Mary Kay Stein; Margaret Schwan Smith; Marjorie A. Henningsen; Edward S. Silver:

Implementing Standards-Based Mathematics Instruction:

A Casebook for Professional Development

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Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development is a rather complex book.

Already the target readership is composite. Firstly, the book addresses practising teachers who want to engage themselves in professional development, maybe in groups together with other teachers, in particular teachers associated with one of the reformed curricula in the USA inspired by the so-called NCTM *Standards* (whether the 1989 or the 1998 version). Secondly, it also addresses in-service teacher trainers and instructors who might choose to base courses on this book. Finally, the potential readership includes mathematics educators at large who are interested in becoming acquainted with the some of the spin-offs of the well-known QUASAR project (Quantitative Understanding: Amplifying Student Achievement and Reasoning) for US urban middle schools (grades 6-8). This project was conducted in the first half of the 1990's at the University of Pittsburgh under the directorship of Edward A. Silver. The authors have all been involved with the research aspects of this project. The book is opened by a thoughtful six-page foreword by Deborah Ball.

Next, the structure of the book is pretty complex as well, which does not prevent it from being impressively clear at the same time. The main subject is what is termed 'challenging tasks', which are seen as a special case of an instructional task, i.e. "a segment of classroom activity devoted to the development of a mathematical idea" (p. 7), and the interest is on the cognitive demands on students to complete such tasks. Thus, the focus of the book is on classroom activity, in particular small group work on challenging tasks, as orchestrated and guided by the teacher, rather than on individual student activity and achievement. The book is divided into two main parts. Part I, "The Mathematical Task Framework", (about 30 pages) offers a brief theoretical outline of basis concepts and terms pertinent to challenging tasks, and tools for teachers to detect, analyse, and classify tasks and activities. Part II, "The Cases", forms the bulk of the book (almost 100 pages) and consists of a presentation and discussion of a number of cases of classroom lessons, typically one or two per case. The fact that the book is meant as an aid to foster professional development is reflected by a section of discussion questions placed after each case presentation, and by sections that are specifically addressed to in-service course instructors.

In Part I the stage is set for analysing challenging mathematical tasks. The main objective is to provide guides and tools for recognising, explaining and categorising tasks according to their cognitive demands. The cognitive demand of a task is defined (p. 11) as the kind and level of thinking required of students to solve it. Thus cognitive demand appears to be an intrinsic characteristic of the task as such without involving the background of the students who are set to work on it. On the other hand, it is emphasised (on pp. 17f) that student backgrounds do in fact enter the issue of determining the cognitive demands of a task. (I would have preferred to see this made part of the definition.) Four levels of tasks are proposed: "Memorisation tasks" and tasks that imply "procedures without connections", i.e. procedures that can be activitated in isolation without requiring meaning to be established for the concepts involved in the task, are said to represent lower level demands, whereas "procedures with connections" and "doing mathematics" are seen as representing higher level demands. Only the latter two levels are in play with regard to challenging tasks. Tasks can be analysed *a priori*, i.e. as they appear in curricular or instructional materials before being implemented in the classroom. They can also be analysed *a posteriori* as they are enacted in the classroom, both as set up by the teacher and as actually dealt with by the students, and in terms of the student learning that results from work on the tasks. Particular emphasis is given in this part, and in fact throughout the book, on whether the level of cognitive demands on a task is maintained throughout the classroom session or whether it declines to a lower level than anticipated and intended at the outset. Attention is paid to factors associated with each of these possible developments, especially factors that are influenced by the teacher's way of setting up, orchestrating, monitoring and controlling the classroom activities.

The first part of the book closes with a chapter on learning from instructional cases, focusing on the question "what is this a case of?", and insisting that each case epitomises a research-based pattern of teaching and learning. In order to learn from cases "...teachers must learn to recognize events as instances of something larger and more generalizable" (p. 34).

Part II consists of the cases. There are six of them, of which two are actually dual cases in that they contrast two different teachers' work on the same mathematical content. The six (eight) cases represent different aspects of middle school mathematics: fractions, decimals and percent (linked to an area model); multiplication of fractions (by means of pattern blocks); mean, median, mode and range of statistical data; multiplication of binomials and monomials (using algebra tiles); organisation and analysis of data (favourite TV programmes and movies); problems solving. Some of the cases serve to illustrate the maintenance of cognitive demands throughout the classroom sesssions, whereas others illustrate decline in cognitive demand in the course of the period.

