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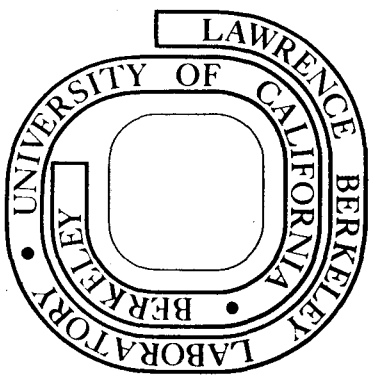

DISCLAIMER

IMPROVED NUMERICAL DISSIPATION FOR TIME INTEGRATION ALGORITHMS IN STRUCTURAL DYNAMICS*

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ABSTRACT

A new family of unconditionally stable one-step methods for the direct integration of the equations of structural dynamics is introduced and is shown to possess improved algorithmic damping properties which can be continuously controlled. The new methods are compared with members of the Newmark family, and the Houbolt and Wilson methods.

CONTENTS

1. INTRODUCTION

In many structural dynamics applications only low mode response is of interest. For these cases the use of implicit unconditionally stable algorithms is generally preferred over conditionally stable algorithms.

Conditionally stable algorithms require that the size of the time step employed be inversely proportional to the highest frequency of the discrete system. In practice this is a severe limitation as accuracy in the lower modes can be attained with time steps which are very large compared with the period of the highest mode.

For unconditionally stable algorithms a time step may be selected independent of stability considerations and thus can result in a substantial saving of computational effort.

In addition to being unconditionally stable, when only low mode response is of interest it is often advantageous for an algorithm to possess some form of numerical dissipation to damp-out any spurious participation of the higher modes. Examples of algorithms commonly used in structural dynamics which possess these properties are Houbolt's method [1], the Wilson θ-method [2] and the Newmark family of methods restricted to parameter values of $\gamma > 1/2$ and $\beta \geq (\gamma+1/2)^2/4$ see [3].

The Newmark family of methods allows the amount of dissipation to be continuously controlled by a parameter other than time step. For example, set $\beta = (\gamma+1/2)^2/4$ and $\gamma > 1/2$; then the amount of dissipation, for a fixed time step, is increased by increasing γ. On the other hand, the dissipative properties of this family of algorithms is considered to be inferior to both the Houbolt and the Wilson methods, since the lower modes are affected too strongly. (It seems all of these algorithms adequately damp the highest modes;

see Bathe and Wilson [2]).

In the Wilson method, θ must be selected greater than or equal to 1.37 to maintain unconditional stability. It is recommended in [2] that $\theta = 1.4$ be employed as further increasing θ reduces accuracy and further increases dissipation; but even for $\theta = 1.4$ the method is highly dissipative. For example, it is suggested in [2] that to accurately integrate a mode, 100 time steps be taken per period, whereas the generally employed rule-of-thumb is ten steps per period for nondissipative algorithms. From this we conclude that the Wilson method is generally too dissipative in the lower modes, requiring a time step be taken that is smaller than that needed for accuracy.

Houbolt's method is even more highly dissipative than Wilson's method and does not permit parametric control over the amount of dissipation present. Thus despite its shortcoming, the Wilson method is considered by many to be the best available unconditionally stable one-step algorithm when numerical dissipation is desired.

Since it seemed that the commonly used unconditionally stable, dissipative algorithms of structural dynamics all possessed some drawbacks, a research effort was undertaken to see if an improved one-step method could be constructed. The requirements of the desired algorithm were delineated as follows:

1. It should be unconditionally stable when applied to linear problems.
2. It should possess numerical dissipation which can be controlled by a parameter other than the time step. In particular, no numerical dissipation should be possible.
3. The numerical dissipaton should not affect the lower modes too strongly.

We have been able to develop an algorithm which achieves the above requirements and this paper is devoted to a description of its properties.

In Section 2 we define and analyze a three-parameter family of algorithms which contains the Newmark family. A new form of dissipation, called α-dissipation, is introduced by way of these algorithms. The new one-parameter family of methods which is advocated here is a subclass contained in the three-parameter family.

In Section 3 the unfavorable algorithmic dissipation possessed by the Newmark family is demonstrated. Furthermore, we show that α-dissipation is similar to linear viscous damping and, in itself, is ineffective in the higher modes. The dissipation of our new algorithms, which consists of a combination of positive Newmark γ-dissipation and negative α-dissipation, is shown to have improved characteristics. Results of a stability analysis of the new family are presented and its algorithmic damping ratio and relative period error are shown to compare favorably with those of the Wilson and Houbolt methods.

The present developments are summarized in Section 4.

