Improved downward continuation of potential field data

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ABSTRACT

A number of techniques for downward continuation of potential field data, some already established in the literature and some novel, are tested and compared on synthetic and measured potential field profiles. A combination of Wiener filtering and Translation-Invariant denoising gives best results on synthetic data with added white noise. A Multiscale Edge Transform followed by a mild low-pass filter, and the ISVD method, prove to be the two most stable and robust approaches on measured data.

INTRODUCTION

Downward continuation is the process whereby, from measurements of potential field data on a plane, we can estimate the field closer to the source. The downward-continued field is sharper, and consequently allows for better resolution of underground rock distribution. The usefulness of this process is limited by the fact that the operation is extremely sensitive to noise. When downward-continued, a signal is amplified exponentially, with an exponent proportional to the spectral frequency. With noise-free data, downward continuation is well defined, provided we do not attempt to continue below the source level (Boschetti et al., 2001). In the presence of noise, the amplification of high frequencies is so strong that it quickly masks the information in the original profile. Low-pass Fourier filtering, while suppressing such noise, also blurs the signal, defeating the purpose of sharpening by downward continuation.

Despite this difficulty, the geophysical community has long been interested in the technique because of its relevance to mineral exploration. A good downward continuation process would provide sharper images, allowing an enhanced interpretation. More importantly, downward continuation is closely linked to numerical inversion in order to reconstruct underground features. Stable downward continuation has the potential to provide a more accurate determination of both horizontal and vertical extents of geological sources (Boschetti et al., 2001).

As explained above, the problems in the downward continuation process are mostly due to the presence of noise. As is the case in other technical fields, the definition of what constitutes noise is very much problem-dependent and subjective. If we aim to reproduce the potential field at ground level, noise is represented by errors in the measurement and in the pre-processing stage. If we wish to reconstruct the field at deeper levels, for exploration purposes, noise is also represented by small, shallow geological sources in which we are not interested or which we cannot accurately model. If we want to obtain information at a continental scale, many exploration targets become noise in the analysis.

In this work, we test and compare a number of techniques for stabilising downward continuation. Some of the techniques are standard tools in the literature. Others have been proposed recently. Finally, two methods are, to our knowledge, novel. We believe that the results emerging from the comparison may be useful for practitioners who need to choose among many available methods.

MULTISCALE EDGE-BASED DOWNWARD CONTINUATION

In Boschetti et al. (2003), we introduce a Multiscale Edge Transform (MET) specifically designed for potential field analysis. This is used in order to remove specific features from a potential field signal. The work is mostly based upon multiscale edge theory developed by Mallat and Zhong (1992). They show that the information necessary to reconstruct a signal is contained in a subset of its wavelet transform. The magnitudes of the wavelet transform at the multiscale edges represent such a subset. Here, an edge is defined as the local extremum of the wavelet transform. Multiscale edges are defined as the collection of edges of the wavelet transform at all scales. An extension of the theory, specifically designed for potential field analysis, can be found in Hornby et al. (1999). For the sake of clarity, we summarise here the MET process:

1) Upward continue the potential to several levels;
2) At each level, calculate the wavelet transform of the field;
3) Pick the multiscale edges as the locations where the wavelet transform has local extrema;
4) Store the position and wavelet magnitude of such edges.

For one-dimensional profiles, the multiscale edges group themselves into strings in the scale-space wavelet domain. In the wavelet literature, these strings are called branches, whilst the collection of branches is called an edge tree. The positions and shapes of branches are strongly related to the locations and shapes of individual features in the profile. The correspondence between features in the profile and edge branches is due to the localization property of wavelets. This suggests that by manipulating some edge branches and reconstructing the profile, the features corresponding to such branches could be modified, thus allowing a signal processing tool to operate locally on specific parts of the image, leaving the rest minimally perturbed.

In Boschetti et al. (2003), we exploit this property by identifying the multiscale edges which correspond to features in

