Improved Polarization Measurements Using a Modified Three-Antenna Technique

ALLEN C. NEWELL, SENIOR MEMBER, IEEE

Abstract—**An improved three-antenna measurement of polarization that greatly reduces the uncertainty due to phase measurement errors is described. This technique is used to calibrate polarization standards and probes used in near-field antenna measurements.**

Introduction

THE THREE-ANTENNA technique is a well-established method for precisely determining antenna polarization parameters [1]–[3]. Axial ratio, tilt angle, and sense of polarization are determined for all three antennas from measurements performed on the three pairs. The measurements consist of determining the amplitude and phase change when one of the antennas is rotated about its axis by 90°. In this orientation, the antennas are cross polarized, the signal level is very low, and accurate phase measurements are difficult. To overcome this difficulty, a modified three-antenna technique was developed which yields significantly higher accuracy and is easier to perform. The phase measurement is reduced to determining only the sign of the phase change, and this can be accomplished where the signal level is farther above the noise level. The modified three-antenna technique produces accurately calibrated polarization standards and near-field probes having axial ratios greater than 40 dB. Such standards are indispensable for measuring antenna systems requiring high polarization purity.

Measurement Technique

Three coordinate systems are used in describing the measurements, one fixed to each of two antennas and one fixed in space to the test range as shown in Fig. 1. Each antenna system is defined with the boresight direction along the z axis and the y axis nominally along the major axis of the polarization ellipse. The exact locations of the axes is not critical, but they must be clearly defined. The reference, or range, coordinate system has its x axis horizontal, its y axis vertical, and its z axis along the center of the test range. The source or transmitting antenna is oriented on the test range so that its axes are coincident with the reference system. The receiving antenna is aligned with its boresight (positive z axis) along the range negative z direction and is rotated about the range z axis into two orientations where measurements are performed. In the first orientation the $+y$ axes of the antennas are in the same direction, and in the second the $-x$ axis of the receiving antenna is in the same direction as the $+y$ axis of the transmitting antenna as shown in Fig. 1. The coupling equations for these two orientations are, respectively,

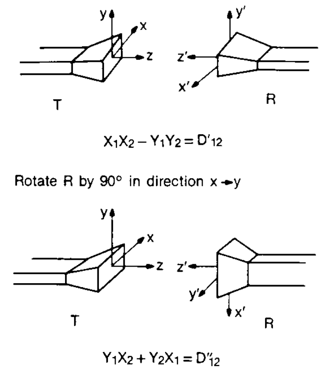

Fig. 1. Orientation of two antennas for determining D'_{12} and D''_{12}.

$$X_1X_2 - Y_1Y_2 = D'_{12} \tag{1}$$

$$X_1Y_2 + X_2Y_1 = D''_{12} \tag{2}$$

where the D's are proportional to the complex received signals, and X_1 and Y_1 are the x and y components of the plane-wave spectrum transmitting characteristic $\mathbf{S}_{10}(\theta, \phi)$ of the source antenna in the direction $\theta = \phi = 0$. X_2 and Y_2 are similar quantities for the receiving antenna. $\mathbf{S}_{10}(\theta, \phi)$ is simply related to the far electric field $\mathbf{E}$ (see [1, eq. (1)]), and since we will only be concerned with ratios of the components, the formulation can be thought of as being in terms of either $\mathbf{E}$ or $\mathbf{S}$.

In (1) and (2) we have assumed that the receiving antenna is reciprocal, and we have expressed all antenna parameters in terms of transmitting properties. This is not necessary to the technique, but it makes the notation more concise and consistent with previous material [2].