2. ANALYSIS

Consider the linear undamped matrix equations of structural dynamics

$$\underset{\sim\sim}{M}\ddot{\underset{\sim}{u}} + \underset{\sim\sim}{K}\underset{\sim}{u} = \underset{\sim}{F} \quad , \tag{1}$$

where $\underset{\sim}{M}$ is the mass matrix, $\underset{\sim}{K}$ is the stiffness matrix, $\underset{\sim}{F}$ is the vector of external forces (a given function of time), $\underset{\sim}{u}$ is the displacement vector and superposed dots signify time differentiation (e.g. $\ddot{\underset{\sim}{u}} = d^2\underset{\sim}{u}/dt^2$ is the acceleration vector). The initial value problem for (1) consists of finding a function $\underset{\sim}{u} = \underset{\sim}{u}(t)$, where $t \in [0,\tau]$, $\tau > 0$, satisfying (1) and the initial conditions:

$$\left.\begin{aligned} \underset{\sim}{u}(0) &= \underset{\sim}{d} \\ \dot{\underset{\sim}{u}}(0) &= \underset{\sim}{v} \end{aligned}\right\} \tag{2}$$

where $\underset{\sim}{d}$ and $\underset{\sim}{v}$ are the given vectors of initial data.

We are interested in obtaining approximate solutions of (1) by one-step difference methods. To this end consider the family of algorithms defined by the following relations:

$$\underset{\sim\sim}{M}\underset{\sim n+1}{a} + (1+\alpha)\,\underset{\sim\sim}{K}\underset{\sim n+1}{d} - \alpha\underset{\sim\sim}{K}\underset{\sim n}{d} = \underset{\sim n+1}{F}, \quad n \in \{0,1,\ldots,N-1\}, \tag{3a}$$

$$\left.\begin{aligned} \underset{\sim n+1}{d} &= \underset{\sim n}{d} + \Delta t\,\underset{\sim n}{v} + \Delta t^2[(1/2-\beta)\,\underset{\sim n}{a} + \beta\underset{\sim n+1}{a}] \\ \underset{\sim n+1}{v} &= \underset{\sim n}{v} + \Delta t[(1-\gamma)\underset{\sim n}{a} + \gamma\underset{\sim n+1}{a}] \end{aligned}\right\}, \quad n \in \{0,1,\ldots,N-1\}, \tag{3b}$$

$$\left.\begin{aligned} \underset{\sim o}{d} &= \underset{\sim}{d} \\ \underset{\sim o}{v} &= \underset{\sim}{v} \\ \underset{\sim o}{a} &= \underset{\sim}{M}^{-1}(\underset{\sim o}{F} - \underset{\sim\sim o}{K}\underset{\sim}{d}) \end{aligned}\right\} \tag{3c}$$

where N is the number of time steps, $\Delta t = \tau/N$, $\underset{\sim n}{d}$, $\underset{\sim n}{v}$ and $\underset{\sim n}{a}$ are the approximations to $\underset{\sim}{u}(t_n)$, $\dot{\underset{\sim}{u}}(t_n)$ and $\ddot{\underset{\sim}{u}}(t_n)$, respectively, in which $t_n = n\,\Delta t$, $\underset{\sim n}{F} = \underset{\sim}{F}(t_n)$, and α, β and γ are free parameters which govern the stability and numerical

dissipation of the algorithm. If $\alpha = 0$ this family of algorithms reduces to the Newmark family. In this case if $\gamma = 1/2$ the algorithms possess no numerical dissipation (in a sense made precise later on) whereas if $\gamma > 1/2$ numerical dissipation is present; if $\beta \geq 1/4(\gamma + 1/2)^2$ the algorithm in question is unconditionally stable. Elaboration on these points and further properties of the Newmark family of algorithms may be found in [3].

To analyze systems such as (1), or equivalently (3a), it is convenient to invoke the property of orthogonality of the eigenvectors and reduce down to a single degree-of-freedom. Employing the obvious notations, the single-degree-of-freedom analogs of (1) and (3a) - (3c) are:

$$M\ddot{u} + Ku = F, \tag{4}$$

$$Ma_{n+1} + (1+\alpha)\, Kd_{n+1} - \alpha Kd_n = F_{n+1}, \quad n \in \{0,1,\ldots,N-1\}, \tag{5a}$$

$$\left.\begin{aligned} d_{n+1} &= d_n + \Delta t v_n + \Delta t^2[(1/2-\beta)\, a_n + \beta a_{n+1}] \\ v_{n+1} &= v_n + \Delta t[(1-\gamma)a_n + \gamma a_{n+1}] \end{aligned}\right\}, \quad n \in \{0,1,\ldots,N-1\}, \tag{5b}$$

$$\left.\begin{aligned} d_o &= d \\ v_o &= v \\ a_o &= M^{-1}(F_o - Kd_o) \end{aligned}\right\} \tag{5c}$$

