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# Improved Implicit Stochastic Optimization Technique for Multireservoir Water Systems under Drought Conditions

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**Abstract:** Drought is a creeping phenomenon, making its onset and end difficult to determine. Damages from droughts can exceed those resulting from any other natural hazard, although it is difficult to assign a monetary value to them. In the Mediterranean area a severe drought period occurred over the years 2000-2002 and economic losses from that drought exceeded 250 million euros in Sardinia (Italy) (source: ENAS Regional Water Authority). Currently, technological developments and environmental modelling tools are improving our ability to more effectively manage water supply systems. Models can provide decision makers with better and more timely data and information. In this paper a methodology is proposed to support water decisions by selecting and evaluating reservoir operating rules based on hydrological scenarios generated from historical records. The methodology includes optimization and simulation tools. In particular, the paper presents some improvements to the traditional Implicit Stochastic Optimization (ISO) approach that overcome some severe limitations affecting previous works. Thanks to the collaboration with Regional Water Authorities in Southern Italy, the proposed methodology has been tested in the Agri-Sinni water system. Specifically this investigation focuses on: 1) Defining the reservoir operating rules based on correlations between releases, storages and inflows in a risk management approach of drought events; 2) Analyzing the significant reduction of computational time the user can get by applying the ISO technique in a GRID computing approach.

**Keywords:** Operating rule; Implicit Stochastic Optimization (ISO); GRID computing; Agri-Sinni water system.

## 1 INTRODUCTION

As a result of continuing developments in the conceptual thinking and mathematical formulations, there are now available a variety of methods for analyzing the operation of multi-reservoir systems. Much of the work has focused on the use of simulation models, but limited use has also been made of optimization models for estimating policies which can then be more accurately evaluated using simulation. Since the late '70s, a number of methods have been used to define the operating policies including LP, dynamic programming (DP), non linear programming, Artificial Neural Networks (ANN), Fuzzy Rule-Based (FRB), or Genetic Algorithm (GA). Labadie (2004) highlighted that optimal period of record solutions given by optimization models need to be post processed and results interpreted in order to obtain operating rules. Yeh (1985), Wurbs (1993) and Labadie (2004) analyzed the gap between theoretical development and practical application. In spite of this growing body of technical literature devoted to deriving optimal policies for scheduling releases for multireservoir systems, many of us continue to observe few agencies, particularly at regional level, that have led to substantial changes in models and techniques for obtaining operating policies that specifies the releases and allocations that should be made in those systems. Simple statistics, tables and figures, or electronic worksheets are largely used worldwide.

Today, the issue of a large participation and integration is a central aspect in the European Water Framework Directive, the most ambitious water policy tool of these days (Castelletti and Soncini Sessa, 2006). The promotion of integration and participation in the river basin management plans requires that all those people that are affected by the actions considered should act as they were decision makers being responsible for choosing the alternative, e.g. the operating policy. This active

involvement permits an immediate perception of the effects of the selected operating policy. Clearly, stakeholders are not likely to accept the results from models developed in the decision making process (e.g. planning and management process in multireservoir systems) if they cannot use these models because of their complexity. A planning and management process intended to force clearer ideas requires the use of useful and meaningful model. It seems to me that the “psychological significance” may be a key aspect for accepting and incorporating model results into the decision regarding new operating policies.

Implicit stochastic optimization (ISO) (Labadie, 2004) is a classical approach for the definition of operating rules in multireservoir systems that greatly enlarges the deterministic optimization approach over a very long representative hydrology (generated or from historical records). ISO is one of the most reliable reservoir modelling techniques today (Simonovic, 1992). Comparison of explicit and implicit stochastic approaches has found implicit approach to give better results (Celeste and Billib, 2009). Most stochastic aspects of the problem can be directly included in the ISO approach and deterministic optimization can be directly applied based on the historical or synthetic records. In addition, ISO can be more detailed and can be solved more quickly than explicit stochastic approaches. In general, ISO makes reservoir operation very simple in practice (Liu et al., 2006) and results tend to be easy to explain (Lund and Ferreira, 1996). However, ISO presents some serious limitations when applied to large systems with a large set of representative hydrological and water resources time series. In this case, the search for appropriate operating rules can be time-, resource- and money-consuming (Karamouz and Houck, 1982; Hiew et al., 1989; Lund and Ferreira, 1996).

