Improved Assessment of Interference Limits in Cellular Radio Performance

Ramjee Prasad, *Senior Member, IEEE,* and Adriaan Kegel

Abstract—**Reuse distance and cluster size are evaluated for cellular mobile radio systems. Rayleigh fading, log-normal shadowing, and area mean power inversely proportional to the fourth power of propagation distance are considered, using the technique of Schwartz and Yeh for determining the probability density function of the sum of several log-normal variables in order to obtain the cochannel interference probabilities. These results are used for analyzing the radio spectrum efficiency, taking also traffic intensity into consideration. Different modulation methods, namely, analog FM with 30- and 12.5-kHz channel spacing, SSB with 5-kHz spacing, and digital modulation with 25-kHz channel spacing, with adequate protection ratios, are compared. An example of the calculation of system bandwidth and cluster size is presented using this procedure.**

I. Introduction

IMPROVED assessment of cochannel interference in cellular systems in order to evaluate the system performance has enjoyed a high priority in mobile radio research during the last decade, e.g., [1]–[10]. Generally, three propagation aspects are considered: random (fast) multipath fading, random (slow) shadowing, and the deterministic UHF ground-wave path loss. A difficult problem arises in developing the corresponding interference model for several fading radio signals.

The probability density function (pdf) of the sum of random signals with log-normal probability distribution is required. An approximate pdf was obtained in [1]–[3] using Fenton's method [11], and in [10] using Wilkinson's approach [12]. However, both methods are unfortunately suitable for small signal variance (σ^2) only, while typical values of σ lie between 6 and 12 dB in mobile channels. An improved technique for approximating the probability distribution of the sum of several random variables with such high variances, based on a method by Schwartz and Yeh [12], has recently been reviewed [13]. The present paper deals with the assessment of capacity for a mobile radio system, in terms of reuse distance, cluster size, and spectrum efficiency based on the concept of required protection ratio. Capacity is calculated by evaluating the probability of cochannel interference of a system exposed to multipath fading and shadowing, using Schwartz and Yeh's technique [12], [13]. The motivation for this study of mobile (CW) telephony are the findings in recent studies of mobile (bursty) packet systems [14], [15] in which it was reported that Fenton's classical method is not well suited for applications in typical mobile environments with high σ.

Spectrum efficiency (E_s), expressed in erlangs per megahertz per square kilometers (erlang/MHz/km^2) is defined as the ratio of carried traffic per cell and the product of bandwidth, number of cells per cluster and cell area. Thus E_s is an important parameter for assessing the frequency requirements of cellular mobile radio systems.

It is necessary to consider whether several cochannel interference signals add coherently or incoherently. Coherent addition of phasors would be more appropriate, if the random phase terms of the individual interferers hardly vary during the receive period; this would seem an impractical proposition for mobile telephony. Incoherent cumulation of cochannel interferers in the analysis of the system performance of cellular mobile radio telephony is a more realistic assumption. However, coherent cumulation of interferers was assumed in the analysis presented in [1], [13] by considering that n Rayleigh-distributed phasors add up to one Rayleigh-distribution phasor, with mean power equal to the sum of local mean interference powers [15], [19]. Therefore, the corresponding pdf for the joint interference power is an exponential distribution. In the case of incoherent cumulation of interferers, with equal mean power, the pdf for the joint interference power would rather be given by the gamma distribution [19]. Studies of the throughput of packet radio in typical mobile channels with multiple incoherent interferers [16] indicate that the coherent interference model gives more optimistic results for the spectrum efficiency than the incoherent model. This result for burst-mode systems is confirmed here for typical continuous-mode mobile radio telephony systems with multipath fading and shadowing.

This paper is organized as follows. Section II formulates the probability of cochannel interference for uncorrelated fading signals. Section III describes the computational results of cochannel interference probability and reuse distance. In Section IV, the effect of cluster size, protection ratio and carried traffic on the spectrum efficiency is discussed. Section V defines the system bandwidth and computational results are presented for different modulation schemes and parameters. Section VI contains the concluding remarks.

II. Formulation of Cochannel Interference Probability

The cellular telephony concept has been described in numerous papers, e.g., [17]. In order to investigate the reuse

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The authors are with the Telecommunications and Traffic Control Group, Delft University of Technology, 2600 GA Delft, The Netherlands.

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distance and spectrum efficiency of a cellular system, it is necessary to determine the cochannel interference probability [1]-[10], defined as

$$F(CI) \triangleq \sum_n F(CI \mid n) F_n(n). \quad (1)$$

Here $F_n(n)$ is the probability of n cochannel interferers being active. $F(CI \mid n)$ is the corresponding conditional cochannel interference probability,

$$F(CI \mid n) \triangleq \text{prob}\,\{p_d / p_n < \alpha\} \quad (2)$$

where p_d is the instantaneous power of the desired signal, p_n is the joint interference power from n active channels, and α is the specified cochannel protection ratio.