Dissipative and dispersive characteristics of the above algorithm can be evaluated in terms of the solution it generates to simple pilot problems in which $F \equiv 0$. In these cases (5a) - (5c) can be succintly written in the recursive form

$$\underset{\sim}{X}_{n+1} = \underset{\sim\sim}{A}\underset{\sim}{X}_n, \quad n \in \{0,1,\ldots,N-1\}, \tag{6a}$$

where

$$\underset{\sim}{X}_n = (d_n, \Delta t\, v_n, \Delta t^2 a_n)^T, \tag{6b}$$

and $\underset{\sim}{A}$ is called the amplification matrix. Stability and accuracy of an algorithm depend upon the eigenvalues of the amplification matrix. The characteristic equation for $\underset{\sim}{A}$ is

$$\det (\underset{\sim}{A} - \lambda \underset{\sim}{I}) = \lambda^3 - 2A_1 \lambda^2 + A_2\lambda - A_3 = 0, \tag{7}$$

where $\underset{\sim}{I}$ is the identity matrix, λ denotes an eigenvalue and

$$\left.\begin{aligned} A_1 &= 1/2 \text{ trace } \underset{\sim}{A} \\ A_2 &= \text{sum of principal minors of } \underset{\sim}{A} \\ A_3 &= \text{determinant } \underset{\sim}{A} \end{aligned}\right\} \tag{8}$$

are invariants of $\underset{\sim}{A}$.

The <u>spectral radius</u> $\rho = \max\{|\lambda_1|, |\lambda_2|, |\lambda_3|\}$, where λ_1, λ_2 and λ_3 are the eigenvalues of $\underset{\sim}{A}$.

The velocities and accelerations may be eliminated by repeated use of (6a) to obtain a difference equation in terms of the displacements:

$$d_{n+1} - 2A_1 d_n + A_2 d_{n-1} - A_3 d_{n-2} = 0, \quad n \in \{2,3,\ldots,N-1\}. \tag{9}$$

Comparison of (9) with (7) indicates that the discrete solution has the representation

$$d_n = \sum_{i=1}^{3} c_i \lambda_i^n, \tag{10}$$

where the c_i's are determined from the initial data.

The explicit definition of $\underset{\sim}{A}$ for the family of algorithms defined by (5) is

$$\underset{\sim}{A} = \frac{1}{D} \begin{bmatrix} 1 + \alpha\beta\Omega^2 & 1 & 1/2 - \beta \\ -\gamma\Omega^2 & 1 - (1+\alpha)(\gamma-\beta)\Omega^2 & 1 - \gamma - (1+\alpha)(\frac{\gamma}{2} - \beta)\Omega^2 \\ -\Omega^2 & -(1+\alpha)\Omega^2 & -(1+\alpha)(1/2-\beta)\Omega^2 \end{bmatrix} \tag{11a}$$

in which

$$\left.\begin{aligned} D &= 1 + (1+\alpha)\ \beta\Omega^2 \\ \Omega &= \omega\ \Delta t \\ \omega &= (K/M)^{1/2} \end{aligned}\right\} \tag{11b}$$

Explicit forms corresponding to (8) and (9), respectively, can be computed from (11):

$$\left.\begin{aligned} A_1 &= 1 - \Omega^2[(1+\alpha)(\gamma+1/2) - \alpha\beta]/2D \\ A_2 &= 1 - \Omega^2[\gamma-1/2 + 2\alpha(\gamma-\beta)]/D \\ A_3 &= \alpha\Omega^2(\beta-\gamma+1/2)/D \end{aligned}\right\} \tag{12}$$

and

$$\frac{d_{n+1} - 2d_n + d_{n-1}}{\Delta t^2} + \frac{\omega\Omega(\gamma-1/2)}{D}\ \frac{d_n - d_{n-1}}{\Delta t} + \frac{\omega^2}{D}\ d_n + \frac{\alpha\Omega\omega}{D}\ \frac{d_n - d_{n-2}}{2\Delta t}$$

$$+ \frac{\alpha\Omega^2(\gamma-\beta)}{D}\ \frac{d_n - 2d_{n-1} + d_{n-2}}{\Delta t^2} = 0\ . \tag{13}$$

Example. Consider the case in which $\beta = 0$ and $\gamma = 1/2$. For these values (12) and (13) become, respectively:

$$\left.\begin{aligned} A_1 &= 1 - (1+\alpha)\Omega^2/2 \\ A_2 &= 1 - \alpha\Omega^2 \\ A_3 &= 0 \end{aligned}\right\} \tag{14}$$

and

$$\frac{d_{n+1} - 2d_n + d_{n-1}}{\Delta t^2} + \alpha\Omega\omega\ \frac{d_n - d_{n-1}}{\Delta t} + \omega^2\ d_n = 0\ . \tag{15}$$

Since $A_3 = 0$ there are only two nontrivial eigenvalues of $\underset{\sim}{A}$. Thus the solution of (15) has the form

$$d_n = \sum_{i=1}^{2} c_i \lambda_i^n, \tag{16a}$$

where

$$\lambda_{1,2} = A_1 \pm (A_1^2 - A_2)^{1/2}. \tag{16b}$$

If $A_1^2 < A_2$ the eigenvalues are complex conjugate and (16a) can be written

$$d_n = \rho^n (d_o \cos \bar{\omega} t_n + c \sin \bar{\omega} t_n), \tag{17}$$

where

$$\left.\begin{aligned} \rho &= A_2^{1/2} \\ \bar{\omega} &= \bar{\Omega}/\Delta t \\ \bar{\Omega} &= \arctan (A_2/A_1^2 - 1)^{1/2} \\ c &= (d_1 - A_1 d_o)/(A_2 - A_1^2)^{1/2} \end{aligned}\right\} \tag{18}$$

