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IMPROVED CURVE FITS FOR THE THERMODYNAMIC PROPERTIES OF EQUILIBRIUM AIR SUITABLE FOR NUMERICAL COMPUTATION USING TIME-DEPENDENT OR SHOCK-CAPTURING METHODS

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16. Abstract

Simplified curve fits for the thermodynamic properties of equilibrium air have been devised for use in either the "time-dependent" or "shock-capturing" computational methods. For the "time-dependent" method, curve fits were developed for $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(e, \rho)$, while for the "shock-capturing" method, curve fits were developed for $h = h(p, \rho)$ and $T = T(p, \rho)$. The ranges of validity for these curve fits are for temperatures up to 25,000 °K and densities from 10^{-7} to 10^{3} amagats. These approximate curve fits may be particularly useful when employed on advanced computers such as the Burroughs ILLIAC IV or the CDC STAR since they avoid a cumbersome table-lookup.

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SUMMARY

Simplified curve fits for the thermodynamic properties of equilibrium air have been devised for use in either the "time-dependent" or "shock-capturing" computational methods. The accuracies of these curve fits are substantially improved over the accuracies of previous curve fits appearing in NASA CR-2134. For the "time-dependent" method, curve fits were developed for $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(e, \rho)$, while for the "shock-capturing" method, curve fits were developed for $h = h(p, \rho)$ and $T = T(p, \rho)$. The ranges of validity for these curve fits are the same as the NASA-ARC RGAS program, namely, temperatures up to 25,000 ^{o}K and densities from 10^{-7} to 10^{3} amagats. These approximate curve fits may be particularly useful when employed on advanced computers such as the Burrough's ILLIAC IV or the CDC STAR since they avoid the cumbersome table-lookup feature of the RGAS program.

NOTATION

a = speed of sound

e = internal energy

h = enthalpy

p = pressure

R = gas constant

T = temperature

$\tilde{\gamma}$ = h/e

ρ = density

Subscript

o = standard conditions

INTRODUCTION

When computing real gas flows using a finite-difference solution of the conservative form of the unsteady Navier-Stokes equations, it becomes necessary to determine pressure as a function of density (ρ) and internal energy (e). This requirement led to the previous study[1] in which two different approaches were developed for the case of equilibrium air. In the first approach, the NASA-Ames RGAS program[2] was modified to allow density and internal energy to be the independent variables. This approach permits a very accurate determination of the thermodynamic properties of air. Unfortunately, the table-lookup feature of the RGAS program is too cumbersome to be effectively employed on advanced computers such as the Burrough's ILLIAC IV or the CDC STAR. For this reason, and also to reduce computation time on conventional serial computers, simpler approximate methods were investigated in the second approach.

In the second approach, simplified curve fits were devised for $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(e, \rho)$. In addition, a simplified curve fit was made for $h = h(p, \rho)$. This latter curve fit is required in the "shock-capturing" method[3]. The ranges of validity for these correlation formulas were the same as the RGAS subroutine, namely, temperatures up to 25,000 ^{o}K and densities from 10^{-7} to 10^{3} amagats. The accuracies of these simplified curve fits were much better than the previous curve fits of Barnwell[4], but they did not approach the accuracy of the modified RGAS program. For this reason, the present study was undertaken to substantially improve the accuracies of the previous curve fits without increasing the required computer time.

CONSTRUCTION OF CURVE FITS

The curve fits were constructed using Grabau-type transition functions[5] in a manner similar to Lewis and Burgess[6] and Barnwell[4]. A transition function of this type can be used to smoothly connect two surfaces $f_1(x, y)$ and $f_2(x, y)$. For y = constant, the Grabau-type transition function (with an inflection point) becomes

$$z = f_1(x) + \frac{f_2(x) - f_1(x)}{1 + \exp\left[K(x - x_o)\right]} \tag{1}$$

where K is the parameter which determines the rate at which z changes from $f_1(x)$ to $f_2(x)$, and x_o is the location of the inflection point as shown in Fig. 1.

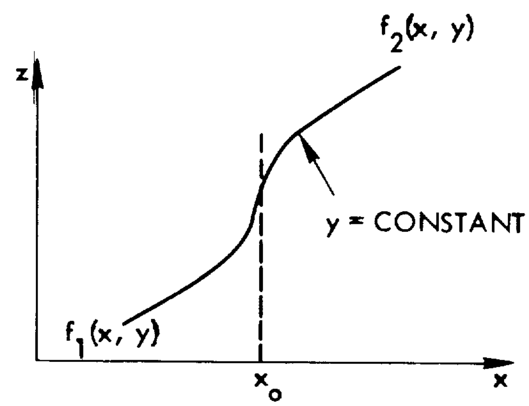

Fig. 1. Grabau-type transition with inflection point.

In the previous study[1], the curve fits were constructed by joining two Grabau-type transition functions with the equation for a perfect gas. In the present study, a substantial improvement in accuracy was achieved by joining together as many as five Grabau-type transition functions with the perfect gas equation as shown in Fig. 2.

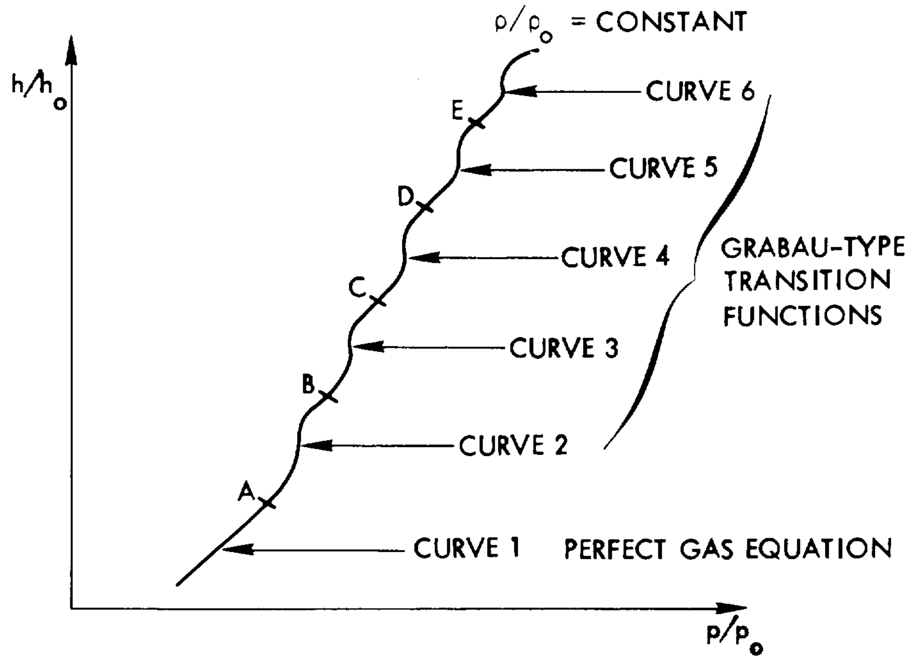

Fig. 2. Example curve fit for $h = h(p, \rho)$.

As in the previous study, the range of the independent variable ρ was subdivided into three separate regions with different coefficients being used in the curve fits for each region (see Fig. 3). The division lines are located at $\rho/\rho_o = 5 \times 10^{-5}$ and $\rho/\rho_o = 5 \times 10^{-1}$.

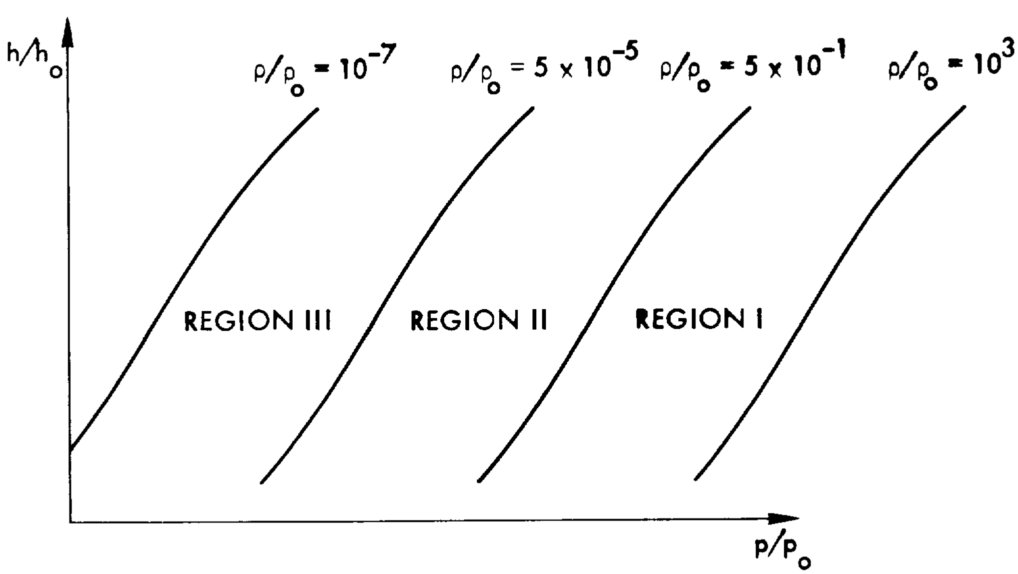

Fig. 3. Division of curve fit range by density.

The coefficients in the equations for $f_1(x, y)$ and $f_2(x, y)$ were determined using a least squares computer program to fit the data from the original NASA RGAS program. The selection of the form of the equations for $f_1(x, y)$ and $f_2(x, y)$ was largely a trial-and-error process. By including more terms, a better curve fit was achieved. In fact, if a sufficient number of terms were retained in $f_1(x, y)$ and $f_2(x, y)$, the accuracy of these curve fits could be made to approach that of the RGAS program, but with little savings in computer time.

