

# Improved heralded single-photon source with a photon-number-resolving superconducting nanowire detector

Samantha I. Davis<sup>1,2</sup>, Andrew Mueller<sup>1,2,3</sup>, Raju Valivarthi<sup>1,2</sup>, Nikolai Lauk<sup>1,2</sup>, Lautaro Narvaez<sup>1,2</sup>, Boris Korzh<sup>3</sup>, Andrew D. Beyer<sup>3</sup>, Marco Colangelo<sup>4</sup>, Karl K. Berggren<sup>4</sup>, Matthew D. Shaw<sup>3</sup>, Neil Sinclair<sup>1,2,5</sup>, and Maria Spiropulu<sup>1,2</sup>

<sup>1</sup>*Division of Physics, Mathematics and Astronomy, California Institute of Technology, 1200 E California Blvd., Pasadena, CA 91125, USA*

<sup>2</sup>*Alliance for Quantum Technologies (AQT), California Institute of Technology, 1200 E California Blvd., Pasadena, CA 91125, USA*

<sup>3</sup>*Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena, CA 91109, USA*

<sup>4</sup>*Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, 50 Vassar St., Cambridge, MA 02139, USA and*

<sup>5</sup>*John A. Paulson School of Engineering and Applied Sciences, Harvard University, 29 Oxford St., Cambridge, MA 02138, USA*

(Dated: December 22, 2021)

We demonstrate real-time single photon heralding from a bulk nonlinearity using an efficient and low-noise photon number-resolving superconducting nanowire detector. A maximum reduction of  $0.118 \pm 0.012$  in the photon  $g^2(0)$  cross-correlation is obtained, indicating a strong suppression of multi-photon emissions. We analytically model our experimental results using a phase-space formalism and obtain excellent agreement. Our experiment, built using fiber-coupled and off-the-shelf components, delineates a path toward engineering ideal sources of single photons.

## I. INTRODUCTION

A challenge in quantum optical science and technology is the realization of an ideal, i.e deterministic, high-fidelity, tunable, and high-rate source of indistinguishable single-photons [1, 2]. One intuitive approach to develop a single-photon source requires coupling an individual quantum emitter to light using a cavity. Significant progress in this regard [3] has been achieved using e.g. quantum dots [4–6], crystal defects [7], or trapped ions [8] and atoms [9], albeit mired with challenges, including fabrication complexity [10, 11] or differing emitter spectra [12–14]. Instead, the strong light-matter coupling offered by solid-state bulk nonlinearities can be used to probabilistically emit photon pairs via laser-driven  $\chi^{(2)}$  and  $\chi^{(3)}$  processes [15], i.e. spontaneous parametric down-conversion (SPDC) and four-wave mixing (SFWM), respectively. Thermal statistics of the emission permit a photon pair to be emitted with low probability (e.g.  $10^{-3}$  in practice [16]). An individual photon (in a signal mode) can be heralded by the detection of a photon (in an idler mode) [1]. Typically this is performed using a threshold detector that discriminates zero from one or more photons. Heralding of photons from optical nonlinearities is scalable, and has enabled tunable and indistinguishable photons with high fidelities and bandwidths [1, 17, 18]. However, there is a non-zero probability to produce multiple pairs. To exclude these pairs, we herald the idler mode from an SPDC process *in real time* using a novel photon number-resolving (PNR) niobium nitride (NbN) superconducting nanowire single-photon detector (SNSPD) [19]. The detector is optimized across several performance metrics [20], namely efficiency  $> 70\%$ , dark count rate of 10 Hz, timing resolution (jit-

ter) of  $< 14$  ps, and a high single-photon discrimination efficiency, defined in Sec. 3. Specifically, we perform a second-order cross-correlation  $g^2(0)$  measurement [21] of the signal mode conditioned on the measured photon number of the idler mode using the number-resolving detector. We operate the detector in two configurations: (i) as a PNR SNSPD, discriminating zero-, one- and multi-photon events, and (ii) as a threshold SNSPD, discriminating zero-photon events from all other events. A  $g^2(0)$  of zero is expected when a single photon pair is generated without any multi-photon events. We measure a maximum reduction of  $g^2(0)$  from  $0.430 \pm 0.009$  to  $0.312 \pm 0.008$  when using PNR detection versus threshold detection, thereby improving the fidelity of the single photon source. Our non-zero  $g^2(0)$  is primarily due to recoverable photon loss. For a fixed  $g^2(0) = 7 \times 10^{-3}$ , we increase the probability to generate a single pair by 25%, from  $4 \times 10^{-3}$  to  $5 \times 10^{-3}$ , which is state-of-the-art [22]. Our results are analytically modeled using a phase-space formalism based on characteristic functions that incorporate all relevant imperfections [23–25].

