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Improved Analytical PID Controller Design for the Second Order Unstable Process with Time Delay

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Abstract: The design of the PID controller cascaded with first order filter has been proposed for the second order unstable time delay processes. The design algorithm is based on the IMC criterion which has single tuning parameter to adjust the performance and robustness of the controller. The setpoint filter is used to diminish the overshoot in servo response. The simulation results of the suggested method are compared with recently published tuning methods to demonstrate the superiority of the proposed method. For the reasonable comparison the controllers are tuned to have the same degree of robustness by the measure of maximum sensitivity (*Ms*). A guideline is also provided for the ease of the selection of closed-loop time constant (λ).

Keywords: Unstable time delay process, PID•filter controller, Disturbance rejection

1. Introduction

The proportional integral derivative (PID) controller algorithm is the most adopted controllers for industrial plants, due to its simplicity and satisfactory performances for a wide range of processes. The cost/benefit ratios provided by the PID controller is difficult to achieve by other controllers.
The numerous important chemical processing units in industrial and chemical practice are open-loop unstable processes that are well known to be difficult to control especially when there exists time delay, such as continuous stirred tank reactors, polymerization reactors and bioreactors are inherently open loop unstable by design. Consequently, there has been much recent interest in the

literature [1-8] on tuning the industrially standard PID controllers for open-loop unstable systems. The effectiveness of internal model control (IMC) design principle has attracted in process industry, which causes many efforts made to exploit the IMC principle to design the equivalent feedback controllers for stable and unstable processes [1-7]. The IMC based PID tuning rules have the advantage of only one tuning parameter to achieve a clear trade-off between closed-loop performance and robustness. The modified IMC methods of two-degree-of-freedom (2DOF) controls such as Lee et al. [2], Yang et al. [7], Wang and Cai [6], Tan et al. [5], Liu et al. [4] have been developed for controlling unstable processes with time delay. In addition, 2DOF control methods based on the Smith-Predictor (SP) had been proposed by Majhi & Atherton [3] and achieved smooth nominal setpoint response without overshoot for first order unstable processes with time delay. It is important to emphasize that the control performance can be significantly enhanced by cascading the PID controller with a lead/lag filter, as given by Eq. (1).

$$G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1+as}{1+bs} \tag{1}$$

where K_c, τ_I and τ_D are the proportional gain, integral time constant, and derivative time constant of the PID controller, respectively, and a and b are the filter parameters. The structure of the PID controller cascaded with a filter was also suggested in several literatures [1,2,4,7,8]. The PID•filter controller in Eq. (1) can easily be implemented in modern control hardware. It is important to emphasize that design principle of the aforementioned tuning methods for the unstable second order with time delay processes is either complicated or the resulting IMC structure is difficult to implement in the real process plant.

Therefore, in the present study a design of the PID•filter for the second order unstable time delay processes and a closed-loop time constant (λ) guidelines has been proposed. Simulation study has been performed to compare the proposed method with recently published PID tuning methods.

2. Controller Design

The IMC based design principle is a powerful method for control system synthesis (Morari and Zafiriou, [1]) and is a well developed methodology which is available in literature [1,2,4-7]. Based on the IMC design methodology, PID•filter has been proposed in the following section.

2.1. Second-Order Delay Unstable Process (SODUP)

$$G_P = G_D = \frac{Ke^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)} \tag{2}$$

where K is the gain, τ the time constant and θ is the time delay. The IMC filter structure exploited is $f=\left(\alpha_2 s^2+\alpha_1 s+1\right)/\left(\lambda s+1\right)^4$. The resulting IMC controller can be obtained as follows

$$q=\frac{\left(\tau_1 s-1\right)\left(\tau_2 s-1\right)\left(\alpha_2 s^2+\alpha_1 s+1\right)}{K\left(\lambda s+1\right)^4} \tag{3}$$