The (fast) amplitude fluctuation of mobile radio signals is described by the Rayleigh density function. The pdf for the signal amplitude r_i of the ith interferer, conditional on its local mean power p_{0i}, is given by

$$f_{ri}(r_i \mid p_{0i}) = \frac{r_i}{p_{0i}} \exp\left(-r_i^2/2p_{0i}\right). \quad (3)$$

The corresponding pdf for the instantaneous power p_i is

$$f_{pi}(p_i \mid p_{0i}) = \frac{1}{p_{0i}} \exp\left(-\frac{p_i}{p_{0i}}\right). \quad (4)$$

The local mean power p_{0i} is itself a (more slowly varying) stochastic variable with the log-normal pdf

$$f_{p_{0i}}(p_{0i}) = \frac{1}{\sqrt{(2\pi)}\,\sigma_i p_{0i}} \exp\left\{-(\ln p_{0i} - m_i)^2/2\sigma_i^2\right\}. \quad (5)$$

Here, σ_i^2 and $\xi_i = \exp(m_i)$ are the logarithmic variance and the area mean power of the interfering signal, respectively.

The amplitude, r_d, instantaneous power, p_d, and local mean, p_{0d}, of the desired signal can also be described by density functions of the forms (3)-(5), respectively, with the following substitutions:

$$r_i \to r_d$$
$$p_i \to p_d$$
$$p_{0i} \to p_{0d}$$
$$\sigma_i \to \sigma_d$$
$$m_i \to m_d$$
$$\xi_i \to \xi_d. \quad (6)$$

The formulation of the conditional cochannel interference probability for three different propagation conditions is described as follows.

1) Rayleigh Fading Only: In the event of incoherent cumulation with equal local mean power, i.e., $p_{0i} = p_0$, the pdf for the joint interference power p_n is obtained by convolving (4) n times and given by gamma distribution [18], [19]

$$f_{p_n}(p_n) = \frac{1}{p_0} \frac{(p_n/p_0)^{n-1}}{(n-1)!} \exp\left(-\frac{p_n}{p_0}\right). \quad (7)$$

The conditional cochannel interference can be derived using (2), (4), (6), and (7)

$$F(CI \mid n) = \int_0^\infty dp_n \int_0^{\alpha p_n} \frac{1}{p_0 p_{0d}} \frac{(p_n/p_0)^{n-1}}{(n-1)!} \cdot \exp\left(-\frac{p_d}{p_{0d}} - \frac{p_n}{p_0}\right) dp_d. \quad (8)$$

After integrating (8) becomes

$$F(CI \mid n) = 1 - \left(\frac{1}{\alpha p_0 / p_{0d} + 1}\right)^n. \quad (9)$$

2) Log-Normal Shadowing Only: The sum of n stochastic independent log-normal variables can be well approximated by another log-normal variable [11], [12]. Schwartz and Yeh [12], [13] derived exact expressions for the mean and variance for the sum of two log-normal variables and then used a recursive approach to approximate the mean, ξ_u, and the variance, σ_u^2, for n variables. Therefore, the pdf for $p_{0u} (= \sum_{i=1}^n p_{0i})$ is of the form (5) with the following substitutions:

$$p_{0i} \to p_{0u}$$
$$\sigma_i \to \sigma_u$$
$$m_i \to m_u$$
$$\xi_i \to \xi_u. \quad (10)$$

Substituting (6) and (10) in (2) yields the conditional cochannel interference probability

$$F(CI \mid n) = \int_0^\infty dp_{0u} \int_0^{\alpha p_{0u}} \frac{1}{2\pi \sigma_d \sigma_u p_{0d} p_{0u}} \cdot \exp\left\{-(\ln p_{0u} - m_u)^2/2\sigma_u^2\right\} \cdot \exp\left\{-(\ln p_{0d} - m_d)^2/2\sigma_d^2\right\} dp_{0d}. \quad (11)$$

Equation (11) simplifies to

$$F(CI \mid n) = \frac{1}{\sqrt{(2\pi)}} \int_x^\infty \exp\left(-t^2/2\right) dt \quad (12)$$

where

$$x \triangleq \left[\ln\left(\xi_d / \alpha \xi_u\right)\right] / \left(\sigma_d^2 + \sigma_u^2\right)^{1/2}. \quad (13)$$

Table I gives values for m_u and σ_u as a function of number of interferers, n, with the initial value of $m_i = 0$ dB.

