Implicit System Specification and the Interface Equation

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This paper investigates a method of systems design via implicit specification and develops mathematical theory for the solution of a particular kind of context equation which could be used to automate the design of interfaces.

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1. INTRODUCTION

In this paper, we initiate a study of implicit system specification.

The general approach is roughly as follows. One is required to design a system which is to interact with a given environment in such a way that the interaction gives rise to a desired form of externally visible behaviour.

Such a problem may be formulated mathematically in a suitable algebraic specification language – in this paper, we use Milner's Calculus of Communicating Systems (hereafter, CCS)[3] – as the problem of solving an equation of the form

$$C[X] \approx q$$

where $C[\,]$ is a context in CCS, representing the environment, and q is a CCS agent, representing the desired externally visible behaviour. A solution for this equation is an agent r such that $C[r] \approx q$. Such a solution may be considered as an abstract description of a system which, by virtue of r satisfying the equation, is 'correct'. If solutions to such equations may be derived mechanically, then we have abolished the need for design and in any case there is no need to verify the system.

An example of such a problem is the design of an interface. Fig. 1 pictures a situation in which two i/o devices called $p1$ and $p2$ are given.

Figure 1.

It is required to construct an interface X (see Fig. 2) so that when linked with $p1$ and $p2$ and internal communication is hidden, the result is indistinguishable from some hypothetical system q.

The system pictured in Fig. 2 could be described by the CCS expression

$$(p1\,|\,X\,|\,p2)\backslash A$$

where A is a set of communications to be internalised. We therefore need to find X such that

$$(p1\,|\,X\,|\,p2)\backslash A \approx q$$

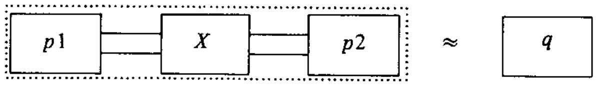

Figure 2.

Setting $p = p1\,|\,p2$, we arrive at an equation of the form

$$(p\,|\,X)\backslash A \approx q$$

which we call an *interface equation*.

In this paper, we investigate the solution of a large class of equations of this form. The solution essentially involves a quotient construction of the kind to be found in elementary algebraic automata theory.

We begin in Section 2 with a brief summary of CCS. Readers already familiar with CCS are advised that the section contains one or two items of notation which are not standard (i.e. not in Milner[3]). In Section 3, we introduce a property called weak determinacy, which is an 'observational' version of the strong determinacy presented in Milner.[3] This property entails that a given equation and solution – which are mostly determined by a triple (p, q, r) – determine a set of triples (p', q', r'), where p', q' and r' belong to the 'state spaces' of p, q and r respectively. From this set of triples we may construct a map from the set of 'states' r' into sets of pairs (p', q'). These matters are dealt with in Sections 4 and 5. Section 6 discovers how 'state transition' between the states of r is reflected in relationships between the sets of $(p', q,)$. From this, we are able in Section 7 to give necessary conditions for a solution in terms of conditions on sets of pairs (p', q'). In Section 8, we show these conditions to be sufficient and present a crude algorithm for generating solutions.

2. A SUMMARY OF CCS

In this section, we shall give a brief outline of some ideas and notation from CCS. For a full account of the Calculus, the reader is referred to Milner.[3]

CCS involves a language of *expressions*, which may be used to define the behaviours of *agents*. An agent's behaviour consists of the making and receiving of communication with its environment or with other agents, this latter providing a means to combine agents into systems of communicating subsystems. Communications are represented by *labels*, which we shall normally write as lower case Greek characters.

We assume a set Λ of labels. To each label λ is associated a unique *complementary label*, denoted $\bar{\lambda}$. If A is a set of labels, we let

$$\bar{A} = \{\bar{\mu}\,|\,\mu \in A\}.$$

Define $\Delta = \Lambda \cup \bar{\Lambda}$.

The complementation operation provides us with a means of determining inter-agent communication. If an agent p may make a λ communication and another agent

q may make a $\bar{\lambda}$ communication, then the two agents combined have the option of handshaking, the result being a special action called τ which denotes a silent, unobservable action.[†]

If $\mu \in \Delta$, then $\bar{\bar{\mu}} = \mu$. We make the convention that $\bar{\tau} = \tau$ and that $\Lambda \cap \bar{\Lambda} = \{\tau\}$.

We also assume a set of *behaviour identifiers*. Roughly speaking, these may be used to name agents.

We now give the rules for forming CCS expressions.

2.1. Definition

(a) NIL is a behaviour expression; it describes an agent that does nothing.

(b) If E is an expression and $\mu \in \Delta$, then $\mu . E$ is an expression: it describes an agent that may perform a μ communication and then behave like E.

(c) If E and E' are expressions then $E+E'$ is an expression: it describes an agent that may non-deterministically choose to behave either like E or like E'.

(d) If E and E' are expressions then $E \mid E'$ is an expression: it describes an agent composed of two agents E and E' communicating through complementary ports.

(e) If E is an expression and $A \subseteq \Delta - \{\tau\}$, then $E \backslash A$ is an expression: it describes an agent which may behave like that described by E except that it may never perform an action which is a member of $A \cup \bar{A}$.

(f) Behaviour identifiers are expressions.[‡]

Finally, if b is a behaviour identifier and E is an expression then we write $b \Leftarrow E$ as an equation signifying that b is to have the behaviour determined by E.

Behaviour expressions determine sequences of communications, representing the visible behaviour of some agent. Formally, we may describe this behaviour by derivations of the form:

$$E_1 \rightarrow^{\mu} E_2$$

to indicate that an agent described by E_1 has the capability of making a μ communication, after which the agent behaves like E_2. We write $E_1 \rightarrow^{\mu}$ to indicate that $E_1 \rightarrow^{\mu} E_2$ for some E_2.

2.2. Definition

Let E, E' E'', E_i, E'_i be expressions, $\mu \in \Delta$ and b a behaviour identifier.

(a) NIL has no actions: $NIL \rightarrow^{\mu} E$ is false for all $\mu \in \Delta, E$.

(b) Guarding: $\mu . E$ always has a μ action:

$$\mu . E \rightarrow^{\mu} E.$$

(c) Summation: the composite agent has the capabilities of both:

$$\frac{E \rightarrow^{\mu} E''}{E+E' \rightarrow^{\mu} E''} \; \frac{E' \rightarrow^{\mu} E''}{E+E' \rightarrow^{\mu} E''}.$$

(d) Communication: the composite may allow its composites to behave independently or handshake, producing a silent (τ) action:

$$\frac{E \rightarrow^{\mu} E''}{E \mid E' \rightarrow^{\mu} E'' \mid E'} \; \frac{E' \rightarrow^{\mu} E''}{E \mid E' \rightarrow^{\mu} E \mid E''}$$

$$\frac{E_1 \rightarrow^{\mu} E'_1 \text{ and } E_2 \rightarrow^{\bar{\mu}} E'_2}{E_1 \mid E_2 \rightarrow^{\tau} E'_1 \mid E'_2}.$$

(e) Restriction: the agent may perform any action which is not indicated by the restriction set:

$$\frac{E \rightarrow^{\mu} E'}{E \backslash A \rightarrow^{\mu} E' \backslash A} \quad \text{if} \quad \{\mu, \bar{\mu}\} \cap A = \varnothing.$$

(f) Assignment: the behaviour identifier is associated with the behaviour of the defining expression:

$$\frac{b \Leftarrow E \text{ and } E \rightarrow^{\mu} E'}{b \rightarrow^{\mu} E'}.$$

2.3. Definition

Let E be an expression. We denote by $E\bullet$ the set of all communications it can make immediately:

$$E\bullet = \{\mu \in \Delta \mid E \rightarrow^{\mu} E' \textit{ some } E'\}.$$

2.4. Definition

We may now define sequences of communications possible to an expression.

Let $s \in \Delta^*$. If E and E' are expressions and $\mu \in \Delta$, then we define:

(a) $E \mapsto^{\Omega} E$ where Ω denotes the null string.

(b) $E \mapsto^{\mu . s} E'$ iff there exists E'' such that $E \rightarrow^{\mu} E''$ and $E'' \mapsto^{s} E'$.

Now, suppose $s \in (\Delta - \{\tau\})^*$. We define $E \Rightarrow^{s} E'$ iff there exists $s' \in \Delta^*$ such that $E \mapsto^{s'} E'$ and $s'|_{\tau} = s$. Here, $s'|_{\tau}$ denotes the string obtained from s' by erasing all τ's.

We shall write $\mathbf{R}(E)$ to denote the *reachability set* of E, that is the set of all expressions that may be derived from E via $\Rightarrow$ derivations:

$$\mathbf{R}(E) = \{E' \mid E \Rightarrow^{s} E', \textit{some } s \in (\Delta - \{\tau\})^*\}.$$

We write $\Lambda(E)$ to denote the set of all communications it may ever possibly make:

$$\Lambda(E) = \{\mu \in \Lambda \cup \bar{\Lambda} \mid \mu \in E'\bullet \textit{ some } E' \in \mathbf{R}(E)\}.$$

Note that if $E \Rightarrow^{s} E'$, then $\Lambda(E') \subseteq \Lambda(E)$.

Finally, we say that E is *rigid* iff $\tau \notin \Lambda(E)$.

We now come to two notions of equality. The first defines two expressions to be equivalent if they are indistinguishable in terms of visible behaviour, that is ignoring τ actions. The second is stricter and takes τ actions into account. Both are equivalence relations. The second is also a congruence relation, that is, for example, if $E_1 \sim E_2$ then $E_1 + E \sim E_2 + E$.

2.5. Definition

Let E_1, E_2 be expressions. We shall say that they are *observationally equivalent* (and write $E_1 \approx E_2$) if $E_1 \approx_n E_2$, for all natural numbers n, where:

(a) We always have $E_1 \approx_0 E_2$.

(b) $E_1 \approx_{n+1} E_2$ iff for all $s \in (\Delta - \{\tau\})^*$.

[†] In the full calculus, the labels represent ports through which data transfer is possible. By convention, unbarred labels represent input ports and barred labels represent output ports. For the sake of simplicity, we shall not consider data transfer here.

[‡] We have omitted one operator, the renaming operator which we shall not use in this paper. See Ref. 3.

(i) If $E_1 \Rightarrow^{s} E'_1$ then there exists E'_2 such that $E_2 \Rightarrow^{s} E'_2$ and $E'_1 \approx_n E'_2$.

(ii) If $E_2 \Rightarrow^{s} E'_2$ then there exists E'_1 such that $E_1 \Rightarrow^{s} E'_1$ and $E'_1 \approx_n E'_2$.

2.6. Proposition

(1) For each n, $\approx_n$ is an equivalence relation.
(2) $\approx$ is an equivalence relation.

2.7. Definition

Let E_1, E_2 be expressions. We shall say that they are *strongly congruent* (and write $E_1 \sim E_2$) if $E_1 \sim_n E_2$, for all natural numbers n, where:

(a) We always have $E_1 \sim_0 E_2$.
(b) $E_1 \sim_{n+1} E_2$ iff for all $\mu \in \Delta$.

(i) If $E_1 \rightarrow^{\mu} E'_1$ then there exists E'_2 such that $E_2 \rightarrow^{\mu} E'_2$ and $E'_1 \sim_n E'_2$.

(ii) If $E_2 \rightarrow^{\mu} E'_2$ then there exists E'_1 such that $E_1 \rightarrow^{\mu} E'_1$ and $E'_1 \sim_n E'_2$.

2.8. Proposition

(1) For each n, $\sim_n$ is a congruence relation.
(2) $\sim$ is a congruence relation.
(3) $E_1 \sim E_2$ implies $E_1 \approx E_2$.

3. WEAK DETERMINACY

We begin with some of the key definitions.

3.1. Definition

q is weakly determinate iff q is weakly-k-determinate for all k. Every q is weakly-0-determinate. If $k > 0$, then q is weakly-k-determinate iff

(a) For all $s \in \Delta^{*}: q \Rightarrow^{s} q'$ implies q' is weakly-k-1-determinate.

(b) For all $s \in \Delta^{*}: q \Rightarrow^{s} q_1$ and $q \Rightarrow^{s} q_2$ implies $q_1 \approx q_2$.

This 'observational' analogue of strong determinacy was considered by Milner [p. 154][3] but not adopted. We have found it a technically useful idea, however. In fact, we have shown[4] that an agent is weakly determinate iff it is observationally equivalent to an agent which is rigid and strongly determinate.

The significance of this is that weak determinacy is the weakest property that an agent q may have in order that it is observationally equivalent to a (not necessarily finite) state machine.

The following sequence of lemmas present some useful consequences of this property.

3.2. Lemma

Suppose q is weakly determinate and $q \Rightarrow^{s} q'$, then q' is also weakly determinate.