The IMC controller in the Eq. (3) is proper. The ideal feedback controller G_c equivalent to the IMC controller can be obtained after the approximation of the dead time $e^{-\theta s}$ by 1/1 Pade expansion as:

$$G_c=\frac{\left(\tau_1 s-1\right)\left(\tau_2 s-1\right)\left[\left(\alpha_2 s^2+\alpha_1 s+1\right)\left(1+\theta s/2\right)\right]}{K\left(\theta+4\lambda-\alpha_1\right)s\left[1+\frac{\left(\alpha_1\theta/2-\alpha_2+2\lambda\theta+6\lambda^2\right)}{\left(\theta+4\lambda-\alpha_1\right)}s+\frac{\left(\alpha_2\theta^2/2+3\lambda^2\theta+4\lambda^3\right)}{\left(\theta+4\lambda-\alpha_1\right)}s^2+\frac{\left(2\lambda^3\theta+\lambda^4\right)}{\left(\theta+4\lambda-\alpha_1\right)}s^3+\frac{\lambda^2\theta^2/2}{\left(\theta+4\lambda-\alpha_1\right)}s^4\right]} \tag{4}$$

The analytical PID formula can be obtained by rearranging Eq. (4) and listed in Table 1. The parameters b in the filter can be obtained by equating the remaining part of the denominator of Eq. (4) with $\left(\tau_1 s-1\right)\left(\tau_2 s-1\right)(bs+1)$. The remaining part of the denominator of Eq. (4) contains the factor of the process pole, filter *(bs+1)* and a high order polynomial terms in *s*. The high order polynomial term in *s* has no impact because it is not in control relevant frequency range. Taking the first derivative of *(bs+1)* and substituting $s=0$, the parameter *b* can be easily obtained and listed in Table 1. The value of α_1 and α_2 is selected so that it cancels out the open-loop unstable pole at $1/\tau_1$ and $1/\tau_2$. This requires $\left[1-Gq\right]\big|_{s=1/\tau_1,\,1/\tau_2}=0$ and $\left[1-\left(\alpha_2 s^2+\alpha_1 s+1\right)e^{-\theta s}/\left(\lambda s+1\right)^4\right]_{s=1/\tau_1,\,1/\tau_2}=0$. The value of α_1 and α_2 is obtained and also listed in Table 1. In Table 1, the PID•filter tuning rule is listed for several process models.

Table 1 PID•filter tuning rule for several process models

Process model	K_C	b	α_1 & α_2
$\frac{Ke^{-\theta s}}{\left(\tau_1 s-1\right)\left(\tau_2 s-1\right)}$	$\frac{\alpha_1}{K\left(\theta+4\lambda-\alpha_1\right)}$	$\frac{\left(\frac{\alpha_1\theta}{2}-\alpha_2+2\lambda\theta+6\lambda^2\right)}{\left(\theta+4\lambda-\alpha_1\right)}+\left(\tau_1+\tau_2\right)$	$\alpha_1=\frac{\tau_1^2\left(\frac{\lambda}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-\tau_2^2\left(\frac{\lambda}{\tau_2}+1\right)^4 e^{\theta/\tau_2}+\left(\tau_2^2-\tau_1^2\right)}{\left(\tau_1-\tau_2\right)}$ $\alpha_2=\tau_1^2\left(\frac{\lambda}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-\left(\frac{\alpha_1}{\tau_1}+1\right)$
$\frac{Ke^{-\theta s}}{\left(\tau_1 s-1\right)\left(\tau_2 s+1\right)}$	$-\frac{\alpha_1}{K\left(\theta+4\lambda-\alpha_1\right)}$	$\frac{\left(\frac{\alpha_1\theta}{2}-\alpha_2+2\lambda\theta+6\lambda^2\right)}{\left(\theta+4\lambda-\alpha_1\right)}+\left(\tau_1-\tau_2\right)$	$\alpha_1=\frac{\tau_1^2\left(\frac{\lambda}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-\tau_2^2\left(-\frac{\lambda}{\tau_2}+1\right)^4 e^{-\theta/\tau_2}+\left(\tau_2^2-\tau_1^2\right)}{\left(\tau_1+\tau_2\right)}$ $\alpha_2=\tau_1^2\left(\frac{\lambda}{\tau_1}+1\right)^4 e^{\theta/\tau_1}-\left(\frac{\alpha_1}{\tau_1}+1\right)$
$\frac{Ke^{-\theta s}}{s\left(\tau s-1\right)}=\frac{K\psi e^{-\theta s}}{\left(\psi s-1\right)\left(\tau s-1\right)}$	$\frac{\alpha_1}{K\psi\left(\theta+4\lambda-\alpha_1\right)}$	$\frac{\left(\frac{\alpha_1\theta}{2}-\alpha_2+2\lambda\theta+6\lambda^2\right)}{\left(\theta+4\lambda-\alpha_1\right)}+\left(\tau+\psi\right)$	$\alpha_1=\frac{\tau^2\left(\frac{\lambda}{\tau}+1\right)^4 e^{\theta/\tau}-\psi^2\left(\frac{\lambda}{\psi}+1\right)^4 e^{\theta/\psi}+\left(\psi^2-\tau^2\right)}{\left(\tau-\psi\right)}$ $\alpha_2=\tau^2\left(\frac{\lambda}{\tau}+1\right)^4 e^{\theta/\tau}-\left(1+\frac{\alpha_1}{\tau}\right)$