TABLE I
STANDARD DEVIATION (σ_u) AND AREA MEDIAN POWER (m_u) FOR $n = 1$ TO 6 INTERFERERS SUBJECT TO LOG-NORMAL SHADOWING. INITIAL VALUE OF $m_i = 0$ dB

	$\sigma_i = 6$ dB, $m_i = 0$ dB		$\sigma_i = 12$ dB, $m_i = 0$ dB	
n	σ_u (dB)	m_u (dB)	σ_u (dB)	m_u (dB)
1	6.00000	0.00000	12.00000	0.00000
2	4.57930	4.57589	9.58168	7.45252
3	3.93286	6.90771	8.39718	11.20030
4	3.53877	8.43359	7.65698	13.61733
5	3.25742	9.56948	7.13423	15.37234
6	3.03997	10.47850	6.73756	16.73845

3) Rayleigh Fading plus Log-Normal Shadowing: In the

TABLE II
STANDARD DEVIATION (σ_n) AND AREA MEDIAN POWER (m_n) FOR $n = 1$ TO 6 INTERFERERS SUBJECT TO RAYLEIGH FADING AND LOG-NORMAL SHADOWING. INITIAL VALUE OF $m_i = 0$ dB

	$\sigma_i = 6$ dB, $m_i = 0$ dB		$\sigma_i = 12$ dB, $m_i = 0$ dB	
n	σ_n (dB)	m_n (dB)	σ_n (dB)	m_n (dB)
1	7.00525	−1.50515	12.53290	−1.50515
2	5.41069	3.51069	10.02766	6.22217
3	4.66320	6.05705	8.79895	10.10659
4	4.20419	7.71282	8.03063	12.61139
5	3.87870	8.93309	7.48808	14.42947
6	3.62925	9.89978	7.07652	15.84395

general case of combined multipath fading and shadowing, considering the individual, stochastically independent signals add incoherently [19], the composite pdf for the (individual) ith interfering signal power can be approximated by the log-normal pdf with logarithmic variance $[\sigma_i^2 + \ln(2)]$ and area median power $\xi_i/\sqrt{2}$ [16]. According to the suggestion of Schwartz and Yeh [12], the composite pdf for the (total) interference sum p_n can then be approximated by a pure log-normal distribution with the appropriate moments

$$f_n(p_n) = \frac{1}{\sqrt{(2\pi)}\sigma_n p_n}\exp\left\{-(p_u - m_n)^2/2\sigma_n^2\right\}. \quad (14)$$

Here $p_u \triangleq \ln p_n$, while $\xi_n (= \exp m_n)$ and σ_n^2 are the area median (interference) power and variance, respectively, of the equivalent log-normal variable. They are found in accordance with the procedure in [12], [13]. Hence, using (2)–(6) and (14) the conditional cochannel interference probability is given as

$$F(CI \mid n) = \int_0^\infty dp_{0d}\int_0^\infty dp_n \int_0^{\alpha p_n} \frac{1}{2\pi\sigma_d\sigma_n p_{0d}^2 p_n} \cdot \exp\left(-\frac{p_d}{p_{0d}}\right)\exp\left[\left\{-(\ln p_n - m_n)^2/2\sigma_n^2\right\} + \left\{-(\ln p_{0d} - m_d)^2/2\sigma_d^2\right\}\right] dp_d. \quad (15)$$

After repeated integration, (15) becomes

$$F(CI \mid n) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-w^2) f(w)\, dw \quad (16)$$

where

$$f(w) = 1 - \exp\left[-\exp\left\{m_n + \alpha_0 - m_d - \left[2(\sigma_d^2 + \sigma_n^2)\right]^{1/2}\right\} w\right] \quad (17)$$

$$\alpha_0 \triangleq \ln \alpha. \quad (18)$$

Table II gives values for m_n and σ_n as a function of the number of interferes, n, with the initial value of $m_i = 0$ dB.

In order to compare the system performance in coherent and incoherent interference conditions, the conditional cochannel interference probability for the coherent case is also derived. To this end, we write the composite pdf for n interfering signals with combined Rayleigh fading and log-normal shadowing for coherent addition using (3)–(6).

$$f_{fs}(p_n) = \frac{1}{\sqrt{(2\pi)}\sigma_u}\int_0^\infty \frac{1}{p_{0u}^2}\exp(-p_n/p_{0u}) \cdot \exp\left\{-(\ln p_{0u} - m_u)^2/2\sigma_u^2\right\} dp_{0u}. \quad (19)$$

The composite pdf for the desired signal with Rayleigh fading and log-normal shadowing is again of the form (19), with the following substitutions:

$$p_n \to p_d$$
$$p_{0u} \to p_{0d}$$
$$\sigma_u \to \sigma_d$$
$$m_u \to m_d. \quad (20)$$

The conditional cochannel interference probability is obtained using (2), (19), and (20)

$$F(CI \mid n) = \frac{1}{2\pi\sigma_d\sigma_u}\int_0^\infty dp_{0d}\int_0^\infty dp_{0u}\int_0^\infty dp_n\int_0^{\alpha p_n} \cdot \frac{1}{p_{0u}^2 p_{0d}^2}\exp\left\{-\left(\frac{p_d}{p_{0d}} + \frac{p_n}{p_{0u}}\right)\right\} \cdot \exp\left[\left\{-(\ln p_{0u} - m_u)^2/2\sigma_u^2\right\} + \left\{-(\ln p_{0d} - m_d)^2/2\sigma_d^2\right\}\right] dp_d. \quad (21)$$