## II. EXPERIMENTAL METHODS

The experimental setup is shown in Fig. 1. Light pulses of  $\sim 600$  ps duration are created by injecting 1540 nm wavelength light from a continuous-wave laser into an intensity modulator (IM). The modulator is driven by an arbitrary waveform generator (AWG) at a rate of 1 MHz, which is the clock rate of the experiment. The pulses are amplified by an erbium doped fiber optic amplifier (EDFA) and then directed to a second harmonic generation module (SHG), which up-converts the pulses

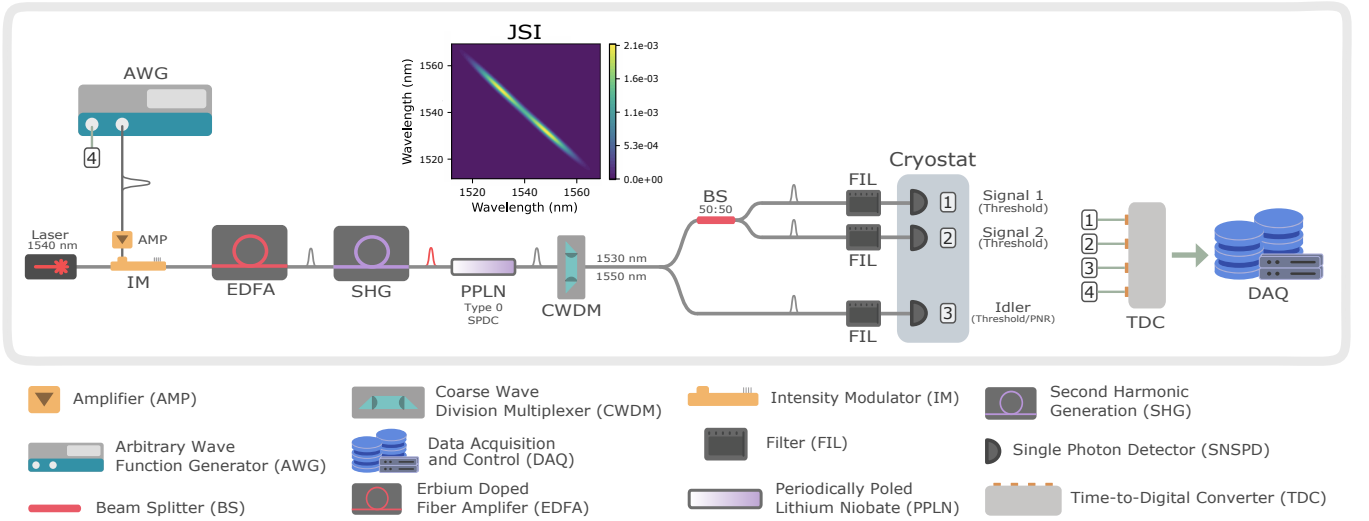


FIG. 1. Experimental setup. AWG - Tektronix AWG7002A, BS - Thorlabs 1550nm fiber optic 50:50 beamsplitter, CWDM - FS one-channel Coarse Wave Division Multiplexing/Optical Add/Drop Multiplexer, EDFA - Pritel Optical Fiber Amplifier, Laser - General Photonics TLS-101, PPLN - Covesion Ruggedized Waveguide, SHG - Pritel Optical Fiber Amplifier/Second Harmonic Generator.

to 770 nm. The 770 nm pulses pump a fiber-coupled type-0 periodically poled lithium niobate (PPLN) waveguide, which produces photon pairs centered at 1540 nm wavelength by way of SPDC. A coarse wave division multiplexer (CWDM) splits the photon pairs into the signal and idler paths, centered at 1530 nm and 1550 nm, respectively, and ascribes them with 13 nm bandwidths. The estimated joint spectral intensity (JSI) with the CWDMs is shown in Fig. 1, corresponding to a multi-mode spectrum with a Schmidt number of  $K = 20.6$  [26]. We vary the average number of pairs  $\mu$  by changing the input power to the SHG module. Light in the signal path is split by a 50:50 beamsplitter (BS) into two paths, labelled as signal 1 and 2. Filters with a total of 60 dB extinction on the idler path and 120 dB extinction on the signal path are used to suppress the unconverted 770 nm pump light. The photons from the signal and idler paths are detected using conventional and PNR SNSPDs, respectively. The readout pulses from the detectors and the 1 MHz signal from the AWG are sent to a time tagger, which is interfaced with custom-made software for real-time analysis and multi-photon event discrimination (see Supplemental Material).

To measure the signal modes, we use two single-pixel tungsten silicide (WSi) SNSPDs, which have timing jitters of  $\sim 50$  ps, efficiencies of  $\sim 80\%$ , and dark count rates below 5 Hz [16]. To measure the idler mode, we use a PNR SNSPD with an active area of  $22 \times 15 \mu\text{m}^2$ , formed by a meander of 100-nm-wide and 5-nm-thick niobium nitride (NbN) nanowires with a 500 nm pitch. The detector employs a differential architecture to cancel the signal propagation delays' contribution to the timing jitter [19] and an impedance matching taper to increase the signal-to-noise ratio and minimize reflections and distortions

[19, 27]. The integration of an impedance-matching taper was also shown to enable photon-number-resolution. The number of incident photons is encoded in the amplitude of the output pulse [19, 28]. A single incident photon, absorbed by the nanowire, induces a single time-dependent resistive hotspot and results in a RF pulse [20]. Multiple incident photons, absorbed by the nanowire at the same time, will induce multiple time-dependent resistive hotspots, increasing the total resistance and, hence, an RF pulse with the amplitude and slew rate that depends on the number of hotspots. In our experiments, rather than measuring the pulse amplitude variation [19, 28], we measure its slew rate variation [29], because it only requires a constant threshold time-tagger, and thus enables real-time readout. We trigger at 90% of the pulse amplitude where the variation in slew rate contributes to variation in time of the registered time-tag. Earlier (later) time-tags arriving in the left (right) bin of Fig. 2 correspond to multi-photon (single-photon) pulses with higher (lower) slew rate.