It is clear from (17) and (18) that the requirement for stable oscillatory response is $A_1^2 < A_2 \leq 1$ or, equivalently, $\Omega < 2/(1+\alpha)$ and $0 \leq \alpha$. With $\alpha = 0$ this algorithm becomes the familiar central difference method which is non-dissipative, i.e., $\rho = 1$. For positive values of α the algorithm is dissipative; the algorithm with $\alpha = 1/9$ has been used successfully in the finite difference work of Aboudi [4] on elastic shock-wave propagation. □

In general $A_3 \neq 0$ for the family of algorithms defined by (5) and therefore the amplification matrix has three nonzero eigenvalues. In this case we define stability in terms of the root condition which requires that $\rho \leq 1$ and double roots (eigenvalues of multiplicity two) satisfy $|\lambda| < 1$; see Gear [5] for further details. If the algorithm in question satisfies the root condition for all $\Omega > 0$, it is said to be unconditionally stable.

It is a standard exercise to show that the algorithms defined by (5) are convergent, i.e., for t_n fixed and $n = t_n/\Delta t$, $d_n \to u(t_n)$ as $\Delta t \to 0$.

A consequence of convergence is that there exists an $\Omega_c > 0$ such that if $0 < \Omega < \Omega_c$ then (7) has two complex conjugate roots $\lambda_{1,2}$, principal roots, and a so-called spurious root λ_3, which satisfy $|\lambda_3| < |\lambda_{1,2}| \leq 1$. Under these circumstances the principal roots of (7) are

$$\lambda_{1,2} = A \pm Bi = \exp\,[\bar{\Omega}(-\bar{\xi} \pm i)], \tag{19}$$

and the solution of (9) may be written in the form

$$d_n = e^{-\bar{\xi}\bar{\omega}t_n}\,(c_1 \cos\bar{\omega}t_n + c_2 \sin\bar{\omega}t_n) + c_3\lambda_3^n, \tag{20a}$$

where

$$\left.\begin{aligned} \bar{\omega} &= \bar{\Omega}/\Delta t \\ \bar{\xi} &= -\ln(A^2+B^2)/2\bar{\Omega} \\ \bar{\Omega} &= \arctan\,(B/A) \end{aligned}\right\} \tag{20b}$$

and the c_i's are defined by the initial data.

As measures of the numerical dissipation and dispersion we consider the algorithmic damping ratio $\bar{\xi}$ and relative period error $(\bar{T}-T)/T$, respectively, where $T=2\pi/\omega$ and $\bar{T}=2\pi/\bar{\omega}$. Note that from (20b), both $\bar{\xi}$ and $\bar{T}$ are defined in terms of the principal roots. Thus these measures of accuracy are defined only for values of Ω such that $0 < \Omega < \Omega_c$. Outside this region accuracy is not an issue and we are concerned only about stability; here ρ is the most important quantity since it provides information about stability and concomitantly about dissipation.

For completeness we note that the logarithmic decrement $\bar{\delta} = \ln[d(t_n)/d(t_n+\bar{T})]$ and amplitude decay function $AD = 1-d(t_n+\bar{T})/d(t_n)$ are also commonly used measures of algorithmic dissipation. Either of these measures determines the other as $AD = 1-\exp(-\bar{\delta})$. As is clear from their definitions, AD and $\bar{\delta}$ can

only be determined from the discrete solution of an initial-value problem, see [2]. This entails post-processing involving approximate interpolation to ascertain consecutive peak values. Since $\bar{\xi}$ is defined in terms of the principal roots, it seems to be the preferable measure of dissipation. For small time steps all three measures are equivalent for practical purposes. This can be seen as follows: First of all, as $\Delta t/T \to 0$, $\bar{\delta} \to 0$; therefore for sufficiently small $\Delta t/T$, the definition of AD implies that $AD \approx \bar{\delta}$. Furthermore, for convergent algorithms the effects of the spurious roots vanish in the limit $\Delta t/T \to 0$. Thus neglecting λ_3 in (20a) yields $\bar{\delta} \approx 2\pi\bar{\xi}$.

The period of the discrete solution $\bar{T}$ can also be determined analytically from (20b), rather than by solving initial-value problems and approximately ascertaining consecutive peak values.

