
Serrano, R.: Fifty years of the Nash program, 1953–2003. Investig Econ **29**, 219–258 (2005)
Shapley, L.S.: A value for *n*-person games. In: Kuhn, H.W., Tucker, A.W. (eds.) Contributions to the Theory of Games II pp. 307–317. Princeton, NJ: Princeton University Press (1953)
Thomson, W.: Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey. Math Soc Sci **45**, 249–297 (2003)
Vidal-Puga, J.: Bargaining with commitments. Int J Game Theory **33**, 129–144 (2004)
Vidal-Puga, J.: A bargaining approach to the Owen value and the Nash solution with coalition structure. Econ Theory **25**, 679–701 (2005)
Vidal-Puga, J., Bergantiños, G.: An implementation of the Owen value. Games Econ Behav **44**, 412–427 (2003)
Winter, E.: The demand commitment bargaining game and snowballing cooperation. Econ Theory **4**, 255–273 (1994)
Young, H.P., Okada, N., Hashimoto, T.: Cost allocation in water resources development. Water Resour Res **18**, 463–475 (1982)