Equation (21) reduces to

$$F(CI \mid n) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\left[\left\{\exp(-u^2)\right\}/ \left\{1 + (\xi_d/\alpha\xi_u)\exp\left[-\left\{2(\sigma_d^2 + \sigma_u^2)\right\}^{1/2} u\right]\right\}\right] du. \quad (22)$$

It is assumed throughout that all cochannel interferers are statistically independent and identically distributed. If, moreover, only interfering signals from the nearest neighboring six cochannel cells are considered, and both the blocking probability B and the number of channels n_c are the same in all cells, $F_n(n)$ can be written as [3]

$$F_n(n) = \binom{6}{n} B^{n/n_c}\left(1 - B^{1/n_c}\right)^{6-n}, \qquad n = 0, 1, \cdots, 6. \quad (23)$$

It has been noted that $F(n)$ can also be expressed in terms of carried traffic a_c per channel [1], [2]

$$F_n(n) = \binom{6}{n} a_c^n (1 - a_c)^{6-n} \quad (24)$$

where $a_c = A_c/n_c$ and A_c is the carried traffic per cell. Generally, (24) appears to be the more accepted representation in traffic analysis [20].

III. REUSE DISTANCE

The (normalized) cochannel reuse distance, R_u, is defined as the ratio of the distance D between the centers of the nearest neighboring cochannel cells and the cell radius R,

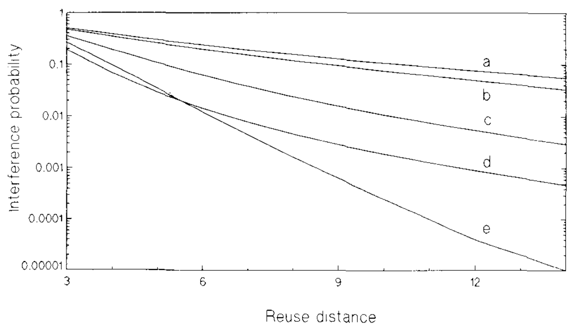

Fig. 1. Cochannel interference probability versus normalized reuse distance for protection ratio $\alpha = 8$ dB, blocking probability $B = 0.02$, carried traffic per cell $A_c = 5$ erlang, and $n_c = 10$ channels. Incoherent cumulation of interferers for: (a) combined Rayleigh fading and shadowing ($\sigma = 12$ dB). (b) shadowing only ($\sigma = 12$ dB). (c) combined Rayleigh fading and shadowing ($\sigma = 6$ dB). (d) Rayleigh fading only ($\sigma = 0$). (e) shadowing only ($\sigma = 6$ dB).

i.e.,

$$R_u \triangleq \frac{D}{R}. \tag{25}$$

The signal-to-interference ratio at a receiving base station located at the cell center is

$$\frac{\xi_d}{\xi_u} = \frac{R^{-\gamma}}{D^{-\gamma}\left[\exp\sum_{i=1}^{n} G_{li}\right)\right]} \tag{26}$$

where γ is the ground-wave propagation path-loss slope (we take $\gamma = 4$), n is the number of cochannel interfering cells (here $n = 6$), and G_{li} is a correction to the area median power which is dependent on n and given by [13, eq. (8)]. Using (25) and (26), R_u can be written as

$$R_u = \left[\frac{\xi_d}{\xi_u}\exp\left(\sum_{i=1}^{n} G_{li}\right)\right]^{1/4}. \tag{27}$$

Now, the cochannel interference probability can be computed as a function of R_u, for the three cases i), ii), and iii). The results are shown in Fig. 1 for $\sigma_i = \sigma_d = \sigma = 6$ and 12 dB. It is confirmed from Fig. 1 that a large reuse distance is essential to avoid excessive values of cochannel interference. It is also observed that shadowing is the predominant factor in determining the reuse distance and the cochannel interference probability, in particular for large σ.

Typical results for cochannel interference probabilities in the event of pure log-normal shadowing with $\sigma = 6$ and 12 dB, $\alpha = 8$ dB, $n_c = 10$, and $B = 0.02$, are shown in Fig. 2, for both Fenton's classical method [11] and Schwartz and Yeh's technique [12], [13]. It is seen from Table III that Schwartz and Yeh's method results in up to 89% higher cochannel interference probabilities for $R_u = 15$.

IV. Spectrum Efficiency

Spectrum efficiency, E_s, is defined as the carried traffic per cell, A_c, divided by the product of bandwidth per channel, W, number of channels per cell, n_c, and cluster size C cells, given a unit cell area of S. Thus

$$E_s \triangleq \frac{A_c}{n_c WCS}\,\text{erlang/MHz/km}^2. \tag{28}$$

Carried traffic can be obtained by

$$A_c \triangleq A(1 - B) \tag{29}$$

where A is the offered traffic per cell in erlang. The blocking probability B is determined using the Erlang-B formula [20], [21]:

$$B = \frac{A^n c}{n_c! \sum_{n=0}^{n_c}\left(\frac{A^n}{n!}\right)}. \tag{30}$$

The cells are assumed to form a cluster of size C, located around a reference cell and repeated around each of its cochannel cells. However, the exact shape of a valid cluster need not be precisely specified [17]. The cluster size is taken on the form

$$C = i^2 + ij + j^2, \qquad i, j \geq 0 \tag{31}$$

with integer i and j.