For $G_P=G_D=\frac{\left(\tau_a s+1\right)Ke^{-\theta s}}{\left(\tau_1 s-1\right)\left(\tau_2 s-1\right)}$, the extra lag filter $Ft=1/\left(\tau_a s+1\right)$ in the $G_c=K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\bullet\frac{1+as}{1+bs}\bullet Ft$ and for every case $\tau_I=\alpha_1$, $\tau_D=\alpha_2/\alpha_1$, $a=0.5\theta$

Remarks: For the SODUP process without any zero, it is observed that the designed value of b is too large to obtain robust performances of the closed-loop system when the parametric uncertainties are large. Based on extensive simulation study that has been conducted on different SODUP processes, it is observed that using a value of *"0.1b"* instead of b gives robust control performances. The recommended *"0.1b"* was also suggested by Seshagiri & Chidambaram [8]. For the processes containing negative zero, the value of b will be as usual without any alteration.

3. Simulation Results

3.1. Example 1. SODUP (Two Unstable Pole)

A widely published SODUP example that has been considered for the comparison (Seshagiri & Chidambaram [8], Liu et al. [4]) is:

$$G_P = G_D = \frac{2e^{-0.3s}}{(3s-1)(1s-1)} \tag{5}$$

Seshagiri & Chidambaram [8] had already demonstrated its superiority over many widely accepted previous approaches in their recently published paper. The proposed method is compared with the Seshagiri & Chidambaram [8] and Liu et al. [4]. For the fair comparison, λ for the proposed method has been adjusted to give the same *Ms* as Seshagiri & Chidambaram [8]. The value of λ in the proposed method has been adjusted as $\lambda = 0.3555$ to achieve *Ms* =3.09 and the corresponding tuning parameters are $K_c = 3.4706$, $\tau_I = 1.5052$, $\tau_D = 1.3633$, $a=0.15$, $b=0.0059$ and $f_R = (0.4516s+1)\big/\left(2.0520s^2+1.5052s+1\right)$ for $b=0.3$. Figs 1(a) and 1(b) show the comparison of the proposed method with Seshagiri & Chidambaram [8] and Liu et al. [4], by introducing a unit step change in the setpoint and the load disturbance, respectively. For the servo response the setpoint filter is used for both the proposed and Seshagiri & Chidambaram [8] methods whereas three control element structure is used for the Liu et al [4].

It is clear from Fig. (1), the proposed method results in the improved load disturbance response. For the servo response all the three methods have almost similar response. It is important to note that the modified IMC structure by Liu et al. [4] has theoretical advantage of eliminating the time delay from the characteristic equation. Unfortunately, this advantage is lost if the process model is inaccurate. Besides, there usually exists the process unmodeled dynamics in real process plant that inevitably tends to deteriorate the control system performance severely. From Fig. (1) it is clear that the proposed methods has a big advantage over the other methods.

3.2. Example 2. SODUP (One Stable Pole)

$$G_P = G_D = \frac{1e^{-0.939s}}{(5s-1)(2.07s+1)} \tag{6}$$

The proposed method has been compared with the Seshagiri & Chidambaram [8] and Tan et al. [5]. In the proposed method the value of λ has been adjusted as $\lambda = 0.9296$ to give $Ms = 2.34$ same as Seshagiri & Chidambaram [8]. Corresponding tuning parameters are $K_c = 6.7051$, $\tau_I = 5.4738$, $\tau_D = 1.333$, $a = 0.4695$, $b = 0.023$ and $f_R = (1.6421s+1)/(7.2966s^2+5.4738s+1)$ for *b=0.3*. Fig. 2 shows the comparison of the proposed method with Seshagiri & Chidambaram [8] and Tan et al. [5], by introducing a unit step change in the setpoint and the load disturbance, respectively. For the servo response, the setpoint filter is used for both the proposed and Seshagiri & Chidambaram [8] methods whereas three control element structure are used for the Tan et al. [5].
It is clear from Fig. (2), the proposed method obtains the improved load disturbance response. For the Tan et al. [5] method both the setpoint and disturbance rejection response is very slow and requires long settling time. The setpoint response of Seshagiri & Chidambaram [8] and the proposed method is almost similar whereas disturbance rejection for proposed method shows best among all the three methods.