EQUATIONS OF CURVE FITS

$p = p(e, \rho)$

For the correlation of $p = p(e, \rho)$, the ratio $\tilde{\gamma} = h/e$ was curve-fitted as a function of e and ρ so that p can be calculated from

$$p = \rho e(\tilde{\gamma} - 1) \tag{2}$$

The general form of the equation used for $\tilde{\gamma}$ was

$$\tilde{\gamma} = a_1 + a_2 Y + a_3 Z + a_4 YZ + a_5 Y^2 + a_6 Z^2 + a_7 YZ^2 + a_8 Z^3 + \frac{a_9 + a_{10} Y + a_{11} Z + a_{12} YZ}{1 + \exp\left[\left(a_{13} + a_{14} Y\right)\left(Z + a_{15} Y + a_{16}\right)\right]} \tag{3}$$

where $Y = \log_{10}(\rho/1.292)$ and $Z = \log_{10}(e/78408.4)$. The units for ρ are kg/m^3 and the units for e are m^2/sec^2. The coefficients a_1, a_2, ..., a_{16} are given in Table A1, Appendix A, for the entire range of e and ρ. It should be noted that many of the terms appearing in Eq. (3) are not used over the entire range of e and ρ.

$a = a(e, \rho)$

An exact expression for the speed of sound in terms of $\tilde{\gamma}$ was derived by Barnwell[4] and may be written as

$$a = \left[e\left\{(\tilde{\gamma} - 1)\left[\tilde{\gamma} + \left(\frac{\partial\tilde{\gamma}}{\partial \log_e e}\right)_\rho\right] + \left(\frac{\partial\tilde{\gamma}}{\partial \log_e \rho}\right)_e\right\}\right]^{1/2} \tag{4}$$

Because of the errors in the approximate expression for $\tilde{\gamma}$, Eq. (3), it was found that a much better correlation for $a = a(e, \rho)$ could be obtained from

$$a = \left[e \left\{ K_1 + (\tilde{\gamma} - 1) \left[\tilde{\gamma} + K_2 \left(\frac{\partial \tilde{\gamma}}{\partial \log_e e} \right)_\rho \right] + K_3 \left(\frac{\partial \tilde{\gamma}}{\partial \log_e \rho} \right)_e \right\} \right]^{1/2} \quad (5)$$

where the coefficients K_1, K_2, and K_3 were determined using the least-squares-best-fit program in conjunction with the NASA RGAS program. The coefficients K_1, K_2, and K_3 are tabulated in Table A1, Appendix A.

$\underline{T = T(e, \rho)}$

In the calculation of $T = T(e, \rho)$, the pressure is first found using Eq. (2), and then the temperature is found from the equation

$$\log_{10}(T/151.78) = b_1 + b_2 Y + b_3 Z + b_4 YZ + b_5 Z^2 + b_6 Y^2 + b_7 Y^2 Z + b_8 YZ^2 + \frac{b_9 + b_{10} Y + b_{11} Z + b_{12} YZ + b_{13} Z^2}{1 + \exp\left[\left(b_{14} Y + b_{15}\right)\left(Z + b_{16}\right)\right]} \quad (6)$$

where $Y = \log_{10}(\rho/1.225)$, $X = \log_{10}(p/1.0134 \times 10^5)$, and $Z = X - Y$. The units for p are newtons/m^2, and the units for T are $^\circ K$. The coefficients $b_1, b_2, \ldots, b_{16}$ are given in Table A2, Appendix A. These coefficients were determined in such a manner as to compensate for the errors incurred in the initial calculation of pressure using Eq. (2).

$\underline{h = h(p, \rho)}$

For the correlation of $h = h(p, \rho)$, the ratio $\tilde{\gamma} = h/e$ was curve-fitted as a function of p and ρ so that h can be calculated from

$$h = (p/\rho)\left(\frac{\tilde{\gamma}}{\tilde{\gamma} - 1}\right) \quad (7)$$

The general form of the equation used for $\tilde{\gamma}$ was

$$\tilde{\gamma} = c_1 + c_2 Y + c_3 Z + c_4 YZ + \frac{c_5 + c_6 Y + c_7 Z + c_8 YZ}{1 + \exp\left[c_9\left(X + c_{10} Y + c_{11}\right)\right]} \tag{8}$$

where $Y = \log_{10}(\rho/1.292)$, $X = \log_{10}(p/1.013 \times 10^5)$, and $Z = X - Y$. The coefficients c_1, c_2, ..., c_{11} are tabulated in Table A3, Appendix A.

<u>$T = T(p, \rho)$</u>

The general form of the equation used for the correlation $T = T(p, \rho)$ was

$$\log_{10}\left(T/T_o\right) = d_1 + d_2 Y + d_3 Z + d_4 YZ + d_5 Z^2 + \frac{d_6 + d_7 Y + d_8 Z + d_9 YZ + d_{10} Z^2}{1 + \exp\left[d_{11}\left(Z + d_{12}\right)\right]} \tag{9}$$

where $Y = \log_{10}(\rho/1.225)$, $X = \log_{10}(p/1.0134 \times 10^5)$, and $Z = X - Y$. The coefficients d_1, d_2, ..., d_{12} are given in Table A4, Appendix A.

For the "time-dependent" method, the three curve fits $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(e, \rho)$ have been placed in a single subroutine named TGAS. The calling sequence and FORTRAN IV listing of this subroutine appear in Appendix B. For the "shock-capturing" method, the curve fits $h = h(p, \rho)$ and $T = T(p, \rho)$ have been placed in separate subroutines, each named TGAS. These subroutines could be combined into a single subroutine, if desired, or could be used in their present forms in the same computer program if one of the subroutines names were changed. The calling sequences and FORTRAN IV listings of these subroutines appear in Appendix C and Appendix D, respectively.

COMPARISONS WITH RGAS PROGRAM

Comparisons of the curve fits $p = p(e, \rho)$, $a = a(e, \rho)$, $T = T(e, \rho)$, $h = h(p, \rho)$, and $T = T(p, \rho)$ with the original RGAS program are shown in Figs. 4, 5, 6, 7, and 8. In order to make the comparisons for the first three curve fits, the following procedure was used. First, p and ρ data were supplied, which allowed the original RGAS program to compute e. Then, this e and the original ρ were inputed into the TGAS subroutine to obtain p, a, and T. Because of this procedure, pressure is plotted as one of the independent variables in Figs. 4, 5, and 6.

In order to assess the relative accuracies of the present curve fits with the RGAS program, 500 data points, along constant density lines ranging from 10^3 to 10^{-7} amagats, were selected for a comparison. The maximum percentage differences between the RGAS and TGAS programs along each constant density line are tabulated in Table 1. The accuracies of the present curve fits are substantially improved over the accuracies of the previous curve fits appearing in NASA CR-2134[1]. The maximum percentage differences for the primary variables $p = p(e, \rho)$ and $h = h(p, \rho)$ were found to be 4.7% and 4.6%, respectively.

A comparison of the relative computer times required for the TGAS subroutines and the NASA RGAS programs on the IBM 360-65 computer are given in Table 2. The new TGAS subroutine for finding $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(e, \rho)$ is 2.68 times faster than the modified NASA RGAS subroutine, as compared with the old TGAS subroutine which was 2.65 times faster. These comparisons do not include the time spent by the RGAS subroutine in reading the tape. The new TGAS subroutine for finding $h = h(p, \rho)$ is 3.80 times faster than the original RGAS program as compared with the old TGAS subroutine which was 3.88 times faster, again

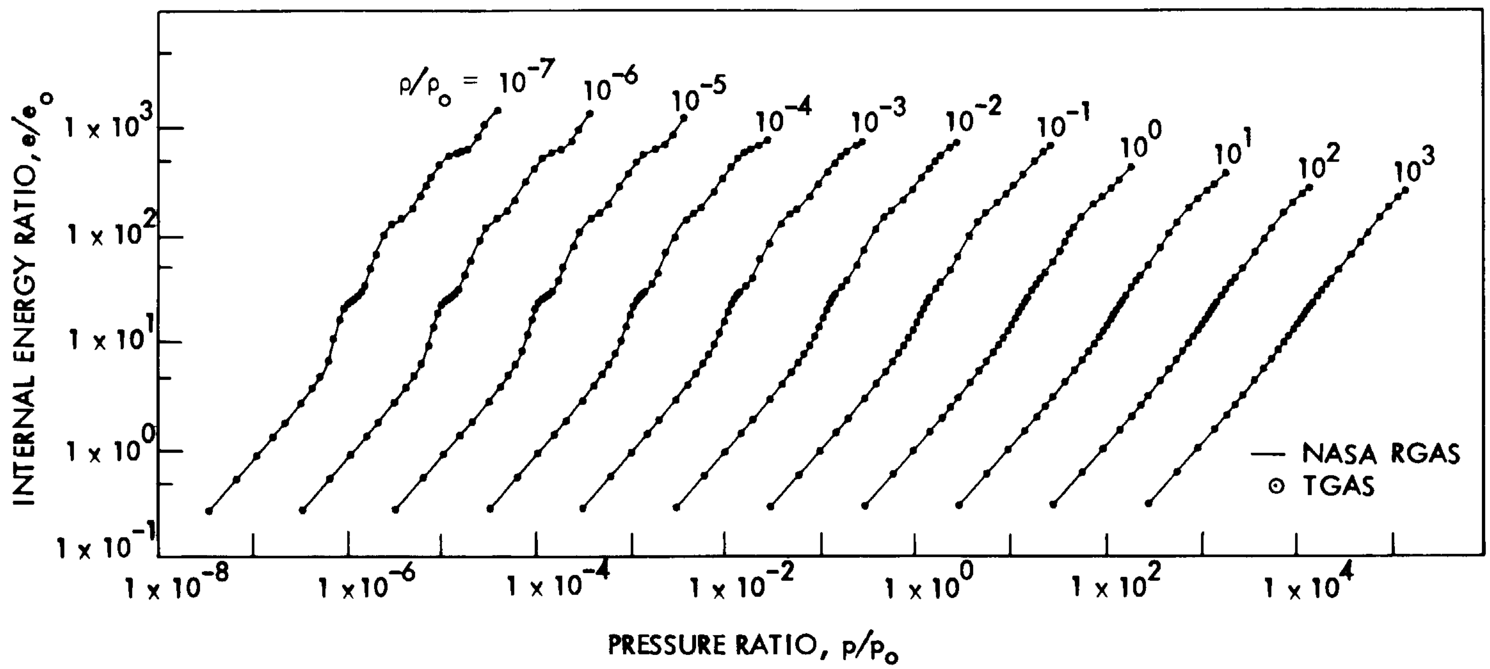

Fig. 4. Comparison of curve fits for $p = p(e, \rho)$.