A more or less uniform scheme is adopted for the

presentation of each case. First comes a brief description of the teacher and the school involved in the case, including some words about his or her background and situation. Then the teacher offers her/his own fairly detailed presentation of the classroom session(s) in the case, with the focus on the setup and the implementation phases of the task, and of her/his reflections on what happened and why. After a number of discussion questions posed by the authors of the book, the authors offer their own comments and analysis ("teaching notes") of the course of events. In two of the cases (the first and the last) possible solution strategies for the task(s) are proposed as well. The teaching notes concentrate on identifying the cognitive levels of the task(s) at issue and the factors responsible for the maintenance, resp. decline, of the initial cognitive demands during the classroom session. Each case is brought to an end by a section on "additional layers of interpretation" which addresses features that are specific to the case.

The book is meant to be a tool for teachers and in-service teacher trainers, not so much a book on research in mathematics education. Yet it is based on research, both theoretical and empirical, but it does not (intend to) put forward and discuss research findings. Instead, we may well consider the book as an instance of applied research. I find the book interesting, well and very carefully written for the intended readership. In fact it also has many observations and points to make that are relevant to the researcher. Generally speaking it is written in a down-to-earth-language and style which makes the book accessible for readers without lot of academic prerequisites. Clearly it is marked by its American origin. It addresses the US scene with its growing tension between "reform quarters", adhering to the Standards, and "traditionalists", a tension which in some places has evolved into the sorts of "math wars" that are most manifest in the case of California but are making their way into other states as well. It also reflects the American scene in that teachers in the US, in contradistinction to what is the case in many countries with more centralised curricula and corresponding surveillance of students' achievements and of teachers, have a considerable amount of freedom in what they teach and how they teach. Even though the book is definitely American, the general influence exerted by developments in the USA on the rest of the world, makes it much less parochial than it might appear at first sight.

Although I find the book worth reading, I also have some objections. As is always the case with categories and subsequent classifications of objects (here tasks) drawn from a continuum, in particular a multidimensional continuum, the issue of the constitution of the categories and of identification of the characteristics of individual objects so as to allow for the classification, is a tricky one. Determining whether a given mathematical task should be classified as, say, being "procedures with connections" or "doing mathematics" is a problem that can often lead to different conclusions on equally sound grounds. But as the categories are supposed to form a stable partition of the universe under consideration, i.e. they are well-defined, exhaustive and mutually disjoint, there is a mismatch between the rigidity of the categorial partition and the inevitable fuzziness of the decisions involved in determining the membership of an individual object. This is a classical problem in empirical work in mathematics education. The problem is aggravated if strong inferences are going to be made from the distribution of objects on the categories, in which case the conclusions can be simply be markedly changed if the objects are classified just a little bit differently. Although the latter problem is not present in the context of the book under review, by virtue of its nature, one can actually argue with the classification of some of the tasks discussed in the book. For instance, it is claimed (pp. 77-78) that the enactment of the task in teacher Fran's class declined into "procedures without connections". The evidence offered for this conclusion is too weak to fully justify it. Also, I would challenge the conclusion on p. 118 that the classroom implementation of a data analysis task contained "no mathematical activity". Against this background I think that the teaching notes go a little too far in providing "the right answer" to the classification issue.

As to the exposition of the book, each case contains a fairly lengthy description of the classroom sessions under consideration, written by each teacher in the first person mode ("I"). It appears as if these presentations are authentic texts produced by the teachers, but it is not stated explicitly whether or not this is the case. If it is not, i.e. if there is the slightest element of editing on the part of the authors, the readers ought to have been informed. Because of the relative homogeneity of these texts I came to suspect that they are not quite verbatim products by the teachers.

A few – minor – objections are of a more mathematical nature. These might not have been worth mentioning if the book werent' addressing teachers some of whom may have a less than solid mathematical background. In a number of places, e.g. p. 12, the authors speak as if numbers and operators are identical. For instance it is stated that ½ = 0.5 = 50%. I find this very misleading, as ½ and 0.5 are two different representations of the same number, whereas 50% is not a number but an operator on numbers and magnitudes. It is true that if 50% operates on the number 1 the result is ½ = 0.5, but if it operates on any other number the outcome is different. I know that the book does not think of ½ and 0.5 as numbers but of operators, they too – one half *of* something – but this is contradictory to the established usage of the number symbols. As it is a main point in the book, especially in the first cases in Part II, to look at students' understanding and interpretation of fractions, I think the conceptual unclarity introduced in this way is rather unfortunate.

Another point is found on p. 28, where it is suggested that it is possible to make a table of all possible rectangles whose perimeters are 24 (in order to find the rectangle with the largest area). It appears – but only very indirectly – that rectangles are supposed to have integer sides.

The figures on pp. 66f are supposed to be composed of regular hexagons, which is crucial to the reasoning, but quite a few of the drawings fall short of displaying regular hexagons.

Apart from these not too severe objections, I find the book well structured, carefully balanced and written, sensitive and reasonable, and rich in thoughtful observations and comments. It provides good food for thought and practice for its composite readership.

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