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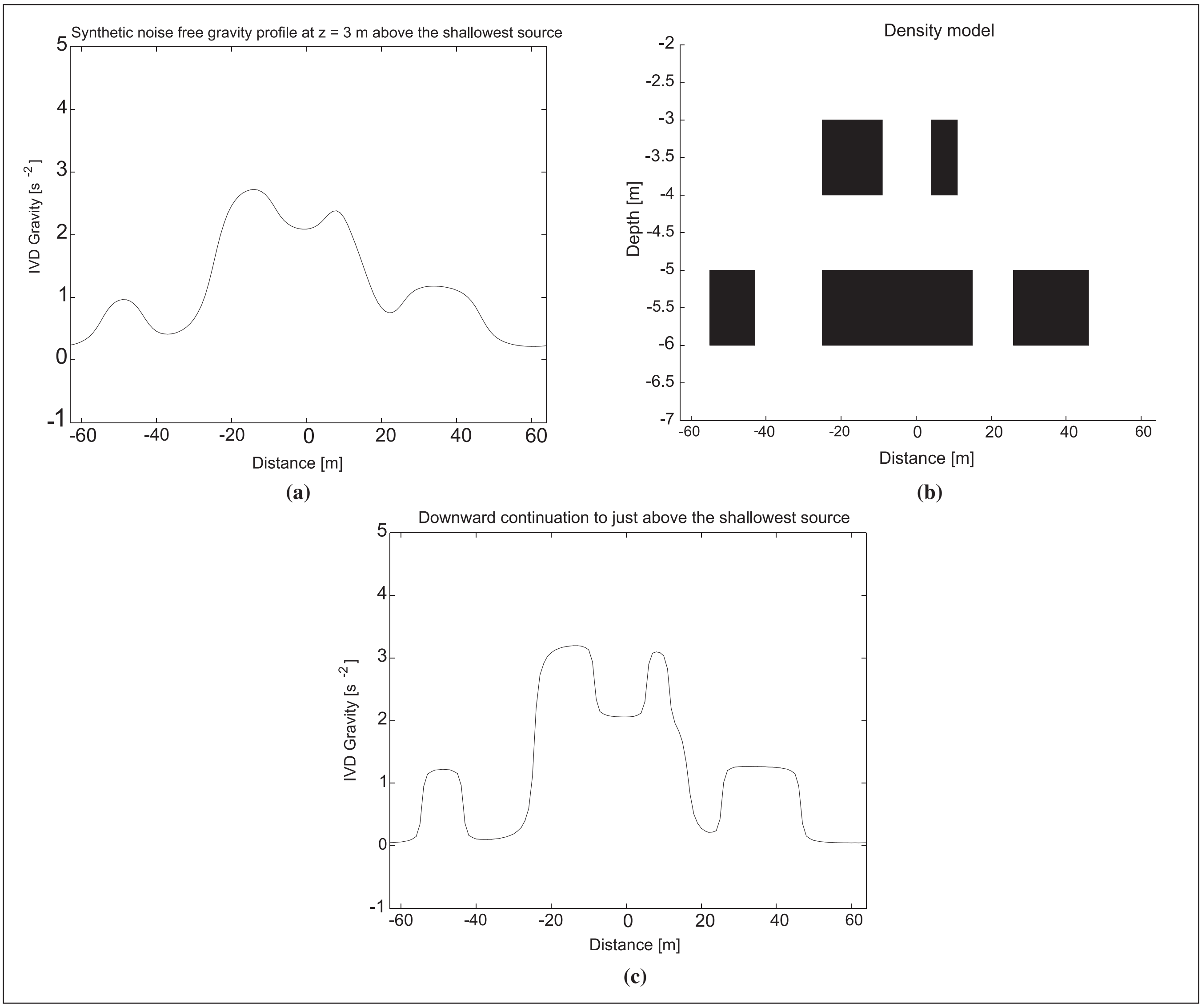

Fig. 1. (a) Synthetic, noise-free profile 3 m above the shallowest source. (b) Density model. (c) Downward continuation to just above the shallowest source.

the signal, and removing those features from a gravity profile by suppressing the corresponding multiscale edges before the inverse transform.

Using the same technique, here we attempt to stabilise the downward continuation operator by removing the edges that are due to spurious oscillations caused by noise amplification. In order to do so, we need a criterion to discriminate between edges due to features in the signal and edges due to spurious noise oscillations. The choice of such a criterion is contestable, but a simple method follows:

1) Build the edge tree from a potential field profile; the edge tree links the edges generated by the same feature within the different scales to give the branches;
2) Perform traditional Fourier-based downward continuation;
3) Pick the edges in the downward-continued profile; among these edges, the ones corresponding to the branches present in the initial edge tree are kept, and the others are deleted;
4) Reconstruct the profile from the remaining edges.

Basically, we prevent features that were not present in the original profile from appearing in the downward-continued profile. The implicit assumption is that if a feature is so weak that it does not appear in the starting profile (i.e., it does not produce edges), the data may not be of sufficient resolution or fidelity to define that feature. This is a debatable position, but not one under scrutiny in this work.

We test the algorithm on the gravity profile in Figure 1a, obtained from the density model in Figure 1b. This is a synthetic, noise-free profile. Consequently, its downward continuation is stable (Figure 1c). If we add the white noise of Figure 2a to the profile and downward-continue, we obtain the profile in Figure 2b. The profile is hardly interpretable. By filtering the spurious multiscale edges, we obtain the profile in Figure 2c. The main features in the profile are clearly sharpened and the profile closely resembles the noise-free downward-continued one. Spurious oscillations are still present, but they are of much smaller amplitude and they do not prevent visual inspection of the signal. If these oscillations are deemed problematic, they can be further suppressed by applying a mild low-pass filter. The result is shown in Figure 2d.