Proof

We show that for all k, q' is weakly-k-determinate. Since q is weakly determinate, it must be weakly-$k+1$-determinate for each k. Since $q \Rightarrow^{s} q'$ and q is weakly-$k+1$-determinate, it follows, by 3.1 (a), that q' is weakly-k-determinate.

3.3. Lemma

Let q be weakly determinate and suppose $p \approx q$ and $p \rightarrow^{\mu} p'$, some $\mu \in \Delta - \{\tau\}$, then there exists q' such that $q \Rightarrow^{\mu} q'$ and $p' \approx q'$.

Proof

Since $p \approx q$ then by 2.5 (b), for all n there exists q_n such that $q \Rightarrow^{\mu} q_n$ and $p' \approx_n q_n$. Since q is weakly determinate, it follows from $q \Rightarrow^{\mu} q_n$ and $q \Rightarrow^{\mu} q_m$ that $q_n \approx q_m$, by 3.1 (b). Thus, if we let $q' = q_1$ then for all n, $q \Rightarrow^{\mu} q'$ and $q' \approx_n q_n$. Thus, for all n, $p' \approx_n q_n \approx_n q'$. Thus, for all n, $p' \approx_n q'$, by 2.6 (1). Thus $p' \approx q'$, by 2.5.

3.4. Lemma

Let q be weakly determinate and suppose $p \approx q$ and $p \rightarrow^{\mu} p'$ and $q \rightarrow^{\mu} q'$, some $\mu \in \Delta - \{\tau\}$, then $p' \approx q'$.

Proof

By 3.3, there exists q'' such that $q \Rightarrow^{\mu} q''$ and $p' \approx q''$. Since q is weakly determinate, from $q \Rightarrow^{\mu} q'$ and $q \Rightarrow^{\mu} q''$ and 3.1 (b), we obtain $q' \approx q''$. Since $p' \approx q''$, we may use 2.6 (2) to conclude that $p' \approx q'$ as required.

3.5. Lemma

Let q be weakly determinate and suppose $p \approx q$ and $p \rightarrow^{\tau} p'$ then $p' \approx q$.

Proof

Since $p \Rightarrow^{\Omega} p'$ and $p \approx q$, it follows that for all n there exists q_n such that $q \Rightarrow^{\Omega} q_n$ and $p' \approx_n q_n$. Since $q \Rightarrow^{\Omega} q$ and q is weakly determinate, we must have $q \approx q_n$, each n, by 3.1 (b). Thus $p' \approx_n q$, all n, and hence $p' \approx q$ by 2.5.

3.6. Proposition

Let q be weakly determinate and suppose $p \approx q$, then p is weakly determinate.

Proof

We argue by induction on k that if $p \approx q$ then p is weakly k determinate.

This is true for $k = 0$, by 3.1.

Suppose true for k and let $p \Rightarrow^{s} p'$. Since $p \approx q$, by 2.5 for each n there exists q_n such that $q \Rightarrow^{s} q_n$ and $p' \approx_n q_n$. Since q is weakly determinate, there exists q' ($= q_1$) such that $q' \approx_n q_n$ for each n. By 2.6 (1), we have that $p \approx_n q'$ for each n and that hence $p' \approx q'$.

By 3.2, q' is weakly determinate. By induction, p' is weakly k determinate. We have shown that if $p \approx q$ and $p \Rightarrow^{s} p'$ then p' is weakly k determinate. Thus, by 3.1, p is weakly $k+1$ determinate. This concludes the induction step and the proof.

4. THE INTERFACE EQUATION

4.1. Definition

An *interface equation* is an expression of the form

$$(p \mid X) \backslash A \approx q$$

where

(1) q is weakly determinate and

(2) $\Lambda(p) \cap \overline{\Lambda(q)} \subseteq \{\tau\}$

(3) $\Lambda(p) \cap \bar{A} = \varnothing$

(4) $\Lambda(q) \cap (A \cup A) = \varnothing$.

4.2. Definition

r is a *solution* to the equation $(p \mid X)\backslash A \approx q$ iff r satisfies

(1) $(p \mid r)\backslash A \approx q$.

(2) $\Lambda(r) \cap \Lambda(p) \subseteq \{\tau\}$.

The constraints are for technical reasons, mostly in order to make the proofs of lemmas work. Weak determinacy, for example, ensures that the implications (I), (II) and (III) below hold. Constraint (2) is needed to ensure that proposition 8.2(1)(c) is true.

Note that if constraint (4) does not hold, then there is an action $\mu \in \Lambda(q)$ such that $\mu \neq \tau$ (since $\mu \in A \cup \bar{A}$) but $\mu \notin \Lambda((p \mid X)\backslash A)$. Accordingly, $q \Rightarrow^{s\mu}$ for some string s, but we cannot have $(p \mid X)\backslash A \Rightarrow^{s\mu}$. Thus, by 2.5(b), 4.2(1) cannot hold and hence there is no solution.

The aim of the rest of this section is to prove that derivations preserve interface equations, that is, if $(p \mid r)\backslash A \approx q$ then

(I) $(p \mid r)\backslash A \rightarrow^{\mu} (p' \mid r')\backslash A$ and $q \rightarrow^{\mu} q'$ implies

$$(p' \mid r')\backslash A \approx q'.$$

(II) $(p \mid r)\backslash A \rightarrow^{\tau} (p' \mid r')\backslash A$ implies $(p' \mid r')\backslash A \approx q$

(III) $q \rightarrow^{\tau} q'$ implies $(p \mid r)\backslash A \approx q'$.

This will enable us to relate the structure of r to that of certain sets of subsets of $\mathbf{R}(p) \times \mathbf{R}(q)$ which become significant when we try to build solutions.

Propositions such as (I), (II) and (III) are not generally true in CCS and will depend on the fact that q is weakly determinate.

4.3. Lemma

Suppose r solves the interface equation $(p \mid X)\backslash A \approx q$ and suppose $(p \mid r)\backslash A \rightarrow^{\mu} (p' \mid r')\backslash A$ and $q \rightarrow^{\mu} q'$ with $\mu \in \Delta - \{\tau\}$, then $(p' \mid X)\backslash A \approx q'$ is also an interface equation and has r' as a solution.

Proof

We check the conditions of 4.1.

(1) q' is weakly determinate, by lemma 3.2. Since $\Lambda(p') \subseteq \Lambda(p)$ and $\Lambda(q') \subseteq \Lambda(q)$ (see 2.4), it follows that

(2) $\Lambda(p') \cap \overline{\Lambda(q')} \subseteq \Lambda(p) \cap \overline{\Lambda(q)} \subseteq \{\tau\}$

(3) $\Lambda(p') \cap \bar{A} \subseteq \Lambda(p) \cap \bar{A} = \varnothing$

(4) $\Lambda(q') \cap (A \cup \bar{A}) \subseteq \Lambda(q) \cap (A \cup \bar{A}) = \varnothing$.

Thus $(p' \mid X)\backslash A \approx q'$ is an interface equation.

Now we check the conditions of 4.2.

(1) $(p' \mid r')\backslash A \approx q'$ follows from lemma 3.4.

(2) Since $\Lambda(p') \subseteq \Lambda(p)$ and $\Lambda(r') \subseteq \Lambda(r)$ it follows that $\Lambda(r') \cap \Lambda(p') \subseteq \Lambda(r) \cap \Lambda(p) \subseteq \{\tau\}$.

4.4. Lemma

Suppose r solves the interface equation $(p \mid X)\backslash A \approx q$ and suppose $(p \mid r)\backslash A \rightarrow^{\tau} (p' \mid r')\backslash A$, then $(p' \mid X)\backslash A \approx q$ is also an interface equation and has r' as a solution.

Proof

$(p' \mid X)\backslash A \approx q$ is an interface equation since

(1) q is weakly determinate by hypothesis and as in the proof of 4.3 we have

(2) $\Lambda(p') \cap \overline{\Lambda(q)} \subseteq \Lambda(p) \cap \overline{\Lambda(q)} \subseteq \{\tau\}$

(3) $\Lambda(p') \cap \bar{A} \subseteq \Lambda(p) \cap \bar{A} = \varnothing$

(4) $\Lambda(q) \cap (A \cup \bar{A}) = \varnothing$ by hypothesis.

Checking the conditions of 4.2, we have

(1) $(p' \mid r')\backslash A \approx q$ follows from lemma 3.5

(2) $\Lambda(r') \cap \Lambda(p') \subseteq \Lambda(r) \cap \Lambda(p) \subseteq \{\tau\}$.

4.5. Lemma

Suppose r solves the interface equation $(p \mid X)\backslash A \approx q$ and suppose $q \rightarrow^{\tau} q'$, then $(p \mid X)\backslash A \approx q'$ is also an interface equation and has r as a solution.

Proof

$(p \mid X)\backslash A \approx q$ is an interface equation since

(1) q' is weakly determinate by 3.2

(2) $\Lambda(p) \cap \overline{\Lambda(q')} \subseteq \Lambda(p) \cap \overline{\Lambda(q)} \subseteq \{\tau\}$

(3) $\Lambda(p) \cap \bar{A} = \varnothing$ by hypothesis

(4) $\Lambda(q') \cap (A \cup \bar{A}) \subseteq \Lambda(q) \cap (A \cup \bar{A}) = \varnothing$.

Now we check the conditions of 4.2.

(1) From the hypothesis, $q \Rightarrow^{\Omega} q'$. But we also have $q \Rightarrow^{\Omega} q$ and so, since q is weakly determinate, it follows by 3.1(b) that $q \approx q'$. Thus $(p \mid r)\backslash A \approx q \approx q'$ and so, by 2.6(b), $(p \mid r)\backslash A \approx q'$ as required.

(2) $\Lambda(r) \cap \Lambda(p) \subseteq \{\tau\}$ by hypothesis.

5. SOLUTION TRIPLES AND *(p,q)* SYSTEMS

In lemmas 4.3 to 4.5, we saw that an equation $(p \mid X)\backslash A \approx q$ and solution r, gave rise to new equations $(p' \mid X)\backslash A \approx q'$ and solutions r'. In this section we look at tuples such as (p, q, r) and (p', q', r') which determine 'equation-solution' triples, the point being that solutions r' are related to sets of pairs (p', q') in a useful sort of way.

We may define a map ϕ from the set of such r' to sets of (p', q') such that (p', q', r') is a 'solution triple'. We shall also (Section 6) define derivations between such sets which makes ϕ into something like a homomorphism. Later, we shall see how solutions may be constructed from such sets and derivations.

First, we set up the means for recursively generating the set of triples.

5.1. Definition

Suppose $p', q', r', p'', q'', r''$ are agents. Define:

(a) $(p', q', r') \rightarrow^{\tau} (p'', q'', r'')$ if

(i) $(p' \mid r')\backslash A \rightarrow^{\tau} (p'' \mid r'')\backslash A$ and $q' = q''$ or

(ii) $q' \rightarrow^{\tau} q''$ and $p' = p''$ and $r' = r''$.

(b) If $\mu \in \Delta - \{\tau\}$, then $(p', q', r') \rightarrow^{\mu} (p'', q'', r'')$ if $(p' \mid r')\backslash A \rightarrow^{\mu} (p'' \mid r'')\backslash A$ and $q' \rightarrow^{\mu} q''$.

We extend → to strings in the obvious way. If $s \in \Delta^*$ and $\lambda \in \Delta$, then we define

(c) $(p', q', r') \Mapsto^{\Omega} (p', q', r')$

(d) $(p', q', r') \Mapsto^{\lambda s} (p'', q'', r'')$ if there exists (p''', q''', r''') such that $(p', q', r') \rightarrow^{\lambda} (p''', q''', r''')$ and $(p''', q''', r''') \Mapsto^{s} (p'', q'', r'')$.

(e) $(p', q', r') \Rightarrow^{s} (p'', q'', r'')$ if there exists $s' \in \Delta^*$ such that $(p', q', r') \Mapsto^{s'} (p'', q'', r'')$ and $s = s'|_{\tau}$.

We may now restate lemmas 4.3, 4.4 and 4.5 in this terminology.

5.2. Lemma

Suppose $(p|X)\backslash A \approx q$ is an interface equation with solution r and suppose $s \in \Delta^*$ and $\lambda \in \Delta$ then

(1) if $(p, q, r) \rightarrow^{\lambda} (p', q', r')$ then $(p'|X)\backslash A \approx q'$ is an interface equation with solution r'.

(2) if $(p, q, r) \Mapsto^{s} (p'', q'', r'')$ then $(p''|X)\backslash A \approx q''$ is an interface equation with solution r''.

(3) if $(p, q, r) \Rightarrow^{s} (p'', q'', r'')$ then $(p''|X)\backslash A \approx q''$ is an interface equation with solution r''.

Proof

(1) By 5.1, we have three cases to consider.