This paper presents some improvements on the traditional ISO approach that overcome limitations of previous works. A combined optimization-simulation approach cycles through different models until further iteration does not further improve the values of system performance. Also, the proposed approach implements the ISO technique in a GRID computing approach instead of a traditional local approach for the definition of reservoir operating policies. A discussion on the significant reduction of computational time saving the user can get by applying the ISO-GRID is also presented. The goal of the paper is to present a flexible and effective tool for selecting and evaluating operating policies for specifying multireservoir releases that could be of any value in different levels of a decision making process. However, the approach is not perfect and new challenges in applying to a real environment are finally discussed.

## **2 IMPLICIT STOCHASTIC OPTIMIZATION APPROACH**

The use of optimization methods in the real-world applications on multi-reservoir system is a hard task, particularly when dealing with hydrologic uncertainty. In applying optimization techniques to define operating rules deterministic and stochastic optimization methods have been developed. It is well known that explicit stochastic approaches often suffer from computational inconvenience (Young, 1967) and cannot be used when there is insufficient statistical information on data estimation to support the model or when probabilistic rules are not available. Deterministic approaches use a specific hydrological series (historic or synthetic record) and permit more detailed and adherent description of systems.

Considering a long historical or synthetically generated hydrologic record, implicit stochastic optimization infers operating rules from deterministic optimization results taking implicitly into account spatial and temporal correlation of unregulated inflows (Hiew et al., 1989). The hindrance of obtaining a set of decision policies closely related to the assumed hydrologic series can be overcome using a streamflow synthesis to provide equally likely future hydrologic series and a multivariate analysis to deduce a unique operating policy from different trajectories for each reservoir. Following Saad and Turgeon (1988), recent applications of implicit stochastic optimization are composed by three modules: streamflow synthesis, deterministic optimization and multivariate analysis. Dynamic Programming (DP) models have been developed to generate operating rules for small-scale systems. Bhaskar and Whitlatch (1980) and Karamouz and Houck (1982) used DP and regression analysis within a simple approach that captures in a general operating rule some of the stochastic aspects of the optimal deterministic operation of DP. Karamouz and Houck (1987) found that implicit stochastic optimization yields better results than explicit approaches when defining long-term performance of a single reservoir. The model proposed by Karamouz and Houck (1982) was generalized to generate operating rules in small-scale multiple reservoir systems through the use of optimization model, regression analysis and simulation model (Karamouz et al., 1992). The model cycles over and over through a simulation model and a DP model in order to refine the operating rules. Because of a large-scale DP model is affected by the dimensionality problem, executing a DP model in many iterations

would become time consuming for an implicit stochastic optimization approach (Mousavi et al., 2005). In general terms, Linear Programming (LP) model has been the most popular technique for solving complex large-scale systems, thanks to the availability of very efficient computer codes supporting million of variables and constraints. When LP allows implicit stochastic optimization models to accurately represent the system behaviour, multiple regression analysis applied to LP optimal solutions, may result in high correlation coefficient without expensive trial and error processes (Hiew et al., 1989).

### 1.1 The Modified ISO approach (MISO)

The proposed Modified ISO model (MISO) consists of four models as shown in Figure 1:

1. A large number of synthetic short equally likely set of series of monthly streamflows upstream each reservoir, are generated using the SAMS-2007 software (Sveinsson et al., 2007);
2. The deterministic LP model WARGI-OPT (Manca et al., 2004) is solved for each set of monthly streamflow series, providing the optimal sequences of monthly releases;
3. A multiple linear regression model is applied to the optimization results. In particular a lag-1 regression model is used in the proposed approach where the dependent variable is the current reservoir release and the independent variables are the current reservoir storage and the monthly streamflow during the previous month;
4. Optimal monthly operating rules conditioned on observable information (current reservoir storage and previous month streamflow) are incorporated into the MODSIM DSS (Ladadie and Larson, 2007) to replace operating rules usually arbitrarily selected by water system managers. MODSIM DSS simulates those operating rules to examine their performance as measured by reliability and vulnerability indexes.

