Improved Phase Factor Computation for the PAR Reduction of an OFDM Signal Using PTS

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***Abstract*—The peak-to-average power ratio (PAR) of an orthogonal frequency-division multiplexing (OFDM) signal can be substantially larger than that of a single carrier system. Partial transmit sequence (PTS) combining can improve the PAR statistics of an OFDM signal. As PTS requires an exhaustive search over all combinations of allowed phase factors, the search complexity increases exponentially with the number of subblocks. In this letter, we present a new algorithm for computing the phase factors that achieves *better* performance than the exhaustive search approach.**

***Index Terms*—OFDM, peak-to-average power ratio.**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) has been proposed for both digital TV broadcasting and high speed wireless networks over multipath channels [1]. The principal drawback of OFDM is that the peak transmitted power can be substantially larger than the average power. Following [2], which first described a block coding technique to reduce the signal peaks, many PAR issues have been studied in the literature (see [3]–[5] as examples among many others).

The PTS [6] approach is a distortionless technique based on combining signal subblocks which are phase-shifted by constant phase factors. Even with the phase factors discretized to 0 and π, PAR_3 can be reduced by more than 4 dB for a 256-subcarrier, quadrature phase shift keyed (QPSK) modulated OFDM system that is partitioned into 16 signal subblocks [throughout the paper we denote by PAR_n a value such that $\Pr(\text{PAR} > \text{PAR}_n) = 10^{-n}$]. If the phase factors are discretized to four levels, for 128 subcarriers and four signal subblocks, PAR_2 is reduced by more than 3 dB. These impressive gains are realized by using what is known as an *optimal binary phase sequence* (OBPS), which was originally suggested in [6]. With this approach, the phase factors are restricted to 0 and π and hence an exhaustive search can be carried out over all combinations of permissible phase factors. A drawback to this approach is that the complexity of the OBPS search increases exponentially with the number of subblocks. In an effort to simplify the PTS method, a recent paper [4] has introduced new algorithms which perform *worse* than the OBPS solution but are much less complex. Note, at this point, that since the OBPS search uses binary quantized phase factors, it does not yield the global optimum solution for PTS.

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The purpose of this paper to present a new algorithm that is able to compute best phase factors for PTS. This algorithm performs *better* than the OBPS solution. For a 256-subcarrier system, PAR_4 can be reduced by approximately 5 dB using the new algorithm. Furthermore, the complexity of the new algorithm does not increase exponentially with the number of subblocks.

II. Phase Factor Computation for PTS

The complex envelope of the transmitted OFDM signal is represented by

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi nt} \tag{1}$$

where $j = \sqrt{-1}$ and $X_n \in \{1, j, -1, -j\}$ (for simplicity, we consider QPSK modulation only). We shall write the input data block as a vector, $\mathbf{X} = [X_0, \ldots, X_{N-1}]^T$. Most PAR-reduction techniques are concerned with reducing $\max|s(t)|$. However, since most systems employ discrete-time signals, the maximum amplitude of LN samples of $s(t)$ is reduced instead, where L is the oversampling factor. The case $L = 1$ is known as critical sampling or Nyquist rate sampling. The case $L > 1$ corresponds to oversampling. Sampling can be implemented by a suitably zero-padded, inverse Fast Fourier transform (IFFT).

For the PTS approach, the input data vector $\mathbf{X}$ is partitioned into disjoint subblocks, as $\{\mathbf{X}_m | m = 1, 2, \ldots, M\}$, and these are combined to minimize the PAR. While several subblock partitioning schemes do exist, we assume the simplest scheme for which the subblocks consist of a contiguous set of subcarriers and are of equal size. Now, suppose that for $m = 1, \ldots, M$, $\mathbf{A}_m = [A_{m1}, A_{m2}, \ldots, A_{m,LN}]^T$ is the zero-padded IFFT of $\mathbf{X}_m$. These are the partial transmit sequences. The objective is thus to combine these with the aim of minimizing the PAR. The signal samples at the output of the PTS combiner can be written as