The case $(p|r)\backslash A \rightarrow^{\mu} (p'|r')\backslash A$ and $q \rightarrow^{\mu} q'$ with $\mu \in \Delta - \{\tau\}$ is covered by lemma 4.3.

The case $(p|r)\backslash A \rightarrow^{\tau} (p'|r')\backslash A$ and $q = q'$ is covered by lemma 4.4.

The case $q \rightarrow^{\tau} q'$ and $p = p'$ and $r = r'$ is covered by lemma 4.5.

(2) follows from (1) and induction on the length of s.

(3) follows from (2) and 5.1 (e).

We collect together the triples that may be 'derived' from (p, q, r).

5.3. Definition

Suppose p, q, r are agents. Let $\mathbf{R}(p, q, r)$ denote the set:

$$\{(p', q', r') \in \mathbf{R}(p) \times \mathbf{R}(q) \times \mathbf{R}(r) \mid (p, q, r) \Mapsto^{s} (p', q', r') \textit{ some } s \in \Delta^*\}.$$

Note that $(p, q, r) \Mapsto^{\Omega} (p, q, r)$ and so $(p, q, r) \in \mathbf{R}(p, q, r)$.

We shall call the elements of $\mathbf{R}(p, q, r)$, *solution triples* because of (2) of the following.

5.4. Corollary

(1) Suppose that $(p', q', r') \in \mathbf{R}(p, q, r)$ and $s \in \Delta^*$ and $(p', q', r') \Mapsto^{s} (p'', q'', r'')$, then $(p'', q'', r'') \in \mathbf{R}(p, q, r)$

(2) Suppose r is a solution,† then for all

$$(p', q', r') \in \mathbf{R}(p, q, r),$$

$(p'|X)\backslash A \approx q'$ is an interface equation with solution r'.

Not every element of $\mathbf{R}(r)$ of a solution r will actually be reached during the simulation of q by $(p|r)\backslash A$. We isolate the set of states, $\mathbf{R}_{p,q}(r)$, which are reached. We may then relate elements of $\mathbf{R}_{p,q}(r)$ to subsets of $\mathbf{R}(p) \times \mathbf{R}(q)$.

5.5. Definition

Define

$$\mathbf{R}_{p,q}(r) = \{r' \in \mathbf{R}(r) \mid (p', q', r') \in \mathbf{R}(p, q, r) \text{ some } p', q'\}.$$

Each element of $\mathbf{R}_{p,q}(r)$ 'appears with' at least one pair $(p', q') \in \mathbf{R}(p) \times \mathbf{R}(q)$. We let $\phi(r')$ denote the set of all such pairs.

Define

$$\phi : \mathbf{R}_{p,q}(r) : \rightarrow P(\mathbf{R}(p) \times \mathbf{R}(q))\dagger$$

by

$$\phi(r') = \{(p', q') \mid (p', q', r') \in \mathbf{R}(p, q, r)\}.$$

From 5.4 and 5.5, we obtain:

5.6. Corollary

Suppose r is in a solution, then if $r' \in \mathbf{R}_{p,q}(r)$ and $(p', q') \in \phi(r')$, then $(p'|X)\backslash A \approx q'$ is an interface equation with solution r'.

We next uncover a characteristic property of sets $\phi(r')$, where $r' \in \mathbf{R}_{p,q}(r)$ and r a solution. It transpires that every such set is closed under three relations, denoted $\rightarrow^{I}, \rightarrow^{\tau,P}$ and $\rightarrow^{\tau,Q}$, that is if $(p', q') \in \phi(r')$ and R is one of the three relations and $(p', q')\,R\,(p'', q'')$, then $(p'', q'') \in \phi(r')$. We then define a relation $\rightarrow_{I\tau}$ to be the pre-order generated by $\rightarrow^{I} \cup \rightarrow^{\tau,P} \cup \rightarrow^{\tau,Q}$.

It now follows that the sets $\phi(r')$ are unions of sets closed under $\rightarrow_{I\tau}$.

Let us define $\rightarrow^{I}, \rightarrow^{\tau,P}$ and $\rightarrow^{\tau,Q}$.

5.7. Definition

Let (p', q'), $(p'', q'') \in \mathbf{R}(p) \times \mathbf{R}(q)$ and $\mu \in \Delta - \{\tau\}$.

(a) Define $(p', q') \rightarrow^{\mu,I} (p'', q'')$ iff $p' \rightarrow^{\mu} p''$ and $q' \rightarrow^{\mu} q''$. Define $(p', q') \rightarrow^{I} (p'', q'')$ iff $(p', q') \rightarrow^{\mu,I} (p'', q'')$ for some μ.

(b) Define $(p', q') \rightarrow^{\tau,P} (p'', q'')$ iff $p' \rightarrow^{\tau} p''$ and $q' = q''$.

(c) Define $(p', q') \rightarrow^{\tau,Q} (p'', q'')$ iff $q' \rightarrow^{\tau} q''$ and $p' = p''$.

We now show that the sets $\phi(r')$ are closed under our three relations.

5.8. Lemma

Suppose r is a solution and that $(p', q') \in \phi(r')$ for some $r' \in \mathbf{R}_{p,q}(r)$ then if $(p', q') \rightarrow^{\mu,I} (p'', q'')$, then $(p'', q'') \in \phi(r')$.

Proof

Since $(p', q') \rightarrow^{\mu,I} (p'', q'')$ it follows that $p' \rightarrow^{\mu} p''$ and $q' \rightarrow^{\mu} q''$, by 5.7 (a). Since $\mu \in q'\bullet$ we have $\mu \notin A \cup \bar{A}$, by 4.1 (4). Since $p' \rightarrow^{\mu} p''$ and $\mu \notin A \cup A$, we have

$$(p'|r')\backslash A \rightarrow^{\mu} (p''|r')\backslash A$$

by 2.2 (d) and 2.2 (e). Thus, $(p'|r')\backslash A \rightarrow^{\mu} (p''|r')\backslash A$ and $q' \rightarrow^{\mu} q''$ and so from 5.1 (b) we may conclude that:

$$(p', q', r') \rightarrow^{\mu} (p'', q'', r'). \qquad (5.8.1)$$

Now $(p', q') \in \phi(r')$ and so by 5.5

$$(p', q', r') \in \mathbf{R}(p, q, r). \qquad (5.8.2)$$

Applying 5.4 (1) to (5.8.1) and (5.8.2) we obtain

$$(p'', q'', r') \in \mathbf{R}(p, q, r)$$

and by 5.5, this implies $(p'', q'') \in \phi(r')$, as required.

5.9. Lemma

Suppose r is a solution and $(p', q') \in \phi(r')$ for some $r' \in \mathbf{R}_{p,q}(r)$. Suppose $(p', q') \rightarrow^{\tau,P} (p'', q'')$, then $(p'', q'') \in \phi(r')$.

† From now on, we shall assume p, q to be fixed and abbreviate 'Suppose $(p|X)\backslash A \approx q$ is an interface equation with solution r' by 'Suppose r is a solution'.

† If X is a set then $P(X)$ denotes the power set of X.

Proof

Since $(p',q') \rightarrow^{\tau,P} (p'',q'')$ it follows that $p' \rightarrow^{\tau} p''$ and $q'' = q'$, by definition 5.7(b). By 2.2(d) and 2.2(e), we have $(p'|r')\backslash A \rightarrow^{\tau} (p''|r')\backslash A$. By 5.1(a)(i),

$$(p',q',r') \rightarrow^{\tau} (p'',q'',r'). \tag{5.9.1}$$

Now $(p',q') \in \phi(r')$ and so by 5.5

$$(p',q',r') \in \mathbf{R}(p,q,r). \tag{5.9.2}$$

Applying 5.4(1) to (5.9.1) and (5.9.2) we obtain

$$(p'',q'',r') \in \mathbf{R}(p,q,r)$$

and by 5.5, this implies $(p'',q'') \in \phi(r')$, as required.

5.10. Lemma

Suppose r is a solution and $(p',q') \in \phi(r')$ for some $r' \in \mathbf{R}_{p,q}(r)$. Suppose $(p',q') \rightarrow^{\tau,Q} (p'',q'')$, then $(p'',q'') \in \phi(r')$.

Proof

Since $(p',q') \rightarrow^{\tau,Q} (p'',q'')$ it follows that $q' \rightarrow^{\tau} q''$ and $p'' = p'$, by definition 5.7(c). By 5.1(a)(ii)

$$(p',q',r') \rightarrow^{\tau} (p'',q'',r'). \tag{5.10.1}$$

Now $(p',q') \in \phi(r')$ and so by 5.5

$$(p',q',r') \in \mathbf{R}(p,q,r). \tag{5.10.2}$$

Applying 5.4(a) to (5.10.1) and (5.10.2) we obtain

$$(p'',q'',r') \in \mathbf{R}(p,q,r)$$

and by 5.5, this implies $(p'',q'') \in \phi(r')$, as required.

5.11. Definition

Let $\rightarrow_{I\tau}$ denote the reflexive transitive closure of $\rightarrow^{I} \cup \rightarrow^{\tau,P} \cup \rightarrow^{\tau,Q}$.

Define $B_{I\tau}(p',q') = \{(p'',q'') \mid (p',q') \rightarrow_{I\tau} (p'',q'')\}$.

5.8, 5.9, 5.10 and transitivity of $\rightarrow_{I\tau}$, now give:

5.12. Proposition

Suppose r is a solution and let $r' \in \mathbf{R}_{p,q}(r)$ and

$$(p',q') \in \phi(r'),$$

then $B_{I\tau}(p',q') \subseteq \phi(r')$.

From 5.12, $\phi(\mathbf{R}_{p,q}(r))$ is a set of unions of sets $B_{I\tau}(p',q')$. This motivates our next definition.

5.13. Definition

Define

$$I(p,q) = \{B_{I\tau}(p',q') \mid (p',q') \in \mathbf{R}(p) \times \mathbf{R}(q)\}.$$

Let $$\Psi(p,q) = \{\bigcup_{B \in X} B \mid X \subseteq I(p,q)\}$$

$\Psi(p,q)$ is the set of all sets that are unions of $\rightarrow_{I\tau}$ classes.

By *(p,q)-system* (or just *system*, if p,q is understood), we mean a set $\mathbf{S} \subseteq \Psi(p,q)$.

5.14. Corollary

Let r be a solution, then $\phi(\mathbf{R}_{p,q}(r)) \subseteq \Psi(p,q)$, that is, $\phi(\mathbf{R}_{p,q}(r))$ is a (p,q)-system.

5.15. Lemma

Suppose $r' \in \mathbf{R}_{p,q}(r)$ and $(p',q') \in \phi(r')$. Suppose

$$\mu \in \Delta - \{\tau\}$$

then if $q' \Rightarrow^{\mu} q''$ and $p' \rightarrow^{\mu} p''$, then $(p'',q'') \in \phi(r')$.

Proof

By hypothesis, we have q_1, q_2 and numbers $n, m \geqslant 0$ such that

$$q' \Mapsto^{\tau^n} q_1 \rightarrow^{\mu} q_2 \Mapsto^{\tau^m} q''. \tag{5.15.1}$$

Since $p' \rightarrow^{\mu} p''$, we may use (5.15.1) together with 5.7 and 5.11 to deduce

$$(p',q') \rightarrow_{I\tau} (p',q_1) \rightarrow_{I\tau} (p'',q_2) \rightarrow_{I\tau} (p'',q'')$$

so $(p',q') \rightarrow_{I\tau} (p'',q'')$. Since $\phi(r') \in \Psi(p,q)$, and $(p',q') \in \phi(r')$, then by 5.12, $(p'',q'') \in \phi(r')$

5.16. Lemma

Suppose $r' \in \mathbf{R}_{p,q}(r)$ and $(p',q') \in \phi(r')$. Suppose

$$\mu \in \Delta - \{\tau\}$$

then if if $q' \rightarrow^{\mu} q''$ and $(p'|r')\backslash A \Rightarrow^{\mu} (p''|r'')\backslash A$, then $(p'',q'') \in \phi(r'')$.

Proof

By hypothesis, we have p_1, p_2, r_1, r_2 and numbers $n, m \geqslant 0$ such that

$$(p'|r')\backslash A \Mapsto^{\tau^n} (p_1|r_1)\backslash A \rightarrow^{\mu} (p_2|r_2)\backslash A \Mapsto^{\tau^m} (p''|r'')\backslash A. \tag{5.16.1}$$

Since $q' \rightarrow^{\mu} q''$, we may use (5.16.1) together with 5.1 to deduce

$$(p',q',r') \Mapsto^{\tau^n} (p_1,q',r_1) \rightarrow^{\mu} (p_2,q'',r_2) \Mapsto^{\tau^m} (p'',q'',r'').$$

Thus, by 5.1,

$$(p',q',r') \Rightarrow^{\mu} (p'',q'',r''). \tag{5.16.2}$$

But, $(p',q') \in \phi(r')$ and so by 5.5, $(p'',q'') \in \phi(r'')$.