The reuse distance and the number of cells per cluster are related by [17], [23]

$$R_u = (3C)^{1/2}. \tag{32}$$

Fig. 3 shows plots for the cluster size and reuse distance determined using (32) and for spectrum efficiency versus reuse distance obtained using (28) and (29), taking $W = 25$ kHz, $B = 0.02$, $A_c = 5$ erlang, $n_c = 10$, and $S = 1$ km^2.

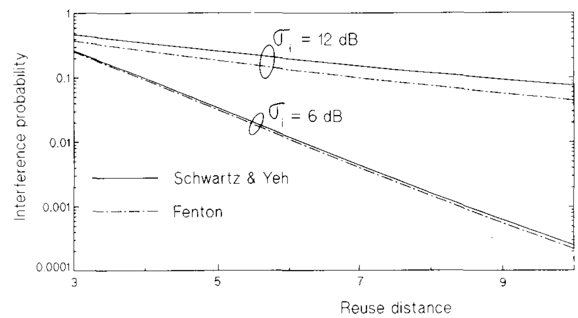

Fig. 2. Cochannel interference probability versus normalized reuse distance for $\alpha = 8$ dB, $A_c = 5$ erlang, $B = 0.02$, and $n_c = 10$ for shadowing only ($\sigma = 6$ and 12 dB), calculated using Fenton's and Schwartz and Yeh's techniques.

TABLE III
Percentage Difference in Cochannel Interference Probability for Log-Normal Shadowing Environment Only, Comparing Schwartz and Yeh's Techniques with Fenton's Method for $\alpha = 8$ dB, $n_c = 10$ and $B = 0.02$

R_u	$\sigma_i = 6$ dB	$\sigma_i = 12$ dB
3	4%	25%
6	9%	47%
9	12%	64%
12	14%	78%
15	15%	89%

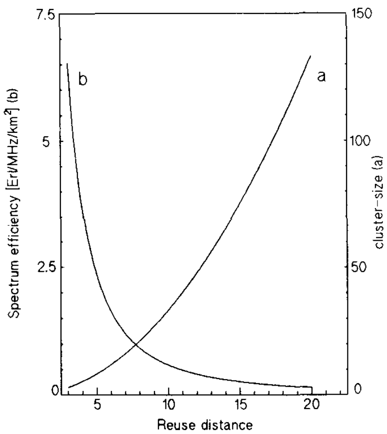

Fig. 3. Influence of normalized reuse distance (R_u). (a) cluster size (C). (b) spectrum efficiency (E_s) for $A_c = 5$ erlang, $B = 0.02$, $W = 25$ kHz, $S = 1$ km^2, and $n_c = 10$.

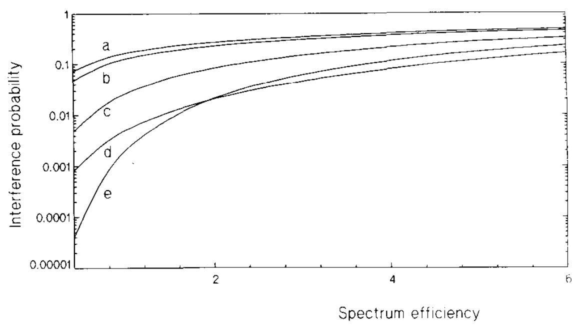

Fig. 4. Cochannel interference probability versus spectrum efficiency (E_s). channel bandwidth $W = 25$ kHz; unit cell area $S = 1$ km^2; other parameters as in Fig. 1.

It can be inferred from Figs. 1 and 3 that the spectrum efficiency can be increased by accepting an increase in the cochannel interference probability; this is seen directly from Fig. 4. The cluster size is seen to play an important role in determining the optimum spectrum efficiency for meeting a certain system requirement: a decrease in cluster size (and hence reuse distance) increases the spectrum efficiency. Referring to Fig. 1, this is equivalent to accepting a higher cochannel interference probability. This conclusion can also be drawn from Fig. 4, which shows the relation between interference probability and spectrum efficiency.

To ascertain the difference between coherent and incoherent cumulation of interference, Table IV compares the corresponding cochannel interference probabilities for $\sigma = 6$ and 12 dB, and different reuse distances. While coherent conditions do allow smaller interference probability, Table IV shows that the difference is rather small (below 10% in the cases considered), and decreases at larger reuse distances. In the following numerical examples, we, therefore, confine ourselves to incoherent cumulation interferes.