For comparison, in Figure 3a we show the results of applying a separation filter (Jacobsen, 1987) and downward continuing the filtered data to the same level as in Figure 1c. The separation filter is designed to remove spectral components due to sources above

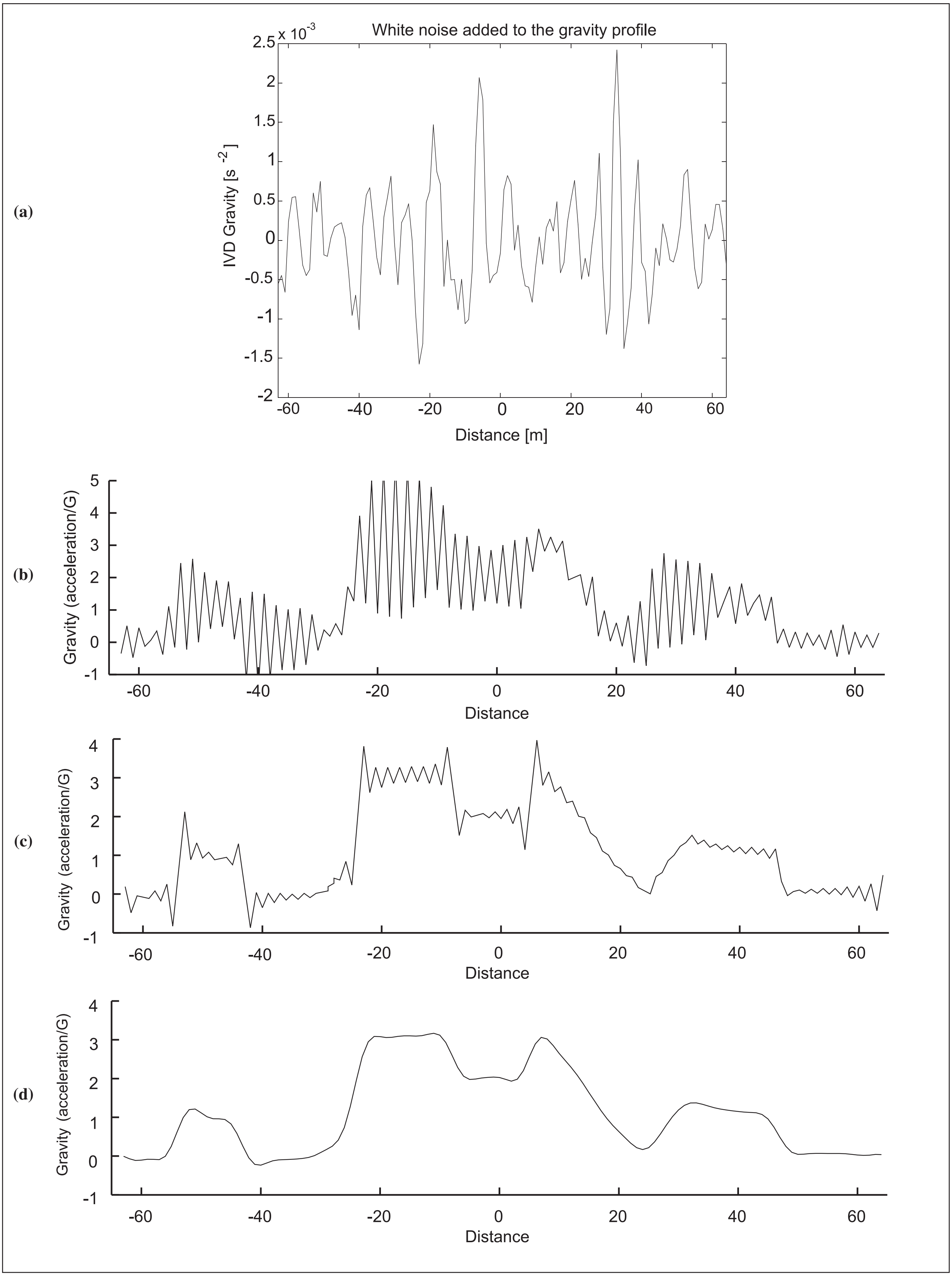

Fig. 2. (a) White noise added to the gravity profile. (b) Standard downward continuation of noisy profile. (c) Downward continuation filtered with the multiscale edge based method by filtering the spurious multiscale edges we obtain the profile in. (d) Suppression of the minor oscillation from profile (c), via standard Fourier filtering.