6. DERIVATIONS INSIDE *(p,q)*-SYSTEMS

We have managed to locate some of the structure of r inside $\mathbf{R}(p) \times \mathbf{R}(q)$. We shall next see how the derivation structure of r is reflected in that of one we may impose on $\Psi(p,q)$.

There are three types of derivation to consider (a) non-τ derivations of which q is also capable, which we shall call *O*-derivations (b) non-τ derivations of which q is not capable, which we shall call *C*-derivations and (c) τ derivations. Each of the three is reflected in some way within $\Psi(p,q)$.

O-derivations

First we consider the case in which $r' \in \mathbf{R}_{p,q}(r)$ has a communication μ, and $\mu \neq \tau$ and $\mu \in \Lambda(q)$. Condition (4) of 4.1 entails that μ will not be restricted by $\backslash A$ and so for any p', $(p' \mid r')\backslash A$ has a μ action. Now, if $(p', q') \in \phi(r')$, then $(p' \mid r')\backslash A \approx q'$ and hence, we must have $q' \Rightarrow^{\mu}$, for each $(p', q') \in \phi(r')$.

Thus if $r' \rightarrow^{\mu}$, then $q' \Rightarrow^{\mu}$ for each $(p', q') \in \phi(r')$. In particular, $\mu \in \Lambda(q)$. Since, by 4.2(2), $\Lambda(r) \cap \Lambda(p) \subseteq \{\tau\}$, we have that $\mu \in \Lambda(q) - \Lambda(p)$ so these are the communications that we consider in this category.

It now turns out that if $r' \rightarrow^{\mu} r''$ and $q' \Rightarrow^{\mu} q''$, then $(p', q'') \in \phi(r'')$. This suggests that we can define derivations between elements of (p, q)-systems in such a way that $r' \rightarrow^{\mu} r''$ implies $\phi(r') \rightarrow^{\mu} \phi(r'')$ – actually, we shall write $\phi(r') \rightarrow^{\mu, O} \phi(r'')$.

O-derivations are introduced formally in definition 6.1. Lemma 6.2 confirms that they have the desired property.

6.1. Definition

Let $(p', q') \in \mathbf{R}(p) \times \mathbf{R}(q)$. We define $(p', q') \rightarrow^{\mu, O} (p'', q'')$ if $p' = p''$ and $q' \Rightarrow^{\mu} q''$ and $\mu \in \Lambda(q) - \Lambda(p)$ and $\mu \neq \tau$.

If $K, K' \in \Psi(p, q)$ then define $K \rightarrow^{\mu, O} K'$ iff for all $(p', q') \in K$ there exists $(p'', q'') \in K'$ such that

$$(p', q') \rightarrow^{\mu, O} (p'', q'').$$

6.2. Lemma

Suppose r is a solution and let $r' \in \mathbf{R}_{p,q}(r)$. Suppose $\mu \in \Lambda(q) - \Lambda(p)$ and $\mu \neq \tau$, then $r' \rightarrow^{\mu} r''$ implies $\phi(r') \rightarrow^{\mu, O} \phi(r'')$.

Proof

Suppose $(p', q') \in \phi(r')$. We shall show that there exists q'' such that $(p', q') \rightarrow^{\mu, O} (p', q'') \in \phi(r'')$, which, in virtue of definition 6.1, will conclude the proof.

Now, $(p', q') \in \phi(r')$, so by 5.6

$$(p' \mid r')\backslash A \approx q' \qquad (6.2.1)$$

Since $\mu \in \Lambda(q)$, then by 4.1(4) we have $\mu \notin A \cup \bar{A}$ and since we also have $r \rightarrow^{\mu} r''$ by hypothesis, it follows from 2.2(d) and (e) that

$$(p' \mid r')\backslash A \rightarrow^{\mu} (p' \mid r'')\backslash A \qquad (6.2.2)$$

But, q' is weakly determinate by 5.6 and 4.1(1) and so from (6.2.1) and (6.2.2) and 3.3, we deduce that there exists q'' such that $q' \Rightarrow^{\mu} q''$.

Thus, for some $n, m \geqslant 0$ and q_1, q_2, we have

$$q' \Mapsto^{\tau^m} q_1 \rightarrow^{\mu} q_2 \Rightarrow^{\tau^n} q''$$

whence, by 5.1

$$(p', q', r') \Mapsto^{\tau^m} (p', q_1, r') \rightarrow^{\mu} (p', q_2, r'') \Mapsto^{\tau^n} (p', q'', r''). \qquad (6.2.3)$$

Since $(p', q') \in \phi(r')$, by 5.5 we have

$$(p', q', r') \in \mathbf{R}(p, q, r). \qquad (6.2.4)$$

We may now apply 5.4(1) to (6.2.3) and (6.2.4) to deduce that $(p', q'', r'') \in \mathbf{R}(p, q, r)$ and hence, by 5.5, that $(p', q'') \in \phi(r'')$ as required.

Finally, since $q' \Rightarrow^{\mu} q''$ and $\mu \in \Lambda(q) - \Lambda(p)$, then $(p', q') \rightarrow^{\mu, O} (p', q'')$, by 6.1, and we are done.

C-derivations

The second kind of action available to r is that not belonging $\Lambda(q)$ and not equal to τ. Supposing μ to be such an action, then it must either belong to $A \cup \bar{A}$ or not. If the latter were the case, then $(p \mid r)\backslash A \Rightarrow^{s\mu}$ for some s but we may never have $q \Rightarrow^{s\mu}$ which in view of 2.5(b) and the fact that r is a solution would be a contradiction to assumption 4.2(1). Thus, this second kind of action belongs to $A \cup \bar{A}$.

Suppose $r' \rightarrow^{\mu} r''$, with $\mu \in A \cup \bar{A}$. Consider all pairs $(p', q') \in \phi(r')$. If we have $p' \rightarrow^{\bar{\mu}}$ for *none* of the p', then the μ communication is redundant, it is suppressed and never accepted. This will certainly be the case if $\mu \in A$, since $p' \rightarrow^{\bar{\mu}}$ would contradict assumption 4.1(3). Thus, we only consider elements $\mu \in \bar{A}$.

If the μ communication is capable of being accepted by p', say in a derivation $p' \rightarrow^{\bar{\mu}} p''$, then we may show that $(p'', q') \in \phi(r'')$. This indicates how we might define derivations of this second type in such a way that $r' \rightarrow^{\mu} r''$ implies that $\phi(r') \rightarrow^{\mu, C} \phi(r'')$. Solutions such that C-communications are always accepted by some p' are defined below to be *irredundant*.

6.3. Definition

Let $(p', q') \in \mathbf{R}(p) \times \mathbf{R}(q)$. We define $(p', q') \rightarrow^{\mu, C} (p'', q'')$ if $q' = q''$ and $p' \Rightarrow^{\bar{\mu}} p''$ and $\mu \in \bar{A}$ and $\mu \neq \tau$.

If $K, K' \in \Psi(p, q)$ then define $K \rightarrow^{\mu, C} K'$ iff

(i) there exists $(p', q') \in K$ and $(p'', q'') \in K'$ such that $(p', q') \rightarrow^{\mu, C} (p'', q'')$.

(ii) for all $(p', q') \in K$ if $(p', q') \rightarrow^{\mu, C} (p'', q'')$, for some $(p'', q'') \in \mathbf{R}(p) \times \mathbf{R}(q)$, then $(p'', q'') \in K'$.

Note, incidentally, the difference between 6.3 and 6.1. In the latter, in order that a derivation between the sets exists, it suffices that there is at least one derivations between pairs. If (p', q') does not have a μ, C derivation, this simply indicates that p' is not capable of communicating via μ with r' and the action is prevented by the restriction, since $\mu \in A \cup \bar{A}$.

We may not have an exact analogy with lemma 6.2 in the case of C derivatives because of the possibility mentioned earlier that there are $r' \in \mathbf{R}_{p,q}(r)$ and $\mu \in A$ such that $r' \rightarrow^{\mu}$ but for no $(p', q') \in \phi(r')$ do we have $p' \rightarrow^{\mu}$: the μ communication is offered but never taken up by p. We isolate the well-behaved solutions.

6.4. Definition

r is *irredundant* w.r.t. p and q iff for all $r' \in \mathbf{R}_{p,q}(r)$ and for all $\mu \in \bar{A}$ if $r' \rightarrow^{\mu}$ then for some $(p', q') \in \phi(r')$, there exists p'' such that $p' \rightarrow^{\bar{\mu}} p''$.

We do not lose anything by concentrating on irredundant solutions because of the following result.

6.5. Theorem

$(p \mid X)\backslash A$ has a solution iff it has an irredundant solution.

Proof

Certainly, if there is an irredundant solution then there is a solution. We prove the converse. Let r be a solution. We construct an irredundant solution.

If $r' \in \mathbf{R}(r)$, then define r'° to be the set of all pairs (λ, r'') such that

(1) $r' \rightarrow^{\lambda} r''$

(2) If $\lambda \in \bar{A}$, then there exists $(p', q') \in \phi(r')$ such that $p' \rightarrow^{\bar{\lambda}}$.

Define

$$r'_{irr} \Leftarrow \sum_{(\lambda, r'') \in r'^{\circ}} \lambda . r''_{\mathrm{irr}}.$$

By 2.2(d) and 2.2(e) and the definition of r'_{irr}, if $(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A$ then $(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A$. We shall prove the converse.

Suppose $(p', q') \in \phi(r')$ and suppose

$$(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A.$$

By 2.2(d), there are three cases to consider.

The first is that $r' = r''$ and $p' \rightarrow^{\lambda} p''$ and $\lambda \notin A \cup \bar{A}$. In this case, from 2.2(d) and 2.2(e) we may readily deduce that $(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A$.

In the second case $r' \rightarrow^{\lambda} r''$ and $p' = p''$ and $\lambda \notin A \cup \bar{A}$, and so $r'_{\mathrm{irr}} \rightarrow^{\lambda} r''_{\mathrm{irr}}$ and hence, by 2.2 again,

$$(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A.$$

The third case is that $\lambda \equiv \tau$ and for some $\mu \in \Delta - \{\tau\}$, $p' \rightarrow^{\bar{\mu}} p''$ and $r' \rightarrow^{\mu} r''$. If $\mu \notin \bar{A}$ then $r'_{\mathrm{irr}} \rightarrow^{\mu} r''_{\mathrm{irr}}$. But if $\mu \in \bar{A}$, then by hypothesis, there exists $(\hat{p}, \hat{q}) \in \phi(r')$ such that $\hat{p} \rightarrow^{\mu}$, and so $(\mu, r'') \in r'^{\circ}$. Hence in both cases $r'_{\mathrm{irr}} \rightarrow^{\mu} r''_{\mathrm{irr}}$ and so, again by 2.2(d) and (e), $(p \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A$.

We have shown that if $(p', q') \in \phi(r')$ then

$$(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A \text{ iff } (p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A. \tag{6.5.1}$$

Next, we shall show that r_{irr} is a solution. Evidently $\Lambda(r_{\mathrm{irr}}) \subseteq \Lambda(r)$ and so $\Lambda(r_{\mathrm{irr}}) \cap \Lambda(p) \subseteq \Lambda(r) \cap \Lambda(p) \subseteq \{\tau\}$ giving 4.2(2). For 4.2(1), we shall actually show that if $(p', q') \in \phi(r')$ then

$$(p \mid r)\backslash A \sim (p \mid r_{\mathrm{irr}})\backslash A. \tag{6.5.2}$$

For, by 2.8(3), (6.5.2) implies that $(p \mid r)\backslash A \approx (p \mid r_{\mathrm{irr}})\backslash A$, which, together with 2.6(2) and the fact that $(p \mid r)\backslash A \approx q$ entails that $(p \mid r_{\mathrm{irr}})\backslash A \approx q$ as required.

Let us prove (6.5.2). We shall prove that if

$$(p', q') \in \phi(r'),$$

then for all k, $(p' \mid r')\backslash A \sim_k (p' \mid r'_{\mathrm{irr}})\backslash A$.

This is certainly true for $k = 0$, by 2.7(a). Let us assume it for k and prove it for $k+1$.

Suppose that $(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A$, then by (6.5.1) $(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A$. There are two cases to consider.

The first case is $\lambda = \tau$. If we define $q'' = q'$, then in this case, by 5.1(a)(i) $(p', q', r') \rightarrow^{\tau} (p'', q'', r'')$ and so $(p'', q'') \in \phi(r'')$.