In the sequel we shall show that the dissipation incurred by positive values of α is not too effective. Its qualitative behavior is the same as that of linear viscous damping; see Hilber [6]. However, by appropriately combining negative α-dissipation with particular values of β and γ a one-parameter family of algorithms with the attributes enumerated in the introduction can be constructed. Specifically, we take $\beta = (1-\alpha)^2/4$ and $\gamma = 1/2 - \alpha$. Then the invariants of the amplification matrix become

$$\left.\begin{aligned} A_1 &= 1 - \Omega^2/2D + A_3/2 \\ A_2 &= 1 + 2A_3 \\ A_3 &= \alpha(1+\alpha)^2\Omega^2/4D \end{aligned}\right\} \qquad (21)$$

where $D = 1 + (1-\alpha)(1-\alpha^2)\Omega^2/4$. Substituting (21) into (7) yields

$$(\lambda - A_3)(\lambda - 1)^2 + \Omega^2\lambda^2/D = 0 \ . \qquad (22)$$

In the limit $\Omega \to 0$, $\lambda_{1,2} \to 1$ and $\lambda_3 \to 0$. (In the Wilson and Houbolt algorithms λ_3 does not vanish in this limit. The significance of this fact does not seem

to be well understood.) On the other hand, in the limit $\Omega \to \infty$, for fixed $\alpha \neq 1$, (22) becomes

$$[(1-\alpha)(1-\alpha^2)\lambda - \alpha(1+\alpha)^2](\lambda-1)^2 + 4\lambda^2 = 0 . \qquad (23)$$

The roots of (23) are real and are depicted in Figure 1 as functions of α. This figure indicates that the proposed algorithm is stable in the limit $\Delta t/T \to \infty$ whenever $-1/2 \leq \alpha \leq 0$. It is clear from Figure 1 that decreasing α below -1/3 increases the spectral radius. Moreover, it was found by numerical experimentation that for small $\Delta t/T$, $\bar{\xi}$ cannot be increased by reducing α below -1/3. Thus we conclude that the range of practical interest is $-1/3 \leq \alpha \leq 0$.

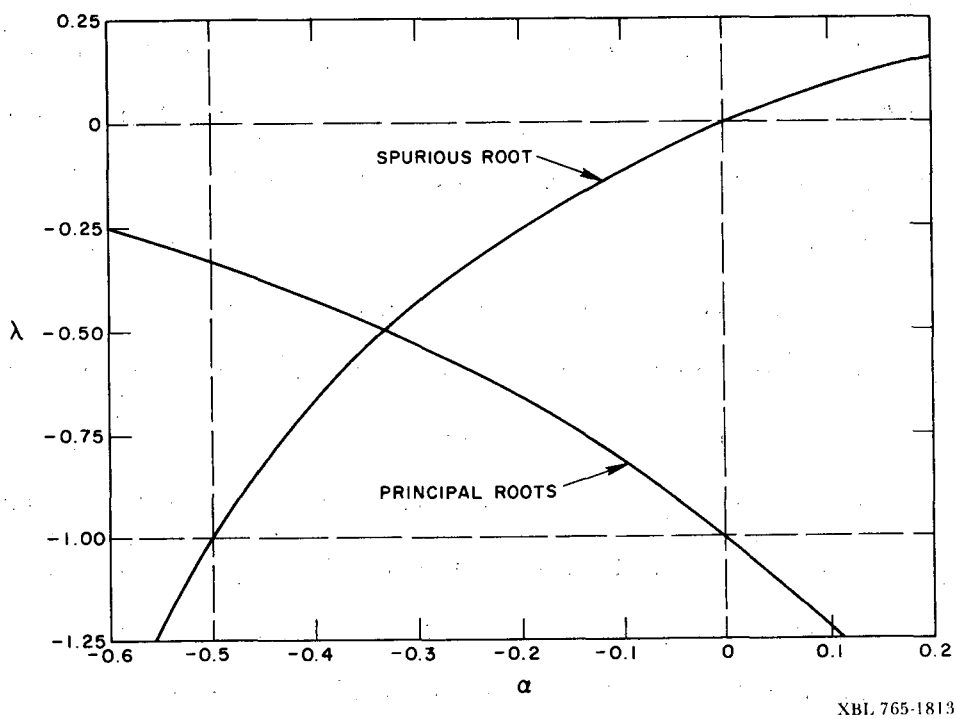

Fig. 1. Eigenvalues of the amplification matrix in the limit $\Delta t/T \to \infty$ versus α.

3. COMPARISON OF DISSIPATIVE ALGORITHMS

Spectral radius is an important measure of stability and dissipation. Figure 2 illustrates the behavior of spectral radii versus $\Delta t/T$ for the following algorithms*:

a. Trapezoidal rule (α=0, β=.25, γ=.5)

b. Trapezoidal rule with α-damping (α=.1, β=.25, γ=.5)

c. Newmark method with γ-damping (α=0, β=.3025, γ=.6)

d. A member of the new family proposed here (α=-.1, β=.3025, γ=.6)

The spectral radii for cases c and d are strictly less than one as $\Delta t/T \to \infty$. This condition insures that the response of higher modes is damped-out. The results for case b indicate why α-damping, in itself, is not an effective dissipative mechanism. For large $\Delta t/T$, cases c and d are identical. However, for small $\Delta t/T$, the spectral radius for case d is closer to one for a larger range of $\Delta t/T$. This is due to the addition of negative α-dissipation. In fact, it was the observation that combining cases b and c would produce an improved spectral radius graph which lead to the present scheme.