The approach in the three-antenna measurement is to repeat this procedure for all three antenna pairs, measure the D_{nm}, and solve the resulting six equations for the linear components X_n and Y_n, the right- and left-circular components R_n and L_n, and obtain gain, complex linear polarization ratio ρ_l, complex circular polarization ratio ρ_c, axial ratio A, and tilt angle τ. These polarization parameters are related by the equations

$$R = \frac{X - iY}{\sqrt{2}} \qquad L = \frac{X + iY}{\sqrt{2}} \qquad X = \frac{R + L}{\sqrt{2}} \qquad Y = \frac{i(R - L)}{\sqrt{2}} \tag{3}$$

$$\rho_l = \frac{X}{Y} \qquad \rho_c = \frac{L}{R} \qquad A = \frac{|R| + |L|}{|R| - |L|} \qquad \tau = \frac{\arg(\rho_c)}{2}. \tag{4}$$

Manuscript received September 24, 1987; revised January 28, 1980.
The author is with the Electromagnetic Fields Division, National Bureau of Standards, Boulder, CO 80303.
IEEE Log Number 8821102.

The linear and circular vector components are, in general, complex quantities, and X and Y do not represent real and imaginary parts. Note also that the measured linear components are those for the reference coordinate system, and all of the quantities in (3) and (4) except A will change if the initial orientation of the antennas is changed.

The time convention used in the theory and measurements is $\exp(-i\omega t)$ which is consistent with the plane-wave scattering matrix theory. The phase measurement instrumentation is set for this convention by observing the direction of phase change when the separation distance between the antennas is increased. For the chosen convention, the phase will increase with distance. The linear and circular polarization ratios are then defined in (4) such that the tilt angle will have the conventional definition in (4), and phases of ρ_l which are positive and less than 180° indicate left-hand elliptical polarization.

For polarization measurements, only the ratios of the components and the received signals are necessary, and we therefore combine (1) and (2) and express the result for the general case of antennas m and n in terms of linear polarization ratios. Taking the ratio of (2) and (1) and dividing numerator and denominator by $Y_1 Y_2$ gives

$$\frac{\rho_{lm}+\rho_{ln}}{\rho_{lm}\rho_{ln}-1}=Q_{nm}=\frac{D''_{nm}}{D'_{nm}}. \tag{5}$$

Substituting the circular components from (3) into (1) and (2) gives the coupling equations in the form

$$R_1R_2+L_1L_2=D'_{12} \tag{6}$$

$$i(R_1R_2-L_1L_2)=D''_{12}. \tag{7}$$

Taking the ratio of these equations we obtain an expression for Q_{nm} in terms of the circular polarization ratios

$$\frac{1-\rho_{cm}\rho_{cn}}{1+\rho_{cm}\rho_{cn}}=-iQ_{nm} \tag{8}$$

and the circular polarization ratios are

$$\rho_{cm}\rho_{cn}=\frac{1+iQ_{nm}}{1-iQ_{nm}}. \tag{9}$$

In the conventional three-antenna measurement, measurement of the three Q_{nm} ratios provides three equations similar to (9) from which we can solve for each individual circular polarization ratio. The major source of error in the polarization parameters obtained from these measurements is the uncertainty in the phase of Q_{nm}. For two nearly linear antennas, the magnitude of Q_{nm} may be -30 to -60 dB and at these levels, multipath, depolarization of scattered signals, and leakage can produce large phase errors. The phase will also change very rapidly as the 90° rotation is approached which makes it necessary to measure the rotation angle quite accurately. The uncertainty in the phase of the Q's can be reduced essentially to zero by the following modification of the measurement.

We begin by deriving a form of (6) that defines the amplitude and phase change due to rotation about the z axis in terms of the polarization ratios. The amplitude and phase of each circular component are represented by

$$L_1=l_1e^{i\tau_1} \qquad L_2=l_2e^{i\tau_2} \tag{10}$$

$$R_1=r_1e^{-i\tau_1} \qquad R_2=r_2e^{-i\tau_2}. \tag{11}$$

The phase of the L component may be chosen arbitrarily and has been set equal to the tilt angle. The phase of the R component is then defined to give the correct phase for the circular polarization ratio as shown in (4). If the source antenna is now rotated about its z axis by the angle ϕ such that its x axis is rotated toward the range y axis, the magnitudes of the circular components will remain the same, but the tilt angles will change by an amount equal to the ϕ rotation. The new circular components are now

$$L_1(\phi)=l_1e^{i(\tau_1+\phi)} \qquad R_1(\phi)=r_1e^{-i(\tau_1+\phi)}. \tag{12}$$