In the second case $\lambda \neq \tau$. Now, by assumption

$$(p', q') \in \phi(r')$$

and so by 5.5 and 5.4(1), $(p' \mid X)\backslash A \approx q'$ is an interface equation with solution r' and in particular q' is weakly determinate. Thus, $(p' \mid r')\backslash A \approx q'$, $(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A$ and q' is weakly determinate and so by 3.3, there exists q'' such that $q' \Rightarrow^{\mu} q''$ and $(p'' \mid X)\backslash A \approx q''$. Thus there exist $m, n \geqslant 0$ such that $q' \mapsto^{\tau^m} q_1 \rightarrow^{\mu} q_2 \Rightarrow^{\tau^n} q''$. Thus, by 5.1,

$$(p', q', r') \mapsto^{\tau^m} (p', q_1, r') \rightarrow^{\mu} (p'', q_2, r'') \mapsto^{\tau^n} (p'', q'', r''). \tag{6.5.3}$$

Since $(p', q') \in \phi(r')$ it follows that $(p', q', r') \in \mathbf{R}(p, q, r)$, and this, together with (6.5.3) and 5.4(2) entails that $(p'', q'', r'') \in \mathbf{R}(p, q, r)$. Thus, by 5.5, $(p'', q'') \in \phi(r'')$.

Thus, in both cases, $(p'', q'') \in \phi(r'')$ and so we may apply the induction hypothesis and conclude that $(p'' \mid r'')\backslash A \sim_k (p'' \mid r''_{\mathrm{irr}})\backslash A$.

From the above and (6.5.1) we may deduce that if $(p', q') \in \phi(r')$ then if $(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A$ then $(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A$ with $(p'' \mid r'')\backslash A \sim_k (p''' \mid r'_{\mathrm{irr}})\backslash A$.

It remains to show that if $(p', q') \in \phi(r')$ then if $(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A$ then $(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A$ with $(p'' \mid r'')\backslash A \sim_k (p''' \mid r'_{\mathrm{irr}})\backslash A$. Then, by 2.7,

$$(p'' \mid r'')\backslash A \sim_{k+1} (p''' \mid r'_{\mathrm{irr}})\backslash A,$$

completing the induction step.

But by (6.5.1), we already know that if $(p', q') \in \phi(r')$ and $(p' \mid r'_{\mathrm{irr}})\backslash A \rightarrow^{\lambda} (p'' \mid r''_{\mathrm{irr}})\backslash A$ then

$$(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A.$$

And we have shown that if $(p', q') \in \phi(r')$ and

$$(p' \mid r')\backslash A \rightarrow^{\lambda} (p'' \mid r'')\backslash A$$

then $(p'' \mid r'')\backslash A \sim_k (p'' \mid r''_{\mathrm{irr}})\backslash A$.

We have shown that r_{irr} is a solution. It remains to show that it is irredundant. We first show that

$$(p', q', r') \in \mathbf{R}(p, q, r) \text{ iff } (p', q', r'_{\mathrm{irr}}) \in \mathbf{R}(p, q, r_{\mathrm{irr}}). \tag{6.5.4}$$

We argue by induction that if $s \in \Lambda^*$ and $lnth(s) = n$† then $(p, q, r) \mapsto^{s} (p', q', r')$ iff $(p, q, r_{\mathrm{irr}}) \mapsto^{s} (p', q', r'_{\mathrm{irr}})$.

This is clearly true when $n = 0$. Suppose that $s' = \lambda s$, where $lnth(s) = n$. We shall show that if $(p, q, r) \mapsto^{s'} (p', q', r')$ then $(p, q, r_{\mathrm{irr}}) \mapsto^{s'} (p', q', r'_{\mathrm{irr}})$.

By 5.1(d), $(p, q, r) \rightarrow^{\lambda} (p'', q'', r'') \mapsto^{s} (p', q', r')$ for some (p'', q'', r''). We shall show that

$$(p, q, r) \rightarrow^{\lambda} (p'', q'', r'') \text{ implies } (p, q, r_{\mathrm{irr}}) \rightarrow^{\lambda} (p'', q'', r''_{\mathrm{irr}}) \tag{6.5.5}$$

and since $(p'', q'', r'') \mapsto^{s} (p', q', r')$ with s of length n, then we may invoke the induction hypothesis to deduce that

$$(p'', q'', r''_{\mathrm{irr}}) \mapsto^{s} (p', q', r'_{\mathrm{irr}}) \tag{6.5.6}$$

(6.5.5) and (6.5.6), together with 5.1(d) entails that $(p, q, r_{\mathrm{irr}}) \mapsto^{s'} (p', q', r'_{\mathrm{irr}})$ as required.

In exactly the same manner, in order to prove that if $(p, q, r_{\mathrm{irr}}) \mapsto^{s'} (p', q', r'_{\mathrm{irr}})$ then $(p, q, r) \mapsto^{s'} (p', q', r')$ we need to show the converse of (6.5.5), namely that

$$(p, q, r_{\mathrm{irr}}) \rightarrow^{\lambda} (p'', q'', r''_{\mathrm{irr}}) \text{ implies } (p, q, r) \rightarrow^{\lambda} (p'', q'', r''). \tag{6.5.7}$$

But in view of 5.1(a) and (b), (6.5.5) and (6.5.7) are easy consequences of (6.5.1) and so (6.5.4) holds.

Finally, suppose $r'_{\mathrm{irr}} \in \mathbf{R}_{p,q}(r_{\mathrm{irr}})$ and suppose $r'_{\mathrm{irr}} \rightarrow^{\mu} r''_{\mathrm{irr}}$ with $\mu \in A$. We must show that there exists

$$(p', q') \in \phi(r'_{\mathrm{irr}})$$

such that $p' \rightarrow^{\mu}$.

By definition, $(\mu, r'') \in r'^{\circ}$ and so $r' \rightarrow^{\mu} r''$ and for some $(p', q') \in \phi(r'), p' \rightarrow^{\mu}$. But by (6.5.4) and 5.5, $\phi(r') = \phi(r'_{\mathrm{irr}})$ and so we are done.

We may now state and prove the analogue of lemma 6.3.

† $lnth(s) = n$ denotes the length of the string s.

6.6. Lemma

Suppose r is a solution which is irredundant w.r.t. p and q. Let $r' \in \mathbf{R}_{p,q}(r)$ and suppose $\mu \in A$ and $\mu \neq \tau$ then $r' \rightarrow^{\mu} r''$ implies $\phi(r') \rightarrow^{\mu, C} \phi(r'')$.

Proof

Since $r' \rightarrow^{\mu} r''$, $\mu \in \bar{A}$ and r is irredundant, there exists $(p', q') \in \phi(r')$ such that $p \rightarrow^{\bar{\mu}} p''$, by 6.4. By 6.3, $(p', q') \rightarrow^{\mu, C} (p'', q')$.

To complete the proof, we show that for every $(p', q') \in \phi(r')$, if $(p', q') \rightarrow^{\mu, C} (p'', q')$ then $(p'', q') \in \phi(r'')$.

Indeed, since $r \rightarrow^{\mu} r''$ and $p \rightarrow^{\bar{\mu}} p''$ then by 2.2(d) $(p' \mid r') \backslash A \rightarrow^{\tau} (p'' \mid (r'') \backslash A$, and so by 5.1(a)

$$(p', q', r') \rightarrow^{\tau} (p'', q', r''). \tag{6.6.1}$$

Since $(p', q') \in \phi(r')$, it follows from 5.5 that

$$(p', q', r') \in \mathbf{R}(p, q, r). \tag{6.6.2}$$

From (6.6.1), (6.6.2) and 5.4(1), we obtain

$$(p'', q', r'') \in \mathbf{R}(p, q, r).$$

Thus, by 5.5, $(p'', q') \in \phi(r'')$ as required.

τ-derivations

Finally, we look at the third type of r derivation, that involving τ actions. The definition and corresponding lemma are straightforward enough to require no gloss.

6.7. Definition

If $K, K' \in \Psi(p, q)$, then define $K \rightarrow^{\tau} K'$ if $K \subseteq K'$.

6.8. Lemma

Suppose r is a solution and suppose $r' \in \mathbf{R}_{p,q}(r)$ then $r' \rightarrow^{\tau} r''$ implies $\phi(r') \rightarrow^{\tau} \phi(r'')$.

Proof

Suppose $(p', q') \in \phi(r')$. We shall show that $(p', q') \in \phi(r'')$.

First note that since $r' \rightarrow^{\tau} r''$ we have, by 2.2(d) and (e)

$$(p' \mid r') \backslash A \rightarrow^{\tau} (p' \mid r'') \backslash A$$

and so by 5.1(a)(i)

$$(p', q', r') \rightarrow^{\tau} (p', q', r''). \tag{6.8.1}$$

Since $(p', q') \in \phi(r')$, by 5.5 we have

$$(p', q', r') \in \mathbf{R}(p, q, r).$$

From this, 5.3 and (6.8.1) it follows that

$$(p', q', r'') \in \mathbf{R}(p, q, r).$$

Thus, by 5.5 $(p', q') \in \phi(r'')$.

Since we are dealing with ⇒ rather than → derivations in some of our work, we will find it useful to construct analogues of the results of this section.

6.9. Definition

Let $K, K' \in \Psi(p, q), \mu \in \Delta - \{\tau\}$ and let X denote either C or O. Then $K \Rightarrow^{\mu, X} K'$ iff there exists

$$K_1, \ldots, K_m, K'_1, \ldots, K'_n \in \Psi(p, q),$$

with $m, n \geqslant 0$ such that $K = K_1, K' = K'_n$ and

$$K_i \rightarrow^{\tau} K_{i+1}, i < m$$

$$K_m \rightarrow^{\mu, X} K'_1$$

$$K'_i \rightarrow^{\tau} K'_{i+1}, i < n.$$

6.10. Proposition

(1) Suppose r is a solution and let $r' \in \mathbf{R}_{p,q}(r)$. Suppose $\mu \in \Lambda(q) - \Lambda(p)$ and $\mu \neq \tau$, then $r' \Rightarrow^{\mu} r''$ implies $\phi(r') \Rightarrow^{\mu, O} \phi(r'')$.

(2) Suppose r is an irredundant solution w.r.t. p and q. Let $r' \in \mathbf{R}_{p,q}(r)$ and suppose $\mu \in \bar{A}$, then $r' \Rightarrow^{\mu} r''$ implies $\phi(r') \Rightarrow^{\mu, C} \phi(r'')$.

Proof

(1) is by 6.2 and 6.8 and (2) is by 6.6 and 6.8.

7. NECESSARY CONDITIONS FOR SOLVABILITY

We have now shown that if a solution r exist, then it determines a (p, q)-system $\phi(\mathbf{R}_{p,q}(r))$ which we may equip with a derivation structure designed to reflect that of r. We shall see in Section 8 that from such a system and derivation structure, we may construct CCS equations. Not all systems determine equations which solve the interface equation, however and so in this section, we isolate properties that r must possess in order to be a solution and translate these properties into properties of $\phi(\mathbf{R}_{p,q}(r))$. This gives us necessary conditions for the existence of solutions. In Section 8 we shall show them to be sufficient. The two properties are those of *I*- and *O*-completeness, defined in 7.1 and 7.3 and shown to hold in lemmas 7.2 and 7.4.

7.1. Definition

Let $\mathbf{S}$ be a (p, q)-system and suppose $K \in \mathbf{S}$. K will be said to be *I-complete* iff for all $(p', q') \in K$, if $p' \rightarrow^{\mu}$ and $\mu \notin A$, and $\mu \neq \tau$ then $q' \Rightarrow^{\mu}$.

7.2. Lemma

Let r be a solution and suppose $K \in \phi(\mathbf{R}_{p,q}(r))$, then K is *I-complete*.

Proof

Let $K \in \phi(\mathbf{R}_{p,q}(r))$, then $K = \phi(r')$ for some $r' \in \mathbf{R}_{p,q}(r)$. Let $(p', q') \in K$, then by 5.6, $(p' \mid r') \backslash A \approx q'$ with q' weakly determinate.

Since $p' \rightarrow^{\mu}$, it follows from 4.1(3) that $\mu \notin \bar{A}$. Since, by hypothesis $\mu \notin A$, we must have $\mu \notin A \cup \bar{A}$. Since $p' \rightarrow^{\mu}$ and $\mu \notin A \cup \bar{A}$ it follows from 2.2(d) and 2.2(e) that $(p' \mid r') \backslash A \rightarrow^{\mu}$.

But, we now have shown that $(p' \mid r') \backslash A \rightarrow^{\mu}$ and $(p' \mid r') \backslash A \approx q'$ and q' is weakly determinate and so we may use 3.3 to infer that $q' \Rightarrow^{\mu}$.