The proposed ISO approach cycles through models 2, 3 and 4 until further iteration does not improve the values of system performance indexes.

One of the challenges of the proposed ISO approach was in trying to satisfy the needs of those at different levels of decision making. This approach can:

- provide a flexible and transparent tool for non-skilled water managers to understand the interrelations among different system components and achieve a consensus-base management policy;
- facilitate a stakeholder involvement in that decision making process: all models included in the tool can be downloaded free through the websites and clear information is available in the operating manuals;
- help stakeholders to develop at home their own models, reaching a common understanding on how the shared water system will be managed in the future based on the proposed management policy.

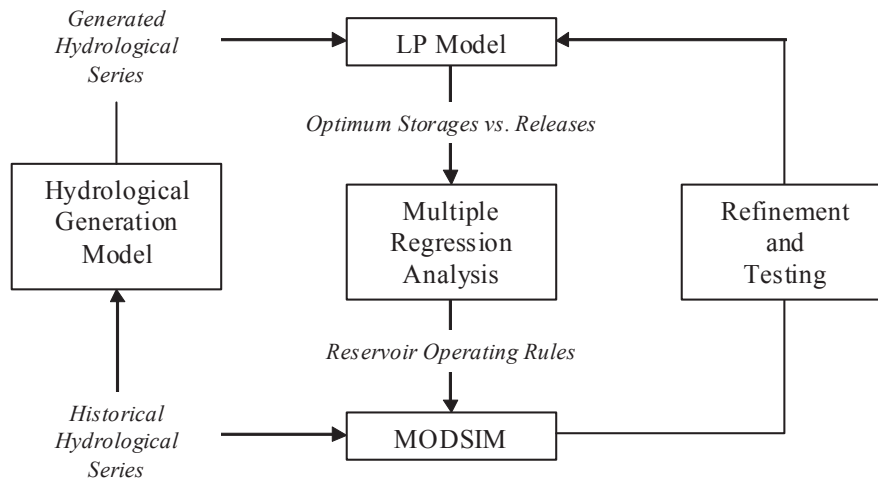


Figure 1. The proposed Modified ISO approach.

Models are relatively easy to use and interactive:

- LP is the most popular and commonly applied optimization method. It can be successfully applied to very large optimization problems, and unlike other optimization methods that require a different problem formulation for each different system and suffer from great computational inconvenience, ready-to-use solution packages are available for LP problems. WARGI-OPT is a LP optimization tool developed by the University of Cagliari. It is intuitive in the data input phase and requires moderate training in resource modelling for an effective use;
- Reservoir operating policies might be presented to stakeholders in the form of regression equations. Linear regression equations enable non-experts to achieve familiarity with reservoir operation, helping them reach a common understanding on future management. This is why regression techniques have been widely used to infer reservoir operating policies. Optimal releases are regressed on presently knowable state variables used to describe the state of the system. The SQ type linear decision rule (LDR) originally proposed by Loucks (1970) is adopted in the proposed tool and was found to be as good as or better than more complex forms in many cases (Bhaskar and Whitlatch, 1980);
- MODSIM DSS is a simple generic simulation model incorporated within interactive graphics-based interface and continuously maintained at the Colorado State University. MODSIM DSS is developed in the .NET Framework that provides a powerful environment for customization without requiring recoding. Reservoir operating policies can be conditioned on hydrologic state variables, whereas customization includes more complex user-defined operating policies. An effective use of MODSIM DSS requires moderate training and experience in resource modelling. MODSIM DSS can be downloaded free through the website (<http://modsim.engr.colostate.edu/>).