For comparison purposes we have plotted in Figure 3 the spectral radii of various schemes versus $\Delta t/T$. The strong dissipation possessed by the Houbolt and Wilson methods is clearly evident. The superiority of the dissipative characteristics of the present scheme over those of the Wilson method can be seen from Figure 3. Consider the case α=-.3; for small $\Delta t/T$ the new algorithm has a spectral radius curve closer to one than does the Wilson method indicating that it is more accurate in the lower modes yet for large $\Delta t/T$ the dissipation is stronger.

The point at which the spectral radius attains its minimum in Wilson's

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*In all cases $\beta = (\gamma+1/2)^2/4$ which insures unconditional stability.

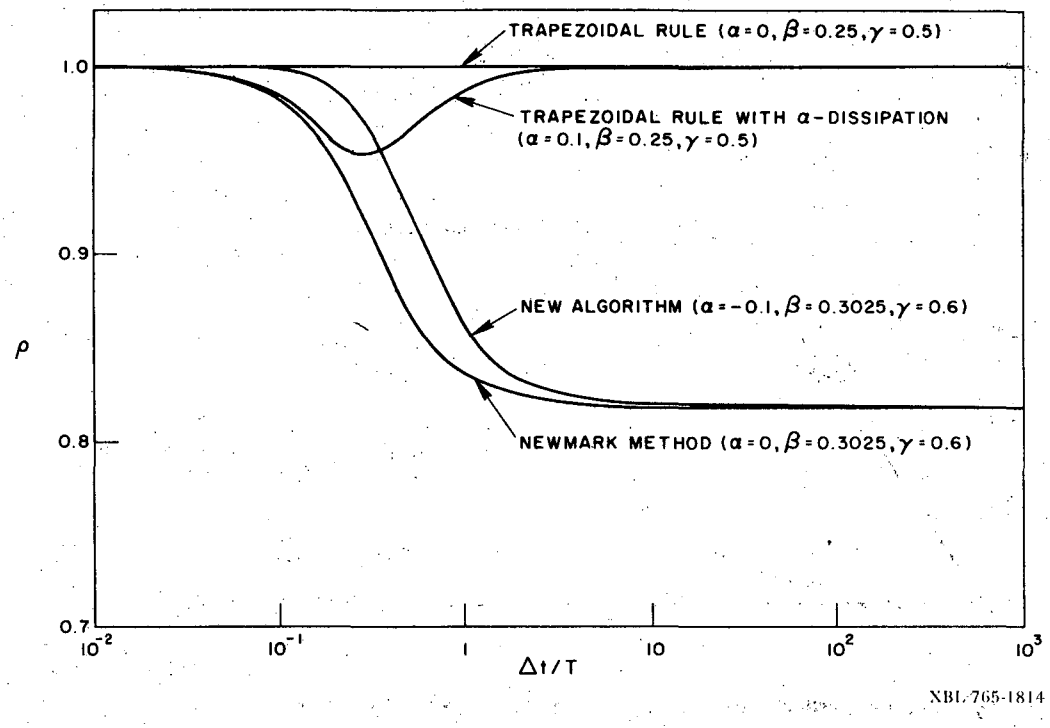

Fig. 2. Spectral radii versus $\Delta t/T$ for new method and Newmark schemes

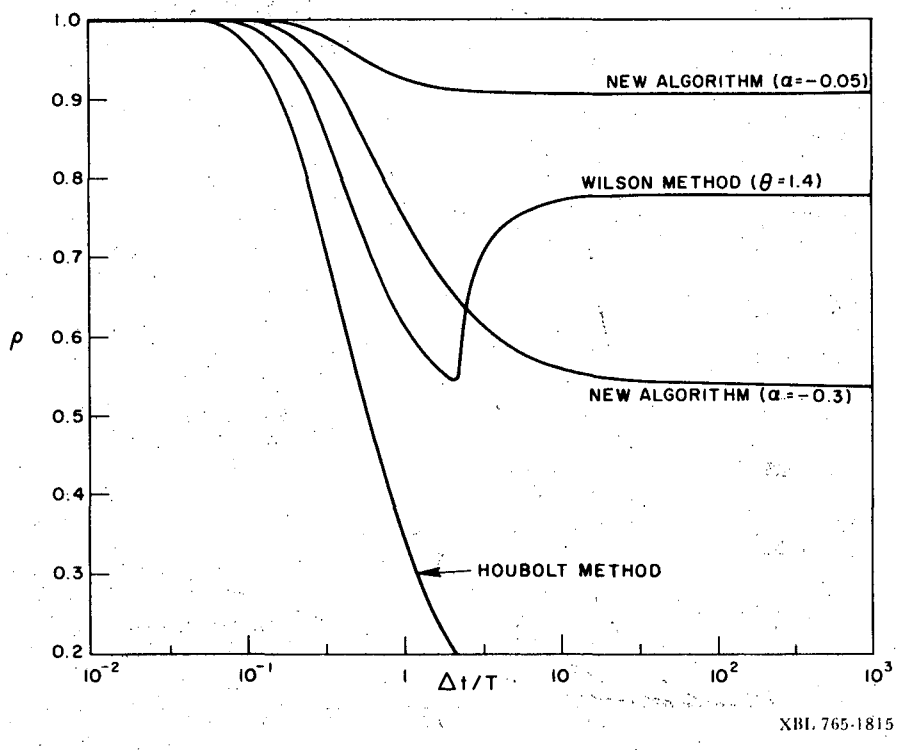

Fig. 3. Spectral radii versus $\Delta t/T$ for new methods and Houbolt and Wilson schemes