7.3. Definition

Let **S** be a (p, q)-system and suppose $K \in \mathbf{S}$. K will be said to be *O-complete* w.r.t. S iff for all $(p', q') \in K$, if $\mu \neq \tau$ and $q' \rightarrow^{\mu} q''$ then there exists $n \geqslant 0$, $\mu_1, \ldots, \mu_n \in A$, $p_0, \ldots, p_n, p'' \in \mathbf{R}(p)$ and $K_0, \ldots, K_n, K'' \in \mathbf{S}$, such that

(1) $p' = p_0 \Rightarrow^{\mu_1} p_1 \Rightarrow^{\mu_2} \ldots \Rightarrow^{\mu_{n-1}} p_{n-1} \Rightarrow^{\mu_n} p_n$

(2) $K = K_0 \Rightarrow^{\bar{\mu}_1, C} K_1 \Rightarrow^{\bar{\mu}_2, C} \ldots \Rightarrow^{\bar{\mu}_{n-1}, C} K_{n-1} \Rightarrow^{\bar{\mu}_n, C} K_n$.

(3) Either

(a) $p_n \rightarrow^{\mu} p''$ and $K = K''$ and $(p'', q'') \in K''$ or

(b) $p_n = p''$ and $K_n \rightarrow^{\mu, O} K''$ and $(p'', q'') \in K''$.

7.4. Lemma

Let r be an irredundant solution and suppose $K \in \phi(\mathbf{R}_{p,q}(r))$ then K is *O-complete* w.r.t. $\phi(\mathbf{R}_{p,q}(r))$.

Proof

Since $K \in \phi(\mathbf{R}_{p,q}(r))$, there exists $r' \in \mathbf{R}_{p,q}(r)$ such that $K = \phi(r')$. Let $(p', q') \in K$, then by 5.6, $(p' \mid r')\backslash A \approx q'$ with q' weakly determinate. By 3.6, $(p' \mid r')\backslash A$ is weakly determinate.

Now, suppose $q' \rightarrow^{\mu} q''$ with $\mu \neq \tau$. Since we also have $q' \approx (p' \mid r')\backslash A$ and $(p' \mid r')\backslash A$ is weakly determinate, then by 3.3, $(p' \mid r')\backslash A \Rightarrow^{\mu} (\hat{p} \mid \hat{r})\backslash A$, some $\hat{p}, \hat{r}$.

Now $(p' \mid r')\backslash A \Rightarrow^{\mu} (\hat{p} \mid \hat{r})\backslash A$ and so by 2.2(d), 2.2(e) and 2.4 there exists $n \geqslant 0$ and $\mu_1, \ldots, \mu_{n-1} \in A, p_0, \ldots, p_{n+1} \in \mathbf{R}(p)$ and $r_0, \ldots, r_{n+1} \in \mathbf{S}$, such that

(1) $p' = p_0 \Rightarrow^{\mu_1} p_1 \Rightarrow^{\mu_2} \ldots \Rightarrow^{\mu_{n-1}} p_{n-1} \Rightarrow^{\mu_n} p_n$, giving (1) of 7.3.

(2) $r' = r_0 \Rightarrow^{\bar{\mu}_1} r_1 \Rightarrow^{\bar{\mu}_2} \ldots \Rightarrow^{\bar{\mu}_1} r_{n-1} \Rightarrow^{\bar{\mu}_n} r_n$

(3) $(p'_n \mid r'_n)\backslash A \rightarrow^{\mu} (p_{n+1} \mid r_{n+1})\backslash A$

(4) $p_{n+1} \Rightarrow^{\Omega} \hat{p}$ and $r_{n+1} \Rightarrow^{\Omega} \hat{r}$.

Define $K'' = \phi(r'')$ and define $K_i = \phi(r_i)$ for

$$i = 0, \ldots, n+1.$$

By 6.10(2)

$\mu_i \in \bar{A}$, by 4.1(2) and 4.1(3) and so by 6.10(2)

$$K = K_0 \Rightarrow^{\bar{\mu}_1, C} K_1 \Rightarrow^{\bar{\mu}_2, C} \ldots \Rightarrow^{\bar{\mu}_{n-1}, C} K_{n-1} \Rightarrow^{\bar{\mu}_n, C} K_n. \quad (7.4.1)$$

Furthermore, $K = \phi(r') = \phi(r_0) = K_0$, so we have (2) of 7.3.

Let us now consider (3). Since $\mu \neq \tau$ by hypothesis, by 2.2(d) there are two cases to consider.

Case 1. $p_n \rightarrow^{\mu} p_{n+1}$ and $r_n = r_{n+1}$. Let $p'' = p_{n+1}$ and let $K'' = \phi(r_n)$. By (1) and (2),

$$(p' \mid r')\backslash A \Rightarrow^{\Omega} (p_n \mid r_n)\backslash A. \quad (7.4.2)$$

Since $q' \rightarrow^{\mu}$, we have $\mu \in \Lambda(q)$ and hence $\mu \notin A \cup \bar{A}$, by 4.1(4). Since $p_n \rightarrow^{\mu} p_{n+1} = p''$ and $\mu \notin A \cup \bar{A}$, then by 2.2(d) and 2.2(e),

$$(p_n \mid r_n)\backslash A \rightarrow^{\mu} (p'' \mid r_n)\backslash A. \quad (7.4.3)$$

From (7.4.2) and (7.4.3), it follows that

$$(p' \mid r')\backslash A \Rightarrow^{\mu} (p'' \mid r_n)\backslash A.$$

Since $q' \rightarrow^{\mu} q''$ by hypothesis, it follows from 5.16 that $(p'', q'') \in \phi(r_n) = K''$.

Thus $p_n \rightarrow^{\mu} p''$ and $K'' = \phi(r_n) = K_n$ and

$$(p'', q'') \in \phi(r_n) = K''.$$

This gives (3)(a) of 7.3.

Case 2. $p_n = p_{n+1}$ and $r_n \rightarrow^{\mu} r_{n+1}$. Again, since $q' \rightarrow^{\mu} q''$, it follows that $\mu \in \Lambda(q)$. Since also $r_n \rightarrow^{\mu} r_{n+1}$ it follows that $\mu \in \Lambda(r)$. Since, by 4.2(2), $\Lambda(p) \cap \Lambda(r) \subseteq \{\tau\}$ and $\mu \neq \tau$ by hypothesis, it follows that $\mu \in \Lambda(q) - \Lambda(p)$. We may now appeal to lemma 6.2 to conclude that $K_n \rightarrow^{\mu, O} K_{n+1}$. Thus, if we define $K'' = K_{n+1}$ and $p'' = p_n$, then we have $p'' = p_n$ and $K_n \rightarrow^{\mu, O} K''$.

Thus, $p_n = p''$ and $K_n \rightarrow^{\mu, O} K''$. Again, since $\mu \in \Lambda(q)$, we have from 4.4(4) that $\mu \notin A \cup \bar{A}$ and hence, from 2.2(d) and 2.2(e) and the fact that $r_n \rightarrow^{\mu} r_{n+1} = r''$, we infer that

$$(p_n \mid r_n)\backslash A \rightarrow^{\mu} (p_n \mid r'')\backslash A. \quad (7.4.4)$$

From (7.4.2) and (7.4.4), it follows that

$$(p' \mid r')\backslash A \Rightarrow^{\mu} (p_n \mid r'')\backslash A.$$

Since $q' \rightarrow^{\mu} q''$ by hypothesis, it follows from 5.16 that $(p'', q'') = (p_n, r'') \in \phi(r'') = K''$.

We have established that $p'' = p_n, K_n \rightarrow^{\mu, O} K''$ and $(p'', q'') \in K''$. This gives (3)(b) of 7.3.

The two lemmas inspire the following definition, which plays a crucial part in our analysis.

7.5. Definition

Let $\mathbf{S} \subseteq \Psi(p, q)$ and let $K \in \mathbf{S}$. We will say that K *compromises* **S** iff either K is not I-complete or K is not O-complete w.r.t. **S**.

7.6. Lemma

Suppose r is an irredundant solution and let

$$\mathbf{S} = \phi(\mathbf{R}_{p,q}(r)),$$

then **S** is a (p, q)-system and for all $r' \in \mathbf{R}_{p,q}(r)$, $\phi(r')$ does not compromise **S**. Furthermore, there exists $K \in \mathbf{R}_{p,q}(r)$ such that $(p, q) \in K$.

Proof

S is a (p, q)-system by 5.14. From 7.2 and 7.4, for all $r' \in \mathbf{R}_{p,q}(r)$, $\phi(r')$ is both I-complete and O-complete w.r.t. **S** and hence does not compromise **S**. If we take $K = \phi(r)$, then $(p, q) \in K$.

Let us give a name to the sort of systems unearthed in 7.6.

7.7. Definition

$\mathbf{S} \subseteq \Psi(p, q)$ is *uncompromised* iff

(1) No $K \in \mathbf{S}$ compromises **S**.

(2) For some $K \in \mathbf{S}$: $(p, q) \in K$.

The following is an easy consequence of 7.6.

7.8. Theorem

Suppose a solution exists, then $\Psi(p, q)$ contains an uncompromised system.

Proof

By 6.5, there exists an irredundant solution r. Let $\mathbf{S} = \phi(\mathbf{R}_{p,q}(r))$ and apply lemma 7.6.

8. SUFFICIENT CONDITIONS FOR SOLVABILITY

In Section 5, we associated 'states' of r with subsets of $\mathbf{R}(p) \times \mathbf{R}(q)$. A derivation structure, imposed on such sets of subsets turned out to mimic that of r. We may use this idea to associate to each system $\mathbf{S}$ a set of equations as follows.

8.1. Definition

Let $\mathbf{S} \subseteq \Psi(p, q)$.

Let K° be the set of all pairs (μ, K'), where $K \to^{\mu, X} K'$, $X \in \{C, O\}$ and $K' \in \mathbf{S}$.

Now define, for each $K' \in \mathbf{S}$

$$r_s(K') \Leftarrow \sum_{(\mu, K'') \in K^{\circ}} \mu . r_{\mathbf{S}}(K'').$$

Note that $r_{\mathbf{S}}(K)$ is *rigid*, that is, there are no τ derivations. This is because, by 6.1 and 6.3, μ, O and μ, C derivations are only defined when $\mu \neq \tau$.

We show that uncompromised systems give rise to solutions. The key idea is that

$$\text{if } (p', q') \in K \in \mathbf{S} \text{ then } (p' \mid r_{\mathbf{S}}(K)) \backslash A \approx q'.$$

To prove this, we assume as an induction hypothesis that if $(p', q') \in K \in \mathbf{S}$, then $(p' \mid r_{\mathbf{S}}(K)) \backslash A \approx_n q'$, and show that the implication holds for $n+1$.

To do this, it suffices to show that

(I) if $(p' \mid r_{\mathbf{S}}(K)) \backslash A \Rightarrow^{s} (p'' \mid r_{\mathbf{S}}(K')) \backslash A$ then there exists q'' such that $q' \Rightarrow^{s} q''$ and $(p'', q'') \in K'$ (for then, $(p'' \mid r_{\mathbf{S}}(K')) \backslash A \approx_n q''$, by induction);

(II) if $q' \Rightarrow^{s} q''$ then there exists p'' such that $(p' \mid r_{\mathbf{S}}(K)) \backslash A \Rightarrow^{s} (p'' \mid r_{\mathbf{S}}(K')) \backslash A$. and $(p'', q'') \in K'$.

These two facts are proved in propositions 8.5 and 8.6 below. The intervening results establish similar properties in the case of a derivation involving a single action. Proposition 8.2 deals with τ actions and propositions 8.3 and 8.4 deal with non-τ actions.

8.2. Proposition

Suppose $\mathbf{S} \subseteq \Psi(p, q)$ and suppose $K \in \mathbf{S}$ and $(p', q') \in K$, then

(1) If $(p' \mid r_{\mathbf{S}}(K)) \backslash A \to^{\tau} (p'' \mid r_{\mathbf{S}}(K')) \backslash A$ then $(p'', q') \in K'$.
(2) If $q' \to^{\tau} q''$, then $(p', q'') \in K$.

Proof of (1)

By 2.2(d), we have three cases to consider.

Case (a): $p' \to^{\tau} p''$ and $K' = K$. By 5.7(b),

$$(p', q') \to^{\tau, P} (p'', q').$$

Thus, by 5.11, $(p', q') \to_{I\tau} (p'', q')$. Since $K \in \Phi(p, q)$ and $(p', q') \in K$ and $(p', q') \to_{I\tau} (p'', q')$ it follows by 5.11 and 5.13 that $(p'', q') \in K = K'$.

Case (b): $p' = p''$ and $r_{\mathbf{S}}(K) \to^{\tau} r_{\mathbf{S}}(K')$. This is impossible, since 8.1 entails that $r_{\mathbf{S}}(K)$ is rigid.