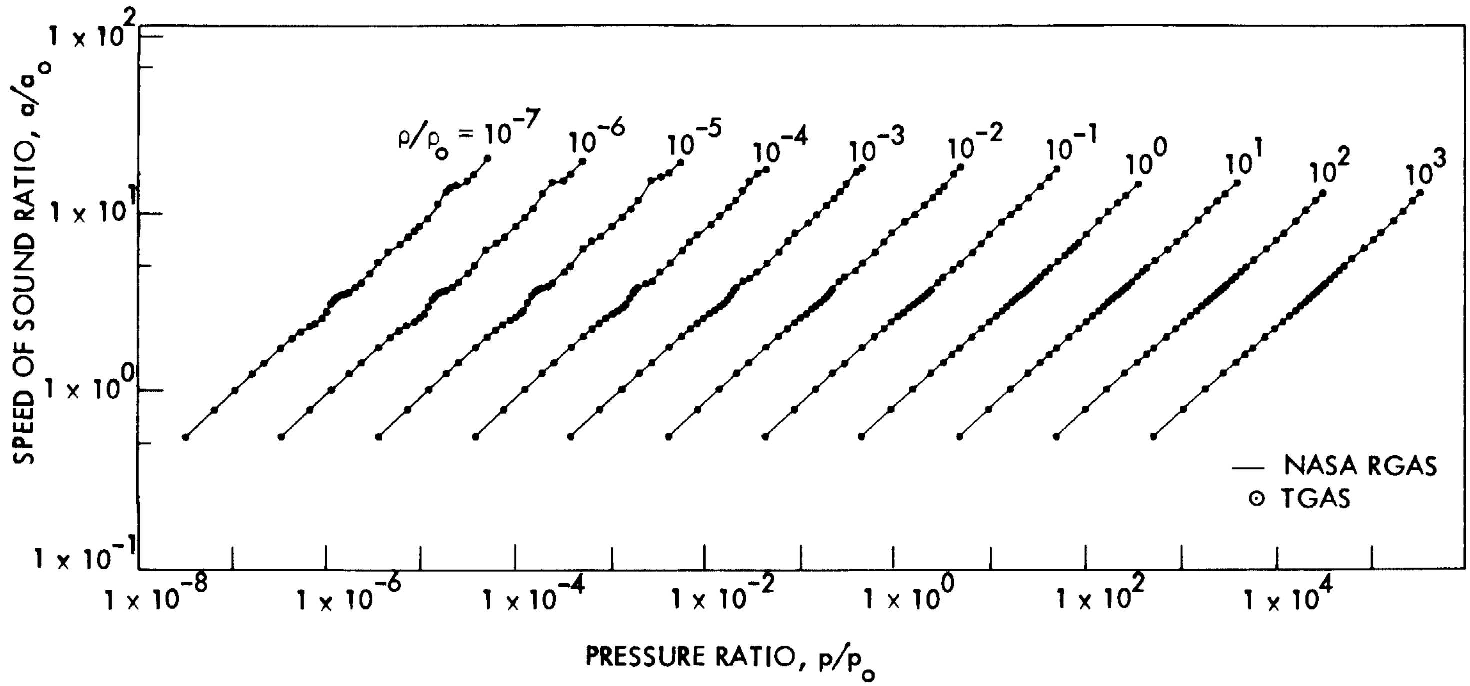

Fig. 5. Comparison of curve fits for $a = a(e, \rho)$.

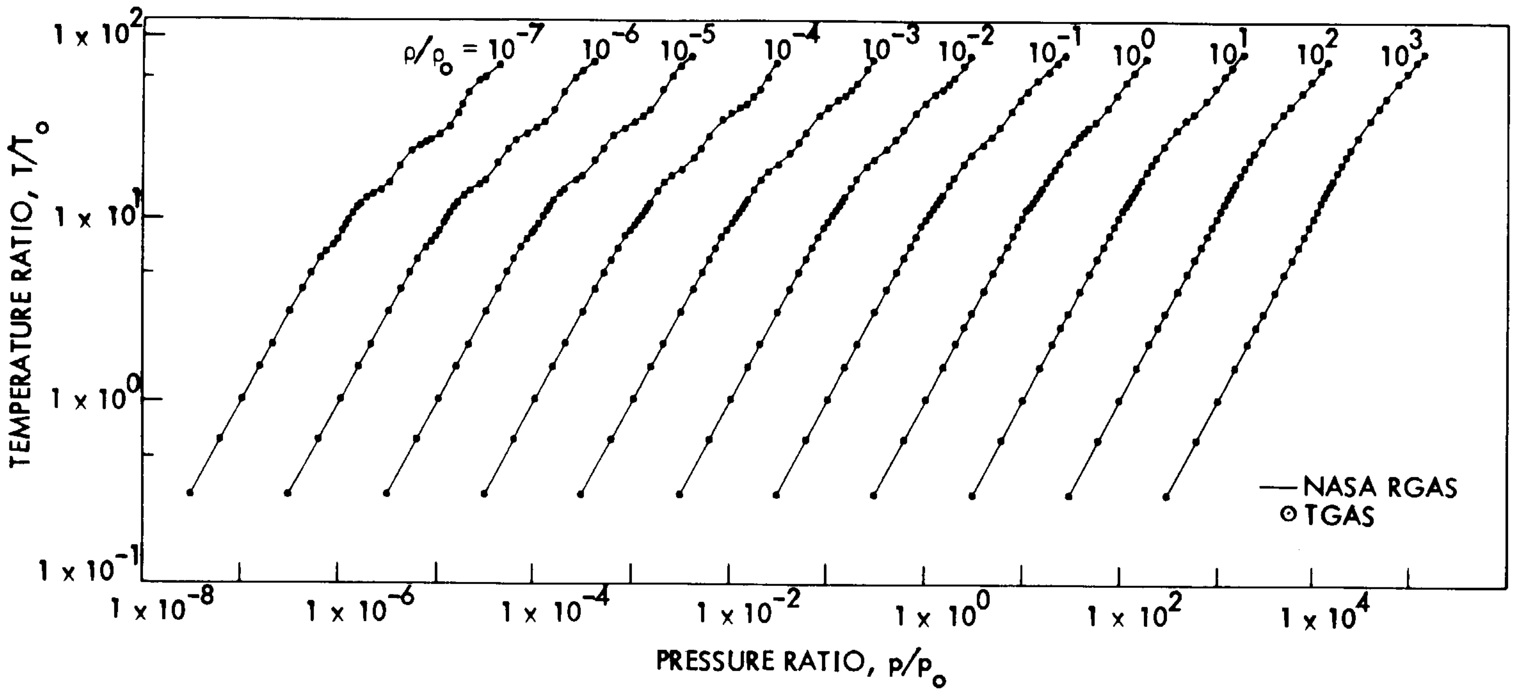

Fig. 6. Comparison of curve fits for T = T(e, ρ).