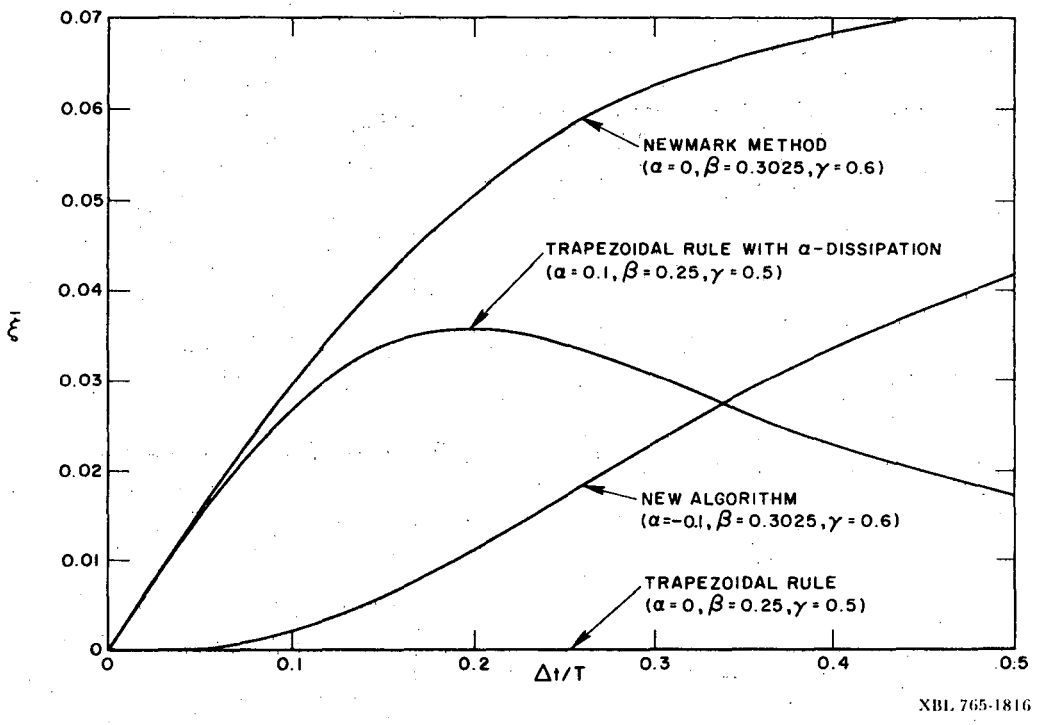

Fig. 4. Damping ratios versus $\Delta t/T$ for new method and Newmark schemes

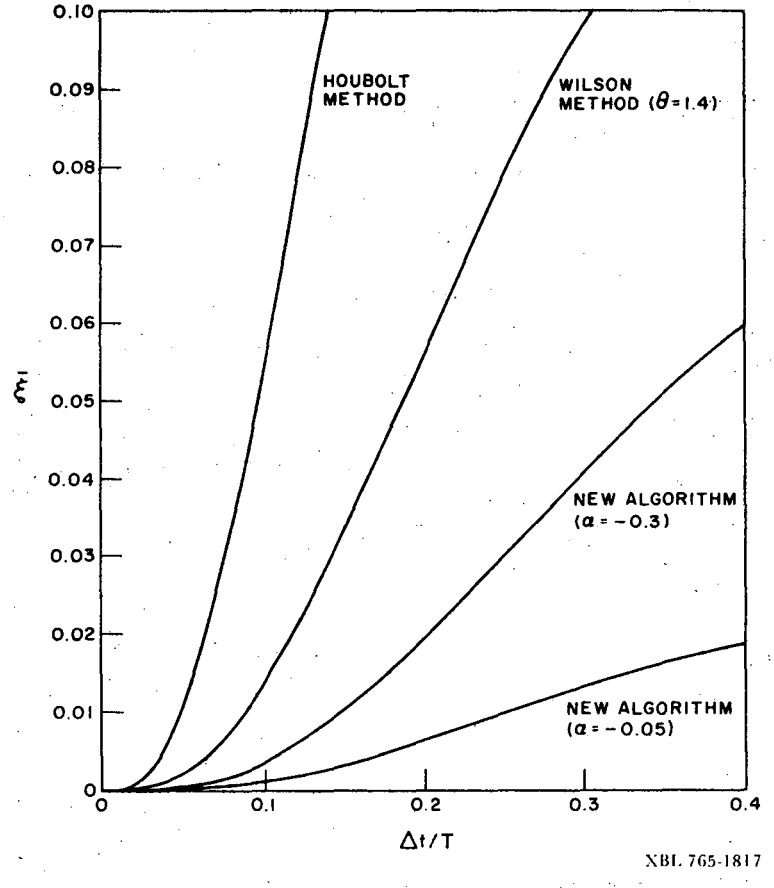

Fig. 5. Damping ratios versus $\Delta t/T$ for new methods and Houbolt and Wilson schemes

method ($\Delta t/T \approx 3$) marks the bifurcation of the complex conjugate principal roots into real roots.

In Figure 4 the damping ratios versus $\Delta t/T$ are plotted for cases a, b, c and d. Desirable properties for an algorithmic damping ratio graph to possess are a zero tangent at the origin and subsequently a controlled turn upward. This insures adequate dissipation in the higher modes and at the same time guarantees that the lower modes are not affected too strongly. Notice that for case c the dissipation ratio curve has positive slope at the origin. This is typical for Newmark γ-damping and is the reason why the Newmark family is felt to possess ineffective numerical dissipation. Case b also possesses this property and, in addition, turns downward at $\Delta t/T$ increases, which further emphasizes the ineffectiveness of α-dissipation. On the other hand, the dissipation ratio for case d has a zero slope at the origin and then turns upward.