## 1.2 The Linear Operating Rule in MISO

In multipurpose multireservoir systems there are sometimes conflicting and sometimes complementary multiple purposes served by the water stored in and released from reservoirs. Operating policies define what should be done for any combination of system state and hydrologic conditions to minimize any necessary deviation from ideal conditions in those systems. According to Loucks and Sigvaldason (1982), reservoir operating rules may include one or more of the following components:

1. Target storage levels or volumes;
2. Multiple zoning;
3. Conditional rule curve.

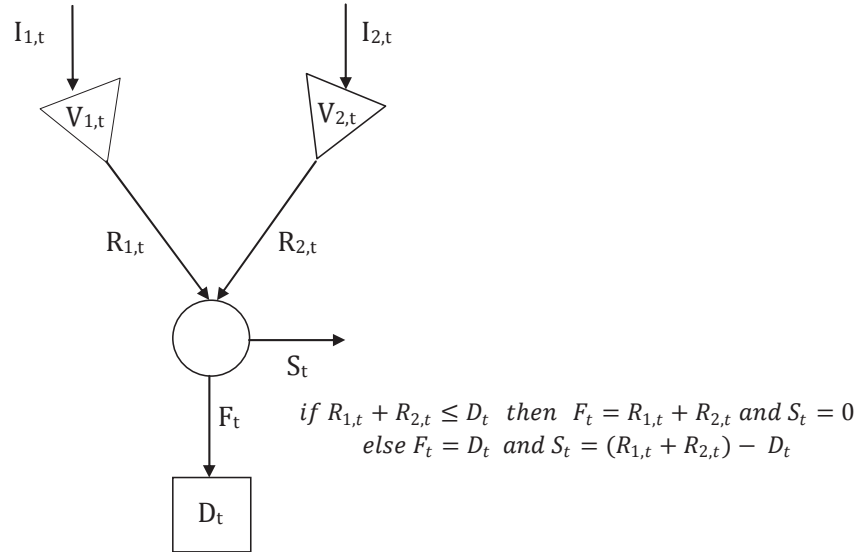
Here, I focus on the conditional rule curve that defines reservoir releases as a function of the existing storage volumes and the expected natural inflows for some future months. In particular, the well-known SQ type linear decision rule, originally proposed by Loucks (1970), applied to reservoir  $j$  at time step  $t$ :

$$R_{j,t} = a_{j,t}V_{j,t} + b_{j,t} \sum_{i=t}^{t+m} I_{j,i} + c_{j,t} \quad (1)$$

where  $R_{j,t}$  is the release from the reservoir,  $V_{j,t}$  is the stored volume in the reservoir at the beginning of the month  $i$ ,  $I_{j,i}$  is the expected natural inflow during the month  $i$  within the estimating  $m$  month period, and  $a_{j,t}$ ,  $b_{j,t}$  and  $c_{j,t}$  are coefficient to be assessed. Specifically, the MISO approach implements the SQ rule in Equation 1 with  $m=1$ .

## 3 APPLICATION TO A SINGLE-PURPOSE RESERVOIR SYSTEM

Before applying the proposed approach for definition of operating policies in multi-purpose multi-reservoir systems, some discussion of operation in single-purpose multiple-reservoirs in parallel may be helpful (Figure 2). Operation of reservoirs in series and in parallel has been extensively discussed in Sulis and Sechi (2013).



**Figure 2.** Configuration of two reservoirs in parallel.

Moreover, Sulis and Sechi (2013) analysed the adopted configuration with five generic simulation models including MODSIM. The system has an urban water demand with a monthly request  $D_t$  of 4.0  $\text{Mm}^3$ . Some statistical parameters of the adopted historical hydrological series 52-year length are reported in Table 1. In this test case, the ISM, the simplest nonparametric model, generated 16 equally likely synthetic series 10-year length at each reservoir selecting blocks of the first 26 years of the historic data. The advantages were that ISM is simple and easy to implement, assumption free, and can reproduce the entire distributional properties of the historic data; the main disadvantage was that only historically observed sequences can be input in WARGI-OPT resulting in 16 simulations with optimal sequences of monthly releases with limited variety.