If the receiving antenna is rotated by the angle ϕ, its components will change also, giving

$$L_2(\phi)=l_2e^{i(\tau_2+\phi)} \qquad R_2(\phi)=r_2e^{-i(\tau_2+\phi)}. \tag{13}$$

Using these components in (6) the transmission equation as a function of rotation angle is

$$D_{12}(\phi)=r_1r_2[e^{-i(\psi+\phi)}+a_1a_2e^{i(\psi+\phi)}] \tag{14}$$

or

$$D'_{12}(\phi)=r_1r_2(1+a_1a_2)\cdot\left[\cos(\psi+\phi)+i\left[\frac{a_1a_2-1}{a_1a_2+1}\right]\sin(\psi+\phi)\right] \tag{15}$$

$$D''_{12}=D'_{12}(\pi/2)=r_1r_2(1+a_1a_2)\cdot\left[-\sin(\psi)+i\left[\frac{a_1a_2-1}{a_1a_2+1}\right]\cos(\psi)\right] \tag{16}$$

where

$$a_1=|\rho_{c1}| \qquad a_2=|\rho_{c2}| \qquad \psi=\tau_1+\tau_2. \tag{17}$$

We next note from (3) and (4) and the definition of τ that $\tau = \pm\pi/2$, if and only if ρ_c is negative and real, $\arg(\rho_l) = \pm\pi/2$, and the major axis of the polarization ellipse is exactly along the y axis of the reference coordinates. This condition can be achieved for the transmitting antenna by a temporary redefinition of the reference coordinate axes. The new y axis is defined to be coincident with the major axis of the transmitting antenna, and therefore

$$\tau'_1=\pi/2 \tag{18}$$

where the prime denotes the new tilt angle with respect to the new reference coordinate system. Primes will be used in the same way on other polarization parameters. From (16) the minimum signal occurs when $\psi' = \pi$, and the major axis of the receiving antenna is then made coincident with the new y axis by rotating to the minimum and then back by 90°. In this modified reference system, both of the tilt angles are 90°, the ρ_c are both negative and real, and the phase of Q'_{mn} is exactly $\pm\pi/2$.

The sign of the phase change can be determined by observing the direction of phase change as the receiving antenna is rotated toward 90°. Using (15) with $\psi' = \pi$, the phase change of D'_{nm} for any rotation angle ϕ is

$$\arg(D'_{nm}(\phi)) = \tan^{-1}\left[\frac{|\rho'_{cn}\rho'_{cm}|-1}{|\rho'_{cn}\rho'_{cm}|+1}\tan(\phi)\right]. \quad (19)$$

Therefore, if the argument of Q'_{nm} is $\pm\pi/2$, the phase change will be positive for all angles between 0 and 90° and will be negative for a $-\pi/2$ argument.

In summary, the measurement procedure is initially to align the antennas with their chosen y axes parallel, set the angle encoders to 0° for this orientation, rotate the receiving antenna in the proper direction about the z axis until a minimum is found, and then rotate 90° in the opposite direction. In this orientation the amplitude is set to 1, the phase to 0°, and the rotation angle $\Delta\phi_{nm}$ is recorded. The receiver is again rotated to the minimum, the direction of phase change during rotation is noted, and the amplitude at the minimum α_{nm} is recorded. Q'_{nm} is then equal to $\pm i\alpha_{nm}$ where the correct sign is equal to the direction of phase change. This procedure is repeated for the other two antenna pairs, and the three equations obtained from (9) are solved for the polarization ratios. For example,

$$\rho'_{cn} = \left[\frac{\rho'_{cn}\rho'_{cm}\cdot\rho'_{cn}\rho'_{ck}}{\rho'_{cm}\rho'_{ck}}\right]^{1/2} \quad (20)$$

$$= \left[\frac{\dfrac{1+iQ'_{nm}}{1-iQ'_{nm}}\cdot\dfrac{1+iQ'_{nk}}{1-iQ'_{nk}}}{\dfrac{1+iQ'_{mk}}{1-iQ'_{mk}}}\right]^{1/2}. \quad (21)$$