Figs. 5 and 6 show interference probability as a function of spectrum efficiency, with protection ratio (Fig. 5) and the carried traffic (Fig. 6) as a parameter. Clearly, the spectrum efficiency can be increased by tolerating a lower value of

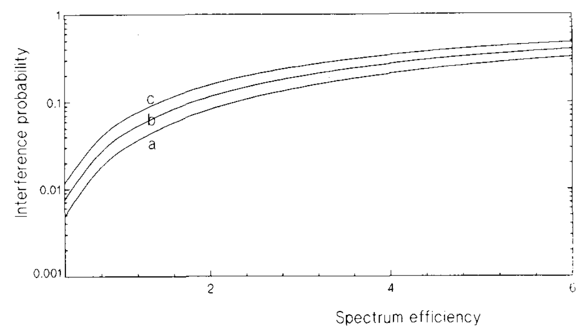

Fig. 5. Cochannel interference probability versus spectrum efficiency for combined Rayleigh fading and shadowing ($\sigma = 6$ dB) using incoherent cumulation of interference with $A_c = 5$ erlang, $B = 0.02$, $n_c = 10$, $S = 1$ km^2, $W = 25$ kHz, and (a) $\alpha = 8$ dB, (b) $\alpha = 10$ dB, (c) $\alpha = 12$ dB.

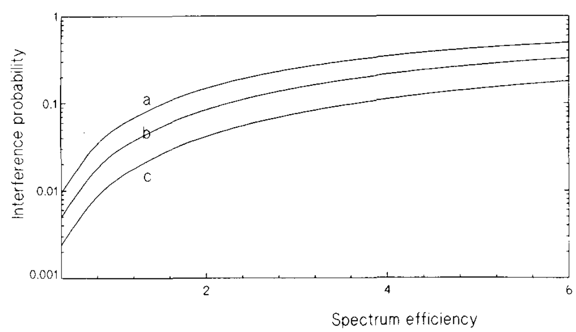

Fig. 6. Cochannel interference probability versus spectrum efficiency for combined Rayleigh fading and shadowing using incoherent cumulation of interference with $n_c = 10$, $S = 1$ km^2, $\sigma = 6$ dB, $W = 25$ kHz, $\alpha = 8$ dB, (a) $a_c = 0.99$ erlang/channel ($B = 0.8$), (b) $a_c = 0.5$ erlang/channel ($B = 0.02$), (c) $a_c = 0.23$ erlang/channel ($B = 0.0001$).

TABLE IV
COCHANNEL INTERFERENCE PROBABILITY FOR THE COHERENT AND INCOHERENT CUMULATION OF COCHANNEL INTERFERERS WITH $\sigma = 6$ AND 12 dB AS A FUNCTION OF REUSE DISTANCE

	$\sigma_i = 6$ dB		$\sigma_i = 12$ dB	
R_u	$F(CI)$ incoherent	$F(CI)$ coherent	$F(CI)$ incoherent	$F(CI)$ coherent
3	0.35867	0.32317	0.50531	0.47402
4	0.19453	0.17840	0.38644	0.36220
5	0.10791	0.10107	0.30083	0.28155
6	0.06236	0.05954	0.23971	0.22285
7	0.03767	0.03655	0.19187	0.17910
8	0.02374	0.02333	0.15501	0.14574
9	0.01555	0.01543	0.12743	0.11995

protection ratio (Fig. 5). More surprising on first sight is the *increase* of spectrum efficiency obtained with a *lower* carried traffic A_c, given a fixed interference probability (Fig. 6). This is caused by the much stronger impact in (28) of the corresponding reduction in the necessary reuse distance R_u (32). Thus a higher blocking probability does not necessarily result in higher spectrum efficiency.

Figs. 7–9 compare the spectral efficiencies of the different modulation methods of interest in cellular telephony shown in Table V. The protection ratio (α) is defined as the minimum value of the wanted (carrier)-to-unwanted (interference) signal ratio at the receiver input such that a specified reception

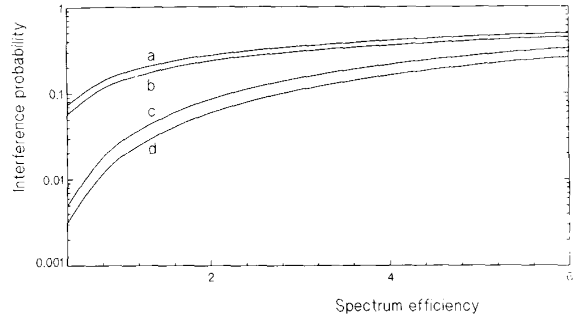

Fig. 7. Cochannel interference probability versus spectrum efficiency for $a_c = 0.5$ erlang/channel, $B = 0.02$, $S = 1\ \text{km}^2$, and $n_c = 10$ for combined fading and shadowing using incoherent cumulation of interference with (a) 25-kHz digital modulation with 8-dB protection ratio and $\sigma = 12$ dB, (b) 12.5-kHz FM with 12-dB protection ratio and $\sigma = 12$ dB, (c) 25-kHz digital modulation with 8-dB protection ratio and $\sigma = 6$ dB, (d) 12.5-kHz FM with 12-dB protection ratio and $\sigma = 6$ dB.