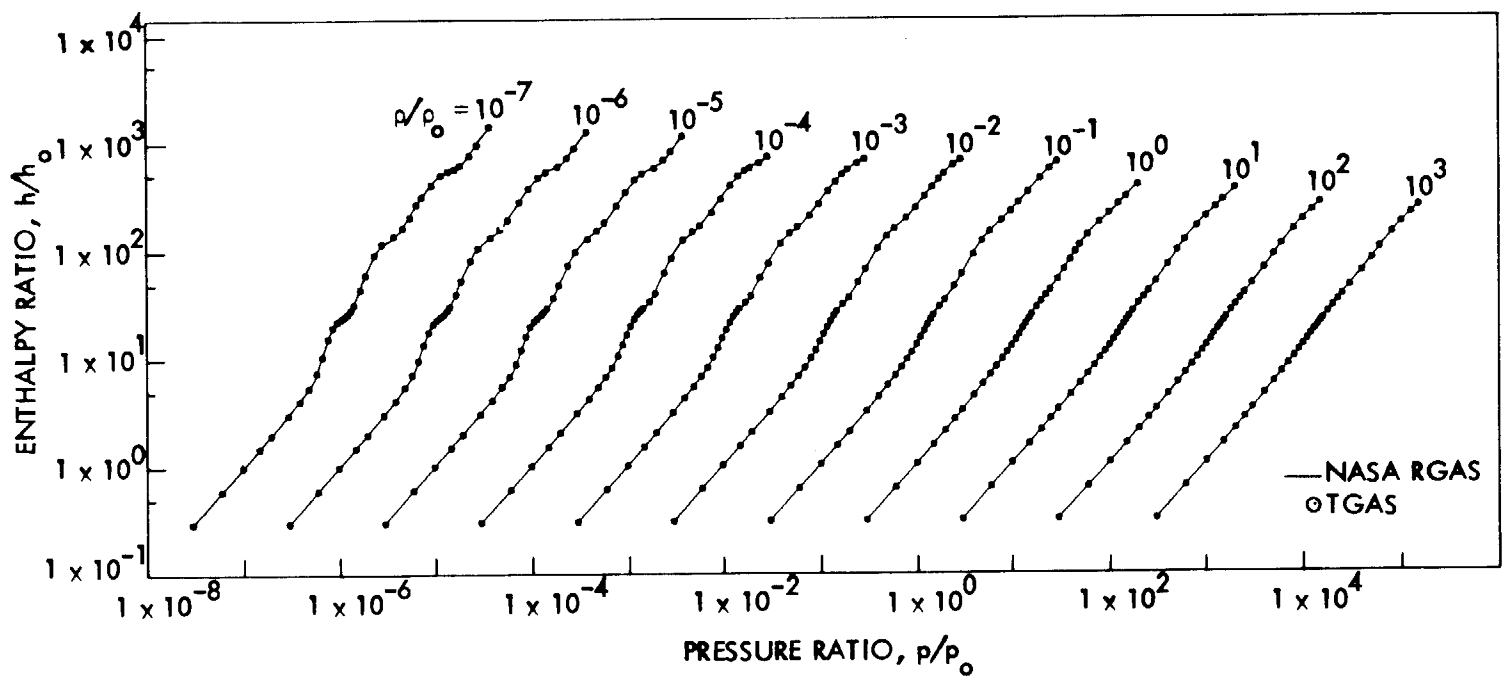

Fig. 7. Comparison of curve fits for $h = h(p, \rho)$.

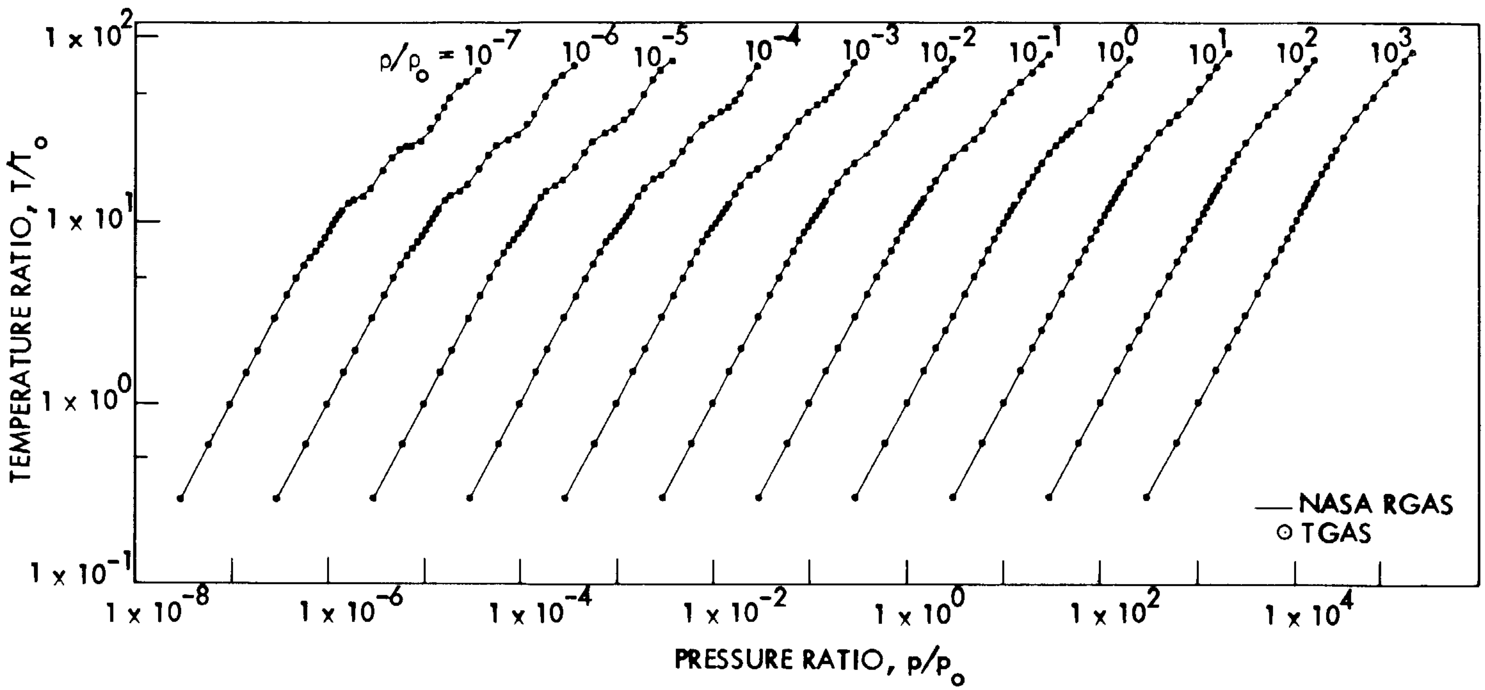

Fig. 8. Comparison of curve fits for $T = T(p, \rho)$.

Table 1. Maximum percentage differences between RGAS and TGAS programs.

Density ratio ρ/ρ_o	Curve Fit: $p = p(e, \rho)$	$a = a(e, \rho)$	$T = T(e, \rho)$	$h = h(p, \rho)$	$T = T(p, \rho)$
10^{3}	2.2%	2.0%	3.9%	1.8%	2.3%
10^{2}	1.3	1.0	2.5	1.4	3.0
10^{1}	1.5	1.3	3.3	1.9	2.6
10^{0}	1.8	1.1	2.9	2.2	2.0
10^{-1}	2.5	2.7	4.4	2.4	2.9
10^{-2}	2.9	1.4	3.0	2.9	1.9
10^{-3}	3.7	3.2	4.1	3.3	2.8
10^{-4}	4.7	2.9	5.3	3.9	2.2
10^{-5}	3.9	3.2	3.1	4.2	2.5
10^{-6}	4.0	3.7	4.0	3.3	4.3
10^{-7}	4.2	5.9	4.4	4.6	3.4

Table 2. Comparison of computer times.

Curve Fit	Number of Data Points	Old TGAS	New TGAS	RGAS	
$p = p(e, \rho)$	5080	6.04 sec	5.98 sec	3.73 sec	(data point 0-1) includes tape read
$a = a(e, \rho)$				16.01 sec	(data points 2-5080)
$T = T(e, \rho)$				19.74 sec	
$h = h(p, \rho)$	5096	2.61 sec	2.67 sec	2.80 sec	(data point 0-1) includes tape read
				10.12 sec	(data points 2-5096)
				12.92 sec	

excluding the tape read time. If there are only a few hundred calls made to these real gas subroutines, then the TGAS subroutines are substantially faster than the RGAS subroutines when the tape read time is included. For instance, if there are 500 calls made to find $p = p(e, \rho)$, $a = a(e, \rho)$ and $T = T(e, \rho)$, then the new TGAS is 8.95 times faster than the modified RGAS subroutine.

Comparisons of the values obtained at the juncture points of adjacent curve fits (see Fig. 2) are shown in Table 3. The maximum deviations between the curve fits at the juncture points of the primary variables $p = p(e, \rho)$ and $h = h(p, \rho)$ are 0.81% and 0.98%, respectively.

The simplified curve fits developed in this study for the thermodynamic properties of equilibrium air allow the user to reduce computer time and storage while maintaining good accuracy. This is particularly true in the "time-dependent" method, since the simplified curve fits could be used until near the end of a calculation when the "steady-state" solution is approached. Then, the modified RGAS subroutine could be used to give more accurate thermodynamic properties for the final steps. Substantial savings in computer time may also result in the "shock-capturing" method since an iterative procedure involving $h = h(p, \rho)$ is required for equilibrium calculations.

Table 3. Comparison of variables at juncture points.