Reservoir	Mean Inflow ( $\text{Mm}^3/\text{year}$ )	Maximum Inflow ( $\text{Mm}^3/\text{year}$ )	Minimum Inflow ( $\text{Mm}^3/\text{year}$ )	Initial Volume ( $\text{Mm}^3$ )	Maximum Capacity ( $\text{Mm}^3$ )
1	126.5	318.6	24.1	131	262
2	15.9	53.6	1.7	160	320

**Table 1.** Statistical parameters of adopted hydrology.

The multiple linear regression model applied to this 16 storage trajectories give the following SQ linear decision rules:

$$R_{1,t} = 0.03V_{1,t} + 0.07I_{1,t} + 0.7 \quad (2)$$

$$R_{2,t} = 0.08V_{2,t} + 0.12I_{2,t} + 0.8 \quad (3)$$

MODSIM implemented the Equations 2 and 3 as operating rules on reservoirs 1 and 2, respectively, then simulating the system over a 26-year time horizon considering the second half (26 years) of the historic hydrological data. Table 2 compares the system performance when no operating rule is implemented in MODSIM and releases are not restricted in all system configurations (Configuration A in Table 2) and when MODSIM implements the Equations 2 and 3 (Configuration B in Table 2). System performance evaluation analysis in the form presented by Sulis and Sechi (2013) includes basic statistics on demand supply and temporal reliability indexes based on different threshold deficits. As expected the SQ rule significantly reduces the maximum values of monthly deficit (from 4  $\text{Mm}^3$  to 2.9  $\text{Mm}^3$ ) and increases their monthly temporal reliabilities for 100% (no supply), 75% and 50% thresholds. This is done at the cost of more frequent low deficit (60.6% versus 88.7% with months without deficits) restrictions in irrigation use (-28% and -21% in temporal reliabilities).

Configuration	Mean deficit (Mm <sup>3</sup> /month)	Maximum deficit (Mm <sup>3</sup> /month)	Minimum deficit (Mm <sup>3</sup> /month)	Reliability			
				100%	75%	50%	0%
A	0.34	4.0	0.0	99.5	89.6	91.3	88.7
B	0.43	2.9	0.0	100.0	100.0	92.0	60.6

**Table 2.** Comparison of system performance values.

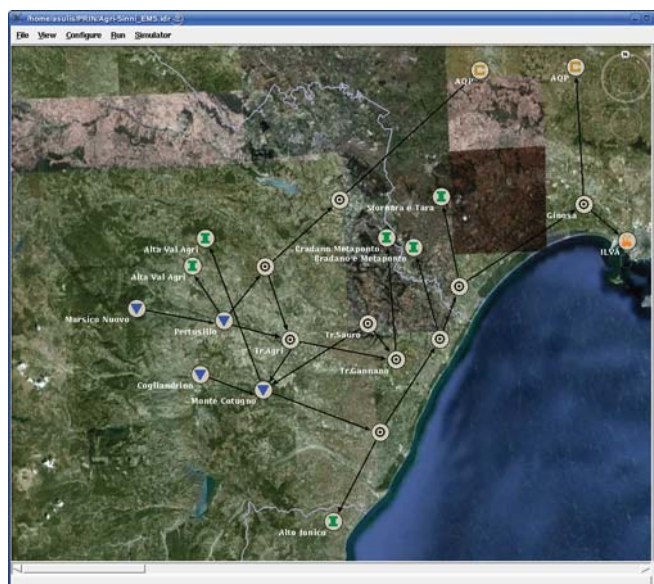
#### 4 APPLICATION TO AGRI-SINNI WATER SYSTEM

The Agri-Sinni water system (Figure 3) is located in Southern Italy and supplies water to the three regions. The reader is referred to Sulis and Sechi (2013) for a deeply description of the system features. Urban, industrial (ILVA in Figure 3), and irrigation demands (C.B. in Figure 3) are 246.5 Mm<sup>3</sup>/yr, 12.6 Mm<sup>3</sup>/yr, and 240 Mm<sup>3</sup>/yr, respectively.

Based on the observed monthly inflows over the period 1983-2005 (Table 3), SAMS generated 20 equally likely synthetic time series.