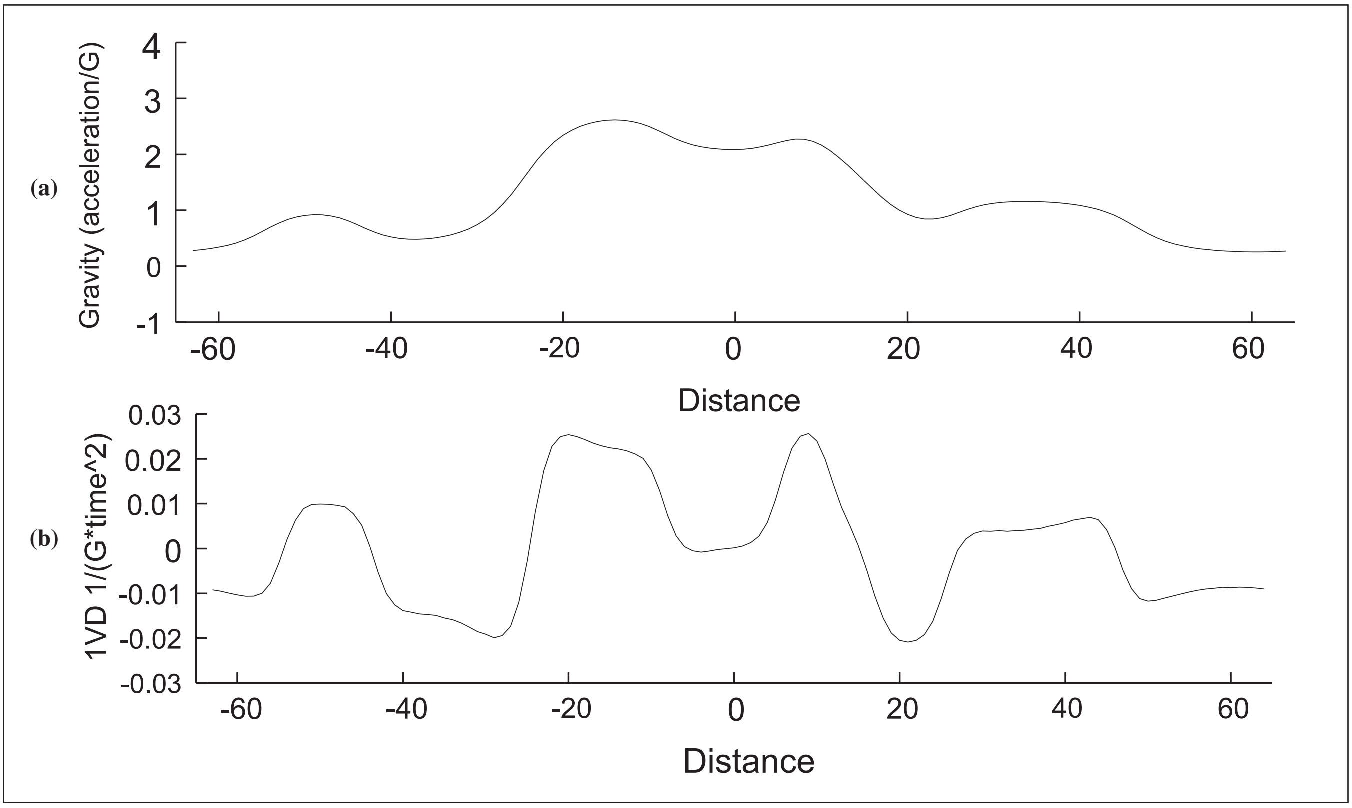

Fig. 3. (a) Application of a separation filter and downward continuation of the filtered data to just above the shallowest source. (b) First vertical derivative.

the level to which we are downward continuing. Another common sharpening operation is the first vertical derivative shown in Figure 3b.

The results in Figure 2d should be considered a significant improvement on the noisy, unfiltered, downward-continued profile of Figure 2b. The main feature of the MET-processed data is the sharpness at the location of the edges in the original profile. Alternative Fourier approaches which use a low-pass filter of some kind before downward continuation will always produce slightly blurred results, as in Figure 3a. We owe our sharper results to the fact that MET filtering of the noise-induced oscillations is not achieved by smoothing the overall profile, but by acting mostly on the oscillations at the positions where they occur. Finally, the first vertical derivative sharpens the image, but removes low-frequency components as it does so, thereby eliminating information regarding the total mass of anomalies. We observed that even the first vertical derivative of the unprocessed noisy data is oscillating, indicating to the reader the extent of our heavy-handedness in adding noise.

OTHER WAVELET-BASED DENOISING TECHNIQUES

A number of wavelet denoising methods have been proposed in the signal processing literature. Most of them involve three steps:

1. Transform the noisy data by the use of an orthogonal wavelet basis;
2. Apply some kind of thresholding to the resulting transform;
3. Inverse transform the signal into the original domain.

A straightforward denoising via an orthogonal wavelet transform usually exhibits local artefacts like Gibbs phenomena. The size of these artefacts is connected intimately to the discretization of both the signal and the wavelet, and in particular, to the spatial alignment between the signal singularities and the wavelet used in the process. Coifman and Donoho (1995) propose a Translation-Invariant (TI) procedure to overcome this problem. It consists of denoising different shifted versions of the same signal and then averaging the results. We tested this idea by applying the TI procedure to our noisy gravity profile. We aimed to remove noise before downward continuation. We used different levels of Gaussian noise, various orthogonal wavelet bases, as well as both soft and hard thresholding. In hard and soft thresholding, all the wavelet coefficients below a given threshold are set to zero, while the others are unmodified for hard thresholding, or shrunk by a value equal to the threshold for soft thresholding. Our results proved to be both data and wavelet dependent. In Figure 4a, a good result is obtained with a symmetric wavelet called Symmlet, while in Figure 4b with another wavelet (asymmetric Daubechies wavelet) the downward continuation exhibits local high frequencies. However, with differently generated noise we obtained exactly the opposite outcome: a good result for the Daubechies wavelet and a noisy downward continuation with the Symmlet wavelet. These results did not prove sufficiently robust to convince us to pursue this avenue.

WIENER FILTER DENOISING

The Wiener filter is the optimal linear filter, in the mean square sense, for the restoration of signals degraded by convolution and additive noise. It is often used to deblur or denoise images. Its calculation requires the assumption that both noise and signal processes are second-order stationary, which is a reasonable assumption in profiles collected over relatively small distances.