### III. MODEL

The second-order cross-correlation of photons in the signal 1 and 2 paths conditioned on the detection of the idler for the threshold and PNR configurations is calculated as

$$g^2(0) = \frac{C_{is_1s_2}C_i}{C_{is_1}C_{is_2}}, \quad (1)$$

where  $C_{is_1s_2}$  is the number of threefold coincidence detection events of photons in the idler and the two signal paths,  $C_i$  is the number of idler detection events, and

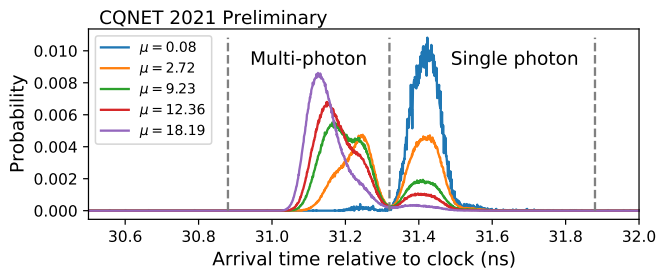


FIG. 2. Probability distribution of the arrival times of detection events by the PNR SNSPD for varied  $\mu$ . The dashed lines define the time bins corresponding to single (right) and multi-photon (left) events. The total number of events are used when operating the SNSPD as a threshold detector.

$C_{i_s_j}$  is the number of idler and signal  $j$  twofold coincidence detection events where  $j = 1, 2$ . We develop a theoretical model that takes into account full multi-photon effects, losses, and multi-mode behavior to determine the dependence of  $g^2(0)$  on  $\mu$ . Analytical expressions for the detection rates of each detector, twofold coincidence rates, threefold coincidence rates, and  $g^2(0)$  as a function of  $\mu$  and efficiencies are derived using a characteristic function-based approach [23]. For our experiment, we construct the symplectic transformation that maps the characteristic function of the state after the CWDMs, which is a multi-mode squeezed vacuum state, to that of the state prior to detection. This is permitted because the states and all relevant experimental operations are Gaussian. The characteristic function is constructed using the Schmidt modes of the JSI (see Supplemental Material) [30].

We calculate the detection probability using

$$\text{Tr}[\hat{\rho}\hat{\Pi}] = \left(\frac{1}{2\pi}\right)^n \int dx^{2n} \chi_\rho(x) \chi_\Pi(-x), \quad (2)$$

where  $\hat{\rho}$  describes an  $n$ -mode state prior to detection,  $\hat{\Pi}$  is the measurement operator corresponding to the detector,  $\chi_\rho(x)$  is the characteristic function of  $\hat{\rho}$  and  $\chi_\Pi(x)$  is the characteristic function of  $\hat{\Pi}$ . Since the measurement operators describing PNR detectors are not Gaussian operators [31], unlike threshold detectors, we cannot immediately evaluate Eq. (2) to find the detection probabilities for the PNR detector. Instead, we model the PNR detector as an effective  $2N$ -port beamsplitter with threshold detectors at each output port [24, 25, 32, 33]. We implement the  $2N$ -port beamsplitter as a network of beamsplitters forming a so-called “binary tree” architecture, which has  $N$  input and output ports, as shown in Fig. 3 for the case  $N = 8$ . To model a PNR detector, photons are injected to an input of the “top-most” beamsplitter of the tree, which corresponds to input 6 in the figure. The detection of photons with the PNR SNSPD is modeled as detection events from any combination of threshold detectors at the output ports of the tree. For an input Fock state  $|n\rangle$ , the probability that

multiple photons arrive to the same output port is negligible when  $N \gg n$ , corresponding to ideal photon number discrimination. In this case, the number of detection events equals the number of input photons. For  $N \sim n$ , the probability of multiple photons arriving to the same output port is non-negligible, corresponding to non-ideal photon number discrimination. In this case, the number of detection events does not equal the number of input photons. Therefore, the depth of the tree  $k = \log_2(N)$ ,