Curve Fit	Density ratio ρ/ρ_o	Point A		Point B		Point C		Point D		Point E	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	10^{3}	1.400	1.393	1.294	1.293	1.228	1.231				
	10^{2}	1.400	1.393	1.287	1.286	1.208	1.210				
	10^{1}	1.400	1.393	1.280	1.279	1.187	1.189				
	10^{0}	1.400	1.393	1.273	1.272	1.166	1.168				
	10^{-1}	1.400	1.395	1.300	1.298	1.185	1.185	1.153	1.152		
$p = p(e, \rho)$	10^{-2}	1.400	1.395	1.284	1.284	1.173	1.167	1.141	1.138		
	10^{-3}	1.400	1.396	1.270	1.271	1.159	1.150	1.127	1.124		
	10^{-4}	1.400	1.396	1.257	1.257	1.144	1.132	1.113	1.110		
	10^{-5}	1.400	1.397	1.259	1.257	1.131	1.139	1.089	1.094	1.125	1.125
	10^{-6}	1.400	1.397	1.245	1.240	1.125	1.128	1.083	1.086	1.115	1.115
	10^{-7}	1.400	1.398	1.233	1.223	1.119	1.117	1.076	1.077	1.105	1.105
	10^{3}	443	438	364	359	314	314				
	10^{2}	443	438	356	352	296	297				
	10^{1}	443	438	349	346	279	279				
	10^{0}	443	438	341	339	260	260				
	10^{-1}	443	437	357	355	267	271	250	252		
$a = a(e, \rho)$	10^{-2}	443	438	346	345	255	258	238	239		
	10^{-3}	443	438	334	334	242	244	224	225		
	10^{-4}	443	438	323	323	227	229	210	211		
	10^{-5}	443	440	294	316	221	226	186	191	218	224
	10^{-6}	443	440	291	304	216	216	179	181	210	212
	10^{-7}	443	441	289	292	211	206	172	171	201	203
	10^{3}	5.73	5.69	14.87	14.95						
	10^{2}	5.73	5.69	14.87	15.03						
	10^{1}	5.73	5.69	14.87	15.02						
	10^{0}	5.73	5.69	14.87	15.00						
	10^{-1}	5.74	5.71	15.42	15.93	42.28	41.98	109.2	106.6		
$T = T(e, \rho)$	10^{-2}	5.74	5.69	15.36	15.56	39.46	39.17	98.56	95.77		
	10^{-3}	5.74	5.67	15.30	15.21	36.82	36.55	88.92	88.26		
	10^{-4}	5.74	5.65	15.25	14.86	34.36	34.10	80.23	82.20		
	10^{-5}	3.79	3.74	16.50	16.50	28.92	28.97	51.05	53.28	126.3	126.9
	10^{-6}	3.79	3.74	16.11	16.14	27.34	27.37	48.99	50.00	119.2	119.8
	10^{-7}	3.79	3.75	15.72	15.79	25.86	25.85	47.00	46.92	112.5	112.1
	10^{3}	1.400	1.404	1.291	1.294	1.253	1.255				
	10^{2}	1.400	1.404	1.282	1.284	1.231	1.233				
	10^{1}	1.400	1.403	1.272	1.274	1.210	1.211				
	10^{0}	1.400	1.403	1.262	1.263	1.188	1.189				
	10^{-1}	1.400	1.397	1.313	1.312	1.203	1.204	1.155	1.155		
$h = h(p, \rho)$	10^{-2}	1.400	1.397	1.287	1.287	1.191	1.183	1.140	1.137		
	10^{-3}	1.400	1.397	1.262	1.262	1.171	1.163	1.124	1.121		
	10^{-4}	1.400	1.397	1.236	1.238	1.148	1.144	1.107	1.106		
	10^{-5}	1.400	1.384	1.292	1.301	1.159	1.156	1.104	1.104		
	10^{-6}	1.400	1.387	1.260	1.259	1.142	1.138	1.093	1.092		
	10^{-7}	1.400	1.389	1.228	1.217	1.125	1.121	1.083	1.080		
	10^{3}	5.720	5.681	14.96	14.91						
	10^{2}	5.720	5.681	14.96	14.96						
	10^{1}	5.720	5.681	14.96	15.01						
	10^{0}	5.720	5.681	14.96	15.06						
	10^{-1}	5.736	5.698	15.42	15.86	42.59	42.09	109.2	109.0		
$T = T(p, \rho)$	10^{-2}	5.736	5.682	15.37	15.57	39.74	39.45	98.77	98.49		
	10^{-3}	5.736	5.665	15.33	15.28	37.07	37.01	89.34	88.95		
	10^{-4}	5.736	5.649	15.28	15.00	34.59	34.73	80.80	80.33		
	10^{-5}	3.810	3.738	18.51	18.47	36.34	35.98	83.50	83.90		
	10^{-6}	3.810	3.738	18.32	18.33	35.60	35.44	82.45	82.45		
	10^{-7}	3.810	3.738	18.13	18.20	34.92	34.94	81.35	80.95		

REFERENCES

1. Tannehill, J. C. and R. A. Mohling, "Development of Equilibrium Air Computer Programs Suitable for Numerical Computation Using Time-Dependent or Shock-Capturing Methods," NASA CR-2134 (September 1972).

2. Lomax, H. and M. Inouye, "Numerical Analysis of Flow Properties About Blunt Bodies Moving at Supersonic Speeds in an Equilibrium Gas," NASA TR R-204 (July 1964).

3. Kutler, P., W. A. Reinhardt, and R. F. Warming, "Numerical Computation of Multishocked Three-Dimensional Supersonic Flow Fields with Real Gas Effects," AIAA Paper 72-702 (June 1972).

4. Barnwell, R. W., "Inviscid Radiating Shock Layers about Spheres Traveling at Hyperbolic Speeds in Air," NASA TR R-311 (May 1969).

5. Grabau, M., "A Method of Forming Continuous Empirical Equations for the Thermodynamic Properties of Air from Ambient Temperatures to 15,000 ^{o}K with Applications," AEDC TN-59-102 (1959).

6. Lewis, C. H. and E. G. Burgess, "Empirical Equations for the Thermodynamic Properties of Air and Nitrogen to 15,000 ^{o}K," AEDC TDR-63-138 (1963).

APPENDICES

APPENDIX A Coefficients for curve fits

Table A1. Coefficients for curve fit p = p(e, ρ) and a = a(e, ρ).

Density Range	Curve Range	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
Y > -0.50	Z ≤ 0.65	1.40000	0	0	0	0	0	0	0	0	0
	0.65 < Z ≤ 1.68	1.45510	-0.000102	-0.081537	0.000166	0	0	0	0	0.128647	-0.049454
	1.68 < Z ≤ 2.46	1.59608	-0.042426	-0.192840	0.029353	0	0	0	0	-0.019430	0.005954
	Z > 2.46	1.54363	-0.049071	-0.153562	0.029209	0	0	0	0	-0.324907	-0.077599
-4.5 < Y ≤ -0.50	Z ≤ 0.65	1.40000	0	0	0	0	0	0	0	0	0
	0.65 < Z ≤ 1.54	1.44813	0.001292	-0.073510	-0.001948	0	0	0	0	0.054745	-0.013705
	1.54 < Z ≤ 2.22	1.73158	0.003902	-0.272846	0.006237	0	0	0	0	0.041419	0.037475
	2.22 < Z ≤ 2.90	1.59350	0.075324	-0.176186	-0.026072	0	0	0	0	-0.200838	-0.058536
	Z > 2.90	1.12688	-0.025957	0.013602	0.013772	0	0	0	0	-0.127737	-0.087942
-7 ≤ Y ≤ -4.5	Z ≤ 0.65	1.40000	0	0	0	0	0	0	0	0	0
	0.65 < Z ≤ 1.50	1.46543	0.007625	-0.254500	-0.017244	0.000292	0.355907	0.015422	-0.163235	0	0
	1.50 < Z ≤ 2.20	2.02636	0.058493	-0.454886	-0.027433	0	0	0	0	-0.165265	-0.014275
	2.20 < Z ≤ 3.05	1.60804	0.034791	-0.188906	-0.010927	0	0	0	0	-0.124117	-0.007277
	3.05 < Z ≤ 3.38	1.25672	0.007073	-0.039228	0.000491	0	0	0	0	0.721798	0.073753
	Z > 3.38	-84.0327	-0.831761	72.2066	0.491914	0.001153	-20.3559	-0.070617	1.90979	0	0

Density Range	Curve Range	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	K_1	K_2	K_3
Y > -0.50	Z ≤ 0.65	0	0	0	0	0	0	0	0	0
	0.65 < Z ≤ 1.68	-0.101036	0.033518	-15.0	0	0	-1.420	0.000450	0.203892	0.101797
	1.68 < Z ≤ 2.46	0.026097	-0.006164	-15.0	0	0	-2.050	-0.006609	0.127637	0.297037
	Z > 2.46	0.142408	0.022071	-10.0	0	0	-2.708	-0.000081	0.226601	0.170922
-4.5 < Y ≤ -0.50	Z ≤ 0.65	0	0	0	0	0	0	0	0	0
	0.65 < Z ≤ 1.54	-0.055473	0.021874	-10.0	0	0	-1.420	-0.001973	0.185233	-0.059952
	1.54 < Z ≤ 2.22	0.016984	-0.018038	-10.0	3.0	-0.025	-2.025	-0.013027	0.074270	0.012889
	2.22 < Z ≤ 2.90	0.099687	0.025287	-10.0	5.0	0	-2.700	0.004342	0.212192	-0.001293
	Z > 2.90	0.043104	0.023547	-20.0	4.0	0	-3.30	0.006348	0.209716	-0.006001
-7 ≤ Y ≤ -4.5	Z ≤ 0.65	0	0	0	0	0	0	0	0	0
	0.65 < Z ≤ 1.50	0	0	0	0	0	0	-0.000954	0.171187	0.004567
	1.50 < Z ≤ 2.20	0.136685	0.010071	-30.0	0	-0.0095	-1.947	0.008736	0.184842	-0.302441
	2.20 < Z ≤ 3.05	0.069839	0.003985	-30.0	0	-0.007	-2.691	0.017884	0.153672	-0.930224
	3.05 < Z ≤ 3.38	-0.198942	-0.021539	-50.0	0	-0.0085	-3.334	0.002379	0.217959	0.005943
	Z > 3.38	0	0	0	0	0	0	0.006572	0.183396	-0.135960

Table A2. Coefficients for curve fit $T = T(e, \rho)$.