By aligning the antennas so that their electrical axes are parallel we have essentially redefined the reference coordinate system to be coincident with the electrical axes of the source antenna. With this redefinition, the linear polarization ratio in the new coordinate system is the reciprocal of the axial ratio in both the new and the original coordinate system:

$$A' = A = 1/|\rho'_l|. \quad (22)$$

The tilt angle with respect to the antenna axes is determined from the measured rotation angles $\Delta\phi_{nm}$ by the equation,

$$\tau_k = 90 + \frac{\Delta\phi_{km}+\Delta\phi_{kn}-\Delta\phi_{mn}}{2}. \quad (23)$$

This technique is especially useful in the calibration of linear polarization standards with very small cross polarization. In this case, the denominator of (5) is very nearly -1 and a quick calculation yields the linear polarization ratios. For instance,

$$\rho'_{l1} \approx -(Q'_{12}+Q'_{13}-Q'_{23})/2. \quad (24)$$

TABLE I
RESULTS OF POLARIZATION MEASUREMENTS

Antenna Combinations	A_1 (dB)	τ_1 (°)	A_2 (dB)	τ_2 (°)	A_3 (dB)	τ_3 (°)	A_4 (dB)	τ_4 (°)
1, 2, 3	67.7	89.62	54.8	89.84	47.1	89.83		
1, 2, 4	73.9	89.60	53.9	89.86			60.0	89.86
1, 3, 4	70.3	89.68			46.8	89.79	61.0	89.80
2, 3, 4			54.4	89.80	47.3	89.77	59.1	89.82

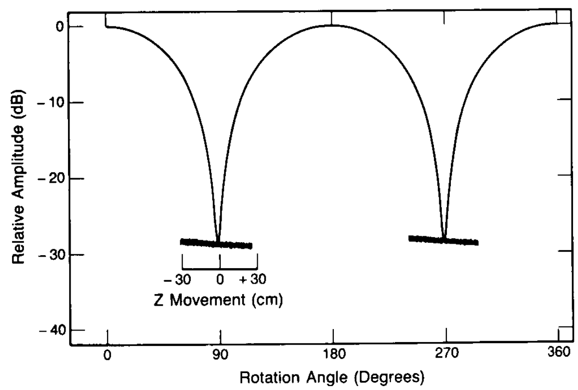

Fig. 2. Sample of measured data showing amplitude as function of rotation angle and z distance.

This technique was used on a set of four Ku-band standard gain horns where all combinations were measured to give three sets of data from which each antenna's polarization could be computed. The consistency of the results shown in Table I indicates the accuracy of the technique.

We have discussed the case where the polarization is measured in the boresight directions (along the antenna's z axes), but the same procedure can be used for any direction. Rotating one or both of the antennas so that a direction other than boresight is along the range z axis, the spherical components in that direction will be determined. Rotation of the receiving antenna in the measurement of Q_{nm} must be about the range z axis which may not be the antenna's z axis in this case.

The accuracy of the results is dependent primarily on the measurement of the magnitude of the Q's, and if care is taken to average the effect of multipath by moving one antenna a small distance in the z direction as illustrated in Fig. 2, the errors in A are on the order of ± 0.05 dB/dB, and about $\pm 0.2°$ in τ.

References

[1] A. C. Newell and D. M. Kerns, "Determination of both polarization and gain of antennas by a generalized 3-antenna measurement method," *Electron. Lett.*, vol. 7, pp. 68–70, Feb. 1971.

[2] A. C. Newell, R. C. Baird, and P. F. Wacker, "Accurate measurement of antenna gain and polarization at reduced distances by an extrapolation technique," *IEEE Trans. Antennas Propagat.*, vol. AP-21, pp. 418–431, July 1973.

[3] E. B. Joy and D. T. Paris, "A practical method for measuring the complex polarization ratio of arbitrary antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-21, pp. 432–435, July 1973.

Allen C. Newell (M'73–M'81–SM'83), for a photograph and biography please see page 733 of this issue.