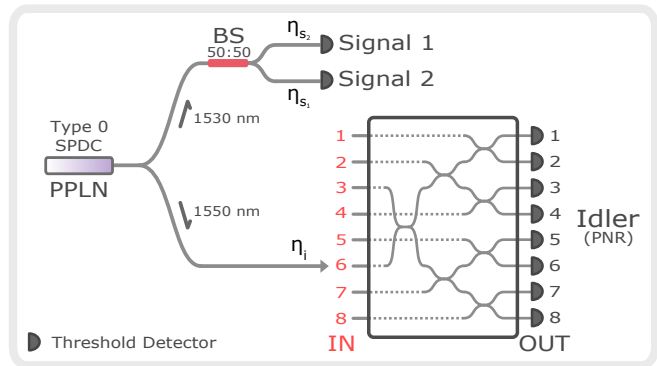


FIG. 3. Schematic of the setup used for theoretical modeling. The PNR detector is modeled as a  $2N$ -port beamsplitter in a binary tree architecture with threshold detectors at the outputs. See the main text for details.

is a figure of merit for photon number discrimination. A schematic of the binary tree model for  $k = 3$  is shown in Fig. 3, where  $\eta_{s_1}$ ,  $\eta_{s_2}$ , and  $\eta_i$  denote the efficiencies (including all coupling and detection losses) of the signal 1, signal 2, and idler paths, respectively. We use the schematic depicted in Fig. 3 and employ characteristic function techniques for Gaussian states to find analytical expressions for single-photon detection probabilities from the PNR SNSPD, as well as twofold and threefold coincidence probabilities of photons in signal 1, signal 2, and idler paths, as a function of the  $\eta_{s_1}$ ,  $\eta_{s_2}$ ,  $\eta_i$  and  $k$ . These expressions are then used to calculate  $g^2(0)$  from Eq. (1). Moreover, we use our model to define the single photon discrimination efficiency of the PNR detector as:

$$\eta_{PNR} = 1 - \frac{1}{2} \text{Tr} \left[ \sqrt{(\hat{\Pi}_1^{exp} - \hat{\Pi}_1^{ideal})^2} \right], \quad (3)$$

where  $\hat{\Pi}_1^{exp}$  and  $\hat{\Pi}_1^{ideal}$  are the experimentally obtained and ideal elements of positive-operator value measure, respectively, corresponding to the single photon measurement outcome of the PNR detector. The  $\eta_{PNR}$  is zero for a threshold detector and one for an ideal PNR detector. For more details, see the Supplemental Material.

## IV. RESULTS

We vary  $\mu$  and measure the corresponding  $g^2(0)$  for threshold and PNR detector configurations. The results

are shown in Fig. 4. We observe suppression of multi-photon events, as indicated by a reduction of  $g^2(0)$ . A maximum reduction of  $0.118 \pm 0.012$  is observed, as shown in the inset of Fig. 4a. To determine  $\mu$ , we fit Eq. (A9) to our measurement results using the threshold configuration, and to determine  $k$ , we fit Eq. (A4) to the single photon rates in the PNR configuration. The  $\eta_{s_1}$ ,  $\eta_{s_2}$  and  $\eta_i$  are calculated from the single and twofold coincidence detection events of the four lowest  $\mu$ . For our experiment,  $k = 2.55$  and the detector efficiency is 0.71, which yields  $\eta_{PNR} = 0.360$ . Thus, the single-photon discrimination of our detector is comparable to that of a pseudo-PNR detector comprised of no more than six threshold detectors each with a detector efficiency of 0.71. The data used to calculate  $g^2(0)$ , along with the fit details, are presented in the Supplemental Material. As seen from Fig. 4, our model agrees with the experimental results. The data for  $\mu \leq 0.015$  is presented in Fig. 4b. For a  $g^2(0)$  of  $7 \times 10^{-3}$  (black line) [22], the crosses and grey lines indicate the corresponding  $\mu$  using threshold and PNR detection, respectively. We observe a 25% improvement in  $\mu$ , from  $4 \times 10^{-3}$  (blue cross) with the threshold configuration, to  $5 \times 10^{-3}$  (green cross) with the PNR configuration.

Next, we estimate the performance of our experiment with future improvements (see Sec. 5). We calculate  $g^2(0)$  using the properties of our PNR detector (purple dashed line) and those of a nearly ideal PNR detector (red dashed line) for a high-efficiency single-mode SPDC source. We assume efficiencies of  $\eta_{s_1} = \eta_{s_2} = \eta_i = 0.87$ , which are the product of the coupling (0.91) and detector (0.96) efficiencies from Refs. [34] and [35], respectively, and are among the best achieved to date. With these upgrades, for a  $g^2(0)$  of  $7 \times 10^{-3}$ , we predict an improved  $\mu = 8.7 \times 10^{-3}$  (purple cross) and  $\mu = 1.4 \times 10^{-2}$  (red cross) using our PNR SNSPD and a nearly ideal PNR detector, respectively.

## V. DISCUSSION

By measuring the idler mode of a spontaneous parametric down-conversion source using a photon-number-resolving nanowire detector, we reduce the  $g^2(0)$  of the signal mode or, likewise, increase the probability to generate a photon. The results and key performance metrics of our experiment are supported by a detailed analytical model which captures multi-photon effects, imperfections, and multiple spectral modes. Using a setup consisting of fiber-coupled and off-the-shelf equipment, we generate photons that can be used in quantum information applications, in particular quantum communications [36, 37].