To simplify our notation we will consider the one-dimension case with zero-mean noise, but the method can easily be extended to 2D and non-zero mean noise.

Let f be our signal degraded by the filter h and the noise n. x is the degraded signal

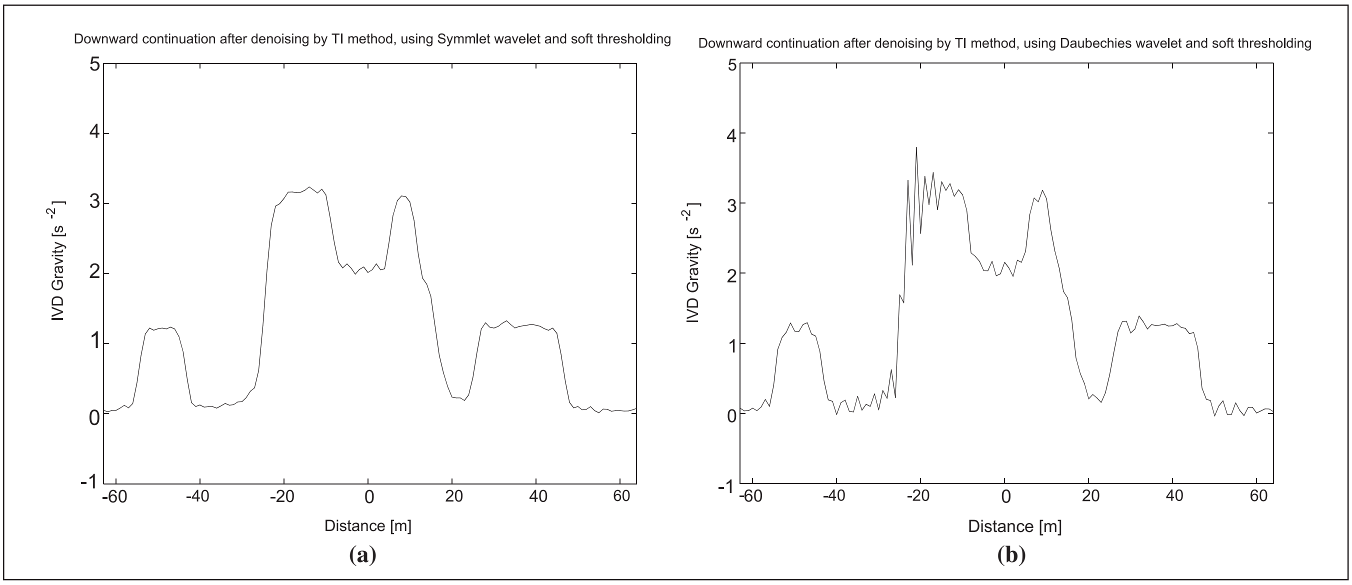

Fig. 4. (a) Downward continuation to the level of the shallowest source after denoising by Translation-Invariant method, using Symmlet wavelet and soft thresholding. (b) Downward continuation after denoising by Translation-Invariant method, using Daubechies wavelet and soft thresholding.

$$x(i) = f * h(i) + n(i) \,, \tag{1}$$

where * denotes convolution. Wiener filters are usually applied in the frequency domain. We write $Y[k]$ and $X[k]$, for the discrete time Fourier transforms (DTFT) of the signal $y(i)$ and $x(i)$, respectively. We also write $H[k]$ for the DTFT of the degradation filter h. Then the DTFT of the restored signal is the product of $X[k]$ with the Wiener filter $W[k]$:

$$\hat{F}[k] = W[k] X[k] \,. \tag{2}$$

The Wiener filter is defined by

$$W[k] = \frac{H^*[k] P_f[k]}{\left|H[k]\right|^2 P_f[k] + P_n[k]} \,, \tag{3}$$

where $P_f[k]$ is the power spectrum of the signal and $P_n[k]$ is the power spectrum of the noise (which is obtained by taking the Fourier transform of the signal and the noise autocorrelation). Let us divide both numerator and denominator by $P_f[k]$:

$$W[k] = \frac{H^*[k]}{\left|H[k]\right|^2 + \dfrac{P_n[k]}{P_f[k]}} \,. \tag{4}$$

Considering the term $\frac{P_n[k]}{P_f[k]}$ as the inverse of the signal-to-noise ratio, the behaviour of the Wiener filter can now be easily explained:

- where the signal is strong, $\frac{P_n[k]}{P_f[k]} \to 0$, the Wiener filter is the inverse filter $H^{-1}[k]$;
- where the signal is weak, $\frac{P_n[k]}{P_f[k]} \to \infty$, the Wiener filter is zero.