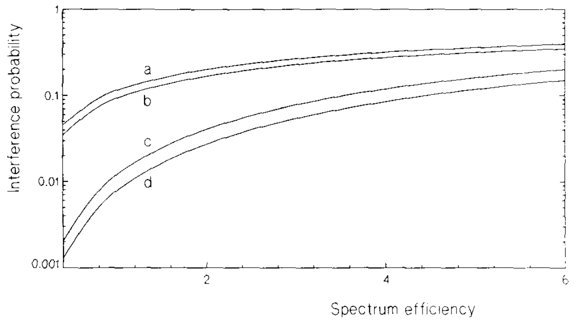

Fig. 8. Cochannel interference probability versus spectrum efficiency for $a_c = 0.5$ erlang/channel, $B = 0.02$, and $n_c = 10$ for combined fading and shadowing using incoherent cumulation of interference with (a) 5-kHz SSB with $\alpha = 18$ dB and $\sigma = 12$ dB, (b) 5-kHz SSB with $\alpha = 16$ dB and $\sigma = 12$ dB, (c) 5-kHz SSB with $\alpha = 18$ dB and $\sigma = 6$ dB, (d) 5-kHz SSB with $\alpha = 16$ dB and $\sigma = 6$ dB.

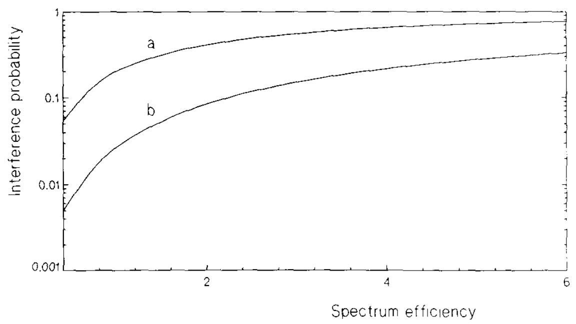

Fig. 9. Cochannel interference probability versus spectrum efficiency for $a_c = 0.5$ erlang/channel, $B = 0.02$, and $n_c = 10$ for combined fading and shadowing using incoherent cumulation of interference with (a) 30-kHz FM with 18 dB protection radio and $\sigma = 6$ dB, (b) 25-kHz digital modulation with 8 dB protection ratio and $\sigma = 6$ dB.

quality of the wanted signal is achieved at the receiver output [27]. For analog FM systems, as used for the U.S. Advanced Mobile Phone Services (AMPS) system and the U.K. Total Access Communication System (TACS), satisfactory operation requires approximately $\alpha = 18$ dB [17], [25]–[28]. The channel spacing in AMPS is 30 kHz, whereas in TACS a closer channel spacing of 25 kHz has been adopted. The Nordic Mobile Telephone (NMT-900) system even uses a 12.5-kHz channel spacing. In the future Pan-European digitally modulated system Groupe Speciale Mobile (GSM), α will be approximately 9.5 dB [25]; here, we have used 8 dB and a channel spacing of 25 kHz (an European standard) for the purpose of computations. For the sake of completeness, the spectrum efficiency is also evaluated (Fig. 8) for SSB. An SSB system with a 5-kHz channel suffers similar subjective effects of cochannel interference as a 25 kHz FM system under real mobile radio field conditions [27]. In both cases, a signal-to-interference ratio of the order of 16 dB is necessary to achieve a fair quality of reception. Therefore, the efficiency of SSB systems is evaluated for $\alpha = 16$ and 18 dB with a 5-kHz channel spacing.

TABLE V
Channel Spacing and Protection Ratio for Different Modulation Methods

Modulation method	Channel spacing	α (dB) Protection ratio	References
Analog FM	12.5 kHz	12	[8], [24]
Analog FM	30 kHz	18	[17], [25]–[27]
Digital modulation	25 kHz	8	[25]
SSB	5 kHz	16 and 18	[8], [24], [27]

The computational results (Figs. 7–9) show that SSB offers the highest efficiency among the modulation techniques considered in this paper. Our efficiency ranking of these methods agree with [8] but also include for 30-kHz spaced FM (AMPS) and the digital GSM.

V. System Bandwidth

System bandwidth represents the total bandwidth required to serve a cluster. Thus it can be expressed as the product of the number of channels per cluster and the bandwidth per channel, i.e.,

$$S_w \triangleq n_c WC = \frac{A_c}{SE_s}\ \text{MHz}. \tag{33}$$

Thus as distinct from the spectrum efficiency, the system bandwidth is an absolute measure of the frequency resources required to meet a certain traffic demand in a given area. The cluster size C and the modulation method influence the system complexity and, hence, the economic resources required to realize the cellular network and the mobile terminals. Thus a judicious trade-off between system options can be based on S_w, C and the modulation parameters.