To realize an ideal single photon source [1], a number of improvements to our experiment must be implemented [38]. First, the Schmidt number of our SPDC source must be decreased from its current value of  $K = 20.6$  to  $K = 1$ . This can be accomplished with either narrower spectral filtering of the pairs, an increase in the pump

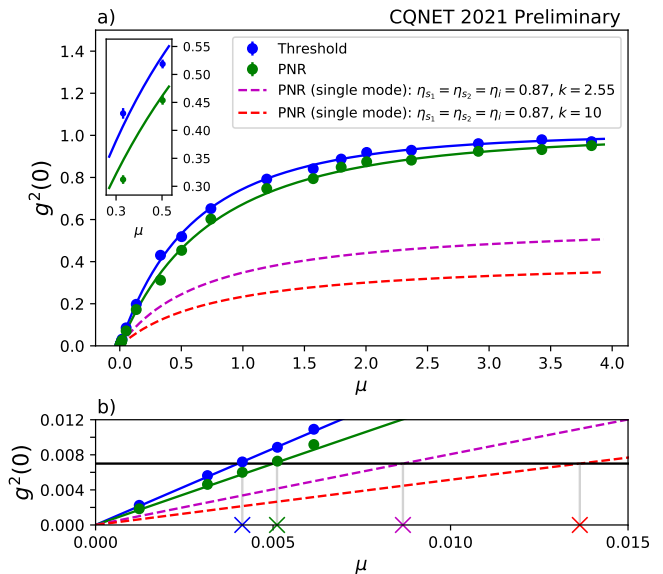


FIG. 4. The cross-correlation  $g^2(0)$  with varied mean photon number for our experiment and using improved heralded single-photon sources. The data using threshold (blue) and PNR (green) configurations are represented by the circular markers. All uncertainties are calculated from Poisson statistics. The blue and green solid lines are obtained from the model using measured  $\eta_i = 0.3280 \pm 0.0110$ ,  $\eta_{s_1} = 0.1802 \pm 0.0063$ ,  $\eta_{s_2} = 0.2210 \pm 0.0077$ . The model predicts  $k = 2.55$  for PNR detection. The inset magnifies the region where the largest reduction in  $g^2(0)$  is measured. The purple and red dashed lines are predictions using an improved source (see main text for details).

pulse bandwidth [39], the use of cavity-enhanced SPDC [40], or by engineering the phase matching function of the nonlinear crystal [41]. A near-unity Schmidt number renders the photons suitable for interference with other independently generated photons in a quantum circuit or network.

Next, the system efficiency should be increased to near unity. Coupling between fibers and devices can be improved with enhanced modal engineering [42] or using anti-reflection-coated free-space components [43]. Alternatively, components could be integrated onto the same chip, for instance using Si- or SiN-on-insulator with SFWM sources [17, 44], or using thin-film lithium niobate [45]. Furthermore, multiplexing strategies must be employed to increase the probability of generating a single pair beyond the theoretical maximum of 25% per mode. Such multiplexing, using, for instance, spatial [46, 47], temporal [34, 48], or frequency modes [49, 50], could also be employed to circumvent loss in the signal mode [51]. This requires on-demand feed-forward mode mapping using switches [52], quantum memories [53], or frequency shifters [54], respectively. Feed-forward requires the real-time readout that our PNR SNSPD provides. If all modes can be mapped, multiplexing can also allow a multi-mode source to be rendered as single mode, i.e.

it effectively decreases its Schmidt number to unity. Our broadband SPDC source is naturally suited for frequency multiplexing, as indicated by the strong frequency correlations in our JSI [55], see Sec. 2. This suggests our measured  $\mu = 5 \times 10^{-3}$  for  $g^2(0) = 7 \times 10^{-3}$  exceeds state-of-the-art SPDC sources using threshold detection, as well as quantum dots [22], accounting for multiplexing. Note that with a system efficiency of 0.87 [34, 35], see Sec. 4, and using 61 multiplexed modes, the probability of our source to generate a single photon approaches unity, while maintaining a  $g^2(0) = 7 \times 10^{-3}$ .

Additional gains can be offered by improvements to the PNR SNSPD. A higher detector efficiency would increase the photon number discrimination efficiency and improve the fidelity of the heralded single photon. This may be achieved through improvements to the optical stack around the nanowire by replacing the gold mirror with a distributed Bragg reflector mirror [56]. Also, the detector reset time of nearly 100 ns restricts the maximum repetition rate of the source to be  $\sim 10$  MHz. An SNSPD with a reduced reset time based on a lower kinetic inductance nanowire material, or integrated with an active quenching circuit [57], would allow for high single-photon generation rates. A multiplexing method based on multiple PNR SNPDSs would also support a high repetition rate.

Beyond single photon sources, extensions of our setup will allow efficient generation of qubits or qudits, or entanglement swapping using PNR SNSPDs [37]. Further uses encompass preparation of heralded photon-number states [58] and non-Gaussian continuous-variable states [59], which are vital resources to realize fault-tolerant photonic quantum computers [60].

During the preparation of our manuscript we became aware of relevant results achieved independently of this work [61].