We would like to apply this Wiener filter to our measurement x, which is a degraded version of the signal f we want to downward continue. If we assume that the only degradation is an additive white noise with a variance σ_n^2, then $P_n[k] = \sigma_n^2$ and $H[k] = 1$. So, the Wiener filter simplifies to:

$$W[k] = \frac{P_f[k]}{P_f[k] + \sigma_n^2} \,. \tag{5}$$

In order to compute this filter we need to estimate the power spectrum of both the original signal f and the noise n. Obviously, we do not know f. We chose to approximate f via the use of the noisy signal x, denoised by the Translation Invariant procedure. Although the TI procedure is not robust enough to solve our downward continuation problem, we believe it might be effective when coupled with the Wiener filter. Also, we estimate the variance of the white noise by using the popular robust median estimator (Donoho, 1994):

$$\sigma_n = \frac{MAD}{0.6745} \,, \tag{6}$$

where MAD is the median of the magnitudes of all the coefficients at the finest decomposition scale of the wavelet transform.

We could now denoise our measurement with the Wiener filter and then downward continue the result. However, the order can be reversed. The Wiener filter proposed has the interesting property of being independent of the level of downward continuation; thus it can be applied either before or after the downward continuation (DC). The Wiener filter corresponding to the degraded downward-continued measurement $DC(x)$ is indeed:

$$W_{DC}[k] = \frac{P_{DC(f)}[k]}{P_{DC(f)}[k] + P_{DC(n)}[k]} \,. \tag{7}$$

The downward continuation can be written in the discrete case as a circular convolution:

$$\mathrm{DC(f)} = dc * f \,, \tag{8}$$

where dc is the inverse DTFT of the well-known downward

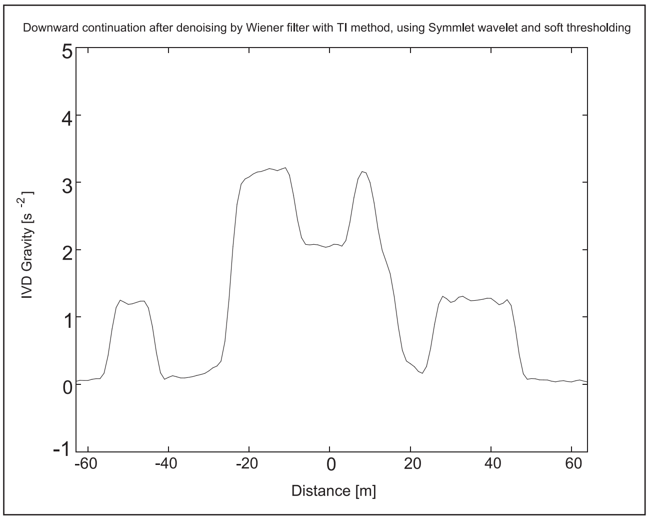

Fig. 5. Downward continuation after denoising by Wiener filter with Translation-Invariant method, using Symmlet wavelet and soft thresholding.

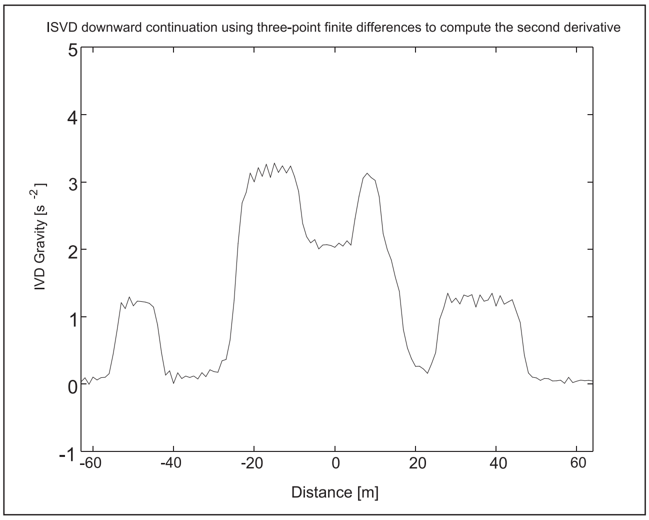

Fig. 6. ISVD downward continuation using three-point finite differences to compute the second derivative, with 10 terms in the Taylor series expansion.

continuation operator used in Fourier domain. From the property $P_{a*h}[k] = |H[k]|^2 P_a[k]$ for any signal process a and filter h, it follows:

$$W_{DC}[k] = \frac{P_f[k]}{P_f[k] + P_n[k]} = W[k] \ . \quad (9)$$

This method gives promising results, as can be seen in Figure 5, and is more robust to different noise realisations than the Translation Invariant procedure alone.

ISVD METHOD

Fedi and Florio (2002) propose the Integrated Second Vertical Derivative as a downward continuation method. It is based upon a representation of the downward-continued field as a sum of vertical derivatives of increasing order via a Taylor series expansion.

$$f_{z_1} = f_{z_2} + \left[\frac{\partial f_z}{\partial z}\right]_{z_2} (z_1 - z_2) + \left[\frac{\partial^2 f_z}{\partial z^2}\right]_{z_2} (z_1 - z_2)^2 + \cdots + \left[\frac{\partial^m f_z}{\partial z^m}\right]_{z_2} (z_1 - z_2)^m \ . \quad (10)$$

The issue becomes that of computing stable vertical derivatives at any order, and the choice of a suitable truncation criterion. The authors propose the use of both Fourier and space domain transformations. The vertical derivatives are computed in three steps:

1. Vertically integrate the field by using a frequency domain operator;

$$\hat{V}_{z_2}(\mathbf{k}) = -\frac{\hat{f}_{z_2}(\mathbf{k})}{2\pi\|\mathbf{k}\|} \ . \quad (11)$$

2. Compute the second vertical derivative via the second horizontal derivatives according to the Laplace equation;
3. Repeat the procedure for all other vertical derivatives by using the Laplace equation starting from the field and its various vertical derivatives.