Density Range	Curve Range	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
$Y > -0.50$	$Z \leq 0.48$	$Log_{10}(p/R\rho T_o)$	0	0	0	0	0	0	0
	$0.48 < Z \leq 1.07$	0.279268	0	0.992172	0	0	0	0	0
	$Z > 1.07$	0.233261	-0.056383	1.19783	0.063121	-0.165985	0	0	0
$-4.5 < Y \leq -0.5$	$Z \leq 0.48$	$Log_{10}(p/R\rho T_o)$	0	0	0	0	0	0	0
	$0.48 < Z \leq 0.9165$	0.284312	0.001644	0.987912	0	0	0	0	0
	$0.9165 < Z \leq 1.478$	0.502071	-0.012990	0.774818	0.025397	0	0	0	0
	$1.478 < Z \leq 2.176$	1.02294	0.021535	0.427213	0.006900	0	0	0	0
	$Z > 2.176$	1.47540	0.129620	0.254154	-0.046411	0	0	0	0
$-7.0 \leq Y \leq -4.5$	$Z \leq 0.30$	$Log_{10}(p/R\rho T_o)$	0	0	0	0	0	0	0
	$0.30 < Z \leq 1.00$	0.271800	0.000740	0.990136	-0.004947	0	0	0	0
	$1.00 < Z \leq 1.35$	1.39925	0.167780	-0.143168	-0.159234	0	0	0	0
	$1.35 < Z \leq 1.79$	1.11401	0.002221	0.351875	0.017246	0	0	0	0
	$1.79 < Z \leq 2.47$	1.01722	-0.017918	0.473523	0.025456	0	0	0	0
	$Z > 2.47$	-45.0871	-9.00504	35.8685	6.79222	-6.77699	-0.064705	0.025325	-1.27370

Density Range	Curve Range	b_9	b_{10}	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}
$Y > -0.5$	$Z \leq 0.48$	0	0	0	0	0	0	0	0
	$0.48 < Z \leq 1.07$	0	0	0	0	0	0	0	0
	$Z > 1.07$	-0.814535	0.099233	0.602385	-0.067428	-0.093991	5.0	-20.0	-1.78
$-4.5 < Y \leq -0.5$	$Z \leq 0.48$	0	0	0	0	0	0	0	0
	$0.48 < Z \leq 0.9165$	0	0	0	0	0	0	0	0
	$0.9165 < Z \leq 1.478$	0.009912	-0.150527	-0.000385	0.105734	0	0	-15.0	-1.28
	$1.478 < Z \leq 2.176$	-0.427823	-0.211991	0.257096	0.101192	0	0	-12.0	-1.778
	$Z > 2.176$	-0.221229	-0.057077	0.158116	0.030430	0	5.0	0	-2.40
$-7.0 \leq Y \leq -4.5$	$Z \leq 0.30$	0	0	0	0	0	0	0	0
	$0.30 < Z \leq 1.00$	0.990717	0.175194	-0.982407	-0.159233	0	0	-20.0	-0.88
	$1.00 < Z \leq 1.35$	-0.027614	-0.090761	0.307036	0.121621	0	0	-20.0	-1.17
	$1.35 < Z \leq 1.79$	-1.15099	-0.173555	0.673342	0.088399	0	0	-20.0	-1.56
	$1.79 < Z \leq 2.47$	-2.17978	-0.334716	0.898619	0.127386	0	0	-20.0	-2.22
	$Z > 2.47$	0	0	0	0	0	0	0	0

Table A3. Coefficients for curve fit h = h(p, ρ).

Density Range	Curve Range	c_1	c_2	c_3	c_4	c_5	c_6
Y > -0.50	Z ≤ 0.30	1.40000	0	0	0	0	0
	0.30 < Z ≤ 1.15	1.42598	0.000918	-0.092209	-0.002226	0.019772	-0.036600
	1.15 < Z ≤ 1.60	1.64689	-0.062155	-0.334994	0.063612	-0.038332	-0.014468
	Z > 1.60	1.48558	-0.453562	-0.152096	0.303350	-0.459282	0.448395
-4.50 < Y ≤ 0.50	Z ≤ 0.30	1.40000	0	0	0	0	0
	0.30 < Z ≤ 0.98	1.42176	-0.000366	-0.083614	0.000675	0.005272	-0.115853
	0.98 < Z ≤ 1.38	1.74436	-0.035354	-0.415045	0.061921	0.018536	0.043582
	1.38 < Z ≤ 2.04	1.49674	-0.021583	-0.197008	0.030886	-0.157738	-0.009158
	Z > 2.04	1.10421	-0.033664	0.031768	0.024335	-0.178802	-0.017456
-7 ≤ Y ≤ -4.5	Z ≤ 0.398	1.40000	0	0	0	0	0
	0.398 < Z ≤ 0.87	1.47003	0.007939	-0.244205	-0.025607	0.872248	0.049452
	0.87 < Z ≤ 1.27	3.18652	0.137930	-1.89529	-0.103490	-2.14572	-0.272717
	1.27 < Z ≤ 1.863	1.63963	-0.001004	-0.303549	0.016464	-0.852169	-0.101237
	Z > 1.863	1.55889	0.055932	-0.211764	-0.023548	-0.549041	-0.101758

Density Range	Curve Range	c_7	c_8	c_9	c_{10}	c_{11}
Y > -0.50	Z ≤ 0.30	0	0	0	0	0
	0.30 < Z ≤ 1.15	-0.077469	0.043878	-15.0	-1.0	-1.040
	1.15 < Z ≤ 1.60	0.073421	-0.002442	-15.0	-1.0	-1.360
	Z > 1.60	0.220546	-0.292293	-10.0	-1.0	-1.600
-4.50 < Y ≤ 0.50	Z ≤ 0.30	0	0	0	0	0
	0.30 < Z ≤ 0.98	-0.007363	0.146179	-20.0	-1.0	-0.860
	0.98 < Z ≤ 1.38	0.044353	-0.049750	-20.0	-1.04	-1.336
	1.38 < Z ≤ 2.04	0.123213	-0.006553	-10.0	-1.05	-1.895
	Z > 2.04	0.080373	0.002511	-15.0	-1.08	-2.650
-7 ≤ Y ≤ -4.5	Z ≤ 0.398	0	0	0	0	0
	0.398 < Z ≤ 0.87	-0.764158	0.000147	-20.0	-1.0	-0.742
	0.87 < Z ≤ 1.27	2.06586	0.223046	-15.0	-1.0	-1.041
	1.27 < Z ≤ 1.863	0.503123	0.043580	-10.0	-1.0	-1.544
	Z > Z 1.863	0.276732	0.046031	-15.0	-1.0	-2.250

Table A4. Coefficients for curve fit $T = T(p, \rho)$.

Density Range	Curve Range	d_1	d_2	d_3	d_4	d_5	d_6
$Y > -0.50$	$0.48 < Z \leq 0.90$	0.27407	0	1.00082	0	0	0
	$Z > 0.90$	0.235869	-0.043304	1.17619	0.046498	-0.143721	-1.37670
$-4.5 < Y \leq -0.5$	$0.48 < Z \leq 0.9165$	0.281611	0.001267	0.990406	0	0	0
	$0.9165 < Z \leq 1.478$	0.457643	-0.034272	0.819119	0.046471	0	-0.073233
	$1.478 < Z \leq 2.176$	1.04172	0.041961	0.412752	-0.009329	0	-0.434074
	$Z > 2.176$	0.418298	-0.252100	0.784048	0.144576	0	-2.00015
$-7 \leq Y \leq -4.5$	$0.30 < Z \leq 1.07$	2.72964	0.003725	0.938851	-0.011920	0	0.682406
	$1.07 < Z \leq 1.57$	2.50246	-0.042827	1.12924	0.041517	0	1.72067
	$1.57 < Z \leq 2.24$	2.44531	-0.047722	1.00488	0.034349	0	1.95893
	$Z > 2.24$	2.50342	0.026825	0.838860	-0.009819	0	3.58284

Density Range	Curve Range	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}
$Y > -0.5$	$0.48 < Z \leq 0.90$	0	0	0	0	0	0
	$Z > 0.90$	0.160465	1.08988	-0.083489	-0.217748	-10.0	-1.78
$-4.5 < Y \leq -0.5$	$0.48 < Z \leq 0.9165$	0	0	0	0	0	0
	$0.9165 < Z \leq 1.478$	-0.169816	0.043264	0.111854	0	-15.0	-1.28
	$1.478 < Z \leq 2.176$	-0.196914	0.264883	0.100599	0	-15.0	-1.778
	$Z > 2.176$	-0.639022	0.716053	0.206457	0	-10.0	-2.40
$-7 \leq Y \leq -4.5$	$0.30 < Z \leq 1.07$	0.089153	-0.646541	-0.070769	0	-20.0	-0.82
	$1.07 < Z \leq 1.57$	0.268008	-1.25038	-0.179711	0	-20.0	-1.33
	$1.57 < Z \leq 2.24$	0.316244	-1.01200	-0.151561	0	-20.0	-1.88
	$Z > 2.24$	0.533853	-1.36147	-0.195436	0	-20.0	-2.47

APPENDIX B

SUBROUTINE TGAS FOR $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(e, \rho)$

The calling statement for this subroutine is

```
CALL TGAS (E, RHO, P, A, T)
```

with

E = Internal energy, m^2/sec^2

RHO = Density, kg/m^3

P = Pressure, newtons/m^2

A = Speed of sound, m/sec

T = Temperature, ^oK

The following logic can be employed when the English system of units is desired:

```
  E1 = E * 0.0929
RHO1 = RHO * 515.4
CALL TGAS (E1, RHO1, P1, A1, T1)
   P = P1 * 0.02088
   A = A1 * 3.281
   T = T1 * 1.80
```