Stations	Mean Inflow (Mm <sup>3</sup> /year)	Maximum Inflow (Mm <sup>3</sup> /year)	Minimum Inflow (Mm <sup>3</sup> /year)	Initial Volume (Mm <sup>3</sup> )	Maximum Capacity (Mm <sup>3</sup> )
Pertusillo	212.1	328.5	118.2	80.0	159.0
Monte Cotugno	277.6	494.1	118.4	278.0	556.0
Cogliandrino	89.76	147.1	33.9	8.0	15.2
Marsico Nuovo	7.82	12.9	2.53	3.0	5.8
Gannano	105.5	389.0	11.7	-	Intake str.
Agri	115.5	241.5	17.9	-	Intake str
Sauro	50.5	101.3	11.9	-	Intake str
Sarmento	84.1	162.1	26.4	-	Intake str

**Table 3.** Statistical parameters of hydrology at Agri-Sinni.



**Figure 3.** Agri-Sinni water system.

Here we briefly focus on the main results of the multiple linear regression model implemented in the MISO model. The lag-1 regression model has been proved to be highly efficient in developing SQ rules for reservoirs whose design features and inflow statistical properties are significant different. Table 4 summarises the coefficients of the adopted rules and the square multiple correlation ( $R^2$ ). The regression analysis resulted in high correlations between optimal releases, reservoir storages and inflows with  $R^2$  between 0.7 and 0.9.

Stations	a	b	c	$R^2$
Pertusillo	0.05	0.03	0.5	0.78
Monte Cotugno	0.06	0.02	0.13	0.91
Cogliandrino	0.12	0.07	0.25	0.85
Marsico Nuovo	0.25	0.38	0.4	0.71

**Table 4.** Parameters of optimal linear rules.

#### 4.1 The GRID computing approach for MISO model

Sulis (2009) demonstrated the effectiveness of ISO approach in a GRID computing environment. GRID computing provides the ability to solve large-scale computation problems using a robust computer network that connects heterogeneous resources and services. The goal is to access those large sets of diverse, geographically distributed resources/services that are collected into a virtual computer for High Performance Computation (HPC), only when they are needed and to scale the problem so that even small home computers can make a useful contribution. While the resource/service allocation problems require an effective dynamic algorithm as presented in Sulis (2009), The architecture of MISO model assures an efficient divisibility of the problem. All independent deterministic LP models in WARGI-OPT represent single jobs organized in a job collection to be submitted to the GRID. The collection submission is composed of a set of operations (authentication, authorization, registration and submission, status control and retrieving) managed by the Workload Manager System, specifically implemented in MISO. The GRID environment was quite "basic" as at that time no tremendous distributed resources were required to solve the 20 jobs for the Agri-Sinni water system analysis. Specifically, it was made of 10 Intel Pentium Dual CPU Computing Elements clocked at 1.60 GHz having 1 GB RAM. Each Computing Element had a Scientific Linux SLC 3.0.8 i386. The GRID included a 100 Mbps LAN. can be (Figure 2). The GRID approach for the MISO model allowed for a significant time saving when compared to a local submission (non-GRID approach). Specifically, the GRID approach reduced the total computational time by approximately 80%, being reduced from 3590 sec to 720 sec in the non-GRID approach.

## 5 CONCLUSIONS AND RECOMMENDATIONS

The paper presents some improvements to the traditional Implicit Stochastic Optimization (ISO) approach that overcome some severe limitations affecting previous works (e.g., poor correlations that invalidate the optimized reservoir operating rules and extensive trial and error processes with little general applicability). The proposed model, called MISO (Modified Implicit Stochastic Optimization), appears to be effective and efficient in defining and testing linear operating rules on a multireservoir system. The aim of the proposed approach was to provide a flexible and transparent tool for non-skilled water managers, also capable of facilitating a stakeholder involvement in that decision making process. However, further research is needed to compared MISO with complex methods largely proposed in the literature (dynamic programming, non linear programming, Artificial Neural Networks, Fuzzy Rule-Based and Genetic Algorithm). Finally, different linear decision rules could lead to more general results.