## VI. ACKNOWLEDGMENTS

We acknowledge partial funding from the Department of Energy BES HEADS-QON Grant No. DE-SC0020376, QuantISED SC0019219, and the AQT Intelligent Quantum Networks and Technologies (INQNET) research program. Partial support for this work was provided by the DARPA DSO DETECT, NASA SCaN and Caltech/JPL PDRDF programs. S.I.D. and A.M. acknowledge partial support from the Brinson Foundation. Part of this research was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with NASA. We acknowledge productive discussions with Kayden Taylor, Sergio Escobar, Daniel Oblak, and Cristian Peña. We are grateful to Jason Trevor for technical assistance.

See Supplement 1 for supporting content.

## Appendix A: Analytical expressions

For a  $2N$ -port beamsplitter realized as a finite depth binary tree, we derive expressions for single, two-fold and three-fold coincidences as a function of the efficiencies and tree depth  $k$ , where  $N = 2^k$ . The equations reduce to the threshold detection case for  $k = 0$ . In the following, we use  $\hat{\Pi}_{\text{off},m}$  and  $\hat{\Pi}_{\text{on},m}$  to denote the measurement operators for a threshold detector at mode  $m$ ,

$$\hat{\Pi}_{\text{off},m} = |0\rangle\langle 0|_m, \quad (\text{A1})$$

$$\hat{\Pi}_{\text{on},m} = \hat{I}_m - |0\rangle\langle 0|_m. \quad (\text{A2})$$

For the PNR detector, we use  $\hat{\Pi}_{\text{on},m} \otimes \hat{\Pi}_{\text{off}}^{\otimes N-1}$  to denote an ‘‘on’’ measurement outcome for a detector at the  $m$ th output and ‘‘off’’ measurement outcomes for the detectors at the remaining  $N - 1$  outputs of the tree.

### 1. Idler detection probability

The idler detection probability  $P_i(\mu, \eta_i, k)$  is calculated as

$$P_i = N \text{Tr} \left[ \rho \left( I_{s_1} \otimes I_{s_2} \otimes \hat{\Pi}_{\text{on},m} \otimes \hat{\Pi}_{\text{off}}^{\otimes N-1} \right) \right], \quad (\text{A3})$$

and is evaluated to

$$P_i = 2^k \left( \prod_s \frac{2^k}{2^k + (2^k - 1)\lambda_s \mu \eta_i} - \prod_s \frac{1}{1 + \lambda_s \mu \eta_i} \right), \quad (\text{A4})$$

where  $\lambda_s$  are the Schmidt coefficients obtained from the joint spectral intensity. The products in the expressions run over all Schmidt coefficients. The Schmidt coefficients for our source are shown in Fig. 5.

### 2. Idler & signal twofold coincidence probability

The idler and signal twofold coincidence probabilities  $P_{is_1}(\mu, \eta_i, \eta_{s_1}, k)$ ,  $P_{is_2}(\mu, \eta_i, \eta_{s_2}, k)$  are calculated as

$$P_{is_1} = N \text{Tr} \left[ \rho \left( \hat{\Pi}_{\text{on},s_1} \otimes I_{s_2} \otimes \hat{\Pi}_{\text{on},m} \otimes \hat{\Pi}_{\text{off}}^{\otimes N-1} \right) \right], \quad (\text{A5})$$

$$P_{is_2} = N \text{Tr} \left[ \rho \left( I_{s_1} \otimes \hat{\Pi}_{\text{on},s_2} \otimes \hat{\Pi}_{\text{on},m} \otimes \hat{\Pi}_{\text{off}}^{\otimes N-1} \right) \right]. \quad (\text{A6})$$

The idler and signal  $j$  twofold coincidence probability  $P_{is_j}(\mu, \eta_i, \eta_{s_j}, k)$  where  $j = 1, 2$  is evaluated to

$$P_{is_j} = 2^k \left( \prod_s \frac{2^k}{2^k + (2^k - 1)\lambda_s \mu} \right. \quad (\text{A7}) \\ \left. - \prod_s \frac{2^{k+1}}{\lambda_s \mu \eta_{s_j} (2^k - (2^k - 1)\eta_i) + 2(2^k + (2^k - 1)\lambda_s \mu \eta_i)} \right. \\ \left. - \prod_s \frac{1}{1 + \lambda_s \mu \eta_i} + \prod_s \frac{2}{2 + 2\lambda_s \mu \eta_i + \eta_{s_j} + \lambda_s \mu (1 - \eta_i)} \right).$$