$$\mathbf{S} = \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{M1} \\ A_{12} & A_{22} & \cdots & A_{M2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1LN} & A_{2LN} & \cdots & A_{MLN} \end{bmatrix} \begin{bmatrix} e^{j\phi_1} \\ e^{j\phi_2} \\ \vdots \\ e^{j\phi_M} \end{bmatrix} \tag{2}$$

where $\mathbf{S} = [S_1(\Phi), \ldots, S_{LN}(\Phi)]^T$ contains the optimized signal samples. We shall write the phase factors as a vector, $\Phi = [\phi_1, \phi_2, \ldots, \phi_M]^T$. The phase factors $\{\phi_k\}$ are chosen to minimize the peak of the signal samples $|S_k(\Phi)|$. So the minimum PAR is related to the problem

minimize

$$\max_{0<k\leq LN} |S_k(\Phi)|$$

subject to

$$0 \leq \phi_m < 2\pi, \qquad m = 1, \ldots, M. \tag{3}$$

A. Suboptimal Exhaustive Search (SES) Algorithms

The phase factors are restricted to a finite set of values and hence (3) is approximated by the problem

minimize

$$\max_{0<k\leq LN} |S_k(\Phi)|$$

subject to

$$\phi_m \in \left\{ \frac{2\pi l}{W} \,|l = 0, \ldots, W-1 \right\}. \tag{4}$$

If the number of rotation angles W is "sufficiently" large, the solution of (4) will approach that of (3). Furthermore, ϕ_1 can be fixed without any performance loss. Now, there are only $M-1$ free variables to be optimized and hence W^{M-1} distinct phase vectors, Φ_i, need to be tested. As such, (4) is solved using W^{M-1} iterations; the ith iteration involves computing LN signal samples, each of which is denoted by $S_k(\Phi_i)$, using (2) and choosing the maximum $|S_k(\Phi_i)|$ value. At the end of each iteration, the phase vector is retained if the current value of $\max|S_k(\Phi_i)|$ is less than the previous maximum. The phase vector that is retained after all the iterations are completed will be an approximation to the global optimal solution of (3).

In SES, the computational load consists of M IFFT's, MLN complex multiplications per iteration, and LN operations of $|\cdot|$. As the computational cost of M IFFT's is fixed for any algorithm, for comparative purposes, we ignore that fixed cost component and define the measure of complexity as

$$N_c = W^{M-1}LN. \tag{5}$$

This measure indicates the total number of operations of $|\cdot|$ and multiplications required. Given that its value increases exponentially with M, SES may not be feasible for $M > 8$.

B. New Algorithm

The motivation for a new algorithm arises from the following observation. For given Φ, we have the ith row of (2) as

$$S_i(\Phi) = A_{1i}e^{j\phi_1} + A_{2i}e^{j\phi_2} + \cdots + A_{Mi}e^{j\phi_M} \tag{6}$$

where A_{ri}, $r = 1, 2, \ldots, M$, are fixed complex numbers dependent only on the input data frame. What choice of Φ will minimize the amplitude of this sum? If we sort $|A_{ri}|$ as

$$|A_{r_1 i}| > |A_{r_2 i}| > \cdots > |A_{r_M i}|,$$

where $\{r_1, \ldots, r_M\}$ is a permutation of $\{1, \ldots, M\}$, and choose

$$\phi_{r_l} = \begin{cases} -\angle A_{r_l i}, & l = 1, 3, \ldots \\ \pi - \angle A_{r_l i}, & l = 2, 4, \ldots \end{cases} \tag{7}$$

where $\angle$ denotes the phase angle of a complex number, then the minimum amplitude sample is given by

$$S_i(\Phi) = |A_{r_1 i}| - |A_{r_2 i}| + |A_{r_3 i}| - \cdots. \tag{8}$$

The phase selection (7) yields nearly always the maximum amount of amplitude cancellation for the ith signal sample. As a result, it is very easy to find Φ that will nearly always minimize the amplitude of a single signal sample. Let $\hat{\Phi}_i$ be the solution (7) that nearly always minimizes $|S_i|$. Each $\hat{\Phi}_i$ can be viewed as a reasonable—but not necessarily the optimal— solution for (3). Our next step is therefore to compute all LN such solutions and choose the one that minimizes the maximum signal samples.