The first step of this algorithm involves a stable integration in the Fourier domain. However, if we use standard Fourier methods to compute second derivatives, we would reintroduce instabilities. We have also verified that the use of cubic splines to calculate the second derivative gives similarly unstable results. The problem can be reduced by using the three-point finite-difference relation

$$f''(n) = f(n+1) + f(n-1) - 2f(n) \ , \quad (12)$$

which is equivalent in the Fourier domain to multiplying the DTFT of f by $-4\sin^2(\pi k)$. Therefore, the high frequencies are less amplified with the three-point finite-difference than with the Fourier domain derivative operator $(2\pi ki)^2$.

This results in a smoothed signal allowing a stable result, but also an undesired smoothing of the important features in the signal (Figure 6).

APPLICATION TO REAL DATA

We have applied the MET, the Wiener-filter-based algorithm, and the ISVD method to real data. We used an airborne magnetic profile collected over the Murray Basin in eastern South Australia, over Cambrian Volcanics rocks. The data were sampled at 90-metre height and the profile interpolated at 6-metre spacing.

Figure 7a shows the magnetic profile. Figure 7b shows a traditional unfiltered downward continuation of 18 metres (equivalent to 3 horizontal spacing intervals). Figure 7c shows the profile downward continued via the Wiener filter. The performance is quite poor. There are at least two reasons for this result. First, the white noise assumption is most likely not correct. Second, the method used to calculate the noise level (Donoho's robust median estimator, introduced above) seriously underestimates the noise level (which is, in turn, partly a consequence of the departure from the white noise assumption). This result shows that this technique might be beneficial only provided we have a satisfactory knowledge of the noise statistics, and, at the same time, it emphasises the importance of such information for successful downward continuation.

Figure 7d shows the profile downward continued via the ISVD method. The result is much better than with the Wiener filter. We still have a few large oscillations, but the profile is now interpretable. Finally, Figure 7e shows the profile downward continued and filtered via the MET technique. The profile is