with

E = Internal energy, ft^2/sec^2

RHO = Density, slugs/ft^3

P = Pressure, lbs/ft^2

A = Speed of sound, ft/sec

T = Temperature, ^oR

Listing of TGAS for $p = p(e, \rho)$, $a = a(e, \rho)$, and $T = T(e, \rho)$

```
      SUBROUTINE TGAS(E,RHO,P,A,T)
      Y2=ALOG10(RHO/1.292)
      Z2=ALOG10(E/78408.4)
      IF(Y2.GT.-.50) GO TO 11
      IF(Y2.GT.-4.50) GO TO 6
      IF(Z2.GT..65) GO TO 1
      GAMM=1.400
      SNDSQ=E*.560
      GO TO 18
    1 IF(Z2.GT.1.50) GO TO 2
      GAMM=1.46543+(.007625+.000292*Y2)*Y2-(.254500+.017244*Y2)*Z2
     A+(.355907+.015422*Y2-.163235*Z2)*Z2*Z2
      GAME=2.304*(-.25450-.017244*Y2+(.711814+.030844*Y2-.489705*Z2)*Z2)
      GAMR=2.304*(.007625+(-.017244+.015422*Z2)*Z2+.000584*Y2)
      A1=-.000954
      A2=.171187
      A3=.004567
      GO TO 17
    2 IF(Z2.GT.2.20) GO TO 3
      GAS1=2.02636+.058493*Y2
      GAS2=.454886+.027433*Y2
      GAS3=.165265+.014275*Y2
      GAS4=.136685+.010071*Y2
      GAS5=.058493-.027433*Z2
      GAS6=-.01427 5+.010071*Z2
      GAS7=EXP(0.285*Y2-30.0*Z2+58.41)
      DERE=-30.0
      DERR=0.285
      A1=.008737
      A2=.184842
      A3=-.302441
      GO TO 15
    3 IF(Z2.GT.3.05) GO TO 4
      GAS1=1.60804+.034791*Y2
      GAS2=.188906+.010927*Y2
      GAS3=.124117+.007277*Y2
      GAS4=.069839+.003985*Y2
      GAS5=.034791-.010927*Z2
      GAS6=-.007277+.003985*Z2
      GAS7=EXP(0.21*Y2-30.0*Z2+80.73)
      DERE=-30.0
      DERR=0.21
      A1=.017884
      A2=.153672
      A3=-.930224
      GO TO 15
```

```
 4 IF(Z2.GT.3.38) GO TO 5
   GAS1=1.25672+.007073*Y2
   GAS2=.039228-.000491*Y2
   GAS3=-.721798-.073753*Y2
   GAS4=-.198942-.021539*Y2
   GAS5=.007073+.000491*Z2
   GAS6=.073753-.021539*Z2
   GAS7=EXP(0.425*Y2-50.0*Z2+166.7)
   DERE=-50.0
   DERR=0.325
   A1=.002379
   A2=.217959
   A3=.005943
   GO TO 15
 5 GAMM=-84.0327+(-.831761+.001153*Y2)*Y2+(72.2066+.491914*Y2)*Z2
  A+(-20.3559-.070617*Y2+1.90979*Z2)*Z2**2
   GAME=2.304*(72.2066+.491914*Y2+(-40.7118-.141234*Y2+5.72937*Z2)
  A*Z2)
   GAMR=2.304*(-.831761+.002306*Y2+(.491914-.070617*Z2)*Z2)
   A1=.006572
   A2=.183396
   A3=-.135960
   GO TO 17
 6 IF(Z2.GT..65) GO TO 7
   GAMM=1.400
   SNDSQ=E*.560
   GO TO 18
 7 IF(Z2.GT.1.54) GO TO 8
   GAS1=1.44813+.001292*Y2
   GAS2=.073510+.001948*Y2
   GAS3=-.054745+.013705*Y2
   GAS4=-.055473+.021874*Y2
   GAS5=.001292-.001948*Z2
   GAS6=-.013705+.021874*Z2
   GAS7=EXP(-10.0*(Z2-1.42))
   DERE=-1.0
   DERR=0.0
   A1=-.001973
   A2=.185233
   A3=-.059952
   GO TO 15
 8 IF(Z2.GT.2.22) GO TO 9
   GAS1=1.73158+.003902*Y2
   GAS2=.272846-.006237*Y2
   GAS3=-.041419-.037475*Y2
   GAS4=.016984-.018038*Y2
   GAS5=.003902+.006237*Z2
   GAS6=.037475-.018038*Z2
   GAS7=EXP((-10.+3.0*Y2)*(Z2-.025*Y2-2.025))
   DERE=3.0*Y2-10.0
   DERR=3.0*Z2+12.15*Y2-20.325
   A1=-.013027
   A2=.074270
   A3=.012889
   GO TO 15
```

```
 9 IF(Z2.GT.2.90) GO TO 10
   GAS1=1.59350+.075324*Y2
   GAS2=.176186+.026072*Y2
   GAS3=.200838+.058536*Y2
   GAS4=.099687+.025287*Y2
   GAS5=.075324-.026072*Z2
   GAS6=-.058536+.025287*Z2
   GAS7=EXP(-10.0*Z2+(5.0*Z2-13.5)*Y2+27.0)
   DERE=5.0*Y2-10.0
   DERR=5.0*Z2-13.5
   A1=.004342
   A2=.212192
   A3=-.001293
   GO TO 15
10 GAS1=1.12688-.025957*Y2
   GAS2=-.013602-.013772*Y2
   GAS3=.127737+.087942*Y2
   GAS4=.043104+.023547*Y2
   GAS5=-.025957+.013772*Z2
   GAS6=-.087942+.023547*Z2
   GAS7=EXP(-20.0*Z2+(4.0*Z2-13.2)*Y2+66.0)
   DERE=-20.+4.0*Y2
   DERR=4.0*Z2-13.2
   A1=.006348
   A2=.209716
   A3=-.006001
   GO TO 15
11 IF(Z2.GT..65) GO TO 12
   GAMM=1.400
   SNDSQ=E*.560
   GO TO 18
12 IF(Z2.GT.1.68) GO TO 13
   GAS1=1.45510-.000102*Y2
   GAS2=.081537-.000166*Y2
   GAS3=-.128647+.049454*Y2
   GAS4=-.101036+.033518*Y2
   GAS5=-.000102+.000166*Z2
   GAS6=-.049454+.033518*Z2
   GAS7=EXP(-15.*(Z2-1.420))
   DERE=-15.
   DERR=0.
   A1=.000450
   A2=.203892
   A3=.101797
   GO TO 15
```

```
13 IF(Z2.GT.2.46) GO TO 14
   GAS1=1.59608-.042426*Y2
   GAS2=.192840-.029353*Y2
   GAS3=.019430-.005954*Y2
   GAS4=.026097-.006164*Y2
   GAS5=-.042426+.029353*Z2
   GAS6= .005954-.006164*Z2
   GAS7=EXP(-15.*(Z2-2.050))
   DERE=-15.
   DERR=0.0
   A1=-.006609
   A2=.127637
   A3=.297037
   GO TO 15
14 GAS1=1.54363-.049071*Y2
   GAS2=.153562-.029209*Y2
   GAS3=.324907+.077599*Y2
   GAS4=.142408+.022071*Y2
   GAS5=-.049071+.029209*Z2
   GAS6=-.077599+.022071*Z2
   GAS7=EXP(-10.0*(Z2-2.708))
   DERE=-10.0
   DERR=0.0
   A1=-.000081
   A2=.226601
   A3=.170922
15 GAS10=1./(1.+GAS7)
16 GAS8=GAS3-GAS4*Z2
   GAS9=GAS8*GAS7*GAS10**2
   GAMM=GAS1-GAS2*Z2-GAS8*GAS10
   GAME=2.304*(-GAS2+GAS4*GAS10+GAS9*DERE)
   GAMR=2.304*(GAS5+GAS6*GAS10+GAS9*DERR)
17 SNDSQ=E*(A1+(GAMM-1.)*(GAMM+A2*GAME)+A3*GAMR)
18 A=SQRT(SNDSQ)
   P=RHO*E*(GAMM-1.)
   X2=ALOG10(P/1.0134E+05)
   Y2= Y2+.0231264
   Z3=X2-Y2
   IF(Y2.GT.-.50) GO TO 29
   IF(Y2.GT.-4.50) GO TO 24
   IF(Z3.GT..30) GO TO 19
   T=P/(287.*RHO)
   RETURN
19 IF(Z3.GT.1.00) GO TO 20
   T=10**(.2718+.00074*Y2+(.990136-.004947*Y2)*Z3+(.990717
  A+.175194*Y2-(.982407+.159233*Y2)*Z3)/(1.+EXP(-20.*(Z3-0.88))))
   GO TO 32
20 IF(Z3.GT.1.35) GO TO 21
   T=10**(1.39925+.167780*Y2+(-.143168-.159234*Y2)*Z3+(-.027614
  A-.090761*Y2+(.307036+.121621*Y2)*Z3)/(1.+EXP(-20.*(Z3-1.17))))
   GO TO 32
21 IF(Z3.GT.1.79) GO TO 22
   T=10**(1.11401+.002221*Y2+(.351875+.017246*Y2)*Z3+(-1.15099
  A-.173555*Y2+(.673342+.088399*Y2)*Z3)/(1.+EXP(-20.*(Z3-1.56))))
   GO TO 32
```

```
22 IF(Z3.GT.2.47) GO TO 23
   T=10**(1.01722-.017918*Y2+(.473523+.025456*Y2)*Z3+(-2.17978
  A-.334716*Y2+(.898619+.127386*Y2)*Z3)/(1.+EXP(-20.*(Z3-2.22))))
   GO TO 32
23 T=10**(-45.0871-9.00504*Y2+(35.8685+6.7922*Y2)*Z3-(6.77699
  A+1.2737*Y2)*Z3*Z3+(-.064705+.025325*Z3)*Y2*Y2)
   GO TO 32
24 IF(Z3.GT..48) GO TO 25
   T=P/(287.*RHO)
   RETURN
25 IF(Z3.GT..9165) GO TO 26
   T=10**(.284312+.987912*Z3+.001644*Y2)
   GO TO 32
26 IF(Z3.GT.1.478) GO TO 27
   T=10**(.502071-.01299*Y2+(.774818+.025397*Y2)*Z3+(.009912
  A-.150527*Y2+(-.000385+.105734*Y2)*Z3)/(1.+EXP(-15.*(Z3-1.28))))
   GO TO 32
27 IF(Z3.GT.2.176) GO TO 28
   T=10**(1.02294+.021535*Y2+(.427213+.006900*Y2)*Z3+(-.427823
  A-.211991*Y2+(.257096+.101192*Y2)*Z3)/(1.+EXP(-12.*(Z3-1.778))))
   GO TO 32
28 T=10**(1.47540+.12962*Y2+(.254154-.046411*Y2)*Z3+(-.221229
  A-.057077*Y2+(.158116+.03043*Y2)*Z3)/(1.+EXP(5.*Y2*(Z3-2.40))))
   GO TO 32
29 IF(Z3.GT..48) GO TO 30
   T=P/(287.*RHO)
   RETURN
30 IF(Z3.GT.1.07) GO TO 31
   T=10**(.279268+.992172*Z3)
   GO TO 32
31 T=10**(.2332605-.056383*Y2+(1.19783+.063121*Y2-.165985*Z3)*Z3+(-.8
  A14535+.099233*Y2+(.602385-.067428*Y2-.098991*Z3)*Z3)/(1.+EXP((
  A5.*Y2-20.)*(Z3-1.78))))
32 T= T*151.777778
   RETURN
   END
```