Case (c): For some $\mu \neq \tau$, $p' \to^{\mu} p''$ and $r_{\mathbf{S}}(K) \to^{\bar{\mu}} r_{\mathbf{S}}(K')$. By 8.1, either $K \to^{\bar{\mu}, C} K'$ or $K \to^{\bar{\mu}, O} K'$. Since $\mu \in \Lambda(p)$, we cannot have $\bar{\mu} \in \Lambda(q)$ by 4.1(2) and thus we cannot have $K \to^{\bar{\mu}, O} K'$. Thus, $K \to^{\bar{\mu}, C} K'$. In particular, $\mu \in A$, by 6.3.

From $p' \to^{\mu} p''$ and $\mu \in A$, we get $(p', q') \to^{\bar{\mu}, C} (p'', q')$ by 6.3, and since we also have $(p', q') \in K$ and $K \to^{\bar{\mu}, C} K'$, it follows from 6.3(ii) that $(p'', q') \in K'$, as required.

Proof of (2)

By 5.7(c), $(p', q') \to^{\tau, Q} (p', q'')$. Thus, by 5.11,

$$(p', q') \to_{I\tau} (p', q'').$$

Since $K \in \Psi(p, q)$ and $(p', q') \in K$ and $(p', q') \equiv_{I\tau} (p', q'')$, it follows by 5.11 and 5.13 that $(p', q'') \in K = K'$.

8.3. Proposition

Suppose $\mathbf{S} \subseteq \Psi(p, q)$ is uncompromised and suppose $K \in \mathbf{S}$ and $(p', q') \in K$. If $\mu \in \Delta - \{\tau\}$ and

$$(p' \mid r_{\mathbf{S}}(K)) \backslash A \to^{\mu} (p'' \mid r_{\mathbf{S}}(K')) \backslash A$$

then there exists q'' such that $q' \Rightarrow^{\mu} q''$ with $(p'', q'') \in K'$.

Proof

By 2.2(d), there are two cases to consider

(1) $p' \to^{\mu} p''$ and $K = K'$. Since $(p', q') \in K$ and $K \in \mathbf{S}$ with $\mathbf{S}$ uncompromised, it follows from 7.4(1), that $q' \Rightarrow^{\mu} q''$. Thus $(p', q') \to_{I\tau} (p'', q'')$ and so $(p'', q'') \in K'$.

(2) $p' = p''$ and $r_{\mathbf{S}}(K) \to^{\mu} r_{\mathbf{S}}(K')$. By 8.1, we have either $K \to^{\mu, C} K'$ or $K \to^{\mu, O} K'$. But if $K \to^{\mu, C} K'$, then by 6.3, $\mu \in \bar{A}$ and if this were the case, then by 2.2(d) and 2.2(e), we could not have $(p' \mid r_{\mathbf{S}}(K)) \backslash A \to^{\mu} (p'' \mid r_{\mathbf{S}}(K')) \backslash A$. Thus $K \to^{\mu, O} K'$.

It follows from 6.1 that there exists $(p''', q'') \in K'$ such that $(p', q') \to^{\mu, O} (p''', q'')$ and in particular that $p' = p'''$ and $q' \Rightarrow^{\mu} q''$. Thus, $q' \Rightarrow^{\mu} q''$ with $(p'', q'') = (p', q'') = (p''', q'') \in K'$.

8.4. Proposition

Suppose $\mathbf{S} \subseteq \Psi(p, q)$ is uncompromised and suppose $K \in \mathbf{S}$ and $(p', q') \in K$. Let $\mu \in \Delta - \{\tau\}$ then if $q' \to^{\mu} q''$ then $(p' \mid r_{\mathbf{S}}(K)) \backslash A \Rightarrow^{\mu} (p'' \mid r_{\mathbf{S}}(K')) \backslash A$ with $(p'', q'') \in K'$.

Proof

Since $\mathbf{S} \subseteq \Psi(p, q)$ is uncompromised, it follows that K is O-complete w.r.t. $\mathbf{S}$, by 7.5. Thus, by 7.3, there exists $n \geqslant 0$, $\mu_1, \ldots, \mu_n \in A$, $p_0, \ldots, p_n, p'' \in \mathbf{R}(p)$ and $K_0, \ldots, K_n, K'' \in \mathbf{S}$, such that

(1) $p' = p_0 \Rightarrow^{\mu_1} p_1 \Rightarrow^{\mu_2} \ldots \Rightarrow^{\mu_{n-1}} p_{n-1} \Rightarrow^{\mu_n} p_n$
(2) $K = K_0 \Rightarrow^{\bar{\mu}_1, C} K_1 \Rightarrow^{\bar{\mu}_2, C} \ldots \Rightarrow^{\bar{\mu}_{n-1}, C} K_{n-1} \Rightarrow^{\bar{\mu}_n, C} K_n$.
(3) Either
 (a) $p_n \to^{\mu} p''$ and $K_n = K''$ and $(p'', q'') \in K''$ or
 (b) $p_n = p''$ and $K_n \to^{\mu, O} K''$ and $(p'', q'') \in K''$.

From (2) and 8.1, we have.

$$r_{\mathbf{S}}(K) = r_{\mathbf{S}}(K_0) \Rightarrow^{\bar{\mu}_1} r_{\mathbf{S}}(K_1) \Rightarrow^{\bar{\mu}_2} \ldots \Rightarrow^{\bar{\mu}_{n-1}} r_{\mathbf{S}}(K_{n-1}) \Rightarrow^{\bar{\mu}_n} r_{\mathbf{S}}(K_n). \quad (8.4.1)$$

From (1) and (8.4.1), using 2.2(d) and 2.2(e), we may see that

$$(p' \mid r_{\mathbf{S}}(K)) \backslash A = (p_0 \mid r_{\mathbf{S}}(K_0)) \backslash A \Rightarrow^{\Omega} (p_n \mid r_{\mathbf{S}}(K_n)) \backslash A. \quad (8.4.2)$$

Now, let us consider the two cases (3)(a) and (3)(b) above.

In case (3)(a), $p_n \to^{\mu} p''$ and $K_n = K''$ and $(p'', q'') \in K''$. Since $q' \to^{\mu}$, we have $\mu \notin A \cup \bar{A}$, by 4.1(4). Thus, $p_n \to^{\mu} p''$ and $\mu \notin A \cup A$ and so from 2.2(d), we obtain

$$(p_n \mid r_{\mathbf{S}}(K_n)) \backslash A \to^{\mu} (p_n \mid r_{\mathbf{S}}(K_n)) \backslash A = (p'' \mid r_{\mathbf{S}}(K'')) \backslash A \quad (8.4.3)$$

with $(p'', q'') \in K''$. Therefore, by (8.4.2) and (8.4.3) we obtain

$$(p' \mid r_{\mathbf{S}}(K)) \backslash A \Rightarrow^{\mu} (p'' \mid r_{\mathbf{S}}(K'')) \backslash A \quad (8.4.4)$$

with $(p'', q'') \in K''$.

In case (3)(b), $p_n = p''$ and $K_n \to^{\mu, O} K''$ and $(p'', q'') \in K''$. By 8.1, $r_{\mathbf{S}}(K_n) \to^{\mu} r_{\mathbf{S}}(K'')$. From this, together, with 2.2(d) and 2.2(e) and the fact established above that $\mu \notin A \cup \bar{A}$, we obtain

$$(p_n \mid r_S(K_n))\backslash A \rightarrow^{\mu} (p_n \mid r_S(K''))\backslash A = (p'' \mid r_S(K''))\backslash A \quad (8.4.5)$$

with $(p'', q'') \in K''$. Therefore, from (8.4.2) and (8.4.3) we obtain

$$(p' \mid r_S(K))\backslash A \Rightarrow^{\mu} (p'' \mid r_S(K''))\backslash A \qquad (8.4.6)$$

with $(p'', q'') \in K''$.

Thus, in both cases, $(p' \mid r_S(K))\backslash A \Rightarrow^{\mu} (p'' \mid r_S(K''))\backslash A$ with $(p'', q'') \in K''$. This completes the proof.

We may now establish the two facts (I) and (II) that we met at the beginning of this section.

8.5. Proposition

Suppose $\mathbf{S} \subseteq \Psi(p, q)$ is uncompromised and suppose $K \in \mathbf{S}$ and $(p', q') \in K$. Suppose,

$$(p' \mid r_S(K))\backslash A \Rightarrow^{s} (p'' \mid r_S(K'))\backslash A$$

then there exists q'' such that $q' \Rightarrow^{s} q''$ with $(p'', q'') \in K'$.

Proof

By 2.4, it suffices to show that if

$$(p' \mid r_S(K))\backslash A \Mapsto^{s} (p'' \mid r_S(K'))\backslash A$$

then there exists s' and q'' such that $q' \Rightarrow^{s'} q''$ with $s' = s|_\tau$ and $(p'', q'') \in K'$. We argue by induction on $lnth(s)$.

If $lnth(s) = 0$ then $s = \Omega$ and we have $p' = p''$ and $K = K'$. We may take $s' = \Omega$ and $q'' = q'$. Accordingly, $(p'', q'') = (p', q') \in K = K'$.

Now suppose the proposition is true when $lnth(s) = n$ and suppose $(p' \mid r_S(K))\backslash A \Mapsto^{s} (p'' \mid r_S(K'))\backslash A$ where $lnth(s) = n+1$. There exists $\hat{s} \in \Delta^*$ and $\lambda \in \Delta$ such that $s = \lambda\hat{s}$. Thus $lnth(\hat{s}) = n$.

By 2.4, there exists $\hat{p}$, $\hat{K}$ such that

$$(p' \mid r_S(K))\backslash A \rightarrow^{\lambda} (\hat{p} \mid r_S(\hat{K}))\backslash A \Mapsto^{\hat{s}} (p'' \mid r_S(K'))\backslash A.$$

If $\lambda = \tau$, then by 8.2(1), $(\hat{p}, q') \in \hat{K}$. We have $(\hat{p} \mid r_S(\hat{K}))\backslash A \Mapsto^{\hat{s}} (p'' \mid r_S(K'))\backslash A$ and $(\hat{p}, q') \in \hat{K}$ and $lnth(\hat{s}) = n$ and so by induction, there exists q'' and s'' such that $s'' = \hat{s}|_\tau$ and $q' \Rightarrow^{s''} q''$ and $(p'', q'') \in K'$. Take $s' = s''$ then $q' \Rightarrow^{s'} q''$ with $s' = s|_\tau$ and $(p'', q'') \in K'$.

If $\lambda \neq \tau$, then by 8.3, there exists $\hat{q}$ such that $q' \Rightarrow^{\lambda} \hat{q}$ and $(\hat{p}, \hat{q}) \in \hat{K}$. Again, by induction, there exists q'' and s'' such that $s'' = \hat{s}|_\tau$ and $\hat{q} \Rightarrow^{s''} q''$ and $(p'', q'') \in K'$. Let $s' = \lambda s''$, then $q' \Rightarrow^{s'} q''$ with $s|_\tau = \lambda\hat{s}|_\tau = (\lambda|_\tau).(\hat{s}|_\tau) = \lambda s'' = s'$ and $(p'', q'') \in K'$.

8.6. Proposition

Suppose $\mathbf{S} \subseteq \Psi(p, q)$ is uncompromised and suppose $K \in \mathbf{S}$ and $(p', q') \in K$. If $q' \Rightarrow^{s} q''$ then there exists p'', K' such that $(p' \mid r_S(K))\backslash A \Rightarrow^{s} (p'' \mid r_S(K'))\backslash A$ and $(p'', q'') \in K'$.

Proof

By 2.4, it suffices to show that if $q' \Mapsto^{s} q''$ then there exists s' and p'' and K' such that $(p' \mid r_S(K))\backslash A \Rightarrow^{s'} (p'' \mid r_S(K'))\backslash A$, $s' = s|_\tau$ and $(p'', q'') \in K'$. We argue by induction on $lnth(s)$.

If $lnth(s) = 0$ then $s = \Omega$ and we have $q'' = q'$. We may take $s' = \Omega$, $p' = p''$ and $K = K'$. Then,

$$(p'', q'') = (p', q') \in K = K'.$$

Now suppose the proposition is true when $lnth(s) = n$ and suppose $q' \Mapsto^{s} q''$ where $lnth(s) = n+1$. There exists $\hat{s} \in \Delta^*$ and $\lambda \in \Delta$ such that $s = \lambda\hat{s}$. Thus $lnth(\hat{s}) = n$.

By 2.4, there exists $\hat{q}$ such that $q' \rightarrow^{\lambda} \hat{q} \Mapsto^{\hat{s}} q''$.