Similar to the SES algorithms for (4), the new algorithm can be applied in LN iterations to obtain a solution for (3); the ith iteration involves computing LN signal samples $S_k(\hat{\Phi}_i)$ using (2) and choosing the maximum of $|S_k(\hat{\Phi}_i)|$. At the end of each iteration, the phase vector is retained if the current value of $\max|S_k(\hat{\Phi}_i)|$ is less than the previous maximum. There are two main differences between the SES algorithms and the new algorithm. First, the number of iterations changes from W^{M-1} to LN. Second, the phase vectors are computed differently. The phase factors from (7) are not restricted to 0 and π, which is the case for SES with $W = 2$. Rather, they are continuous variables between 0 and 2π.

As with (5), we define the measure of complexity as

$$N_c = LN \times LN. \tag{9}$$

The first LN denotes the number of iterations and the second denotes the number of operations per iterations. As well, for $W = 2$, comparison of (5) and (9) reveals that the new algorithm is more complex than SES for small M but less complex for large M (> 8).

III. Results

To justify the new algorithm *vis-a-vis* the SES approach, it is necessary to demonstrate two things. First, we must demonstrate that the PAR reduction achieved with the new algorithm is better and its complexity less than or similar to that of SES. Second, we must also demonstrate that, if the phase factors used in the new algorithm are quantized, the resulting performance loss will be small. This is particularly relevant if coherent demodulation is to be employed. These two issues are studied by simulation. In the results which follow, 10^5 OFDM signals are generated in each case. The transmitted signal is oversampled by a factor of 4 ($L = 4$). All results are for 256-subcarrier and QPSK-modulated systems.

Fig. 1 compares the performance of the two algorithms as function of M, the number of subblocks. For $M = 2, 4$, the new algorithm performs more than 1 dB better than the OBPS solution. For $M = 8$, the performance gain is about 0.5 dB. For $M = 16$, both the algorithms perform nearly equally. However, in this case, the OBPS search requires 2^{15} operations per iteration. As this would take up an enormous amount of computer time, the results for the $M = 16$, OBPS curve are shown for a limited, random search of the phase factor space. We performed only 500 trials and in [4] it was observed that 2000 trials would result in performance which was essentially equivalent to the OBPS. If the entire 2^{15} combinations were tested at each iteration, we would expect the $M = 16$, OBPS curve to improve somewhat (i.e., it should be better than the new algorithm). Note that on the basis of (5) and (9), the OBPS search is 32 times more complex than the new algorithm for $M = 16$. We also tested the execution speed ratio for the two algorithms executed by Matlab on a 900 MHz Pentium machine. For $M = 16$, the OBPS algorithm was 20 times slower than the new algorithm. Thus, in this

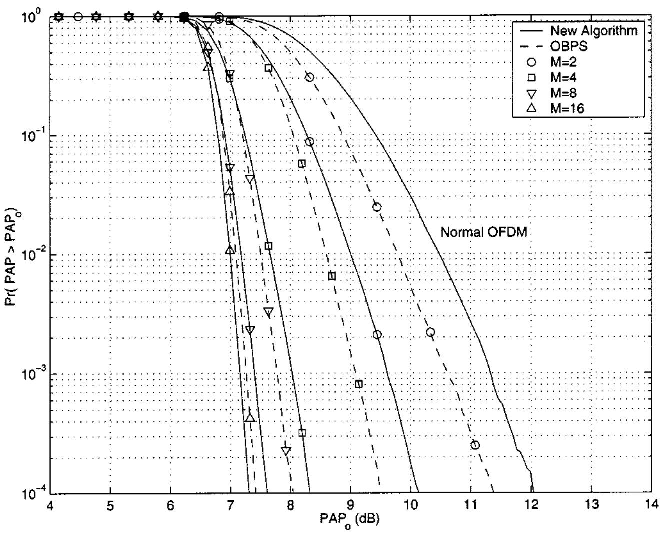

Fig. 1. Comparison of the new algorithm and the OBPS algorithm.