APPENDIX C

SUBROUTINE TGAS FOR $h = h(p, \rho)$

The calling statement for this subroutine TGAS is

```
CALL TGAS (P, RHO, H)
```

with

P = Pressure, newtons/m^2

RHO = Density, kg/m^2

H = Enthalpy, m^2/sec^2

The following logic can be employed when the English system of units is desired:

```
P1 = P/0.02088
RHO1 = RHO * 515.4
CALL TGAS (P1, RHO1, H1)
H = H1/0.0929
```

with

P = Pressure, lbs/ft^2

RHO = Density, slugs/ft^3

H = Enthalpy, ft^2/sec^2

Listing of TGAS for h = h(p, ρ)

```
      SUBROUTINE TGAS(P,RHO,H)
      Y2= ALOG10(RHO/1.292)
      X2= ALOG10(P/1.013E+05)
      Z3= X2-Y2
      IF (Y2 .GT. -.50) GO TO 10
      IF (Y2 .GT. -4.50) GO TO 5
      IF (Z3 .GT. .398) GO TO 1
      H= (P/RHO)*3.50
      RETURN
    1 IF (Z3 .GT. 0.870) GO TO 2
      GAS1=1.47003+.007939*Y2
      GAS2=.244205+.025607*Y2
      GAS3=-.872248-.049452*Y2
      GAS4=-.764158+.000147*Y2
      GAS5=EXP(-20.00*(Z3-0.742))
      GO TO 14
    2 IF (Z3 .GT. 1.270) GO TO 3
      GAS1= 3.18652+.137930*Y2
      GAS2= 1.89529+.103490*Y2
      GAS3= 2.14572+.272717*Y2
      GAS4= 2.06586+.223046*Y2
      GAS5=EXP(-15.00*(Z3-1.041))
      GO TO 14
    3 IF (Z3 .GT. 1.863) GO TO 4
      GAS1= 1.63963-.00100436*Y2
      GAS2= .303549-.0164639*Y2
      GAS3= .852169+.101237*Y2
      GAS4= .503123+.0435801*Y2
      GAS5=EXP(-10.00*(Z3-1.544))
      GO TO 14
    4 GAS1= 1.55889+.0559323*Y2
      GAS2= .211764+.0235478*Y2
      GAS3= .549041+.101758*Y2
      GAS4= .276732+.0460305*Y2
      GAS5=EXP(-15.00*(Z3-2.250))
      GO TO 14
    5 IF (Z3 .GT. .300) GO TO 6
      H= (P/RHO)*3.50
      RETURN
    6 IF (Z3 .GT. 0.980) GO TO 7
      GAS1=1.42176-.000366*Y2
      GAS2=.083614-.000677*Y2
      GAS3=-.005272+.115853*Y2
      GAS4=-.007363+.146179*Y2
      GAS5=EXP(-20.00*(Z3-0.860))
      GO TO 14
    7 IF (Z3 .GT. 1.380) GO TO 8
      GAS1=1.74436-.035354*Y2
      GAS2=.415045-.061921*Y2
      GAS3=-.018536-.043582*Y2
      GAS4=.0443534-.049750*Y2
      GAS5=EXP(-20.00*(X2-1.04*Y2-1.336))
      GO TO 14
```

```
  8 IF (Z3 .GT. 2.040) GO TO 9
    GAS1=1.49674-.021583*Y2
    GAS2=.197008-.030886*Y2
    GAS3=.157738+.009158*Y2
    GAS4=.123213-.006553*Y2
    GAS5=EXP(-10.00*(X2-1.05*Y2-1.895))
    GO TO 14
  9 GAS1=1.10421-.033664*Y2
    GAS2=-.031768-.024335*Y2
    GAS3=.178802+.017456*Y2
    GAS4=.080373+.002511*Y2
    GAS5=EXP(-15.00*(X2-1.08*Y2-2.650))
    GO TO 14
 10 IF(Z3.GT..300) GO TO 11
    H= (P/RHO)*3.50
    RETURN
 11 IF(Z3.GT.1.15) GO TO 12
    GAS1=1.42598+.000918*Y2
    GAS2=.092209+.002226*Y2
    GAS3=-.0197721+.036600*Y2
    GAS4=-.0774694+.043878*Y2
    GAS5=EXP(-15.00*(Z3-1.040))
    GO TO 14
 12 IF(Z3.GT.1.600) GO TO 13
    GAS1=1.64689-.0621547*Y2
    GAS2=.334994-.0636120*Y2
    GAS3=.0383322+.0144677*Y2
    GAS4=.0734214-.0024417*Y2
    GAS5=EXP(-15.00*(Z3-1.360))
    GO TO 14
 13 GAS1=1.48558-.453562*Y2
    GAS2=.152096-.303350*Y2
    GAS3=.459282-.448395*Y2
    GAS4=.220546-.292293*Y2
    GAS5=EXP(-10.00*(Z3-1.600))
 14 GAS10= 1./(1.+GAS5)
    GAMM=GAS1-GAS2*Z3-(GAS3-GAS4*Z3)*GAS10
    H=(P/RHO)*(GAMM/(GAMM-1.))
    RETURN
    END
```

APPENDIX D

SUBROUTINE TGAS FOR T = T(p, ρ)

The calling statement for this subroutine TGAS is

```
CALL TGAS (P, RHO, T)
```

with

P = Pressure, newtons/m^2

RHO = Density, kg/m^3

T = Temperature, ^oK

The following logic can be employed when the English system of units is desired:

```
P1 = P/0.02088
RHO1 = RHO * 515.4
CALL TGAS (P1, RHO1, T1)
T = T1 * 1.80
```

with

P = Pressure, lbs/ft^2

RHO = Density, slugs/ft^3

T = Temperature, ^oR

Listing of TGAS for T = T(p, ρ)

```
      SUBROUTINE TGAS(P,RHO,T)
      Y2=ALOG10(RHO/1.225)
      X2=ALOG10(P/1.0134E+05)
      Z3=X2-Y2
      IF(Y2.GT.-.50) GO TO 28
      IF(Y2.GT.-4.50) GO TO 23
      IF(Z3.GT..30) GO TO 19
      T=P/(287.*RHO)
      RETURN
   19 IF(Z3.GT.1.07) GO TO 20
      T=10**(2.72964+.003725*Y2+(.938851-.01192*Y2)*Z3+(.682406+
     A.089153*Y2-(.646541+.070769*Y2)*Z3)/(1.+EXP(-20.*(Z3-0.82))))
      GO TO 32
   20 IF(Z3.GT.1.57) GO TO 21
      T=10**(2.50246-.042827*Y2+(1.12924+.041517*Y2)*Z3+(1.72067+
     A.268008*Y2-(1.25038+.179711*Y2)*Z3)/(1.+EXP(-20.*(Z3-1.33))))
      GO TO 32
   21 IF(Z3.GT.2.24) GO TO 22
      T=10**(2.44531-.047722*Y2+(1.00488+.034349*Y2)*Z3+(1.95893+
     A.316244*Y2-(1.01200+.151561*Y2)*Z3)/(1.+EXP(-20.*(Z3-1.88))))
      GO TO 32
   22 T=10**(2.50342+.026825*Y2+(.838860-.009819*Y2)*Z3+(3.58284+
     A.533853*Y2-(1.36147+.195436*Y2)*Z3)/(1.+EXP(-20.*(Z3-2.47))))
      GO TO 32
   23 IF(Z3.GT..48) GO TO 24
      T=P/(287.*RHO)
      RETURN
   24 IF(Z3.GT..9165) GO TO 25
      T=10**(.281611+.990406*Z3+.001267*Y2)
      GO TO 31
   25 IF(Z3.GT.1.478) GO TO 26
      T=10**(.457643-.034272*Y2+(.819119+.046471*Y2)*Z3+(-.073233-
     Q.169816*Y2+(.043264+.111854*Y2)*Z3)/(1.+EXP(-15.*(Z3-1.28))))
      GO TO 31
   26 IF(Z3.GT.2.176) GO TO 27
      T=10**(1.04172+.041961*Y2+(.412752-.009329*Y2)*Z3+(-.434074-
     A.196914*Y2+(.264883+.100599*Y2)*Z3)/(1.+EXP(-15.*(Z3-1.778))))
      GO TO 31
   27 T=10**(.418298-.252100*Y2+(.784048+.144576*Y2)*Z3+(-2.00015-
     A.639022*Y2+(.716053+.206457*Y2)*Z3)/(1.+EXP(-10.*(Z3-2.40))))
      GO TO 31
   28 IF(Z3.GT..48) GO TO 29
      T=P/(287.*RHO)
      RETURN
   29 IF(Z3.GT..90) GO TO 30
      T=10**(.27407+1.00082*Z3)
      GO TO 31
   30 T=10**(.235869-.043304*Y2+(1.17619+.046498*Y2-.143721*Z3)*Z3+
     A(-1.3767+.160465*Y2+(1.08988-.083489*Y2-.217748*Z3)*Z3)/
     A(1.+EXP(-10.*(Z3-1.78))))
   31 T=T*273.2
   32 T=T/1.8
      RETURN
      END
```