### 3. Idler & signal 1 & signal 2 threefold coincidence probability

The idler, signal 1, and signal 2 threefold coincidence probability  $P_{i,s_1,s_2}(\mu, \eta_{s_1}, \eta_{s_2}, \eta_i, k)$  is calculated as

$$P_{i,s_1,s_2} = N \text{Tr} \left[ \rho \left( \hat{\Pi}_{\text{on},s_1} \otimes \hat{\Pi}_{\text{on},s_1} \otimes \hat{\Pi}_{\text{on},m} \otimes \hat{\Pi}_{\text{off}}^{\otimes N-1} \right) \right], \quad (\text{A8})$$

and is evaluated to

$$\begin{aligned} P_{i,s_1,s_2} = & 2^k \left( \prod_s \frac{2^k}{(2^k + (2^k - 1)\lambda_s \mu \eta_i)} - \prod_s \frac{2^{k+1}}{\lambda_s \mu \eta_{s_1} (2^k - (2^k - 1)\eta_i) + 2(2^k + (2^k - 1)\lambda_s \mu \eta_i)} \right. \\ & - \prod_s \frac{2^{k+1}}{\lambda_s \mu \eta_{s_2} (2^k - (2^k - 1)\eta_i) + 2(2^k + (2^k - 1)\lambda_s \mu \eta_i)} + \prod_s \frac{2^{k+1}}{\lambda_s \mu (\eta_{s_1} + \eta_{s_2}) (2^k - (2^k - 1)\eta_i) + 2(2^k + (2^k - 1)\lambda_s \mu \eta_i)} \\ & - \prod_s \frac{1}{1 + \lambda_s \mu \eta_i} + \prod_s \frac{2}{2 + 2\lambda_s \mu \eta_i + \eta_{s_1} \lambda_s \mu (1 - \eta_i)} + \prod_s \frac{2}{2 + 2\lambda_s \mu \eta_i + \prod_s \eta_{s_2} \lambda_s \mu (1 - \eta_i)} \\ & \left. - \prod_s \frac{2}{2 + 2\mu \lambda_s \eta_i + (\eta_{s_1} + \eta_{s_2}) \mu \lambda_s (1 - \eta_i)} \right). \end{aligned} \quad (\text{A9})$$

With these expressions, we can derive analytical expressions for the  $g^{(2)}$ ,

$$g^{(2)}(0) = \frac{P_{i,s_1,s_2} P_i}{P_{i,s_1} P_{i,s_2}} = \frac{C_{i,s_1,s_2} C_i}{C_{i,s_1} C_{i,s_2}}. \quad (\text{A10})$$

For a more detailed discussion of the derived formulas, see the Supplemental Material.

#### Appendix B: Single Photon Discrimination Efficiency

The element of positive-operator value measure corresponding to the single photon outcome for a phase independent PNR detector can be described by

$$\hat{\Pi}_1 = \sum_{n=0}^{\infty} c_n |n\rangle \langle n|, \quad (\text{B1})$$

where  $c_n$  are the matrix elements corresponding to the representation of the operator in the photon number basis. For an ideal PNR detector,  $c_1 = 1$  and  $c_{n \neq 1} = 0$ .

From our model, we obtain  $\hat{\Pi}_1$  for our PNR detector using  $k = 2.55$  and efficiency of  $\eta = 0.71$  (see Supplemental Material),

$$\begin{aligned} \hat{\Pi}_1^{exp} \approx & 0.418 |1\rangle \langle 1| + 0.293 |2\rangle \langle 2| + 0.156 |3\rangle \langle 3| \\ & + 0.0744 |4\rangle \langle 4| + 0.0336 |5\rangle \langle 5| + 0.0147 |6\rangle \langle 6|. \end{aligned} \quad (\text{B2})$$

The single photon discrimination efficiency, which is defined in (3) as the trace norm between the ideal  $\hat{\Pi}_1^{ideal}$  and the normalized  $\hat{\Pi}_1^{exp}$ , evaluates to

$$\eta_{PNR} = 1 - \frac{1}{2} \sum_n |c_n^{exp} - c_n^{ideal}|. \quad (\text{B3})$$

For our detector, we calculate  $\eta_{PNR} = 0.360$ .

#### Appendix C: Additional Data

The joint spectral intensity (JSI) of the source is shown in Fig. 5a and the eigenvalues  $\lambda_s$  from the Schmidt decomposition of the JSI are shown in Fig. 5b.

The data used to calculate the  $g^2(0)$  in the threshold and PNR detection configurations are shown in Fig. 6 (see below).

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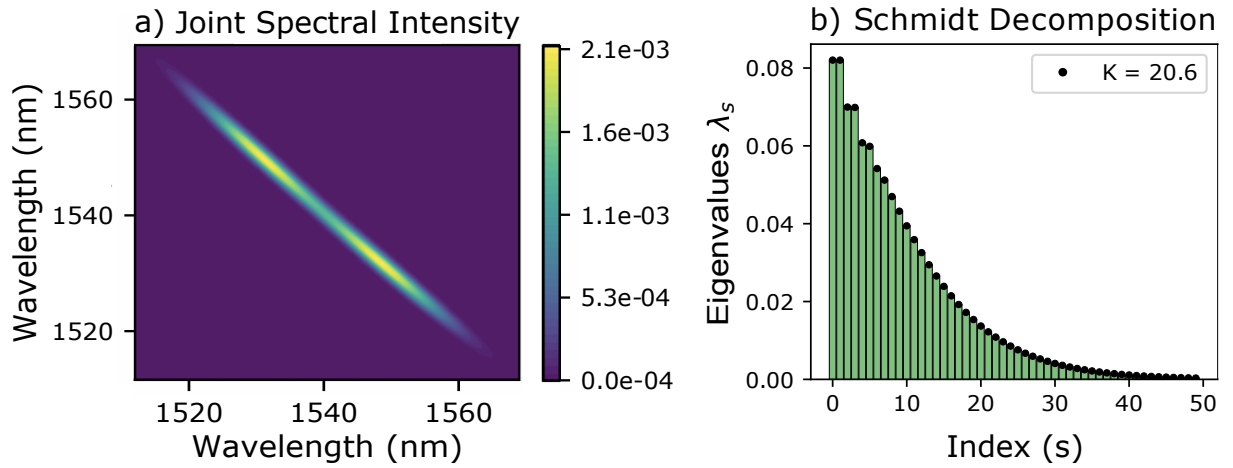