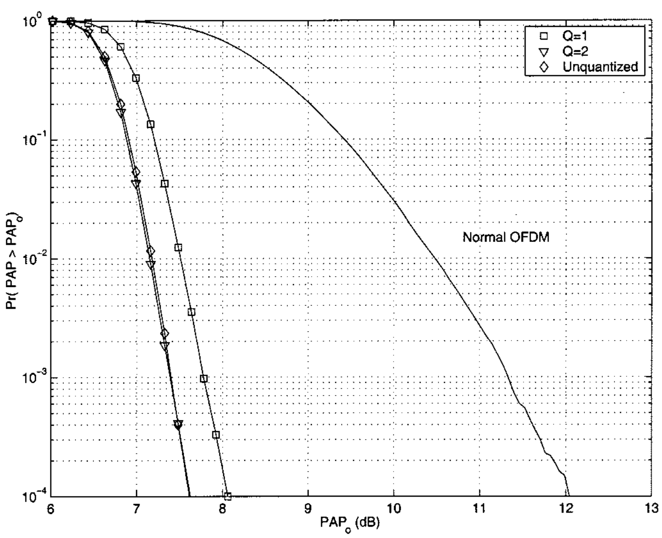

Fig. 2. The effect of quantization on the new algorithm, for $M = 8$.

particular case and for a similar level of performance, the new algorithm is less complex.

Next, we consider quantization of the phase factors in the new algorithm to Q bits. That is, the phase factors (8) are rounded off to the nearest element in the set $\{k\pi/2^{Q-1}|k = 0, 1, \ldots, 2^{Q-1}\}$. The minimum overhead required to transmit all the phase factors to the receiver is then $(M-1)Q$ bits. Fig. 2 shows results for $Q = 1$, 2 and ∞. Even $Q = 1$ achieves a performance level within 0.4 dB of the unquantized case ($Q = \infty$). Performance degradation for the $Q = 2$ case is negligible.

Finally, we look at the effect of the oversampling factor. If L is increased, improved performance can be expected for the new algorithm. Of course, this occurs at an increasing level of complexity. Fig. 3 evaluates the performance of the new algorithm as a function of the oversampling factor, L. Increasing L beyond 4 seems to bring very little improvement in performance.

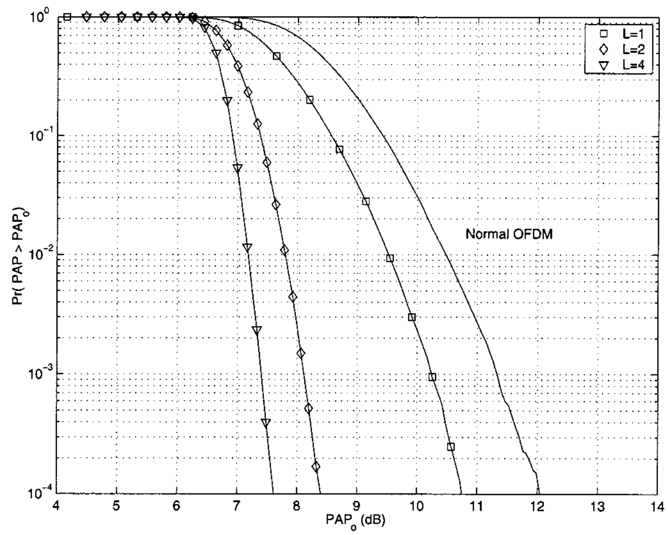

Fig. 3. The effect of oversampling factor for $M = 8$.

IV. CONCLUSIONS

In conclusion, we observe that the PAR-reduction problem for OFDM has received a great deal of attention recently. In this paper, a new algorithm for computing a good set of phase factors for PTS combining has been developed. This algorithm performs better than the OBPS search for small M. As the number of subblocks increases, the performance difference between the two algorithms tends to zero, while the complexity of the OBPS solution increases exponentially. The effect of 2-bit quantization on the performance of the new algorithm is negligible.

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