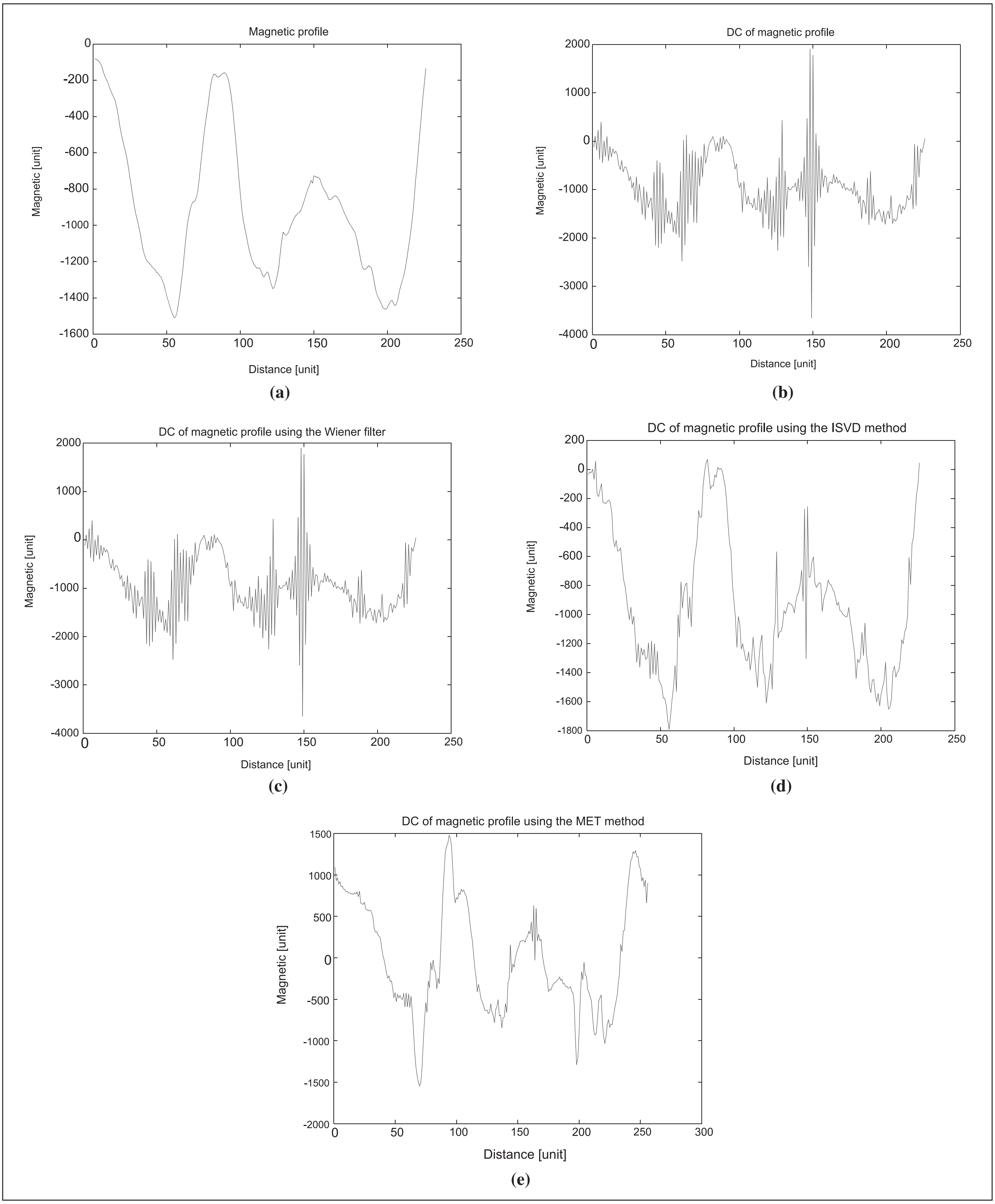

Fig. 7. Test on real data. (a) Real magnetic profile. (b) Standard unfiltered downward continuation. (c) Downward continuation via Wiener filter and TI denoising. (d) Downward continuation via ISVD method. (e) Downward continuation via MET method.

cleaner, the oscillations smaller and fewer. The profile 'looks' somehow more realistic, and it presents features that are not obvious in the ISVD profile (in particular the two sharp lows, and the much smaller high near sample 100). The drawback of using real data in testing algorithm performance is that we are unable to judge whether these different features should be attributed to 'real' signal components or to algorithm artefacts. We are thus unable to draw a final judgement on the relative performance of ISVD and MET method on this data set. However, both offer a considerable improvement over standard downward continuation and the Wiener filter method.

As a last note, the reader should not be surprised about the good performance of the MET technique on real data, compared to poorer performance on noisy synthetic data. The reason is quite simple. The real data set (as measured, not downward continued) has far more edges than the synthetic set. The MET algorithm works by constraining the reconstruction at edge locations, and the larger number of edges results in a better performance. More edges also better control the artefacts due to the distortion in the wavelet transform imposed by removing spurious edges.

DISCUSSION

According to our tests, Wiener filtering, ISVD and MET techniques have the potential to usefully improve the downward continuation process, and more work on each algorithm is justified. Our experiments highlighted a few important points. First, Wiener downward continuation depends crucially on noise statistics. Without reliable information, this method is doomed to fail on real data sets. Should better noise statistics be available, we believe its potential is high, as demonstrated by the performance on synthetic tests.

The ISVD method offers a major improvement in dealing with the instabilities induced by Fourier processing. More understanding is required to properly select the truncation threshold of the Taylor series in the calculation of the vertical derivatives. This process also can benefit by better understanding of noise statistics.

From an algorithm perspective, the MET method is the one which offers more hope for improvement, for example through the manner in which the wavelet transform of the signal is reconstructed from the edges which are left untouched (the ones we do not remove, i.e., the ones we believe are *not* due to noise). Currently, edge removal forces quite a strong distortion in the original wavelet transform, which, in turn, introduces some oscillations in the signal. A less brutal edge removal, together with an allowance for the remaining edges to adjust their positions and intensities slightly, may considerably improve the technique. For a more in-depth analysis of the MET algorithm see Boschetti et al. (2003).

Finally, it should be noted that a level of subjectivity is inherent in the judgement of what makes a good downward continuation algorithm. When we attempt to downward continue below the source level, we should expect oscillations in the profile to occur which are not strictly a consequence of 'noise'. This is explained in detail in Boschetti et al. (2001). Whether we want such oscillations removed depends on what we define as 'noise' and as 'source', as well as on whether we want to know if we have continued below the source level itself. Both issues depend on the purpose of the data analysis. Consequently, a brute force algorithm, able to always suppress all oscillations, may not necessarily be the best choice. In this view, the MET algorithm, which is more 'local' than the Wiener and ISVD method, offers the option to act selectively on specific features of the profile, without the need to impose the same amount of filtering on all features. This may lead to a more flexible algorithm which allows for more control by the user, who can chose where and how to manipulate the profile, depending on the purpose of the downward continuation exercise.

CONCLUSION

A stable downward continuation procedure remains one of the most difficult hurdles in potential field analysis, with no clear breakthrough foreseeable in the immediate future. We have proposed two new methods, Multiscale Edge Transform followed by a mild low-pass filter, and a combination of Wiener filtering and Translation-Invariant denoising, both of which deserve further study.

Downward continuation has strong links to the numerical inversion of potential field data. We envisage the use of the algorithms presented here in conjunction with an anomaly removal algorithm. This would incorporate a partial downward continuation to the level of the shallowest sources, followed by individual removal of these sources before further downward continuation. The procedure could be implemented within a visualisation package in which specific features are selected and processed in real time.

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