In Figure 5 damping ratios for various values of α in the present scheme are compared with those for the Wilson and Houbolt methods. The continuous control of numerical dissipation possible in the present scheme is evident and the graphs show that the proposed family of algorithms possesses the desirable numerical dissipation characteristics cited previously.

Finally, in Figure 6 the relative period error is plotted versus $\Delta t/T$ for the various cases.

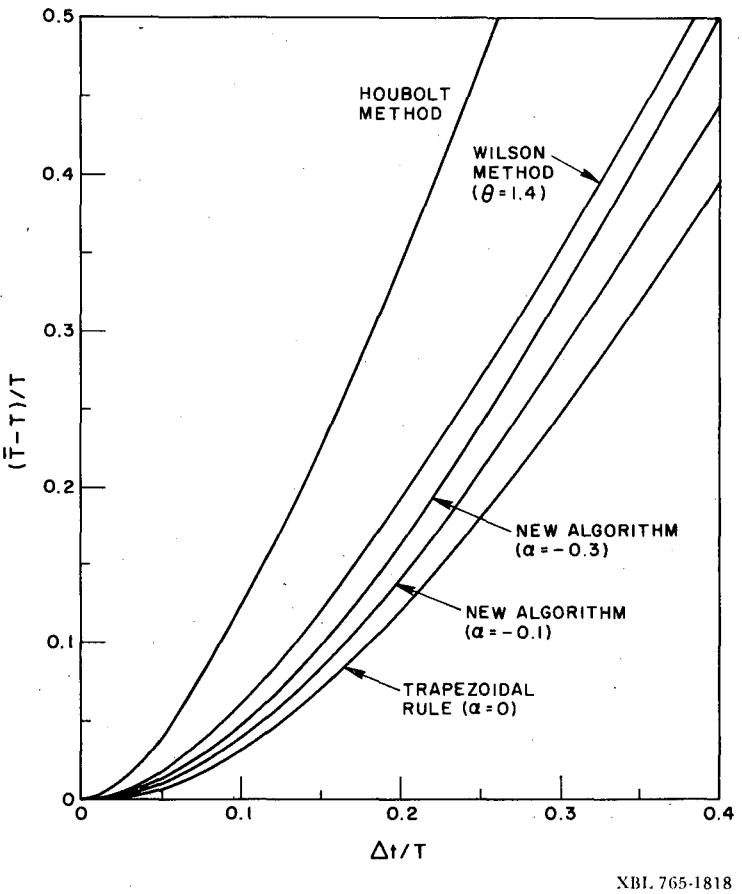

Fig. 6. Relative period error versus $\Delta t/T$ for new methods and Houbolt and Wilson schemes

4. CONCLUSIONS

A new family of unconditionally stable one-step algorithms for structural dynamics has been developed which possesses improved algorithmic damping properties that can be continuously controlled. In particular, no damping is possible. It is shown that there are members of the new family which are more accurate in the lower modes than the Wilson method, yet are more strongly dissipative in the higher modes. The new methods involve commensurate storage when compared with the Newmark and Wilson methods, and are no more difficult to implement.

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