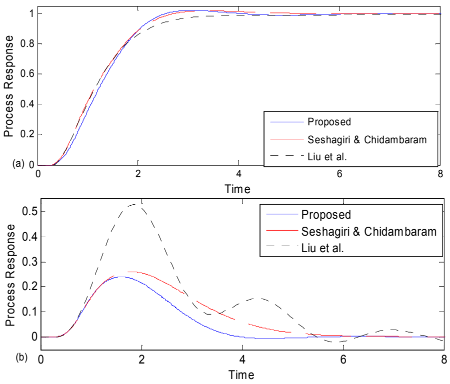

Fig. 1 Simulation results for Example 1

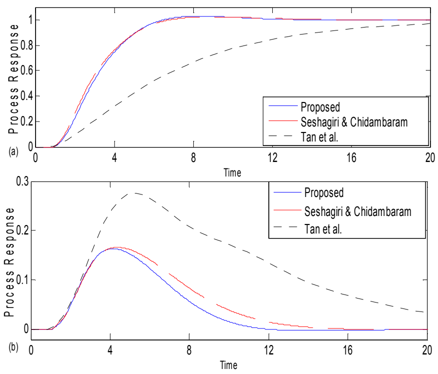

Fig. 2 Simulation results for Example 2

3.3. Closed-loop time constant (λ) guidelines

Only the closed-loop time constant λ is the user-defined tuning parameter in the proposed tuning rule. It is directly related to the performance and robustness of the proposed tuning method, which is why it is important to have some λ guidelines in order to provide both a fast and robust performance for a desirable range of θ/τ ratio. Based on extensive simulation studies, it is observed that the starting value of λ can be considered to be equal as process time delay, which can give robust control performance. If not, the value should be increased carefully until both the nominal and robust control performances are achieved.

4. Conclusions

A simple design method of the analytical PID•filter tuning has been proposed for the several second-order unstable processes based on the IMC principle. Two important representative processes have been considered for the simulation study to show the superiority of the proposed method. The simulation conducted for the fair comparison when each controller was tuned to have the same degree of robustness by the measure of *Ms* value, where the proposed method show a clear advantage. The closed-loop time constant λ guideline was also suggested for the ease of selection.

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References

1. M. Morari, and E. Zafiriou, Robust Process Control, Prentice-Hall: Englewood Cliffs, NJ, (1989).
2. Y. Lee, J. Lee, S. Park, PID Controller Tuning for Integrating and Unstable Processes with Time Delay, Chem. Eng. Sci. 55, (2000), 3481-3493.
3. D. P. Majhi, and Atherton, Obtaining Controller Parameters for a New Smith Predictor using Autotuning, Automatica 36, (2000), 1651–1658.
4. T. Liu, W. Zhang and D. Gu, Analytical Design of Two-Degree-of-Freedom Control Scheme for Open-loop Unstable Process with Time Delay, J. Process Control, 15, (2005), pp. 559–572.
5. W. Tan, H. J. Marquez, and T. Chen, IMC Design for Unstable Processes with Time Delays, J. Process Control, 13, (2003), 203–213.
6. Y. G. Wang and W. J. Cai, Advanced Proportional-Integral-Derivative Tuning for Integrating and Unstable Processes with Gain and Phase Margin Specifications, Ind. Eng. Chem. Res. 41, (2002), 2910–2914.
7. X. P. Yang, Q. G. Wang, C. C. Hang and C. Lin, IMC-Based Control System Design for Unstable Processes, Ind. Eng. Chem. Res., 41, (2002), 4288–4294.
8. R. A. Seshagiri, and M. Chidambaram, Enhanced Two-Degree-of- Freedom Control Strategy for Second-Order Unstable Processes with Time Delay, Ind. Eng. Chem. Res., 45, (2006), 3604-3614.