If $\lambda = \tau$, then by 8.2(2), $(p', \hat{q}) \in K$. Since $\hat{q} \Mapsto^{\hat{s}} q''$ and $(p', \hat{q}) \in K$ and $lnth(\hat{s}) = n$, then by induction there exists p'' and K' and s' such that

$$(p' \mid r_S(K))\backslash A \Rightarrow^{s'} (p'' \mid r_S(K'))\backslash A$$

and $s' = s|_\tau$ and $(p'', q'') \in K'$.

If $\lambda \neq \tau$, then by 8.4, there exists $\hat{p}$ and $\hat{K}$ such that $(p' \mid r_S(K))\backslash A \Rightarrow^{\lambda} (\hat{p} \mid r_S(\hat{K}))\backslash A$ and $(\hat{p}, \hat{q}) \in \hat{K}$. Again, by induction, there exists p'', K' and s'' such that $s'' = \hat{s}|_\tau$, $(p \mid r_S(\hat{K}))\backslash A \Rightarrow^{s''} (p'' \mid r_S(K'))\backslash A$ and $(p'', q'') \in K'$. Let $s' = \lambda\hat{s}$, then $(p' \mid r_S(K))\backslash A \Rightarrow^{s'} (p'' \mid r_s(K'))\backslash A$ with $s|_\tau = (\lambda|_\tau).(\hat{s}|_\tau) = \lambda s'' = s'$ and $(p'', q'') \in K'$.

8.7. Theorem

Suppose $\mathbf{S} \subseteq \Psi(p, q)$ is uncompromised and suppose $K_0 \in \mathbf{S}$ is such that $(p, q) \in K_0$, then

$$(p \mid r_S(K_0))\backslash A \approx q.$$

Proof

Define $E(n)$ to be the following predicate:

$$\text{If } (p', q') \in K \in \mathbf{S} \text{ then } (p' \mid r_S(K))\backslash A \approx_n q'.$$

Certainly $E(0)$ is true, by 2.5(a). Suppose $E(n)$ and suppose $(p', q') \in K \in \mathbf{S}$, then

If $q' \Rightarrow^{s} q''$, then by 8.6, $(p' \mid r_S(K))\backslash A \Rightarrow^{s} (p'' \mid r_S(K'))\backslash A$ with $(p'', q'') \in K'$. By $E(n)$: $(p'' \mid r_S(K'))\backslash A \approx_n q''$.

Thus if $q' \Rightarrow^{s} q''$, then $(p' \mid r_S(K))\backslash A \Rightarrow^{s} (p'' \mid r_S(K'))\backslash A$ with $(p'' \mid r_S(K'))\backslash A \approx_n q''$.

Similarly, if $(p' \mid r_S(K))\backslash A \Rightarrow^{s} (p'' \mid r_S(K'))\backslash A$ then we may use 8.5 and $E(n)$ to deduce that $q' \Rightarrow^{s} q''$ with $(p'' \mid r_S(K'))\backslash A \approx_n q''$.

But by 2.5(a) these two statements imply that $(p'' \mid r_S(K'))\backslash A \approx_{n+1} q''$.

Thus, given $E(n)$, we have shown that if $(p', q') \in K \in \mathbf{S}$, then $(p'' \mid r_S(K'))\backslash A \approx_{n+1} q''$. In other words, $E(n)$ implies $E(n+1)$.

By induction, $E(n)$ holds for all n and so by 2.5,

$$\text{If } (p', q') \in K \in \mathbf{S} \text{ then } (p' \mid r_S(K))\backslash A \approx q'.$$

In particular, $(p \mid r_S(K_0))\backslash A \approx q$ as required.

We may now state our main theorem.

8.8. Theorem

There exists r solving $(p \mid X)\backslash A \approx q$ iff $\Psi(p, q)$ contains an uncompromised system $\mathbf{S}$.

If $\mathbf{S}$ is an uncompromised system and $(p, q) \in K_0 \in \mathbf{S}$, then $r_S(K_0)$ is a solution. Furthermore, $r_S(K_0)$ is rigid.

Proof

If r solves $(p \mid X)\backslash A \approx q$ then there exists an uncompromised $\mathbf{S}$, by 7.8.

Conversely, if $\mathbf{S}$ is uncompromised, then there exists $K_0 \in \mathbf{S}$ such that $(p, q) \in K_0$. By 8.7, $(p \mid r_S(K_0))\backslash A \approx q$ and so $r_S(K_0)$ is a solution.

8.9. Corollary

An interface equation $(p \mid X)\backslash A \approx q$ has a solution iff it has a rigid solution.

Proof

Certainly if the equation has a rigid solution, then it has a solution.

Conversely, if it has a solution, then $\Psi(p, q)$ contains an uncompromised system $\mathbf{S}$, by 8.8. Also by 8.8, if $(p, q) \in K_0$ for $K_0 \in \mathbf{S}$, then $r_{\mathbf{S}}(K_0)$ is a rigid solution.

9. SOLVING INTERFACE EQUATIONS

We now sketch an algorithm for solving interface equations.

9.1. Step 1

Let $\mathbf{S}_0 = I(p, q)$.

Repeat the following. Beginning with $B_{I\tau}(p, q)$, examine each $B_{I\tau}(p', q')$ and remove it from $\mathbf{S}_0$ if it is not I-complete. Continue to do this until either

(a) $B_{I\tau}(p, q) \notin \mathbf{S}_0$ and in this case let $\mathbf{S} = \varnothing$ and go to step 4, or

(b) All of the $B_{I\tau}(p', q')$ remaining in $\mathbf{S}_0$ are I-complete.

The rationale behind this step is the following fact. If $B_{I\tau}(p', q') \subseteq K$ and $B_{I\tau}(p', q') \subseteq K$ is not I-complete, then neither is K and hence K cannot belong to an uncompromised (p, q) system.

Let $\mathbf{S}_1 = \{\bigcup_{B \in X} B | X \subseteq \mathbf{S}_0\}$.

By construction, any $\mathbf{S} \subseteq \mathbf{S}_1$ will be I-complete.

9.2. Step 2

Let $\mathbf{S}$ denote the set of all $K \in \mathbf{S}_1$ such that $B_{I\tau}(p, q) \subseteq K$.

If $K \in \mathbf{S}$ and $K' \in \mathbf{S}_1 - \mathbf{S}$ with $K \rightarrow^{\mu, X} K'$ some μ and X, then let $\mathbf{S} = \mathbf{S} \cup \{K'\}$. Repeat this step until no more such K' may be found.

The point behind this is that we need not consider agents which are not reachable from any possible 'starting state'.

By construction $\mathbf{S}$ will contain all I-complete sets K such that for some $K_0 \in \mathbf{S}$, $(p, q) \in K_0$ and for some $s \in (\Delta - \{\tau\})^*$, $e_{\mathbf{S}}(K_0) \Rightarrow^s r_{\mathbf{S}}(K)$.

9.3. Step 3

If $K \in \mathbf{S}$ is not O-complete w.r.t. $\mathbf{S}$, then let $\mathbf{S} = \mathbf{S} - \{K\}$. Repeat this step until no more such K may be found or no remaining K contains (p, q). In this latter case, let $\mathbf{S} = \varnothing$ and go to step 4.

Note that I-completeness is a local property, a property of K which does not depend on the rest of $\mathbf{S}$. It follows that on termination of step 3, the resulting system will either be empty or no remaining K contains (p, q) or will be both I- and O-complete and some K contains (p, q).

9.4. Step 4

If $\mathbf{S} = \varnothing$, then report failure. Otherwise select $K \in \mathbf{S}$ such that $(p, q) \in K$. $r_{\mathbf{S}}(K)$ is a solution of the interface equation.

10. CONCLUSIONS AND FUTURE WORK

In this paper, we have presented some preliminary results concerning the solution of context equations. We would like to stress two things:

(1) The work has a strong flavour of traditional applied mathematics about it in the sense that it attempts to provide a mechanism by which a practical problem may be transformed into a mathematical problem and solved with the aid of mathematical theory.

(2) The solution technique described here is likely to be somewhat expensive in general. Our main point is that the equations considered here are solvable in principle.

This is not unusual in applied mathematics. Consider the case of differential equations. Firstly, there is no solution technique which applies to all of them. However, there are a number of subclasses which have great practical importance and for which solution techniques do exist and which are capable of being applied by persons who are not themselves mathematicians.

This suggests a number of possible extensions, all of which are under investigation.

(1) There are almost certain to be other problems which are capable of being formulated in the fashion of the interface equation. We seek input from workers in the field so that we may broaden the scope of application of a developing theory. With this in mind, we have presented an informal overview of our approach.[7]

(2) We must certainly attempt to improve the efficiency of our solution method.

(3) We must also look to practical application to identify *practically* useful classes of interface equation, for which solutins may be simpler. This problem may be approached from two ends; by seeking out 'easy' subclasses from a purely theoretical point of view and by identifying classes of system to which the method might be applied.

(4) From purely theoretical considerations, we would like to be able to tackle a larger class of context equations. Some initial work in this area has been reported.[5, 6]

(5) Finally, the work must be made useful to – and usable by – the non-theoretician. This is the case with the 'useful' parts of the calculus, which has over the centuries acquired a notation and body of technique which make it accessible to scientists and engineers. We are investigating the possibility of developing software tools based on theory of this kind.

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Announcements

4–8 December 1989

Toulouse '89, Second International Workshop on Software Engineering and its Applications

The success of the First Toulouse International Workshop on Software Engineering and its Applications in December 1988 was such that the workshop could not be allowed to be a one-off event. Thus, Toulouse '89 will bear witness to the emergence of software engineering as a major activity in the field of software development.

The extent to which software engineering is being integrated into the professional world may be gauged by the increasing number of companies, from whatever domain, that make use of its techniques, together with the availability of software engineering tools in the marketplace. It was already apparent at Toulouse '88 that a new technology was taking its first steps, and it is in this perspective that Toulouse '89 should confirm the inevitable evolution of this development.

The power of new generations of computers and workstations is both boosting traditional approaches and allowing the new technologies, resulting from a decade of research, to become operational.

The tools of a profession

Whatever the application domain, be it management, scientific or industrial, software engineering technology is based on the union of three broad categories: software tools, necessary to improve production process efficiency, methods, providing an indication of consistency, and discipline in software development and organization which from human factors to management of the technology, ensures both technical and economic success.

Toulouse '89 confirms its role as a major event dedicated to software engineering and its applications in the three fields of management, industry and science. Maintaining the choice of Toulouse, the birthplace of the Airbus and Hermes, as the host town reinforces the international nature of the workshop.

Three complementary approaches will allow participants to evaluate all aspects of the latest software engineering products and techniques:

- a technical conference presenting the papers selected by the International Program Committee
- a series of tutorials on software engineering techniques
- an exhibition of commercially available products

TUTORIALS AND PANEL SESSIONS

The technical conferences will be complemented by tutorials and panel sessions. Given by internationally renowned experts and held on 4 and 5 December 1989, these tutorials will provide an introduction to the different tools and techniques used in software engineering, in-depth coverage of specific techniques, or the state of the art in particular fields of application.

EXHIBITION

From 6 to 8 December 1989 an exhibition covering several thousand square metres will be held to present commercially available products relating to the topics covered at the workshop. In addition to this, there will be demonstrations of advanced prototypes.

GENERAL INFORMATION

Date

Tutorials: 4 and 5 December 1989
Conference and Exhibition: 6, 7 and 8 December 1989

Location

Palais des Congrès, Parc des Expositions, Rond-Point Michel Benech, 31000 Toulouse – France. Tel: (33) 61 25 21 77

Registration Fee (VAT incl.)

Regular fee	FF 4500
University fee	FF 2700
Student fee	FF 500
Tutorials	
2 days	FF 3300
$1\frac{1}{2}$ day	FF 2600
1 day	FF 1700
$\frac{1}{2}$ day	FF 950

Languages

English and French will be the official working languages. Simultaneous translation will be provided.

Accommodation

Hotel reservations should be made directly by each participant with the local agency below. All requests should be addressed to:

Promo Toulouse, Donjon du Capitole, 31000 Toulouse – France. Tel: (33) 61 21 92 32.
Telex: 531 508.

Workshop Chairman

Jean-Claude Rault, EC2, Nanterre

International Program Committee Chairman

Michel Galinier, IGL Technology, Paris

The complete list of the International Program Committee will be published later

Organised by:

EC2, 269-287, rue de la Garenne, 92000 Nanterre, France. Tel: (33.1) 47 80 70 00.
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17–20 September 1990

Information '90, Third International Conference, Bournemouth International Centre

Information '90, a major international conference, sponsored by Aslib (the Association for Information Management), COPOL (the Council of Polytechnic Librarians), the Institute of Information Scientists, the Library Association and the Society of Archivists, will be held at the Bournemouth International Centre from 17 to 20 September 1990.

The conference will bring together world experts in the library and information field to ensure that **Information '90** will be the biggest and best event of its kind ever staged.

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