To illustrate the typical differences between cellular system options, Table VI shows spectrum efficiency, system bandwidth and cluster size for the two FM systems, one digital modulation system (DM) and two SSB systems dealt with in Figs. 7–9. The cluster size C is approximated by the nearest valid number, according to (31). The Table clearly reveals the relative merits of the different systems, in terms of frequency demands and cellular system complexity.

VI. Conclusions

Cochannel interference probabilities for mobile radio systems exposed to realistic propagation impairments have been

TABLE VI
SPECTRUM EFFICIENCY E_S, SYSTEM BANDWIDTH S_w AND CLUSTER SIZE C FOR DIFFERENT TYPES OF MODULATION TECHNIQUES WITH $n_c = 10$, $B = 0.02$, $a_c = 0.5$ ERLANG/CHANNEL, AND $F(CI \mid n) = 10^{-1}$ IN A COMBINED FADING AND SHADOWING ENVIRONMENT WITH $\sigma = 6$ dB

Type of modulation	E_s (erlang/MHz/km^2)	S_w (MHz)	C
FM_{30}	0.6	8.4	28
DM_{25}	2.2	2.25	9
$FM_{12.5}$	2.8	2	16
SSB_{18}	3.6	1.4	28
SSB_{16}	4.5	1.25	25

calculated using Fenton's classical method and compared with our values, obtained by Schwartz and Yeh's technique. Our results indicate significantly higher interference probabilities, up to 89% more than suggested by the classical method. This indicates that earlier results for cochannel interference calculations in cellular radio systems based on Fenton's method are optimistic. The use of Schwartz and Yeh's technique is recommended for mobile scenarios in which the variance of the shadowing lies between 6 and 12 dB.

Earlier studies of cochannel interference probability in a cellular mobile network were carried out assuming coherent cumulation of interference. Now, results are also available for the more realistic assumption of incoherent cumulation of interfering signals with independent Rayleigh fading, log-normal shadowing and UHF ground-wave path loss. Computed results show that coherent cumulation of interferers leads to somewhat lower values of cochannel interference probability, i.e., a slightly higher spectrum efficiency than with incoherent cumulation. However, for large reuse distances the differences become minute.

A detailed assessment of the spectrum efficiency and system bandwidth of general cellular systems has been given. It is confirmed that the spectrum efficiency is higher for lower protection ratio, and that a tradeoff has to be made between cluster size and cochannel interference probability to achieve the desired spectrum efficiency. In particular, it cannot be assumed that higher carried traffic (higher blocking probability) always leads to better spectral efficiencies, because of the associated increase in cluster size required to maintain a given interference probability.

In practical terms, a design trade-off will more often be between the overall system bandwidth (absolute bandwidth required to provide a certain service) and the associated network complexity (in terms of cluster size, modulation type, etc.). The approach described in this paper would assist planners and designers in obtaining a more realistic assessment of different system options, especially in the many cases where the shadowing of mobiles cannot be assumed to be moderate.

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Ramjee Prasad (M'88–SM'90) was born in India on July 1, 1946. He received the B.Sc. (Engg.) degree from Bihar Institute of Technology, Sindri, India, in 1968, and the M.Sc. (Engg.) and Ph.D. degrees from Birla Institute of Technology (BIT), Ranchi, India, in 1970 and 1979, respectively.

He joined BIT as Senior Research Fellow in 1970 and became Associate Professor in 1980. During 1983–1988 he was with the University of Dar es Salaam (UDSM), Tanzania, where he became Professor in Telecommunications at the Department of Electrical Engineering in 1986. Since February 1988, he has been with the Telecommunications and Traffic Control Systems Group, Delft University of Technology, The Netherlands, where he is actively involved in the area of mobile communications. While he was with BIT, he supervised many research projects in the area of microwave and plasma engineering. He was project responsible at UDSM for the collaborative project "Satellite Communications for Rural Zones" with Eindhoven University of Technology, The Netherlands. He has published over 70 technical papers. His current research interest lies in mobile communications, indoor wireless communications, and telematics.

Dr. Prasad is a Fellow of the Institution of Electronics and Telecommunication Engineers, India.

Adriaan Kegel was born in The Netherlands on November 4, 1932. He received the M.Sc. degree from Delft University, The Netherlands, in 1972.

He joined Philips in 1954 where he worked on the development of microwave link equipment as a Radio Engineer. In 1964 he joined Delft University of Technology. In 1970 he became Chairman of the "Working Group Indonesia," which developed the so-called "Electronic Blackboard." For this development the working group received an Award from the Scientific Radio Foundation Veder. After completion of the Indonesian Project in 1974, he continued his research work on the coding and transmission of graphical information. This work led to the development of several systems (e.g., Vidibord) and a number of publications. His recent interest is in research and education on mobile communication systems.

Mr. Kegel is a member of The Netherlands Electronics and Radio Society and the Federation of Telecommunication Engineers of the European Community.