FIG. 5. a) Joint spectral intensity of source, normalized by the area. b) Eigenvalues obtained from the Schmidt decomposition of the JSI.

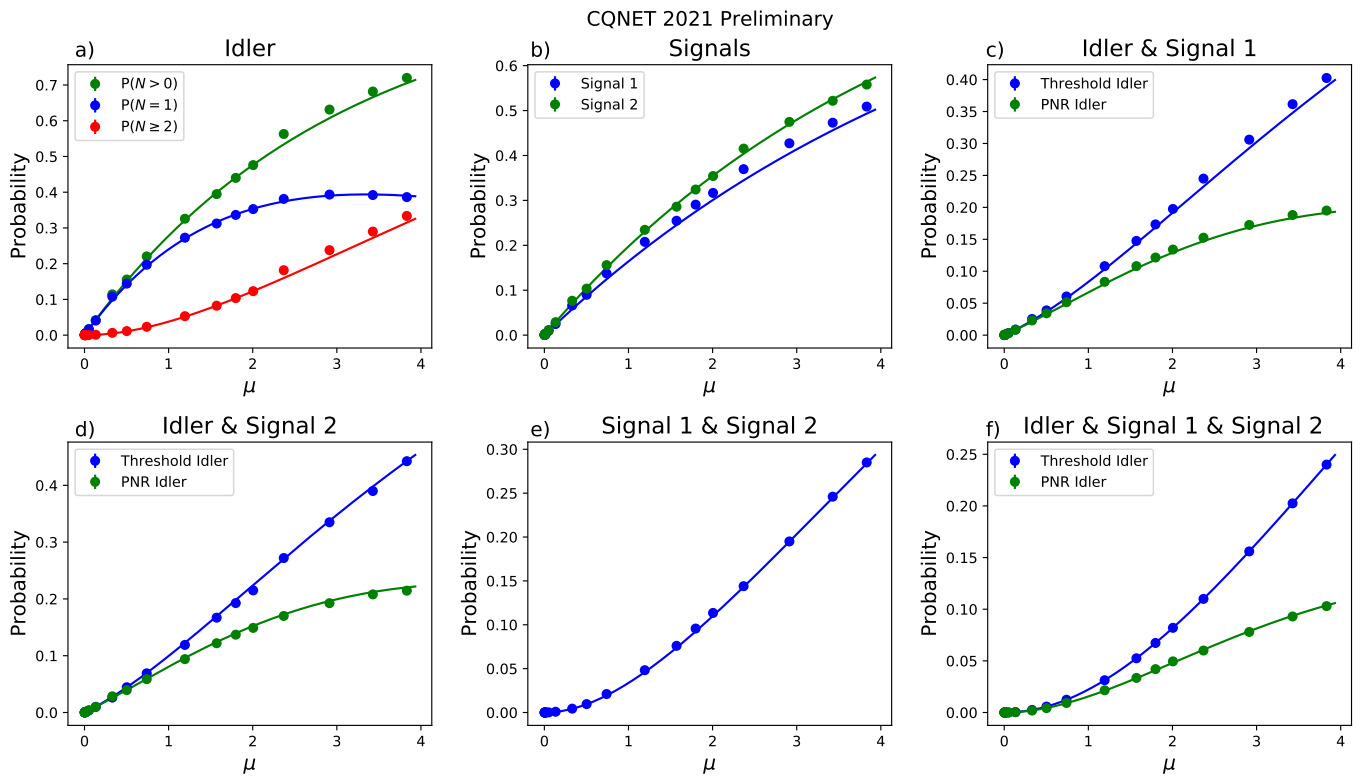


FIG. 6. a) Idler detection probabilities, b) signal 1 and signal 2 detection probabilities, c) signal 1 and idler twofold coincidence detection probabilities, d) signal 2 and idler twofold coincidence detection probabilities, e) signal 1 and signal 2 twofold coincidence detection probabilities, and f) threefold coincidence detection probabilities as a function of  $\mu$ . In a),  $P(N > 0)$  denotes the probability of at least one photon in the threshold configuration,  $P(N = 1)$  denotes the probability of single photon detection from the single photon bin of the PNR configuration, and  $P(N \geq 2)$  denotes the probability of multi-photon detection from the multi-photon bin of the PNR configuration (see Fig. 2).