## ACKNOWLEDGMENTS

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## REFERENCES

- Bhaskar, N.R., Whitlatch Jr., E.E., 1980. Derivation of Monthly Reservoir Release Policies. *Water Resources Research* 16(6), 987-993.
- Castelletti, A., Soncini-Sessa, R., 2006. A procedural approach to strengthening integration and participation in water resource planning. *Environmental Modelling and Software*, 21(10), 1455-1470.
- Celeste, A.B., Billib, M., 2009. Evaluation of stochastic reservoir operation optimization models. *Advances in Water Resources*, 32, 1429-1433.
- Hiew, K., Labadie, J., Scott, J., 1989. Optimal Operational Analysis of the Colorado-Big Thompson Project. In: Labadie et al. (Eds.), *Computerized Decision Support Systems for Water Managers*. ASCE, Reston, Va., pp. 632-646.
- Karamouz, M., Houck, M.H., 1982. Annual and Monthly Reservoir Operating Rules. *Water Resources Research*, 18(5), 1337-1344
- Karamouz, M., Houck, M.H., 1987. Comparison of Stochastic and Deterministic Dynamic Programming for Reservoir Operating Rule Generation. *Water Resources Bulletin* 23(1), 1-9.
- Karamouz, M., Houck, M.H., Delleur, J.W., 1992. Optimization and Simulation of Multiple Reservoir System. *Journal of Water Resources Planning and Management*, ASCE, 118(1), 71-81.
- Labadie, J.W., 2004. Optimal Operation of Multireservoir Systems: State-of-the-Art Review. *Journal of Water Resources Planning and Management* 130(2), 93-111.
- Labadie, J.W., Larson, R., 2007. MODSIM 8.1: River basin management decision support system. User manual and documentation. Colorado State University.
- Loucks, D.P., 1970. Some comments on linear decision rules and change constraints. *Water Resources Research*, 6(2), 668-671.
- Loucks, D.P. and O.T. Sigvaldason, The operation of multiple reservoir systems, In *Multiple-reservoir operation in North America*, Z. Kaczmarek & J. Kindler (eds.), International Institute for Applied Systems Analysis, Laxemburg, Austria, 1982.
- Lund, J.R., Ferreira, I., 1996. Operating Rule Optimization for Missouri River Reservoir System. *Journal of Water Resources Planning and Management*, 122(4), 287-295.
- Manca, A., Sechi, G.M., Sulis, A., Zuddas, P., 2004. Complex Water Resources System Optimization Tool Aided by Graphical Interface. In: LLiong, Phoon, Babovic (Eds.), *Proceedings of 6th International Conference on Hydroinformatics*. World Scientific Publishing Company, Singapore, pp. 1059-1066.
- Mousavi, S.J., Ponnambalam, K., Karray, F., 2005. Reservoir Operation Using a Dynamic Programming Fuzzy Rule-Based Approach. *Water Resources Management*, 19, 655-672.
- Saad, M., Turgeon, A., 1988. Application of Principal Component Analysis to Long-Term Reservoir Management. *Water Resources Research* 24(7), 907-912.
- Simonovic, S.P., 1992. Reservoir systems analysis: closing gap between theory and practice, *Journal of Water Resources Planning and Management*, 118(3), 262-280.
- Sveinsson, O.G.N., Salas, J.D., Lane, W.L., Frevert, D.K., 2007. *Stochastic Analysis, Modeling, and Simulation (SAMS) – Version 2007 – User’s Manual*. Technical Report No.11. Colorado State University.
- Sulis, A., 2009. GRID Computing Approach for Multireservoir Operating Rules with Uncertainty. *Environmental Modelling and Software*, 24(7), 859-864.
- Sulis, A., Sechi, G.M., 2013. Comparison of Generic Simulation Models for Water Resource Systems. *Environmental Modelling and Software*, 40, 214-225.
- Wurbs, R.A. (1993). Reservoir-System Simulation and Optimization Models. *Journal of Water Resources Planning and Management*, 119(4), 455-472.
- Yeh, W.W.-G. (1985). Reservoir Management and Operation Models: A State-of-the-Art-Review. *Water Resources Research*, 21(12), 1797-1818.
- Young, G.K., 1967. Finding reservoir operating rules. *Journal of Hydraulic Division*. ASCE, 93(HY6), 297-319.